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TIME AND FREQUENCY MEASUREMENTS

RELATIVISTIC EFFECTS ON MOVING CLOCKS

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Using the general theory of relativity, we obtain practical formulas for calculating relativistic effects on clocks moving in the anomalous gravitational field of the Earth. We examine compensation methods for these effects and determine the compensation error using the orbital parameters of the motion path for clocks in GLONASS/GPS systems
Keywords: *moving clocks, relativistic effects, gravitational potential,compensation methods.*

The problem of relativistic effects on moving clocks has been studied both in domestic and in foreign publications [1–7]. However, the results are applicable only for short routes and for bounded frequency stability standards for moving clocks. With increasing stability standards there is an increase in the requirements to account for relativistic effects, especially in global movements influenced by the anomalies of the Earth's gravity field along the travel route. In this regard, it is necessary to consider the relativistic effects from the standpoint of the general theory of relativity (GR) in the measurement of time and frequency, as well as to assess the precision limits in accounting and compensating for these effects using synchronous orbital data for clocks moving in GLONASS/GPS systems.

Frequency and Time in Moving Clocks in GR. We derive the change in proper (directly measured) time for a observer of frequency and time, moving relative to the Earth's surface using a geocentric coordinate system rotating with the Earth (ITRF – International Terrestrial Reference Frame) with unit vectors **i**, **j**, **k** along the axes *XYZ*, respectively. For this, we use the general expression for the square of the space-time interval ds [8]:

$$ds^2 = g_{00}c^2dt^2 + 2g_{0\alpha}dx^\alpha cdt + g_{\alpha\beta}dx^\alpha dx^\beta, \tag{1}$$

where c is the velocity of light; dt is a an infinitesimal interval of coordinate time t which does not depend on the parameters of the gravitational field; g_{00}, $g_{0\alpha}$, $g_{\alpha\beta}$ are the components of the four-dimensional metric tensor g_{ik} of the gravitational field in the selected frame of reference; dx^α, dx^β are spatial infinitesimal intervals; here and below, α, β = 1, 2, 3 and i, k = 0, 1, 2, 3.

The interval ds is invariant with respect to the choice of the reference system and therefore does not alter the values of any instantaneously chosen nonrotating inertial reference system [8]. Therefore, in an inertial frame:

$$ds^2 = g_{00}c^2dt^2 + g_{11}dx^2 + g_{22}dy^2 + g_{33}dz^2,$$

where $g_{\alpha\alpha} = 1$, $g_{00} = -1$. For two events at the same spatial point ($dx = dy = dz = 0$), $ds^2 = g_{00}c^2dt^2 = -c^2d\tau^2$, where $d\tau$ is the system proper time. As a result, the relation (1) becomes [8]:

$$-c^2d\tau^2 = \gamma_{\alpha\beta}dx^\alpha dx^\beta - \left[\sqrt{-g_{00}}\,cdt - (g_{0\alpha}/\sqrt{-g_{00}})dx^\alpha\right]^2, \tag{2}$$

where $\gamma_{\alpha\beta} = g_{\alpha\beta} - g_{0\alpha}g_{0\beta}/g_{00}$ is a three-dimensional metric tensor.

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Coordinate velocity of the observer with respect to origin of the chosen reference system is

$$V_{\mathrm{c}}^{\alpha} = dx^{\alpha} / dt; \quad V_{\mathrm{c}}^{2} = V_{\alpha} V^{\alpha} = \gamma_{\alpha\beta} V^{\alpha} V^{\beta}. \tag{3}$$

Using (3) and relation (2), we obtain the conversion factor of the proper time τ relative to the coordinate time t for an observer moving in the gravitational field with coordinate velocity V_{c}:

$$d\tau / dt = \left\{ \left[\sqrt{-g_{00}} - g_{0\alpha} V_{\mathrm{c}}^{\alpha} / (c\sqrt{-g_{00}}) \right]^{2} - V_{\mathrm{c}}^{\alpha} / c^{2} \right\}^{0.5} = \theta. \tag{4}$$

