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HIGH-Q FACTOR OPTICAL WHISPERING-GALLERY MODE MICRORESONATORS AND THEIR USE IN PRECISION MEASUREMENTS

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Basic data concerning the use of optical dielectric whispering-gallery mode microresonators are presented. Their properties, fabrication technologies, and ranges of application as well as the most important recent advances are described. Such microresonators may be used in highly stable sources of microwave signals and optical frequency comb oscillators, making it possible to link together optical and radio frequency standards.
Keywords: *optical microresonators, whispering gallery type modes.*

Optical dielectric whispering-gallery mode microresonators (WGMM) possess a number of unique properties, such as high Q factor and stability of frequencies, high field concentration in a small volume within the microresonator and at the surface, application over a broad range of wavelengths, and the use of linear and nonlinear dielectrics. Because of these properties, they may be used as efficient converters of small variations of environmental parameters into a variation of the frequency of the mode, measured with a high degree of precision. Such WGMMs were first demonstrated at the Department of the Physics of Vibrations of the Division of Physics of the Moscow State University in 1989 [1]. A whispering gallery mode may be represented as an optical wave that propagates near the inner surface of an axially symmetric dielectric body so that the angle of incidence to the interface exceeds the angle of total internal reflection (Fig. 1). The resonance in such a representation corresponds to the fact that an integral number of wavelengths $m\lambda$ may be contained in a single revolution ($\sim 2\pi Rn$, where R is the radius of the WGMM and n the index of refraction). In actual WGMMs, the value of m is usually quite large (in a quartz microsphere $m = 1000$ with $R = 70$ μm at a wavelength $\lambda = 0.63$ μm). Whispering-gallery modes, which possess a simple field distribution with a single maximum in both the radial and the meridional directions and high concentration of the electromagnetic field, are called fundamental modes. The frequency spacing between the fundamental modes, or region of free dispersion, is determined by the expression $\Delta\nu = c/(2\pi Rn)$, where c is the speed of light.

A survey of the basic properties of WGMMs and of early experimental studies of the microresonators is presented in the monograph [2]. The first WGMMs were fabricated from fused quartz in the form of microspheres with diameters of tens to hundreds of micrometers formed at the end of a fiber stretched to a required thickness as it was being heated to the melting point under the effect of forces of surface tension. The Q-factor Q of a WGMM does not depend on its linear dimen-

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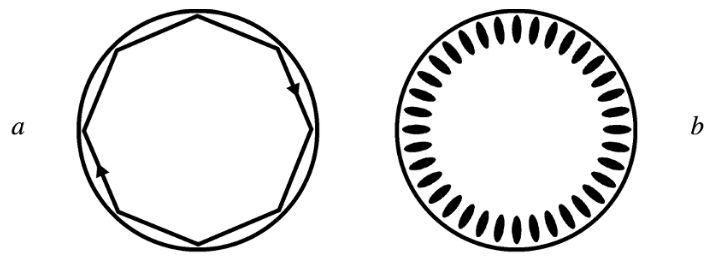

Fig. 1. Propagation of laser radiation within a disk whispering-gallery mode microresonator in an approximation of geometric optics (*a*) and the distribution of the strength of the optical wave in a microresonator at resonance (*b*).

sions but is instead determined basically by the losses in the material and on the surface of the device. The maximal Q-factor obtained in quartz microspheres is around 10^{10}, which is close to the theoretical limit of the fundamental losses in quartz at the wavelength employed [3].

There is a constantly growing interest in the subject of optical WGMMs today in connection with their potential use in different microphotonics devices. Fused quartz possesses a low coefficient of optical cubic nonlinearity and, as it is an isotropic material, does not possess a quadratic coefficient of nonlinearity and is practically insensitive to external fields. Therefore, WGMMs made of fused quartz may be used only in passive devices, such as filters, discriminators, displacement sensors, chemical and biological sensors, etc. The technology of fabricating WGMMs from different crystalline materials was a revolutionary development in the field of WGMMs. In particular, in WGMMs fabricated from crystalline fluorite (CaF_2), $Q > 10^{11}$ [4, 5]. The method of fabricating disc crystalline WGMMs from polished spheres of diameter 4.8 mm from lithium niobate ($LiNbO_3$) by joining together the upper and lower spherical sections was the first method of fabrication [6, 7]. The Q-factor of such microresonators $Q = 3{\cdot}10^6$–$3{\cdot}10^7$. It is precisely these studies, which were begun at the Jet Propulsion Laboratory and continued at the University of Southern California (Los Angeles, United States), which should be considered the first demonstrations of crystalline WGMMs and optical modulators as well as microwave receivers based on such microresonators. The firm of OEWaves, Inc. (United States) is a leader in the development and introduction of different types of devices based on WGMMs.

