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A general formula for the drag on a sphere placed in a creeping unsteady micropolar fluid flow

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Abstract In the present work, we investigate the creeping unsteady motion of an infinite micropolar fluid flow past a fixed sphere. The technique of Laplace transform is used. The drag formula is obtained in the physical domain analytically by using the complex inversion formula of the Laplace transform. The well known formula of Basset for the drag on a sphere placed in an unsteady viscous fluid flow and that of Ramkissoon and Majumdar for steady motion in the case of micropolar fluids are recovered as special cases. The obtained formula is employed to calculate the drag force for some micropolar fluid flows. Numerical results are obtained and represented graphically.

Keywords Drag force · Micropolar fluid · Unsteady motion · Laplace transform

1 Introduction

The theory of micropolar fluids has been introduced by Eringen in 1964 as a subclass of a general type of fluids, namely, microfluids [1]. These microfluids physically represent fluids with microstructure in which each macro-volume element contains micro-volume elements which can move and deform independently of the motion of the macro-volume element [1–3]. The micropolar fluids possess only micro-rotational effects and micro-rotational inertia. From the physical point of view viscous fluids containing suspended, randomly oriented, rigid micro-elements may be modeled by micropolar fluids [4]. These micropolar fluids have many physical models such as animal bloods [5], bubbly fluids [6], liquid crystals [7] and granular fluids [8]. Mathematically, micropolar fluids have only six degrees of freedom, three for translation of macro-element and three for microrotation of micro-elements.

In the literature, steady micropolar fluid flow problems have been considered extensively. Ramkissoon and Majumdar [9] derived an elegant formula for the drag experienced by an axially symmetric body in the slow steady flow of a micropolar fluid. In [10], Palaniappan and Ramkissoon rederived the drag formula obtained by Ramkissoon and Majumdar [9] using a more rigorous mathematical approach. Hoffmann, Marx and Botkin [11] deduced a formula for the drag acting on the surface of a sphere moving with constant velocity in a micropolar fluid with non-zero boundary conditions for the microrotations. Shu and Lee [12] derived new fundamental solutions for micropolar steady fluid flow and obtained the drag on a sphere translating in it. In [13], Hayakawa discussed the slow steady motion of micropolar fluid flows around a sphere and a cylinder. Sherief et al. [14] discussed the slow motion

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of a rigid sphere perpendicular to two infinite parallel plane walls in a micropolar fluid using collocation technique.

As mentioned above it can be seen that the steady micropolar fluid flows have been discussed by several authors, while the unsteady motion has received less attention in spite of the importance of discussing unsteady flows especially for small times. Lakshmana Rao and Bhujanga Rao discussed the rectilinear oscillation of a sphere along a diameter in a micropolar fluid in [15]. Charya and Iyengar [16] obtained a general formula for the drag experienced by an axisymmetric body oscillating rectilinearly along its axis of symmetry in an incompressible micropolar fluid. A general expression for the force exerted on a sphere executing longitudinal oscillation, with small amplitude, in an incompressible micropolar fluid is obtained by Sran in [17]. The unsteady flow due to non-coaxial rotations of a disk with the effect of slip condition is investigated in [18]. The problem of unsteady Couette flow of a micropolar fluid with the slip boundary condition is discussed in [19]. To the author's knowledge, no formula for the drag on a sphere moving with a general non-uniform speed has been obtained yet.

In this work, we investigate the creeping unsteady motion of an incompressible micropolar fluid flow past a fixed rigid sphere. The solution of the problem is obtained and the resultant drag force of the fluid acting on the surface of the sphere is calculated as well in the Laplace transform domain. The complex inversion formula of the Laplace transform is used together with contour integration to get the drag force in the physical domain. The obtained analytical formula of the drag force is applied to some examples of micropolar fluid flows.

