# **Motorcycle brake squeal: experimental and numerical investigation on a case study**

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**Abstract** Since, nowadays, the NVH performance of a vehicle is one of the most important priorities for the market, the noise radiating from disc brakes is considered a source of considerable discomfort and customer dissatisfaction. Squeal is an example of noise, caused by vibrations induced by friction forces, in which the vibration modes of the brake disc are coupled to those of the friction pads or of the caliper. In this work a case study, in which the squeal phenomenon was detected after changing the supplier of the disc pads, is presented. A test bench was purposely developed to investigate the squeal phenomenon; tests at different rotating speeds and pressures in the brake circuit were carried out employing different friction pads. The experimental apparatus appeared capable of reproducing the phenomenon observed in real practice and to investigate the effect of operating parameters and different components on the onset of instability. Friction tests and geometric analysis of the friction pads were also carried out to complete the investigation. At the same time, a finite element (FE) complex eigenvalue parametric analysis was performed on the brake assembly. The different propensity of the pads to squeal was attributed to differences in their geometry and in their friction coefficients. The FE analysis, confirmed the experimental observations and indicated possible

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design improvements to increase the stability of the system.

**Keywords** Disc brake · Noise · Squeal · Motorcycle · Experimental analysis · Finite elements · Complex modal analysis

## **1 Introduction**

To date, comfort and low noise characteristics are considered very important in the automotive field; vehicles are no longer judged solely on the basis of engine performance or aesthetic considerations. "Squeal" (typical brake noise, also called "whistle") represents, in this context, a crucial aspect because, being considered one of the most irritating and annoying noises, it is one of the main reasons of returns under warranty. Conventionally, two types of squeal are identified depending on the frequency range: "low frequency" (1–4 kHz) and "high frequency squeal" (3–20 kHz).

The literature is vast, covering a very wide time range; there are analytical, numerical, and experimental studies. From recent articles on the state of the art [\[1](#page-10-0), [2\]](#page-10-1) it appears that, despite the resources used, general rules for the elimination of squeal have not yet been defined, because its occurrence is intermittent or sometimes even seemingly random. The repeatability of tests involving squeal is notoriously poor;

in fact, the squeal phenomenon, depends on the conditions of friction between the disc and the pads, which in turn depend heavily on the environmental and operating conditions (state of the surfaces, temperature, pressure, presence of moisture etc.) which are difficult to keep under control. The first analytical studies, based on very simple models, based on very simple models, reduced to a few degrees of freedom  $[3, 4]$  $[3, 4]$  $[3, 4]$ , have shown that fluctuations of the friction forces and contact pressure, related to displacements of the disc, can induce instability in the system. These studies suggested that the onset of squeal depends on the coefficient of friction between pads and disc, on the mass and stiffness parameters of the system and on the contact geometry between pads and pistons.

Later, more representative analytical models of disc or drum brakes have been proposed [[5\]](#page-10-4); they showed that "brake squeal" can be promoted by the coupling of two natural modes of two components of the system, having very close natural frequencies; such analyses also showed that the onset of unstable vibrations can occur even for low values of the friction coefficient for particular values of the pad stiffness.

In several works based on lumped parameters models other factors that influence squeal have been investigated, such as the pad geometry [[6\]](#page-10-5), the contact parameters and the piston position [[7\]](#page-10-6). Finite element (FE) codes, instead, make it possible to create very detailed and realistic brake models. It may also be possible to perform complex eigenvalue analyses in the frequency domain and dynamic analyses of transients in the time domain [\[8](#page-10-7)]. The first type of analysis allows for a rapid detection of the potentially unstable vibration modes of the system, and this also allows to make structural changes to the design (in a virtual environment) and quickly check their effects [\[9](#page-10-8), [10](#page-10-9)]. This method, though based on the simplifying hypothesis of the system linearity, provides a conservative estimate of the unstable modes [\[11](#page-10-10)]. The alternative consists in a fully dynamic analysis of the transient, with explicit FE codes, which however often involves a too costly calculation time for the typical vehicle development timing.

