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MECHANICAL TESTS AND CONSTRUCTION OF ANALYTICAL MODELS OF THE BEHAVIOR OF MATERIALS UNDER SUPERPLASTIC CONDITIONS. Part II*

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Different test methods are described to determine the mechanical properties of materials in the superplastic state. The flow stress depends on strain and strain rate, the structural parameter of the materials, and temperature. The rheology and mechanics of superplastic deformation are discussed. The methods have been checked for reliability and produced good results in testing titanium alloys and constructing mathematical models as part of an order submitted by the company EADS (Airbus). The information given on the test methods and the subsequent approximation of materials' mechanical properties is of considerable interest for making reliable predictions of the deformation of materials during shaping operations.
Keywords: *mechanical properties of materials, superplasticity, modeling, mechanical tests.*

Performing Tests and Principles of Calculation of the Parameters

Tests with a stepped change in rate. In tensile tests, the specimen is loaded at a rate within the range v_0 = 1–5 mm/min until stable plastic flow begins. Then a minimum deformation rate is established. When the strain of the specimen reaches 2–3%, the deformation rate is increased stepwise by a factor within the range 1.5–2. In certain cases, it is more appropriate to conduct tests in which the deformation rate is decreased over a number of steps. It is useful to employ these types of tests to evaluate the superplasticity of materials in which structural changes (formation of the grain structure after shaping and/or grain growth, for example) that take place during an initial testing period carried out with a low rate of deformation result in intensive strain-hardening or softening of the material. Testing such materials with the use of a stepped increase in deformation rate can produce exaggerated values of the rate-sensitivity index m when the deformation rate is low. Two-phase titanium alloys are among the materials that exhibit this type of behavior. Tests of the kind just described yield a file of data that includes values for the forces and specimen lengths which correspond to stable flow at each value of cross-head speed. The main characteristics which are calculated from this data file – the true flow stress σ and deformation rate $\dot{\varepsilon}$ – are determined from the formulas

$$\sigma = P_i L_i / W, \tag{1}$$

$$\dot{\varepsilon} = v_i / L_i, \tag{2}$$

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where W is the volume of the working part of the specimen, mm^3; v_i is deformation rate, mm/min; P_i is the deforming force, N; and L_i is the running length of the specimen, mm.

In the case of tension, L_i is determined as

$$L_i = L_0 + D_i - P_i/K_{\text{s–m}}, \tag{3}$$

where L_0 is the initial length of the specimen; $K_{\text{s–m}}$ is the stiffness of the specimen–machine system, N/mm; and D_i is the displacement of the cross-arm (mm) and is calculated from the formula

$$D_i = v_i\tau_i. \tag{4}$$

Here, τ_i is the length of time over which the specimen is deformed at the ith rate, sec.

The stiffness of the specimen–machine system $K_{\text{s–m}}$ is determined from the slope of the force-displacement curve on a linear loading section with the rate v_0. The criterion chosen for stable flow of the specimen material with the cross-arm moving at a specific speed is the moment during the deformation process when the load stops rapidly increasing (or decreasing) and the force either remains stable or begins to slowly decrease due to a decrease in the cross-sectional area of the specimen. Values of force and specimen length for the given cross-arm speed are recorded in the data file before the testing proceeds to the next rate of deformation in tension v_{i+1}.

Tests performed with a constant deformation rate. In tests conducted with a constant deformation rate, the deformation rate $\dot{\varepsilon}$ – being one of the main characteristics of superplasticity – continuously decreases with an increase in the length of the specimen in accordance with Eq. (2). In connection with this, it is better to perform tests with a constant strain rate in order to analyze the plasticity of the material and the strain-hardening/softening processes that take place. Such testing regimes are easily realized with the use of a system that employs a computer to control the testing machine. In that case, if a certain value is chosen for strain rate $\dot{\varepsilon}$, then the initial velocity of the cross beam v_0 that corresponds to the initial length of the specimen L_0 is also determined. Strain rate is reduced as the specimen continues to be loaded in tension, and the actual strain rate is calculated from Eqs. (2)–(4). The speed of the cross-head is corrected in order to keep the strain rate constant. This process is repeated until either the prescribed elongation of the specimen is reached or until it ruptures. The stepwise change in cross-head speed should maintain the specified average strain rate with an accuracy of at least ±(0.3–1)%. The results obtained from tests conducted with a constant strain rate allow the data file of force and specimen-length values to be used to calculate the flow stress σ (Eq. (1)) and the true strain