Through the use of the newly developed technology, it is possible to fabricate crystalline WGMMs of substantially lesser diameter while maintaining a rather high Q-factor of around 10^8 [8].

Fabrication of Optical Crystalline Whispering-Gallery Type Mode Microresonators. These types of microresonators are fabricated by means of diamond turning from disk or elongated cylindrical blanks that are mounted on the spindle of a lathe with small drifts of the axle. A method based on the use of a handmade micro-lathe with air cushions and computer-controlled feed of the diamond tool is described in [9]. WGMMs made of CaF_2 excised in this way had $Q = 10^7$. Manual asymptotic polishing of the smoothened generator of the surface of the disk on the same lathe by means of diamond powders, polishing paste, suspensions, or films with successively decreasing grain size is needed to achieve higher values of Q. Careful cleaning of the surface from residues of the abrasive must be performed following each stage of polishing. This method of fabrication may be applied to different crystals. To obtain the highest Q-factors, a blank of CaF_2 is subjected to repeated annealing at a temperature of 650°C for several days [6]. Annealing makes it possible to eliminate the residual stress in the crystal and the associated birefringence as well as increase the mobility of defects that have risen to the surface. A similar procedure was previously used to obtain high-Q disk WGMMs from leucosapphire in the microwave range. Following each annealing, the blank is repolished. As a result of a thrice-repeated procedure, $Q = 3{\cdot}10^{11}$ at a wavelength 1.55 μm in a disk WGMM 4.5 mm in diameter and 0.5 mm in thickness, which corresponds to a sharpness of (2.1 ± 10^7), which is ten times greater than in the case of super-mirrors of Fabry–Perot resonators.

Magnesium fluoride (MgF_2) has become increasingly popular as a material for optical micro- and mini-resonators. It has low optical losses ($Q > 10^9$) and, unlike CaF_2 [10], positive coefficient of temperature dependence of the index of refraction, which is important for passive stabilization of the frequency of optical combs. Though both materials are centrally symmetric and, consequently, do not possess field-quadratic polarization coefficients, cubic nonlinearity in combination with high Q-factor produces a variety of frequency shift effects. These types of WGMMs have been studied in order to create stable microwave oscillators, converters, and optical combs with a broad potential range of applications. A WGMM fabricated in similar fashion from leucosapphire Al_2O_3 has a Q-factor $6.2 \cdot 10^8$.

$LiNbO_3$ has drawn the greatest interest of all the materials with quadratic nonlinearity and optoelectronic effect. The index of refraction $n = n_{ex} - n_{ex}^3 E r_{33}/2$ varies in this material when a stress is applied along the *z*-axis of the crystal (E is the field strength; $n_{ex} = 2.14$, the index of refraction of an extraordinary wave; and $r_{33} = 30.8$ pm/V, the electro-optic coefficient of $LiNbO_3$) and, consequently, the resonance frequency varies.

The geometry of WGMMs and the technique of polishing exert a well-defined influence on the optical Q-factor and efficiency of the bond with crystalline WGMMs. Disk WGMMs are fabricated either from cylindrical blanks or from flat plates. WGMMs are produced from disk blanks of radius $R = 1$–3 mm that have been preliminarily cut out of a plate by means of a tubular drill. The lateral surface of the disk is polished, imparting to it a rounded form with radius of curvature R' equal to the radius of the disk. An effort was then made so that the equator of the rounded disk surface would be close to one-half the height of the plate. The condition of a spherical generatrix is not necessary, and optimization of the curvature of the lateral surface in order to attain the required structure of the transverse modes is possible. By imparting to the lateral surface a toroidal form (radius of curvature equal to one-half the height) or even a biconical form (radius of curvature much less than the thickness), it becomes possible to substantially thin out the spectrum while preserving a high Q-factor.