In this work, done in collaboration between the Department of Civil and Industrial Engineering of the University of Pisa and Piaggio & C SpA, a real case study is presented. The validation of a new type of pads on a Piaggio vehicle indicated a quite evident

squeal problem of the front brake at very low speed. That suggested a more in-depth investigation of the influence of the new pads on the onset of squeal. Firstly a test bench was purposely developed in order to reproduce the phenomenon; then, a thorough experimental investigation on the operating parameters and a comparative study on the employed friction pads was carried out to assess the causes of its occurrence. At the same time, a finite element (FE) complex eigenvalue parametric analysis was performed on the brake assembly both to support the experimental investigation and to suggest possible design improvements to increase the stability of the system.

#### **2 On road tests**

Specific tests were carried out on a specially instrumented vehicle: the acoustic signal was acquired by using a microphone placed close to the brake; vehicle speed, engine rotational speed, pad temperatures and the pressure of the hydraulic circuit were also monitored. Figure [1](#page-2-0) shows the trend of the parameters in a typical test sequence. In the graph, in particular, one can see the different temperature of the two pads due to the different exposure to ventilation, the pad temperature rise of approximately 10  $\degree$ C at the end of braking and the pressure variation during braking reaching a maximum of about 2.5 bar.

From the graphs of Fig. [2](#page-2-1) the phenomenon of noise detected on the brake, which appears below 20 km/h, after a brief warm-up phase, is quite evident. Performing an FFT analysis of the microphone signal of the entire sequence, it is possible to highlight the characteristic frequencies; in particular, there is a main frequency at 3900 Hz, with the  $2 \times$  frequency evident at 7800 Hz and another important frequency with a peak at 5100 Hz.

However, looking at the spectrogram of Fig. [3](#page-2-2) and by comparison with Fig. [1](#page-2-0) one can note that the noise phenomenon appears at the various frequencies in different time intervals apparently related to particular values of pressure and temperature (greater than 75  $\degree$ C); both quantities, indeed, influence the frictional force directly or indirectly (via the contact stiffness and the coefficient of friction). As shown in [[10\]](#page-10-9), temperature has also a non-negligible effect on the elastic modulus of the pad material. By comparing Figs. [3](#page-2-2)

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<span id="page-2-1"></span>





**Fig. 3** Spectrogram of the acoustic signal of the road test sequence with pads A

<span id="page-2-2"></span>and [1](#page-2-0) it seems that the noise at 3900 Hz and 7800 Hz occurs at a pressure of about 2 bar, while the noise at 5100 Hz builds up for a slightly higher pressure, approximately 2.5 bar, for lower values of temperature, reappearing at 2 bar at the end of braking where the temperature is higher.

#### **3 Laboratory investigations**

# 3.1 Brake bench tests

In order to perform lab tests on the real brake system, the bench shown in Fig. [4](#page-3-0), developed for similar tests [[12\]](#page-10-11), was suitably modified to reproduce the ac-



 $(a)$ 





 $(c)$ 

<span id="page-3-0"></span>**Fig. 4** (**a**) Test bench view; (**b**) details of the electric motor support and load cell used to determine the applied torque; (**c**) view of microphone and pressure transducer used in lab tests

tual geometry of the system; it is composed of three main parts: the brake disc support that allows the rotation of the disc around its axis, the brake caliper support and the transmission system.

The driving torque is provided by an asynchronous motor, supplied by an inverter, having a nominal torque of 7.4 Nm. The transmission system consists of two pulleys and a toothed belt and is characterized by a 3:1 ratio.

The driving torque can be obtained on the basis of the reaction load, measured by a load cell placed underneath the motor support; on the other side the motor support is constrained by a revolute joint, obtained by ball bearings (Figs. [4](#page-3-0)a and [4b](#page-3-0)).