We introduce a moving observer of frequency and time with a proper time scale τ_{m}, as well as a stationary (base) observer with proper time scale τ_0. The connection between the intervals τ_{m} and τ_0 is established under the condition that for both observers the coordinate time intervals are the same: $dt_{\mathrm{m}} = dt_0 = dt$. In addition, the transition from the coordinate velocity of the moving observer V_{c}^{α} to his velocity V^{α}, measured in units of time τ_0 by the base observer, uses the relation $V_{\mathrm{c}}^{\alpha} = V^{\alpha}\theta_0$. As a result, we obtain

$$d\tau_{\mathrm{m}} = (\theta_{\mathrm{m}} / \theta_0) d\tau_0, \tag{5}$$

where $\theta_0 = \sqrt{-g_{00}}$ is the transformation coefficient for the time scale τ_0 determined from (4) for $V_{\mathrm{c}}^{\alpha} = 0$; θ_{m} is the transformation coefficient of the time τ_{m} relative to the coordinate time t for the moving observer, defined by formula (4):

$$\theta_{\mathrm{m}} = \left\{ \left[\sqrt{-g_{00}^{\mathrm{m}}} - \mathbf{GV}\theta_0 / (c\sqrt{-g_{00}^{\mathrm{m}}}) \right]^{2} - (\mathbf{V}\theta_0 / c)^{2} \right\}^{0.5},$$

where $\mathbf{G}$ is the vector potential of the gravitational field with components $G_{\alpha} = g_{0\alpha}$, defined along the route of the moving observer; $\mathbf{V} = \mathbf{i}V_x + \mathbf{j}V_y + \mathbf{k}V_z$ is the speed of the moving observer with axis components V_x, V_y, V_z, which generally vary along the route of the moving observer; g_{00}^{m}, the time component of the four-dimensional metric tensor, also varies along the route of the moving observer.

The time interval $\Delta\tau_{\mathrm{m}}$ recorded by a moving clock is determined by integrating relation (5) over the time interval $\Delta\tau_0$; the relativistic effect of divergence between the time intervals recorded by the stationary and moving clocks is determined by the expression

$$\delta\tau_{\mathrm{r}} = \Delta\tau_{\mathrm{m}} - \Delta\tau_0 = \int_{\tau_{01}}^{\tau_{02}} (\theta_{\mathrm{m}}(\tau_0) / \theta_0) d\tau_0 - \Delta\tau_0, \tag{6}$$

where τ_{01}, τ_{02} are the initial and final endpoints of the integration interval over the time scale τ_0, and $\Delta\tau_0 = \tau_{02} - \tau_{01}$.

Since the phase of the electromagnetic signal is invariant relative to the choice of the reference system (observation point) [8], the phase shift of the output signal generated by the base observer can be written in the form of $d\Psi = \omega_{\mathrm{c}} dt = \omega_0 d\tau_0$, where ω_{c}, ω_0 are coordinate and proper frequencies, respectively. On the other hand, the same phase shift can be expressed in terms of the proper frequency ω_{m} generated by the moving observer in the form $d\Psi = \omega_{\mathrm{c}} dt = \omega_{\mathrm{m}} d\tau_{\mathrm{m}}$. If the coordinate frequency and the intervals of coordinate time are equal then the phase shift in both cases will be equal. As a result, we obtain $\omega_0 d\tau_0 = \omega_{\mathrm{m}} d\tau_{\mathrm{m}}$. Hence, using relation (5), we find the relation between the natural proper cyclic frequencies of the moving f_{m} and base f_0 observers:

$$f_{\mathrm{m}} = f_0 \theta_0 / \theta_{\mathrm{m}}.$$

Next, we find the ratio of the mutual relativistic effect of the frequency shift of oscillators with the moving and base observers:

$$\delta f_{\mathrm{r}} / f_0 = (f_{\mathrm{m}} - f_0) / f_0 = \theta_0 / \theta_{\mathrm{m}} - 1. \tag{7}$$

The generalized expression (7) determines the effect of the relativistic frequency shift due to the difference in the flow of time at two points of the coordinate system in a frame rotating with the Earth. This effect in the gravitational field of the Earth, predicted by Einstein, is sometimes called the redshift.

Calculating the Effects on Frequency and Time in Moving Clocks. To associate formulas (4)–(7) with the parameters of Earth's rotation and its gravitational field, we provide as the basis for further calculations the metric tensor of a non-rotating earth reference frame ITRF, which has the following components [9]:

$$g_{00} = -(1 - 2\varphi/c^2); \quad g_{\alpha\beta} = (1 + 2\varphi/c^2); \quad g_{0\alpha} = 0, \tag{8}$$

where φ is the potential of the gravitational field created by the distribution of the mass density of the Earth.