Optical polishing of flat $LiNbO_3$ surfaces is a routine procedure in industry though polishing of lateral rounded surfaces with optical quality is not a standard practice. The best results have been achieved through the use of a special treatment of the lateral surfaces. A Q-factor greater than $4 \cdot 10^6$ at a wavelength of 1550 nm has been obtained in these types of disk WGMMs. The process of polishing WGMMs begins with thinning the disk to a required width by means of 30 μm and 5 μm corundum powders. The required curvature is then imparted to the lateral surface of WGMMs through the use of corundum powders with grain size 30, 9, and 5 μm, with powders of cerium oxide (2.5 μm) even being used for optical polishing. The quality of the lateral surface is monitored using a microscope with $150^{\times}$ lens by means of the method of Nomarskii interference-contrast microscopy. Following polishing, no roughness is observed along the equatorial surface of the disk. The quality of the polishing is confirmed by means of measurements of the optical Q-factor.

WGMMs from another promising crystalline nonlinear optical material, β-barium borate (β-BaB_2O_4), were first demonstrated in [11]. The advantage of β-barium borate as compared to $LINbO_3$ lies in its lower index of refraction ($n_{ex} \approx 1.54$, $n_O \approx 1.66$ in the visible and near-infrared region), which makes it possible to use prismatic coupling elements that are in greater supply (prisms of leucosapphire were used in [11]). Moreover, β-barium borate is transparent over a broad band of wavelengths from the ultraviolet to the infrared band. In the course of operation, Q-factors of WGMMs $1.5 \cdot 10^8$, $3.8 \cdot 10^8$, and $7.4 \cdot 10^8$ at wavelengths of 370, 980, and 1560 nm, respectively, were obtained. β-barium borate has a significantly lower electro-optic coefficient ($r_{11} = 2.7$ pm/V) than does $LiNbO_3$. A new optical material, SBN ($SrBaNb_2O_6$, $r_{33} = 1400$ pm/V, $n_O = 2.31$, $n_{ex} = 2.27$, $\varepsilon_{32} = 900$), which also exhibits rather low optical losses, is of interest from the point of view of the electro-optic effect, though such WGMMs have not yet been created.

Relationship between Sources and Receivers of Radiation with Disk Crystalline Whispering-Gallery Mode Microresonators. A number of different coupling elements with the operating principle based on the effect of disturbed total internal reflection have been developed [2] to excite whispering gallery modes. There are two methods of realizing a coupling with WGMMs that are the most frequently used today, first, by means of a prism and, second, by means of an elongated fiber. A prismatic coupling element had already been used in the first spherical WGMMs [1]; moreover, a total internal reflection prism (Fig. 2) in which the excited beam is focused on the inner plane at the site of reflection was used. Excitation of modes is possible once a WGMM is brought into contact with the prism if the approximate relation of phase synchronism is satisfied, i.e., $\sin \theta_i = n_r/n_p$, where θ_i is the angle of reflection within the prism and n_r and n_p the indices of refraction of the microresonator and the prism, respectively. The strict condition $n_r > n_p$ must be satisfied in order that the condition of phase synchronism holds. The condition is easily satisfied for WGMMs made of quartz, fluorite, and magnesium fluorite, though the index of refraction of $LiNbO_3$ at a supply wavelength of the order of 1500 nm $n_{ex} = 2.14$ and $n_O = 2.23$, which sharply limits

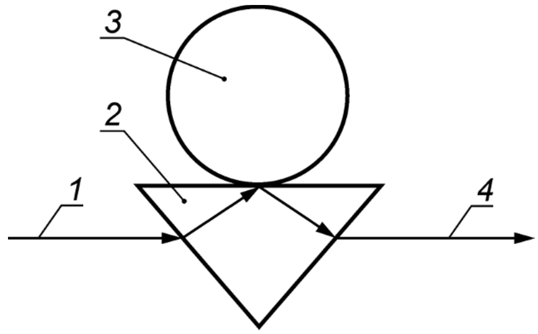

Fig. 2. Use of a prism as a coupling element between a microresonator and laser beam: *1*, *4*) incident and transmitted beam, respectively; *2*) prism; *3*) microresonator.

the choice of material that may be used for the prism. Even in the first studies [6], microprisms made of artificial crystalline diamond (n_p = 2.4) had been used for coupling radiation sources and receivers with WGMMs made of $LiNbO_3$.