$$\varepsilon = \ln(L_i/L_0). \tag{5}$$

Then the dependence of the strain-hardening coefficient γ on deformation is constructed to analyze the strain-hardening processes. The strain-hardening coefficient is calculated from the formula

$$\gamma = d\ln\sigma/d\varepsilon. \tag{6}$$

The method used to conduct tests for relaxation of the load makes it possible to construct the dependence of the flow stress and the index m on strain rate for small values of additional strain (1–5%). This capability of the method distinguishes it from the approach in which such curves are plotted with a discontinuous (stepped) change in tensile loading rate. The cumulative strain reaches 40–70% in the latter case, and the structure of the material could undergo changes as a result of such loading. Thus, the loads that are obtained by the relaxation method can serve as a criterion for evaluating the structural state of the material and/or changes in its strain state during tensile loading. This makes it promising to conduct tests with a strain rate that is optimum for the specified temperature. In tests of this type, the specimen is loaded until it is elongated by a prescribed amount and is then tested for load relaxation. This approach makes it possible to compile a file of data on the rheological characteristics of the material and to use those characteristics to predict the conditions during superplastic forming (SPF).

To resolve the problem being discussed in this article, the specimen is subjected to tension at a constant strain rate until a prescribed amount of elongation is realized. The specimen is then unloaded to zero force and again loaded at a high rate that is deliberately chosen to correspond to the upper boundary of the rate interval in which superplastic behavior takes place. When stable flow is established at a strain rate of 1–5%, the cross-arm is stopped and the reduction in force is recorded in the specimen–machine system (the relaxation process is tracked). During the period when the specimen is loaded at a high rate until the cross-arm is stopped, the total deformation D_0 is composed of the elastic deformation of the specimen–machine system D_e and the specimen's plastic deformation D: $D_0 = D_e + D$. This total strain remains constant over time, while the elastic deformation of the specimen–machine system is converted into plastic deformation of the specimen. As a result,

$$\dot{D}_e + \dot{D} = 0. \tag{7}$$

From here, the rate of plastic deformation of the specimen during relaxation is:

$$\dot{D} = -\dot{D}_e = -\dot{P}/K_{s-m}, \tag{8}$$

where $\dot{P}$ is the rate of decrease in the force on the specimen during relaxation.

Assuming that the stiffness of the specimen–machine system K_{s-m} and the volume of the specimen's working part W are constant during testing, we can calculate the length of the working part at each point of the relaxation curve by using the formula

$$L_i = L_{i+1} - (P_i - P_{i+1})/K_{s-m}. \tag{9}$$

In this case, the rate of deformation of the specimen is:

$$\dot{\varepsilon}_i = \dot{D}_i/L_i = -\dot{P}_i/(K_{s-m}L_i), \tag{10}$$

while the flow stress at each point of the relaxation curve is:

$$\sigma_i = P_i L_i/W. \tag{11}$$

Since the specimen is removed from the clamps after the relaxation test and its working length can be measured precisely, specimen length, strain rate, and stress intensity (Eqs. (9)–(11)) are calculated in the opposite direction, i.e., from the final point on the relaxation curve to its initial point. This makes it possible to avoid errors in determining specimen length that are usually caused by reversing the direction of movement of the cross-arm during the tests. As in other tests, the stiffness coefficient of the specimen–machine system K_{s-m} is calculated based on the slope of the linear section of the force-displacement curve during the stage in which the specimen is loaded before the cross-arm is stopped.