The pressure inside the braking system is measured by a pressure transducer (Fig. [4c](#page-3-0)), just after the brake pump actuated by the lever arm. During the tests the acoustic noise was recorded by a microphone (Fig. [4c](#page-3-0)).

Future improvements of the test bench include a cooling system for replicating the effect of ventilation on the braking system, a system for detecting the pad temperature and the use of proximity sensors to detect the disc displacements perpendicular to its mid-plane.

Piaggio provided three types of pads, indicated in the following as A, B and C (Fig.  $5$ ), purchased from different suppliers.

Based on the experimental observation on the vehicle, the first series of tests was carried out by imposing a linearly descending speed profile in order to identify the velocity ranges where the squeal phenomenon typically occurs. A series of tests was carried out from the maximum value of 50 rad s<sup>-1</sup>, corresponding to a vehicle speed of about 40 km/h, to the minimum value of 10 rad  $s^{-1}$ , which corresponds to a vehicle speed of about 8 km/h. In these preliminary tests, the pressure in the brake system was kept constant during the ramp and tests at different operating pressures (1–3 bars with increments of 0.5 bar) were carried out.

Pads B did not show, for any of the tested conditions, any significant squeal phenomena. Pads C showed a general higher noise level compared to pads B, and, in the tests at 3 bar, generated a faint whistle. Pads A, instead, showed a noise level significantly higher than the others; the phenomenon of squeal occurred in tests at different pressures at speeds below 20 rad s<sup> $-1$ </sup> (in the range 10 to 20 km/h of vehicle longitudinal speed). Even on the bench, as in tests on the vehicle, the first "cold" braking showed little noise compared to the following "warm" ones, indirectly confirming the possible dependence from the surface temperature detected during the road tests. In particular, for pads A, with a pressure of 2 bar, noise appeared

<span id="page-4-0"></span>

**Fig. 5** Three types of tested pads, from left to right: *A*, *B*, *C*



**Fig. 6** Pad A: spectrogram of the brake acoustic signal at varying  $\omega$ , (a)  $p = 1.5$  bar and (b)  $p = 2$  bar

<span id="page-4-1"></span>at a frequency above 6000 Hz (Fig. [6](#page-4-1)) while at 1.5 bar the frequency of the phenomenon was around 4000 Hz with lower peaks above 6000 Hz and at 8000 Hz. For lower pressure values, at about 1 bar there was no apparent occurrence of noise, while increasing the pressure to 2.5 bar, the phenomenon was still present but attenuated.

A second series of tests was then carried out at constant pressure and speed of rotation focusing on a narrower range of the parameters (10, 15, 20 rad s<sup>-1</sup> for speed and 2, 3 bar for pressure). This type of tests was mainly directed to pads A and C, which had shown

the squeal problem. In the case of pads C, noise, in any case mild, was found in the tests with the braking pressure of 3 bar while, for pads A, a much more pronounced squeal phenomenon was found with a pressure of 2 bar (Fig. [7](#page-5-0)) at 4000 Hz and 8000 Hz. Higher speed values did not lead to significant changes in the characteristic frequencies of squeal, but to a more widespread noise level in the spectrum that appeared less defined.

The bench tests confirmed that for the considered brake system and pads, the phenomenon of squeal occurs at different frequencies, depending on the oper-



**Fig. 7** Pad A: spectrogram of the brake acoustic signal for  $\omega = 15$  rad s<sup>-1</sup> and  $p = 2$  bar

<span id="page-5-0"></span>ating conditions. In particular, such frequencies were observed around 4000 Hz, and its multiple, in agreement with the findings of the on road tests; on the contrary, the frequency of 5100 Hz, which was present in on road test, was not detected, while a vibration at 6400 Hz was observed in lab tests (Fig. [6\)](#page-4-1); such differences were attributed to possible different stiffness characteristics of the caliper support.