Zinc selenide ZnSe (n_p = 2.4), which, unlike diamond, produces noticeable losses in the visible range (orange tint), which may make it difficult to tune the system with the use of visible-range lasers, is an alternative material for coupling prisms. A segment of one-half an analogous disk made of the same material as the WGMM may also be used as a coupling element. In turn, radiation from a laser or fiber may be introduced into the prism. Effective coupling by means of this method has still not been achieved in an experiment. Despite the fact that the index of refraction of fused quartz is much less than that of $LiNbO_3$, a coupling between radiation sources and receivers with disks made of $LiNbO_3$ by means of a stretched optical fiber was obtained. More than 30% of the power could be introduced into such a microresonator 1 mm in thickness. A coupling element based on a stretched fiber is simple to use under laboratory conditions, though it does not assure sufficient rigidity of the structure and is sensitive to acoustic noise, which reduces the range of use of the method in instrument construction. The coupling systems considered in [6], which are based on integrated waveguides of $LiNbO_3$, which support coupling at several tens of percent, are of greater interest.

Integrated Ring and Disk Whispering-Gallery Mode Microresonators. Optical high-Q crstyalline WGMMs are similar in many respects to optical integrated ring and disk WGMMs fabricated by means of standard methods of lithography and etching. Such WGMMs usually have diameter in the range 10–200 μm and thickness in the range 1–50 μm. The difference between the modes of disk WGMMs (wave reflected only from the outer lateral surface) from those of ring waveguide WGMMs (wave reflected from the outer and inner surfaces) is not essential from the standpoint of most applications. The principal advantage of integrated optical WGMMs is the fact that they are easily integrated with other microphotonic devices, in particular, with waveguide coupling elements on a single optical microcircuit. The main drawback of such WGMMs is the limited choice of materials that may be used for fabrication and the Q-factor $Q = 10^3$–10^4, which is basically due to the roughness of the surface. Microdisk WGMMs made of indium phosphide (InP) have been fabricated by means of methods of optical lithography and joining of substrates. The diameter of these devices is in the range 4–12 μm and the devices are linked to a comb waveguide produced on the same substrate either vertically or horizontally. Optical filters, switches, and modulators with Q = 7000 are possible applications. Semiconductor microdisks may be used as active switches and modulators, since their index of refraction and losses may be changed by injecting free carriers and, correspondingly, the optical Q-factor and resonance wavelength will be determined by the current through the external electrodes. Q = 8500 in analogous ring modulators made of indium gallium arsenide (InGaAs). An integrated ring WGMM 12 μm in diameter made of silicon (Q = 39000) exhibited even better parameters. In integrated ring WGMMs made of InP produced by means of deep etching (height of waveguide 5 μm) Q = 90000, though the oscillogram presented in the article attests to a value roughly one-third as great. Due to high contrast, the indices of refraction with the surrounding air both in the case of a WGMM and in the case of waveguides and comb semiconductor structures are possibly not optimal from the point of view of obtaining maximal Q-factor and coupling efficiency. Buried ring WGMMs made of InP/InGaAs with $Q = 10^5$ have recently been demonstrated.

With the further development of technologies, the drawback of low Q-factors of integrated ring WGMMs will be overcome and the achieved life in the field of high-Q crystalline WGMMs will be realized in an integrated implementation. Microresonators with wedge edge made of fused quartz produced by means of photolithography and etching without the use of manual operations with $Q > 10^8$ [12] as well as silicon disk and ring waveguide WGMMs with $Q = 5 \cdot 10^6$ and $Q = 1.4 \cdot 10^5$, respectively, have already been demonstrated [13, 14].

Existing ring WGMMs made of $LiNbO_3$ possess maximal $Q \sim 4 \cdot 10^3$ [15]. With a radius of 100 μm, a shift in the resonance frequency by 0.1 nm upon the application of a 100-V voltage (~0.14 GHz/V), which corresponds to modulation of the output signal by roughly 30%, has been demonstrated in such a WGMM. It should be noted that through the use of experimentally proven integrated waveguides made of $LiNbO_3$ with losses of the order of 0.1 dB/cm [16], we may hope for the creation of integrated WGMMs from this material with $Q = 4 \cdot 10^6$ at a wavelength of 1.55 μm and, correspondingly, an improvement in the modulation parameters by a factor of 10^3.