The mathematical analysis of the data obtained in the different types of tests includes calculation of several parameters in the form of derivatives. Among these parameters are the index m, which is determined from the results of tests conducted with a stepped change in tension rate or tests conducted with relaxation of the load. In relaxation tests, the strain rate $\dot{\varepsilon}$ is determined from the rate of decrease in the load $\dot{P}$ during relaxation. In tests performed with a constant strain rate, the strain-hardening coefficient γ is also a derivative of the logarithm of stress with respect to the strain. Smoothing spline functions are used in all of these cases [1] and then differentiation is performed.

The test results are represented in the form of tables and graphs illustrating the dependences of the flow stress and the index m on strain rate for different temperatures and degrees of deformation. The temperature dependence of relative elongation is constructed for the optimum strain rates. The graphs are used to determine the temperature and rate intervals in which superplasticity is exhibited (the range of strain rates and test temperatures in which the index $m > 0.3$ and relative elongation > 200%).

A structural analysis of the investigated material is performed after annealing it for a length of time that corresponds to the time period over which specimens are heated to the corresponding test temperature. The average size of the particles

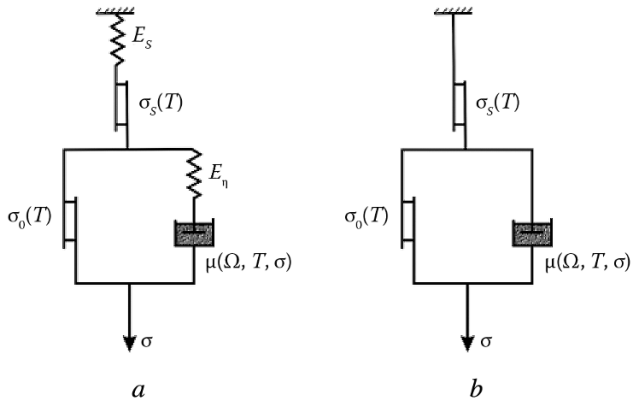

Fig. 1. Variants of SP media with (*a*) and without (*b*) allowance for elastic deformation.

of the α- and β-phases are determined along with the λ-volumetric content of the β-phase. The accuracy with which the parameters of the structure are determined should be no lower than 5% for a level of probability of 0.95.

Rheological and Mechanical Laws of Superplastic Deformation.

As researchers have generalized empirical data on the laws that govern the deformation of metals and alloys in the superplastic state, it has become possible to progress from a qualitative discussion of the physical aspects of superplasticity to a quantitative description of the processes involved in superplastic deformation (SPD). This not only helps make better use of superplasticity in production processes but also broadens general representations of the behavior of superplastic media.

From the standpoint of rheology, superplastic behavior by metals and other materials and the advantages of making practical use of superplastic deformation (low flow stresses, exceptionally high ductility and formability) can readily be attributed to the ability of the corresponding materials to undergo viscous flow. All of the earlier physical and rheological models of SPD and the models that have recently been developed are based on the dependence of stress on strain rate [2–6] – which is known to be a fundamental characteristic of viscous materials. The level of superplasticity that is exhibited is evaluated quantitatively by means of the index *m*, which characterizes the sensitivity of the flow stress to strain rate in the equation

$$\sigma_u = K\dot{\varepsilon}_u^m, \tag{12}$$

where σ_u is flow-stress intensity; $\dot{\varepsilon}_u$ is strain-rate intensity; and *K* is a proportionality factor.

A simple analysis [7] shows that the higher the value of the index that characterizes the rate sensitivity of the flow stress, the more clearly the above-noted advantages of SPD are manifest. The index $m < 0.2$ for ordinary metals and alloys during hot plastic deformation, while $m > 0.3$ for superplastic materials under optimum conditions. Some materials behave as linearly viscous (Newtonian) fluids, and in this case $m > 1.0$. The coefficient *K* is a thermally activated and structurally sensitive parameter. It is linked to the shear viscosity of the material by the following equation:

$$\mu_v = \sigma_u / 3\dot{\varepsilon}_u = K\dot{\varepsilon}_u^{m-1}/3. \tag{13}$$

It follows from Eq. (13) that shear viscosity decreases with an increase in shear rate if the specimen material demonstrates nonlinear viscosity ($m < 1$), while it becomes independent of strain rate if the material acts as a linearly viscous fluid ($m = 1$).