## 3.2 Friction and geometric pad characterization

Additional experimental characterization of the pads was carried out on a pin on disc tribometer with the brake disc in place of the tribometer disc and the pad fixed on a suitable interface in place of the pin (Fig. [8](#page-5-1)). The load was applied adding masses on top of the "pin" that can slide vertically with negligible friction in a bore of a support fixed on a load cell that was used to measure the tangential friction force A jet air dryer was used to heat the disc and a pirometer to measure its temperature.

The tests were aimed at measuring the friction coefficient of the three pads and to investigate the eventual different response to temperature, speed and load variations. The test apparatus did not allow for a continuous variation and acquisition of the parameters, therefore tests were carried out: (a) at room temperature (30 °C) and at about 50 °C, (b) at 7.2 rad s<sup>-1</sup>, 2.2 rad s<sup>-1</sup>, 0.6 rad s<sup>-1</sup>, (c) with 85 N and 135 N of applied load. Such values were chosen in order to cover the ranges interested by the phenomenon, compatibly with the available experimental apparatus.

The friction coefficients that were obtained for the pads at room temperature for pads B and C were very



**Fig. 8** Pad and disc fixed on the tribometer for friction tests

<span id="page-5-1"></span>similar, while pad A showed a somewhat higher friction. Figure [9](#page-6-0) depicts the influence of temperature on friction. As it can be observed, at about  $50^{\circ}$ C, the friction coefficient increased significantly with respect to the room temperature value.

Tests conducted at different speeds showed a slight dependence of the friction coefficient on sliding speed,

<span id="page-6-0"></span>

**Fig. 9** Friction coefficient of the three pads at 30 ◦C and 50 ◦C at 7.2 rad s<sup> $-1$ </sup> (average value and standard deviation)

<span id="page-6-1"></span>

Fig. 10 Friction coefficient of the three pads as function of disc rotational speed; tests at room temperature

with a general decrease of friction with speed (Fig. [10\)](#page-6-1). Increasing the load, and thus the contact pressure, also produced a decrease of the friction coefficient of about  $10-15$  %.

An analysis on the geometric characteristics of the pads was also carried out. Pad A, which resulted to produce the most evident squeal noise, indeed, had some significant differences in the geometry with respect to the other pads, which, on the contrary, resulted to be very similar.

In particular, the friction material portion of pad A was slightly smaller, in the disc radial direction, than that of the other pads (Fig. [5](#page-4-0)), and curved at the inner side. In addition, the length of the steel plate that supports the friction material portion was about 0.5 mm shorter, in the disc tangential direction, than that of the other pads. That means a loose fit of the pad in the caliper mount and the possibility of pad rattling.

#### **4 FEM analysis**

## 4.1 Component modal analysis

To better understand the sources of the squeal vibration frequencies detected on the brake object of the study, a modal analysis was carried out to identify the vibration modes of the system components with AN-SYS FE software. Three-dimensional models of the bodies involved were created with a 3D CAD software. The models were then discretized using tetrahedral elements (SOLID187). The disc was fully constrained on circular rings around the holes of the 6 screws that fix it to the rim. Table [1](#page-7-0) lists the first 30 eigenfrequencies of the disc with the corresponding type of vibration mode.

Figure [11](#page-7-1) shows the more interesting mode shapes for the disc, namely those close to the frequencies detected in both the bench trials and on the vehicle. Two of them are the frequencies very close to 3900 Hz corresponding to the main peak of the noise. Also two pairs of vibration modes, 23, 24 and 26, 27, each pair with very close eigenfrequencies, correspond to the noise peak around 5100 Hz, and at 6400 Hz, respectively.