Besides $LiNbO_3$, optoelectronic polymers have also recently been considered for use in modulators based on ring WGMMs. These types of polymers have been previously used to create ordinary Mach–Zehnder-type modulators. Ring WGMMs with radius 300 μm made of the optoelectronic polymer CLD1/APC (r_{33} = 36 pm/V) and coupling waveguides made of the polymer SU-8 fabricated by means of optical lithography have been created.

Resonance Optical Modulators. Traditional amplitude waveguide modulators based on $LiNbO_3$ with Mach–Zehnder interferometer formed by two parallel arms of optically buried waveguides in an $LiNbO_3$ substrate 1–2 cm in length formed by diffusion of titanium or by some other method are widely used in modern optical communication lines. Phase-matched 50-Ω coplanar electrodes are used to increase the overlapping of the optical and electrical fields, while constant-voltage electrodes are used for control. The characteristic voltage of total modulation V_π is usually around 5 V. This type of modulator may possess a band exceeding 40 GHz [17] with the efficiency of convergence of the microwave signal in such modulators not very high.

Electrical and optical resonances or both simultaneously may be used in resonance optical modulators to increase interaction. Through the use of electrical resonance, the amplitude of the electrical field may be increased, while through the use of optical resonance the interaction length and time may be increased, though in both cases the modulation band decreases, which is determined by the greatest optical Q-factor of all the operational modes. The conception of resonance modulation was first proposed in 1963 [18] using the example of modulation of light in a Fabry–Perot resonator.

Optical Modulators and Microwave Receivers Based on Crystalline Disks. Devices capable of receiving, transforming, and processing signals in the millimeter and centimeter ranges are required for microwave honeycomb communication systems and other networks of personal communication systems. It is possible that this role may be fulfilled by optoelectronic modulators based on the interaction of electromagnetic waves in nonlinear optical WGMMs.

An approach by means of which microwave and light fields may be linked in a WGMM was demonstrated in 2000 [6]. In these studies, the effective resonance interaction between several optical WGMMs and the microwave mode is achieved as a consequence of careful development of the form of the microwave resonator linked to optical toroidal WGMMs, as a result of which double resonance is achieved. A new type of optoelectronic modulator as well as a photonic receiver of microwave radiation was proposed and realized on the basis of such interaction [7]. A survey of the contemporary state in the field of optical disk modulators and receivers is presented in [19].

A diagram of such a modulator is presented in Fig. 3. In this case, both the optical and the microwave field are resonance fields, hence the sensitivity is increased [7]. A half-wave band resonator in the form of a half-ring deposited on the surface of a crystalline disk was used as the microwave resonator. Its resonance frequency is tuned as a function of the length of the half-ring and is made equal to the region of free dispersion. A diamond prism is used to realize optical excitation of the WGMM, with two different prisms sometimes being used for input and output of the radiation. A ring microwave band resonator may be used in place of a half-ring, half-wave band electrode, though tuning of its resonance frequency represents a more complicated process and requires a preliminary exact calculation. Such an electrode assures better overlapping with the microwave field than a half-ring or disk electrode; a sensitivity –85 dBc (3 pW with signal-to-noise ratio 1 and frequen-

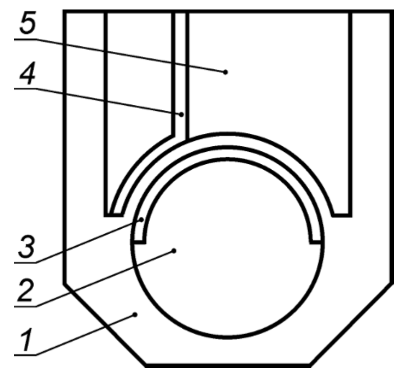

Fig. 3. Diagram of modulator based on a toroidal resonator made of lithium niobate: *1*) substrate; *2*) optical disk microresonator; *3*) metallic band – coupling element; *4*) waveguide (coupling input); *5*) waveguide insulation.

cy 14.6 GHz) was demonstrated in precisely such a configuration. A ring electrode assures better overlapping especially in the case of low-frequency modulation within the optical band. Placement of the resonator not on a planar, but instead on a cylindrical grounded electrode corresponding to the diameter of the WGMM also improves overlapping of the microwave and optical fields. More complex configurations of the electrodes are possible [20].