In solving practical problems, we ignore the component defined by the angular momentum of the Earth due to its small value [10]. Below we use a rectangular coordinate system rotating around the axis OZ, and in this new system we compute the space-time interval (1), obtaining the following components of the metric tensor:

$$g_{00} = -(1 - 2\Phi/c^2); \quad g_{\alpha\beta} = \delta_{\alpha\beta}(1 + 2\varphi/c^2); \quad g_{0\alpha} = G_\alpha; \quad \mathbf{G} = [\mathbf{\Omega R}]/c,$$

where the square brackets denote the vector product; $\mathbf{\Omega} = \mathbf{k}\Omega_z$ is angular velocity vector of the Earth directed along the axis OZ; $\Omega_x = \Omega_y = 0$; $c\mathbf{G} = -\mathbf{i}\Omega_y + \mathbf{j}\Omega_x$; $\mathbf{R} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$ is the radius vector of the point in question with current coordinates x, y, z; $\delta_{\alpha\beta}$ is the unit tensor of second rank; $\Phi = \varphi + \varphi_\Omega$ is the full gravitational potential at a given point in the vicinity of the Earth, which is the sum of the actual potential of the gravitational field of the Earth φ and the potential of the force field induced by the Earth's rotation, $\varphi_\Omega = 0.5\mathbf{G}^2 = 0.5[\mathbf{\Omega R}]^2$, where $\mathbf{\Omega} = 7.29 \cdot 10^{-4} c^{-1}$ is the angular velocity of the Earth.

The potential of the gravitational field of the Earth can be most fully described by an expansion in terms of spherical functions [11]. In this case, it is convenient to separate the normal φ_n and anomalous $\Delta\varphi_{an}$ components. The full gravitational potential Φ in this case can be represented as

$$\Phi = \varphi_n + \varphi_\Omega + \Delta\varphi_{an},$$

where $\varphi_n = (\mu/\rho)[1 - J_2(R_e/\rho)^2 P_2(\sin\psi)]$ is the normal component of potential defined by the zeroth harmonic and the second zonal harmonic expansion with coefficient $J_2 = 1.0826 \cdot 10^{-3}$; $P_2(\sin\psi) = (3/2)\sin^2\psi - 1/2$ is a Legendre polynomial; ψ is the geocentric latitude; $\mu = 3.986 \cdot 10^{14}$ m^3/sec^2 is a geocentric gravitational constant; $\rho = (x^2 + y^2 + z^2)^{1/2}$ is the geocentric distance of the point in question; $R_e = 6.378 \cdot 10^6$ m is Earth's equatorial radius (semi-major axis of the reference ellipsoid);

$$\varphi_\Omega = 0.5\Omega^2(x^2 + y^2) = 0.5\Omega^2\rho^2\cos^2\psi; \tag{9}$$

$$\varphi_{an} = (\mu/\rho)\left[-\sum_{n=3}^{\infty} J_n (R_e/\rho)^n P_n \sin\Psi + \sum_{n=2}^{\infty}\sum_{m=1}^{n} (R_e/\rho)^n (C_{nm}\cos(m\lambda) + S_{nm}\sin(m\lambda)) P_{nm}\sin\Psi\right]$$

is the anomalous component of the field, including the zonal harmonics from the third, as well as sectorial and tesseral harmonics of the potential expansion in spherical harmonics with coefficients C_{nm}, S_{nm}; $P_{nm}\sin\psi$ are associated Legendre polynomials; λ is the longitude of the point in question.

Based on (8)–(9), the scale transformation coefficients can be expressed as

$$\theta_0 = 1 - \Phi_0/c^2; \quad \theta_m = 1 - (\Phi_m/c^2) - c^{-2}[\mathbf{\Omega R}_m]\mathbf{V} - V^2/2c^2, \tag{10}$$

where $\mathbf{R}_m$ is the radius vector of the position of the moving observer.

For clocks moving over the surface of the earth, the value of the excluded terms, of order of $1/c^3$ and higher, does not exceed 10^{-18}.