2 Formulation of the problem

The field equations governing an isothermal incompressible micropolar fluid flow are given by [2]

$$\operatorname{div}\mathbf{q} = 0, \tag{2.1}$$

$$(\lambda + 2\mu + \kappa)\operatorname{grad}\operatorname{div}\mathbf{q} - (\mu + \kappa)\operatorname{curl}\operatorname{curl}\mathbf{q} + \kappa\operatorname{curl}\boldsymbol{\nu} - \operatorname{grad} p + \rho\mathbf{F} = \rho\dot{\mathbf{q}}, \tag{2.2}$$

$$(\alpha + \beta + \gamma)\operatorname{grad}\operatorname{div}\boldsymbol{\nu} - \gamma\operatorname{curl}\operatorname{curl}\boldsymbol{\nu} + \kappa\operatorname{curl}\mathbf{q} - 2\kappa\boldsymbol{\nu} + \rho\mathbf{C} = \rho j\dot{\boldsymbol{\nu}}, \tag{2.3}$$

where the two vectors $\boldsymbol{q}$ and $\boldsymbol{\nu}$ are representing the velocity and micro-rotation vectors, respectively. The body forces and body couples per unit mass are denoted by the two vectors $\boldsymbol{F}$ and $\boldsymbol{C}$. Also, p, ρ and j are denoting fluid pressure, fluid density and micro-inertia, respectively. λ and μ are the ordinary viscosity parameters of the classical viscous fluids and the constant κ is the new translational viscosity coefficient which can be termed as micropolarity parameter. The remaining constants α, β and γ are termed gyro-viscosity coefficients. These material constants have to satisfy the inequalities [2].

$$\begin{aligned} &2\mu + \kappa \geq 0, \quad \kappa \geq 0, \quad 3\lambda + 2\mu + \kappa \geq 0, \\ &\gamma \geq 0, \quad \gamma \geq |\beta|, \quad 3\alpha + \beta + \gamma \geq 0. \end{aligned} \tag{2.4}$$

Moreover, a superposed dot, appeared in Eqs. (2.2) and (2.3), indicates material differentiation.

The stress and couple stress tensors are given by the following constitutive relations [3]

$$t_{ij} = -p\delta_{ij} + (2\mu + \kappa)e_{ij} + \kappa\varepsilon_{ijk}(\omega_k - \nu_k), \tag{2.5}$$

$$m_{ij} = \alpha\nu_{r,r}\delta_{ij} + \beta\nu_{i,j} + \gamma\nu_{j,i}, \tag{2.6}$$

where ε_{ijk} is the usual alternating tensor and δ_{ij} denotes the Kronecker delta function.

The deformation rate tensors e_{ij} and ω_k are defined by

$$e_{ij} = \frac{1}{2}(q_{i,j} + q_{j,i}), \qquad \omega_k = \frac{1}{2}(\operatorname{curl}\mathbf{q})_k.$$

Assume that a rigid sphere of radius "a" is placed in a an unbounded micropolar fluid that starts to move unsteadily with a rectilinear non-uniform velocity $U(t)$ along the diameter $\theta = 0$ as represented in Fig. 1. Then the motion is axially symmetric. Working with the spherical polar coordinates (r, θ, ϕ), therefore the velocity and microrotation vectors have the forms

$$\begin{aligned} &\mathbf{q} = \big(u(r,\theta,t), v(r,\theta,t), 0\big) \quad \text{and} \\ &\boldsymbol{\nu} = \big(0, 0, \omega(r,\theta,t)\big). \end{aligned} \tag{2.7}$$

If the fluid initially is at rest, then the initial condition becomes

$$\mathbf{q}(r,\theta,t) = 0, \qquad \boldsymbol{\nu}(r,\theta,t) = 0 \quad \text{at } t = 0. \tag{2.8}$$

At the time moment $t = 0^+$, the fluid is set in motion by applying a time dependent speed $U(t)$ away from the sphere along the diameter $\theta = 0$. Thus

$$\begin{aligned} &\mathbf{q}(r,\theta,t) = U(t)(\cos\theta\hat{e}_r - \sin\theta\hat{e}_\theta), \\ &\boldsymbol{\nu}(r,\theta,t) = 0 \quad \text{as } r \to \infty, \end{aligned} \tag{2.9}$$

where $\hat{e}_r$ and $\hat{e}_\theta$ are the unit vectors along radial and transverse directions.