The sensitivity of a micro-disk modulator is proportional to the optical and microwave Q-factors and the overlapping of the optical and microwave fields and is inversely proportional to the thickness of the disk. Modulators with disks up to 100 μm in thickness and unloaded Q-factor up to $5.7 \cdot 10^8$ have been demonstrated.

The central frequency of the region of free dispersion of modulation in such devices has been substantially increased from 7–9 GHz to 14.6–35 GHz and recently to 100 GHz [21].

Optical modulators based on disk WGMMs may be used both as radio frequency or microwave receivers that transfer the signal into the optical range, where it may be processed and demodulated. For this purpose, it is only necessary to add a corresponding antenna, hence in published studies optical modulators for the microwave range are sometimes called photonic microwave receivers [22].

In early experiments, the sensitivity of the disk WGMMs of receivers reached 160 μW (at a voltage of 69 mV) at a frequency of 7.56 MHz. In 2008, a sensitivity of 100 pW at a frequency of 35 GHz was attained as a result of a decrease in the band to 5 MHz. In 2011, a sensitivity of 100 pW at a frequency of 35 GHz was achieved as a result of a decrease in the band to 5 MHz. In 2011 a sensitivity of 3 pW (~85 dBc) in a 60-MHz band at a frequency of 14.6 GHz that is still a record-breaking level today was attained.

Antennas and Antenna Receivers Based on Disk Whispering-Gallery Mode Microresonators. One of the potential applications of optical modulators based on disk WGMMs consists in rejection of the use of metallic electrodes in a microwave resonator. The WGMM is placed within a dielectric microwave resonator, which may be used as a receiving dielectric resonance antenna [22]. In a dielectric resonance antenna functioning at a resonance microwave, the field is concentrated near the center. If a WGMM is placed in the slot of a dielectric resonance antenna so that overlapping of the optical and microwave fields is maintained, effective modulation may be achieved. Through the use of such a circuit, the receiving device may be protected from the action of powerful microwave pulses. The antenna is designed for the $TM_{011+\delta}$ mode and is fabricated from $BaTiO_3$ composite ceramic with dielectric permittivity of the order of 100 and a WGMM 3.3 mm in diameter made of $LiNbO_3$. The microwave field is conducted to the device by means of a horn antenna. The measured sensitivity of the antenna to the free-space field is 112 dBc/Hz.

The sensitivity of such antennas was investigated theoretically in [23]. It was shown that the sensitivity of a dielectric antenna is no higher than the sensitivity of a metallic resonator. The increase in sensitivity with the use of optical dielectric horn antennas of concentrators with WGMMs situated in the near-field region reaches roughly 160 dBc/Hz.

Let us next consider the use of semi-elliptic dielectric lens antennas also based on the effect of total internal reflection as antennas for linkage with WGMMs. In [24], it was proposed that an array of optical WGMMs fabricated from optoelectronic materials and related to optical waveguides be used in antennas and arrays by inserting them into the focal plane of the receivers, which makes it possible to transform the received microwave signal into an optical signal for subsequent transmission and processing. A proposal for potential implementation is not contained in the patent. It was, however, proposed in the patent that WGMMs be used in the receiver both as receivers and as optical filters partially tuned to the electrical voltage.

Optoelectronic Oscillators. Spectrally pure signals in the range 1–100 GHz are needed for communication, navigation, and detection and ranging. The appearance of wideband optical communication lines suggests that it is possible that networks operating at speeds up to 160 Gb/sec and consisting of numerous channels separated by frequencies into several gigahertz may be created in the future. The realization of these potentialities depends on sources capable of supporting high-frequency signals with low phase noise without which error-free, high-speed systems cannot be realized. In precisely the same way, high-efficiency radar systems require oscillators with very low phase noise for the purpose of distinguishing weak signals.