For the rectangular earth coordinate system ITRF, formula (7) for the fractional relativistic frequency shift takes the final form:

$$\frac{\Delta f_{\mathrm{r}}}{f_0}=-\frac{\varphi_0^n-\varphi_{\mathrm{m}}^n}{c^2}-\frac{\Delta\varphi_0^{an}-\Delta\varphi_{\mathrm{m}}^{an}}{c^2}-\frac{\Omega^2}{2c^2}\left[(x_0^2+y_0^2)-(x_{\mathrm{m}}^2+y_{\mathrm{m}}^2)\right]+\frac{V^2}{2c^2}+\frac{\Omega^2}{c^2}(x_{\mathrm{m}}V_y-y_{\mathrm{m}}V_x), \tag{11}$$

where the quantities φ_{m}^n, $\Delta\varphi_{\mathrm{m}}^{an}$, x_{m}, y_{m} and the velocity vector of the moving clock $\mathbf{V}=\{V_xV_yV_z\}$ are functions of the time τ_0. For this reason, the relativistic frequency shift of the moving clocks is also time dependent.

In (11), the first three terms define the gravitational frequency shifts caused by the difference in the gravitational potentials of the normal, anomalous and centrifugal field of the Earth between stationary and moving clocks; the fourth term describes the 2nd order Doppler effect. The physical meaning of the last term is easier to establish on the basis of the formula for θ_{m} (cf. (10)):

$$\frac{\delta f_{\Omega}}{f_0}=\frac{1}{c^2}[\mathbf{\Omega R}_{\mathrm{m}}]\mathbf{V}=\frac{\mathbf{\Omega}}{c^2}[\mathbf{RV}]=\frac{\mathbf{\Omega}}{c^2}\left[\mathbf{R}\frac{d\mathbf{R}}{d\tau_0}\right]=\frac{2\mathbf{\Omega}}{c^2}\frac{d\mathbf{S}_{\mathrm{m}}}{d\tau_0}, \tag{12}$$

where $d\mathbf{S}_{\mathrm{m}}$ is area element enclosed between the initial and final positions of the position vector $\mathbf{R}_{\mathrm{m}}$ and the path vector $d\mathbf{R}$ of the moving reference. The effect of shifting the frequency defined by formula (12) is the time derivative of the current phase shift caused by the Sagnac effect [13]. The frequency shift is positive ($f_{\mathrm{m}}\geq f_0$) when the scalar product $[\mathbf{\Omega R}_{\mathrm{m}}]\mathbf{V}$ is positive. In this case, the azimuth of motion is within 0–180° (the mobile frequency reference moves eastward).

Relativistic effect of time shift is determined from relations (6) and (10):

$$\Delta\tau_{\mathrm{r}}=\Delta\tau_{\mathrm{r}}^0-\frac{1}{c^2}\int_{\tau_{01}}^{\tau_{02}}\left[(\varphi_{\mathrm{m}}^n+\varphi_{\mathrm{m}}^{an})+\frac{\Omega^2}{2}(x_{\mathrm{m}}^2+y_{\mathrm{m}}^2)+\frac{1}{2}V^2\right]d\tau_0-\frac{2\Omega S_{\nabla}}{c^2}, \tag{13}$$

where

$$\Delta\tau_{\mathrm{r}}^0=(\Delta\tau_0/c^2)\left[(\varphi_0^n+\varphi_0^{an})+0.5\Omega^2\cos^2\Psi_0(x_0^2+y_0^2)\right]$$

is a constant offset determined by the position of the base clock;

$$S_{\nabla}=\frac{1}{2}\int_{xy}(xdy-ydx)$$

is the area of the triangle with vertices 0, $\mathbf{R}_{\mathrm{m}}$, $\mathbf{R}_0$ projected to the equatorial plane. The integrand in (13) is a function of time, since $\rho_{\mathrm{m}}=\rho_{\mathrm{m}}(\tau_0)$, $\mathbf{V}=\mathbf{V}_{\mathrm{m}}(\tau_0)$. The last term of (13) determines the Sagnac effect which also varies in time.

Calculating the instantaneous value of the relativistic frequency shift from (11), (12) is accomplished by using the current coordinate data and speed obtained from the GLONASS/GPS navigation apparatus (NAP). The same data are used to compute the current onboard time offset of moving clocks by numerical integration of expression (13). In this case, the integration step is chosen equal to or greater than the information sampling interval of the navigation receiver.