The pad is constrained to move in the direction perpendicular to the disc surface, guided on the side faces of the fixing plate and on the buttonhole; the following indicative values were used for the pad friction material properties [\[13](#page-10-12)]: a Young's modulus (*E*) of 372 MPa, a Poisson's ratio of 0.25, a density of  $2.77 \times 10^{-6}$  kg/mm<sup>3</sup>. Actually the literature data (e.g. [\[10](#page-10-9), [11](#page-10-10)]) vary in wide ranges as far as *E* is concerned. However the friction material has usually a much lower  $E$  than the backing steel plate so that it does not have a great influence on pad eigenfrequencies and modes. In fact, even though the macro geometry of the three pads was slightly different, no significant differences were found in eigenfrequencies and modes, mainly related to the backing steel plate. Figure [12](#page-8-0) shows pad A first mode of vibration corresponding to about 3900 Hz.

A modal analysis was also carried out on the caliper and its real support and on the bench support showing the expected differences in their calculated eigenfrequencies due to the extremely simplified geometry of the bench support, compared to the real one.

**Table 1** Disc first eigenfrequencies

<span id="page-7-0"></span>

Mode No.	Frequency [Hz]	Main displacement
1	642.0	Out of plane
2	660.8	Out of plane
3	661.5	Out of plane
4	749.4	Out of plane
5	749.6	Out of plane
6	900.4	Out of plane
7	1142.9	Out of plane
8	1496.3	Out of plane
9	1497.0	Out of plane
10	2141.1	Out of plane
11	2141.5	Out of plane
12	2954.1	Out of plane
13	2982.7	Out of plane
14	3912.8	Out of plane
15	3913.7	Out of plane
16	4191.8	In-plane
17	4229.6	Out of plane
18	4325.7	Out of plane
19	4327.7	Out of plane
20	4336.1	Out of plane
21	4338.4	Out of plane
22	4429.3	Out of plane
23	5172.6	Out of plane
24	5173.3	Out of plane
25	6279.3	Out of plane
26	6447.7	Out of plane
27	6489.9	Out of plane
28	6612.7	Out of plane
29	6617.1	Out of plane
30	7068.5	In-plane

#### 4.2 Complex modal analysis

In addition to the simple modal analysis of the single components and supports, a modal analysis combined with a non-linear pre-stress analysis was carried out, taking into account the sliding friction between the parts in contact and disc rotation. Contact elements were used and that produced an unsymmetric stiffness matrix. The eigensolver for problems of this kind, available within the FE code [\[14](#page-10-13)], is based on the "QR damped" method and in particular on the QR algorithm; the calculated eigenvalues are complex with the imaginary parts that represent the damped natural



<span id="page-7-1"></span>**Fig. 11** Natural vibration modes Nos. 14 and 15

frequencies of the system, while the real parts, if positive, indicate the unstable modes. To reduce the calculation time, a subsystem formed by the components more involved in the vibratory motion, that is the disc, the pads and the pistons, was studied; thus, the support and its related natural frequency were not simulated. These components were modeled and constrained as in the previous analyses, except for the pistons, which are bound to move perpendicularly to the plane of the disc and are subjected to pressure on the inner face.

A first parametric analysis was performed varying the coefficient of friction. From the results it was found that actually the real part of the eigenvalues, for some natural modes, takes on values greater than zero for coefficients of friction as low as 0.2 (Fig. [13a](#page-8-1)). Higher friction coefficients promote unstable vibrations in agreement with experimental observations. In the present case it is the mode No. 17 of the system (Fig. [13](#page-8-1)b), with a frequency of approximately 3900 Hz, corresponding to the frequency at which ex-



**Fig. 12** Natural mode No. 1 of pad A

<span id="page-8-0"></span>perimentally there was the worst and persistent noise. This is a natural mode of the system characterized by out-of-plane displacements of the disc, coupled to bending of the pad plates. The complex eigenvalue analysis also identified a second unstable frequency at 5700 Hz, corresponding to mode No. 27 of the system that affects mainly pistons and pads while the disc is not involved; that seems to exclude its relation to squeal. Since the model does not include the support some squeal frequencies observed in on-road and lab tests might be not reproduced.