An optoelectronic oscillator is a device that produces a spectrally pure signal at a frequency of several gigahertz and by means of which many of the constraints that are inherent to ordinary electronic devices may be overcome [25–27]. A continuous laser serves as the source of the luminous energy in an optoelectronic oscillator. Before being transformed in a fast photodiode into an electrical signal, the radiation of the laser travels through a modulator and a device that stores the optical energy, for example, a long optical fiber or resonator. The high-frequency electrical signal at the output of the photodiode is amplified, filtered, and then fed to a modulator, which closes a feedback loop. If the total amplification exceeds the linear losses in the feedback loop, the system begins to generate oscillations at a frequency determined by the filter.

Through the use of energy-storage optical elements, it becomes possible to achieve exceptionally high Q-factors and, consequently, generate spectrally pure signals in the optical range, since the noise characteristics of the oscillators are determined by the energy storage time or Q-factor. In particular, a delay in a fiber 100 meters in length yields a microsecond storage time, which corresponds to a Q-factor of the order of a million for a generation frequency of 10 GHz, which exceeds the values obtained with ordinary dielectric microwave oscillators [28]. A fiber delay line makes possible generation over a broad range of frequencies without the usual decrease in Q-factor at high frequencies. Thus, spectrally pure signals at frequencies of up to 43 GHz limited only by the band of the modulator and receiver have been demonstrated.

In an ordinary optoelectronic oscillator [25], a long fiber delay line supports a set of microwave frequencies superposed on an optical wave. A narrow-band electronic filter is needed to achieve stable single-mode generation in the electronic part of the feedback of the optoelectronic oscillator. The central frequency of such a filter determines the effective frequency of the optoelectronic oscillator. Though such an approach also assures a high-frequency signal of a required spectral purity, the overall dimensions of the resulting optoelectronic oscillator prove to be very unwieldy, since kilometer-length fiber delay lines are required. Moreover, coils with long fiber delay lines are very sensitive to changes in the environment, and, therefore, such an optoelectronic oscillator does not produce an output signal with high long-term precision and stability. The use of WGMMs in the form of microspheres has been proposed for use in optoelectronic oscillators as an alternative to fiber delay lines. The properties of optoelectronic oscillators with high-Q whispering gallery mode in place of an electronic filter and delay line have been considered in detail in [29], where it is shown that with such an approach practically any type of lasing line may be selected through adjustment of the WGMM.

In an optoelectronic oscillator, the modulator is one of the basic elements that consume energy. Both wideband modulators based on Mach–Zehnder interferometers and spatial resonator narrowband microwave modulators require from one to several watts of microwave power to assure substantial modulation. This means that either a high gain factor of the photocurrent must be achieved or powerful lasers used.

Optoelectronic oscillators with resonance modulator based on WGMMs were proposed and fabricated in [30]. The device is characterized by a low threshold and low energy consumption. The drawbacks of the device include low saturation threshold and low output power, as well as the fact that optical noise may be transferred to the microwave signal. Thus, resonance and ordinary optoelectronic oscillators possess nonoverlapping characteristics and may be equally useful, depending on the required applications.

On the basis of the foregoing discussion, WGMMs may be used in optoelectronic oscillators twice, first, as a delay line and, second, as a modulator. Different circuits of such optoelectronic oscillators with two or one WGMMs are considered in [31]. With this type of practical application, the dimensions of the device may be radically reduced to the dimensions of a microchip. The micro-optoelectronic oscillators that have been commercially produced by the firm of OEwaves function at a frequency of 35 GHz and maintain a low level of phase noise, 108 dBc/Hz at an offset of 10 kHz. The stationary optoelectronic oscillators produced by the firm that function at a frequency of 8–10 GHz assure a level of phase noise of 163 dBc/Hz at an offset of 10 kHz, and in a compact version, 140 dBc/Hz.

A 16-channel optical source for data transmission systems with channels separated by a frequency of 12 GHz with high contrast of the optical spectrum 28 dB and based on an optoelectronic oscillator linked to high-Q WGMMs has also been demonstrated by the firm of OEwaves [32, 33].

Optical Combs. Whispering-gallery mode microresonators with Kerr nonlinearity may be used to generate optical pulses with stable high repetition frequency or as stable microwave oscillators when optical pulses are fed to a high-frequency receiver.