Methods for Compensating Gravitational-Relativistic Effects. Representing (6) in the form $\Delta\tau_{\mathrm{m}}=\Delta\tau_0+\Delta\tau_{\mathrm{r}}$ and adding to both sides of the expression the corrective term $-\Delta\tau_{\mathrm{r}}^*$ we obtain

$$\Delta\tau_{\mathrm{m}}^*=\Delta\tau_0+\delta\tau_{\mathrm{r}}, \tag{14}$$

where $\Delta\tau_{\mathrm{m}}^*=\Delta\tau_{\mathrm{m}}-\Delta\tau_{\mathrm{r}}^*$ is the corrected time of the moving clock; $\delta\tau_{\mathrm{r}}=\Delta\tau_{\mathrm{r}}-\Delta\tau_{\mathrm{r}}^*$ is the time compensation error due to relativistic effects; if it is negligible, it follows from (14) that we have $\Delta\tau_{\mathrm{m}}^*=\Delta\tau_0$, i.e., the corrected time of the moving clock coincides with the base clock.

Corrective term $-\Delta\tau_r^*$ to the current time of a moving clock can be added continuously along the path or once at the end of the path. In the first case, the process of calculating the corrections according to the NAP data and their digital input are carried out continuously, so along the motion path there is maintained a continuous synchrony between the moving and stationary time observers. In the second case, the compensation task is accomplished at the end of the trajectory using the navigation data accumulated during the motion.

We represent (8) in the form $f_m = f_0 + \Delta f_r$, take account the correction $-\Delta f_r^*$, and find

$$f_m^* = f_0 + \delta f_r, \tag{15}$$

where $f_m^* = f_m - \Delta f_r^*$ is the corrected value of the mobile frequency reference; $\delta f_r = \Delta f_r - \Delta f_r^*$ is the error in the correction (compensation) of the relativistic effects on frequency. Upon reaching the required compensation accuracy, when $\delta f_r = 0$, from (15) we have $f_m^* = f_0$, which signifies the equality of the corrected moving frequency reference and the stationary frequency reference. The consequence of this is the synchronization of the moving and stationary time references. Indeed, using (5), (7), and (15) the results of correcting the moving frequency reference can be expressed as: $d\tau_m = (f_0/f_m^*)d\tau_0 = (1 - \delta f_r/f_0)d\tau_0$; if $\delta f_r = 0$, this leads to the coincidence of the flow of time at the moving and stationary observers: $d\tau_m = d\tau_0$. Of course, for the full alignment of both time measurements an initial synchronization is required.

A compensation method using frequency correction is useful for synchronizing stationary references. Indeed, for $V = 0$, according to (11), the relativistic frequency shift is constant, since it is determined only by constant gravitational potentials at the distribution points of the references. Accordingly, the frequency correction is also constant and is entered only once.

The second possibility for using this method occurs when there is a simultaneous constancy of the difference in potentials and the squares of the velocities along the trajectory of the moving clock. In (10), if we go from a rotating to a non-rotating geocentric system by using the relation $[\mathbf{\Omega R}_i] + \mathbf{V}_i = \mathbf{V}_{nr}$ ($i = 0$; $\mathbf{V}_{nr}$ is velocity of clock in the non-rotating system), we obtain formula (11) in the form

$$\Delta f_{gr}/f_0 = -(\varphi_0 - \varphi_m)/c^2 - [(V_{nr}^2)_0 - (V_{nr}^2)_m]/2c^2. \tag{16}$$

The condition that the right-hand side of (16) remain constant is satisfied when placing clocks onboard spacecraft moving in a circular orbit. This option is typical for space communications systems, remote Earth sensing, satellite navigation and others. Moreover, for a strictly circular orbit, the condition of constant displacement (cf. (16)) holds regardless of the spatial orientation and height of the orbit. Compensation of relativistic differences between airborne and ground times using a constant frequency offset is realized specifically for onboard satellite clocks of the GLONASS/GPS systems [14].