The second phase of the parametric analysis considered the effects of the variation of the Young's modulus of the friction material that can be quite different among the pads. Moreover the literature review had shown the possible influence of temperature with a decrease of *E* for increasing temperature, related to the composite nature and microstructure of the friction materials  $[10]$  $[10]$ . So, starting from a basic configuration with a coefficient of friction equal to 0.45 and Young's modulus equal to 372 MPa (configuration that will be used in all subsequent analyses) the latter was varied from a minimum value of 0*.*75*E* to a maximum



<span id="page-8-1"></span>**Fig. 13** (**a**) Variation of the real part of the eigenvalue corresponding to mode No. 17 as function of the friction coefficient; (**b**) system natural mode No. 17 for  $f = 0.6$ 

of 2*E*. From the obtained results it is clear how, in this case, increasing the Young's modulus of the material reduces the instability of the system (Fig. [14a](#page-9-0)). This, in some extent, supports the observation that the squeal was more pronounced after some preliminary warm up braking and could justify the different behavior of the tested pads.

In order to get some design indications from this numerical tool, with reference to the initial configuration, the effects of other design parameters, such as the longitudinal position of the pistons, were also evaluated. As shown in Fig. [14](#page-9-0)b, by moving the axis of the piston towards the leading edge of the pad, the real parts of the eigenvalues decrease. The maximum displacement is bound to the width of the pad pressed by the piston.

To investigate the effect of the disc thickness, it was varied from 3.5 to 5 mm. For each configuration positive real part eigenvalues were found, even though for 3.5 and 4.5 mm the modulus of such eigenvalues was close to 1.



<span id="page-9-0"></span>**Fig. 14** Variation of the real part of the eigenvalue corresponding to mode No. 17 as function of (**a**) the friction material elastic modulus, (**b**) the longitudinal position of the piston

Analyses were performed also on different disc geometries (Fig. [15](#page-9-1)), starting from the original one: (a) with large fillet radii between the bolted junctions to the wheel rim, (b) with a buttonhole above each bolt hole, (c) decreasing the number of disc bolts from 6 to 5. The solutions with buttonholes and the one with 5 bolts showed no natural modes associated with eigenvalues with positive real part, while the filleted geometry did not bring significant improvements.

## **5 Discussion and conclusions**

In this work the phenomenon of squeal of a real brake of a motorcycle was analyzed by using different methods: tests on the actual vehicle, bench testing and complex modal analysis on a finite element model.

The tests carried out on the vehicle, performed by Piaggio, showed the noise characteristics, permitting, in particular, the identification of the frequencies and operating conditions of vehicle speed, engine rotational speed, pad temperatures and pressure of the hydraulic circuit, at which squeal occurred. These first results showed the different propensity to squeal of different friction pads and made it possible to plan the



<span id="page-9-1"></span>**Fig. 15** Modified disc geometries: (**a**) fillets, (**b**) buttonholes, (**c**) 5 bolts

laboratory tests and to set up the experimental equipment capable of reproducing the phenomenon and to compare the noise characteristics. Such a bench is versatile and could be used also to test different disc brake solutions. In particular, it made it possible to reproduce the squeal phenomenon which was observed on the vehicle, even if one of the frequencies that was present in on road tests was shifted in lab tests. Moreover it was possible to analyze the influence of pressure and rotating speed on the onset of squeal.

The laboratory investigation revealed that the pad showing the highest noise had a highest friction coefficient compared to the others, while the quietest pad had the lowest. In addition, it was observed that the friction coefficient had a significant increase with temperature in the investigated range, and a slight decrease with load and sliding speed.

The finite element complex modal analysis of the brake system composed of the disc, pads and pistons identified the frequency of an unstable mode corresponding to that of the noise recorded on the vehicle and on the bench, confirming that the phenomenon is due to the coupling of two vibration modes of the disc and the pad.