Earlier models of SPD were based on representations in which there was a single mechanism that governed deformation. Those models described superplastic flow in uniaxial tension based on traditional representations of high-temperature creep as the flow of a nonlinearly viscous fluid:

$$\sigma_u = K\dot{\varepsilon}_u^m \quad \text{or} \quad \dot{\varepsilon}_u = C\sigma_u^n, \tag{14}$$

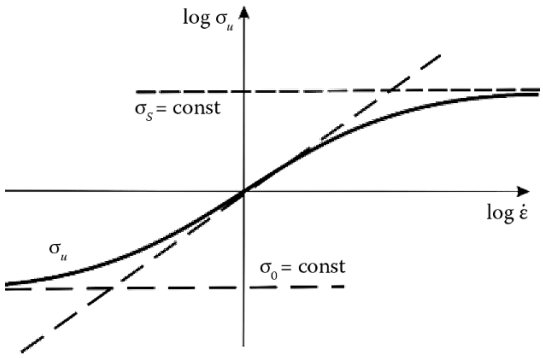

Fig. 2. Dependence of the flow stress of the SP medium on strain rate.

where $n = 1/m$; K and C are coefficients that depend on the structural state of the material, temperature, and the mechanism of deformation. With a known degree of accuracy and within a certain narrow range of strain rates, Eq. (12) describes either an S-shaped curve of superplastic behavior $\sigma_u = \sigma_u(\dot{\varepsilon}_u)$ or the inverse curve $\dot{\varepsilon}_u = \dot{\varepsilon}_u(\sigma_u)$. In this case, the choice of function depends on the types of tests being performed on the material. However, the strain-hardening index m varies appreciably in relation to the deformation rate $\dot{\varepsilon}_u$. This seriously complicates the mathematical modeling of SPD processes, especially bulk forming and sheet forming. Those two operations are characterized by a high degree of transience and highly nonuniform strain rates over the volume of the semifinished product.

To circumvent the problem just mentioned, Smirnov [3] proposed a rheological model of elastoviscoplastic (EVP) media that describes the rheological parameters of superplastic materials within a wide range of strain rates. This model of EVP media is effective for solving scientific and technical problems that involve the shaping of metals in the superplastic state. In one of the simplified variants of this model, a set of viscoelastic elements is replaced by an element that exhibits nonlinear viscosity [8]. Figure 1 presents variants of the model of SP media.

The equation in this model that links the flow stresses and the strain rates in uniaxial tension has the form:

$$\sigma_u = \sigma_S \frac{\sigma_0 + k_v \dot{\varepsilon}_u^{m_v}}{\sigma_S + k_v \dot{\varepsilon}_u^{m_v}},$$

or

$$\dot{\varepsilon}_u = \left(\frac{\sigma_S}{k_v} \frac{\sigma_u - \sigma_0}{\sigma_S - \sigma_u} \right)^{1/m_v}, \qquad (15)$$

where $\dot{\varepsilon}_u$ represents values of strain-rate intensity; σ_u represents the stresses in uniaxial tension; σ_0 is the threshold stress, which is the stress that corresponds to strain rates close to zero; σ_S is the yield point with high strain rates; and k_v and m_v are parameters of a nonlinearly viscous element composed of an SP medium (Fig. 2).

One advantage of the rheological model of SP media is that its coefficients σ_0, σ_S, k_v, and m_v are invariant in relation to changes in strain rate over a wide range of values.

Model (15) well describes ultrafine-grained alloys (UFGAOs). Nevertheless, there are still several issues that need to be discussed in regard to the nature of the superplastic state in these alloys. Among those issues are the problems that make it difficult to use the above-examined model for the computer modeling of SPD processes. Those problems are encountered when attempting to explain the nature of the main rheological parameters of the model and determine their values: specifically, the threshold stress σ_0, rate-sensitivity index m_v, and yield point σ_S. The accuracy with which these values are determined has a substantial effect on the accuracy with which theoretical curves of $\sigma_u(\dot{\varepsilon}_u)$ describe experimental data.

The subsequent development of the model of SP media was concerned mainly with quantitatively accounting for such key parameters as the degree of deformation and the structure of the material.