Compensation accuracy of the relativistic effects is determined by the accuracy of their calculation from (11), (13), which, in turn, is determined by the accuracy of the navigation data. Errors of modern widely-used navigation receivers, for example, of the type TrimbleGeoExplorer 6000 GeoXH, are quite small: in determining velocity, the mean square random error is $\sigma_v = 5$ cm/sec; for determining a coordinate in differential mode, $\sigma_x = 1$ m. The data sampling rate is 1 Hz. Under these conditions, we find the relative error of calculation of the most significant components of the effects under consideration, whose moduli are determined by the potential and the square of the velocity: $\Delta f_r^\varphi = f_0\mu/(c^2\rho)$, $\Delta f_r^v = f_0V^2/(2c^2)$. The corresponding relative random errors are determined by

$$\frac{\sigma(f_r^\varphi)}{f_0} = \frac{\Delta f_r^\varphi}{f_0}\frac{\sigma(\rho)}{\rho}; \qquad \frac{\sigma(f_r^V)}{f_0} = \frac{\Delta f_r^V}{f_0}\frac{2\sigma(V)}{V}.$$

From these relations at a point on the equator with $\rho = X_e = 6.378\cdot10^6$ m, $\sigma_x = 1$ m, we obtain $\sigma(f_r^\varphi)/f_0 \approx 10^{-16}$, for $V = 400$ m/sec, $\sigma(V) = 5$ cm/sec, $\sigma(f_r^V)/f_0 \approx 10^{-16}$. The error in the computation of time shifts over a 24-hour period in both cases does not exceed 9 psec. The error in computing the other components of formulas (11), (13), comprising ΩR and V, are much lower since their values are significantly smaller.

The accuracy of compensating for other gravitational effects on time and frequency offset error is determined by the compensation error in the second zonal harmonic of the normal field and the length of the series expansion of the anomalous potential in the form of spherical functions (9). The second zonal harmonic potential φ_{p2} is maximal near the pole ($\psi = 90°$), and this causes a relative displacement y of the Earth's surface which attains $\varphi_{p2}/c^2 \approx 7.5\cdot10^{-13}$. With the precision of the navigation software established above, the calculation error of this effect does not exceed 10^{-18}.

The number of terms in the expansion of modern models of the anomalous potential reaches hundreds or even thousands. One of the advanced models of Earth's gravitational field EGM2008 contains $n \times m = 2159 \times 2159$ terms in the expansion [15]. However, in the solution of specific practical problems, this series can be significantly truncated as a result of dropping negligible terms in the expansion. Thus, the coefficients of the third and fourth zonal harmonics are, respectively, $J_{30} = -2.5\cdot10^{-6}$, $J_{40} = -1.6\cdot10^{-6}$. The corresponding relative gravitational effects are $-2\cdot10^{-15}$ and $-1.1\cdot10^{-15}$, and cause the difference in the times over a 24-hour period in the range of about –17 psec and –10 psec, respectively, and can be calculated from the navigation data. With increasing index of the zonal harmonic, its coefficient decreases. Beginning with the 13th zonal harmonic, these effects are approximately one order of magnitude smaller than the effects of the third and fourth zonal harmonics and they can be ignored without losing precision. The coefficients of the zonal and tesseral harmonics are also quite small and decrease with increasing index and order: for example, $C_{22} = 1.57\cdot10^{-6}$, $S_{22} = -0.9\cdot10^{-6}$, etc.

It should be noted that the relative instrumental error of modern transportable quantum clocks transported over a one-day interval in transport mode is $1\cdot10^{-14}$, and in stationary mode, $(2–4)\cdot10^{-15}$ [12]. In the near future, this figure may be reduced to $1\cdot10^{-15}$. Thus, the residual relativisitic error is significantly less than the instrumental error.

Conclusion. Practical formulas for calculating the relativistic effects of shifts in frequency and time in clocks moving over global distances in the anomalous gravitational field are derived using the general theory of relativity. The relative value of uncompensated terms does not exceed 10^{-18}. Compensation of the effects is possible along the trajectory as a result of current navigation data available in the GLONASS/GPS systems, or at the end of a clock trajectory using accumulated navigation data. In the first case, there is continuous synchronization in time of the moving and stationary clocks. Compensation effects may be performed by digitally correcting the time scale and applying a frequency shift to the master oscillator. Compensation accuracy, through the use of modern navigation GLONASS/GPS receivers operating in differential mode, as well as the use of modern global models of the anomalous gravitational field of the Earth, gives an estimated relative random error of no more than 10^{-16}.

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