Hyperparametric optical oscillations, which are referred to in fiber optics as modulation instability, are based on a four-wave shift between two pump photons, a signal and an idler wave, and lead to an increase in initiated vacuum fluctuations of the optical side bands in the signal and the idler waves due to the pumping power. Hyperparametric generation differs from parametric generation based on $\chi^{(2)}$ nonlinearity which links three photons and requires maintaining the conditions of phase synchronism for very different frequencies, which in unidirectional propagation, may be fulfilled only in birefringent materials. In contrast, hyperparametric generation is based on $\chi^{(3)}$-type nonlinearity, which links four photons, and the conditions of phase synchronism are imposed on close, nearly degenerate optical frequencies and, therefore, may be fulfilled in most materials in both codirectional and counter-directional propagation of waves. Once the threshold of hyperparametric generation has been exceeded, it may develop in cascade fashion, accompanied by the appearance of newer side spectral bands and gradual population of new frequencies from the spectrum of the modes of a WGMM due to a four-wave shift.

Investigations of hyperparametric generation have reached a new level in recent years due to the development of the technology of crystalline and integrated WGMMs. Generation was first observed experimentally in crystalline WGMMs [34]. In particular, it was proposed that the narrow-band beat signal between optical pumping and the side bands generated in a high-Q WGMM be used as a secondary frequency standard [34].

The phase stability of a frequency signal of the same pumping power grows as the Q-factor of the modes of a WGMM increases. For the beat signal, there exists a phase stability limit (minimum phase diffusion) which does not depend on either the pumping power or the Q-factor of the modes. Using the fact that the Q-factor of a WGMM may exceed 10^{10} (width of resonance line of several kilohertz) [35], it has been shown that the Allan variation, which determines the stability of oscillations, may be less than 10^{-12} sec$^{-1/2}$ in optical pumping of less than a milliwatt. The generation threshold for reasonable parameters of an experiment may be at the level of a microwatt.

The theory of hyperparametric generation in WGMMs is considered in [36]. Expressions were obtained for the instability threshold and phase fluctuations in a difference microwave signal in a receiver possessing sufficiently high stability, which may be employed in secondary frequency standards. A photon microwave oscillator at a frequency of 8.5 GHz with noise level –120 dBc/Hz at an offset of 100 kHz from the carrier frequency was experimentally demonstrated; moreover, it is also assumed that the parameters may be improved even more substantially.

Hypergeometric generation from the point of view of the production of only two side frequencies around the pumping mode was considered in these studies. The basic focus today is on the production of optical combs in WGMMs. Frequency combs are formed with pumping at a resonance frequency near the frequency at which null total dispersion of a WGMM and the material is attained as a consequence of numerous nonlinear processes that begin with hypergeometric generation at symmetric neighboring modes with subsequent four-wave interaction between the formed modes. Moreover, in the regime of fully developed generation, frequency spacing between the modes of the comb is equalized and stabilized, and synchronization of the modes occurs. In monochromatic pumping, the resulting optical spectra may contain hundreds or thousands of lines with repetition period corresponding to a region of free dispersion that overlaps the optical range for hundreds of nanometers. The experimental display of the generation of optical combs in a WGMM in the case of continuous pumping is one of the most fascinating and most promising achievements of recent years. Such optical combs have been demonstrated in microspheres, microtoroids, crystalline disk CaF_2 and MgF_2 WGMMs, and integrated Si_3N_4 whispering-gallery mode ring microresonators. Studies designed to obtain stable optical combs with broad optical spectrum, narrow line of frequency beats, and low phase noise have recently been carried out. A current survey of studies on the subject of optical combs in WGMMs in 2011 is presented in [37].

Conclusions. Our survey enables us to assess the potential for the use of whispering-gallery mode microresonators in the creation and improvement of measuring instruments. The use of WGMMs opens new horizons in the area of miniaturization of devices, increases in the precision of measurements, the creation of sensors and microelectronics devices, and improvements in the standards base. Among the important problems that have to be solved in order to achieve widespread use of devices based on WGMMs we may include the development of integrated technology that will make it possible to fabricate microresonators with high Q-factor, coupling elements, and auxiliary components in a unified production process.

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