Representations on the equicohesive state of materials were used to construct a rheological model of SP media that accounts for the structure factor in the form:

$$\dot{\varepsilon}_u = \dot{\varepsilon}_{\text{eq}} \exp\left[\alpha\Omega^{\beta}(\sigma_u - \sigma_{\text{eq}})\right]\left(\frac{\sigma_u - \sigma_0}{\sigma_S - \sigma_u}\right)^{1/m_v}, \tag{16}$$

where σ_{eq} and $\dot{\varepsilon}_{\text{eq}}$ are the stress and strain rate that correspond to the equicohesive state. The parameters of the *equicohesive state* σ_{eq} and $\dot{\varepsilon}_{\text{eq}}$ can be determined from the point of intersection of two or more $\sigma_u(\dot{\varepsilon}_u)$ curves of an ultrafine-grained alloy corresponding to different grain sizes. For most such alloys, $\sigma_{\text{eq}} \approx 0.5\sigma_S$. In Eq. (16), Ω is a thermally activated structural parameter that depends on average grain size in the given alloy; α and β are parameters that characterize the extent to which the structural parameter affects the strain rate.

Equation (16) describes a wide range of empirical data for each specific alloy regardless of the structural state it is in. The equation describes these data with a very high degree of accuracy and is recommended for use in designing technologies for superplastic deformation processes (superplastic forming, forming combined with diffusion welding, etc.) in which the structural parameters are the decisive factors.

If it is necessary to quantitatively describe superplasticity within a narrower range of strain rates that is consistent with the maximum value of the strain-hardening index m, the following equation is usually used for the dependence of flow-stress intensity σ_u on strain-rate intensity $\dot{\varepsilon}_u$:

$$\sigma_u = k_0 \exp[Q_{\text{d}}/(RT)]\dot{\varepsilon}_u^m \Omega^f, \tag{17}$$

where k_0 is a proportionality factor that depends on the chemical composition of the alloy; Q_{d} is the apparent activation energy for superplastic deformation; R is the universal gas constant; T is the absolute temperature at which deformation takes place, K; m is the strain-hardening index; Ω is the generalized structural parameter; and f is an index that characterizes the structure-dependent strain-hardening of the material.

There is one more variant [9] that can be used to approximate the mechanical properties of the material. This variant has parameters that depend on temperature and makes allowance for strain-hardening. It is used for processes with a specified strain rate that is roughly constant and it appears as follows:

$$\sigma = A(T)(\dot{\varepsilon}_{\text{opt}}(T))^m \varepsilon^n D_0(T) K_{\text{shift}}(T). \tag{18}$$

Here, T is temperature; the first component $A(T)(\dot{\varepsilon}_{\text{opt}}(T))^m$ is the temperature dependence of the flow stress in relation to the strain rate; m is the index that characterizes the sensitivity of the stress to the strain rate. The second component (ε^n) is the parameter that is used to calculate strain-hardening. The third component provides an estimate of the initial structural state of the alloy $D_0(T)$ and introduces a correction for the shift $K_{\text{shift}}(T)$ at the level of flow stress seen when the structure begins to stabilize. The parameter D_0 is normalized based on the average arithmetic dimensions of the particles of the α and β phases:

$$D_0 = ((L_\alpha L_\beta)^{0.5} L_\beta / L_\alpha)/((L_\alpha + L_\beta)/2), \tag{19}$$

since this parameter is designed to correct the data in cases in which there are differences between the composition of different heats or the technologies used for sheet production.

The below relation is also quite commonly used to describe the flow stress in relation to strain and strain rate at a specified temperature:

$$\sigma_u = A\dot{\varepsilon}^{m(\varepsilon)}. \tag{20}$$

Conclusion. Different models can be chosen to describe the superplastic deformation of a material, the choice depending on the duration of the deformation process, the degree of accuracy needed in the mathematical modeling, and other

factors. It is very important that the algorithms used in performing the calculations allow a solution to be obtained for different approximations of the physical relations that link stress intensity with strain-rate intensity, strain intensity, temperature, and the dimensions of the structural units of the material.

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