A sensitivity study showed that a higher friction coefficient and a lower Young's modulus of the friction material increase the propensity to instability. The first effect was confirmed by the experimental evidence of the pads' behavior. The second did not have a direct confirmation due to the lack of mechanical data of the analyzed materials. However, since both parameters are related to temperature, as the former showed to increase with temperature and the latter to decrease, it means that higher temperatures increase the propensity to instability and that can also explain why squeal was observed experimentally only after warming up the system. As the friction coefficient is also related to load and speed, as shown by the lab tests, the influence of these two parameters on squeal can be indirectly assessed.

The geometrical analysis of pads and caliper highlighted that the noisy pad had a loose fit in the caliper with respect to the others and that could allow pad rattle and exalt vibrations. Though this phenomenon is acknowledged by manufacturers and many patents have been issued to counteract it, to the authors' knowledge systematic laboratory tests on the effects of geometric tolerances on squeal onset have not been reported. The employed test bench reproducing as closely as possible the real brake system and operating conditions made it possible to point out these effects.

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#### <span id="page-10-3"></span><span id="page-10-2"></span>**References**

- 1. Kinkaid NM, O'Reilly OM, Papadopoulos P (2003) Automotive disc brake squeal: a review. J Sound Vib 267(1):105–166
- <span id="page-10-4"></span>2. Ouyang H, Nack W, Yuan Y, Chen F (2005) Numerical analysis of automotive disc brake squeal: a review. Int J Vehicle Noise Vibr 1(3/4):207–231
- <span id="page-10-5"></span>3. North MR (1972) A mechanism of disc brake squeal. In: Proc of 14th int automobile technical congress of FISITA, pp 1/9–1/15
- <span id="page-10-6"></span>4. Millner N (1978) An analysis of disc brake squeal. SAE technical paper 780332
- <span id="page-10-7"></span>5. Chowdhary HV, Bajaj AK, Krousgrill CM (2001) An analytical approach to model disc brake system for squeal prediction. In: Proc of DETC'01/VIB21560. ASME, Pittsburg, PA, pp 1–10
- <span id="page-10-8"></span>6. Watany M, Abouel-Seoud S, Saad A, Abdel-Gawad I (1999) Brake squeal generation. SAE technical paper 1999- 01-1735
- <span id="page-10-9"></span>7. El-Butch AM, Ibrahim IM (1999) Modelling and analysis of geometrically induced vibration in disc brakes considering contact parameters. SAE technical paper 1999-01-0143
- <span id="page-10-10"></span>8. AbuBakar AR, Ouyang H (2006) Complex eigenvalue analysis and dynamic transient. Int J Vehicle Noise Vibr 2:143– 155
- <span id="page-10-11"></span>9. Liu P et al (2007) Analysis of disc brake squeal using the complex eigenvalue method. Appl Acoust 68:603–615
- <span id="page-10-12"></span>10. Triches MJ, Gerges SNY, Jordan R (2008) Analysis of brake squeal noise using the finite element method: a parametric study. Appl Acoust 69:147–162
- <span id="page-10-13"></span>11. Massi F, Baillet L, Giannini O, Sestieri A (2007) Brake squeal: linear and nonlinear numerical approaches. Mech Syst Signal Process 21:2374–2393
- 12. Carmignani C, Forte P, Rustighi E (2006) Experimental simulation of the sharpening process of a disc blade and analysis of its dynamic response. J Sound Vib 297:649–663
- 13. Giangreco D, Virzì Mariotti G (2006) Studio termomeccanico di un freno a tamburo per veicoli pesanti. Paper presented at XXXV CONGRESSO AIAS, Ancona, paper no 8: pp 1–10
- 14. Hebbes M (2012) Breakthrough in brake squeal prediction helps to eliminate noise problems early in design process. White paper, ANSYS Resource Library, pp 1–6