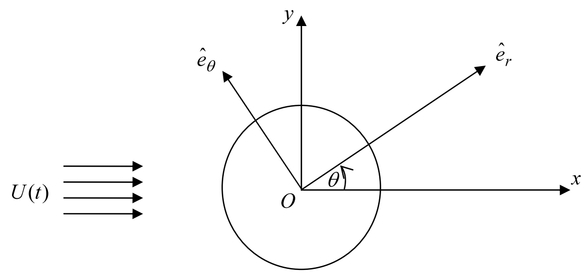

Fig. 1 The geometrical sketch

On the surface of the sphere, the boundary conditions are given by

$$\mathbf{q}(r,\theta,t)=0, \qquad \boldsymbol{\nu}(r,\theta,t)=0 \quad \text{on } r=a. \tag{2.10}$$

The spin inertia appearing in the equation of motion is given by [3]

$$j=\frac{2\gamma}{2\mu+\kappa}. \tag{2.11}$$

The relation (2.11) is assumed to permit the field equations to recover the classical theory of viscous fluids as a special case when the microrotation vector coincide with the angular velocity and the microstructure effects be neglected.

If the body forces and body couples are assumed to be absent, then the governing equations (2.1)–(2.3) in view of (2.11) reduce to

$$-(\mu+\kappa)\,\mathrm{curl}\,\mathrm{curl}\,\mathbf{q}+\kappa\,\mathrm{curl}\,\boldsymbol{\nu}-\mathrm{grad}\,p = \rho\frac{\partial \mathbf{q}}{\partial t}, \tag{2.12}$$

$$-\gamma\,\mathrm{curl}\,\mathrm{curl}\,\boldsymbol{\nu}+\kappa\,\mathrm{curl}\,\mathbf{q}-2\kappa\boldsymbol{\nu} = \frac{2\gamma\rho}{2\mu+\kappa}\frac{\partial \nu}{\partial t}, \tag{2.13}$$

where the inertial terms of Eqs. (2.2) and (2.3) are neglected since we are considering the creeping motion.

3 Solution in the Laplace transform domain

Here, we apply the integral Laplace transform defined by

$$\bar{F}(r,\theta,s)=\int_0^\infty e^{-st}F(r,\theta,t)\,dt, \tag{3.1}$$

to the governing equations (2.12) and (2.13), with the aid of initial conditions (2.8), to obtain

$$-(\mu+\kappa)\,\mathrm{curl}\,\mathrm{curl}\,\bar{\mathbf{q}}+\kappa\,\mathrm{curl}\,\bar{\boldsymbol{\nu}}-\mathrm{grad}\,\bar{p} = \rho s\bar{\mathbf{q}}, \tag{3.2}$$

$$-\gamma\,\mathrm{curl}\,\mathrm{curl}\,\bar{\boldsymbol{\nu}}+\kappa\,\mathrm{curl}\,\bar{\mathbf{q}}-2\kappa\bar{\boldsymbol{\nu}} = \frac{2\gamma\rho s}{2\mu+\kappa}\bar{\boldsymbol{\nu}}. \tag{3.3}$$

From the equation of continuity (2.1), the velocity components can be represented in terms of the stream function $\bar{\Psi}(r,\theta,s)$ as follows

$$\bar{u}=\frac{-1}{r^2\sin\theta}\frac{\partial\bar{\Psi}}{\partial\theta}, \qquad \bar{v}=\frac{1}{r\sin\theta}\frac{\partial\bar{\Psi}}{\partial r}. \tag{3.4}$$

Hence, the radial and transverse components of Eq. (3.2) can be represented as

$$-\frac{\partial\bar{p}}{\partial r}+\frac{\kappa}{r^2\sin\theta}\frac{\partial}{\partial\theta}(r\sin\theta\,\bar{\nu}) - \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\big((\mu+\kappa)L_{-1}-\rho s\big)\bar{\Psi}=0, \tag{3.5}$$

$$-\frac{\partial\bar{p}}{\partial\theta}-\frac{\kappa}{\sin\theta}\frac{\partial}{\partial r}(r\sin\theta\,\bar{\nu}) + \frac{1}{\sin\theta}\frac{\partial}{\partial r}\big((\mu+\kappa)L_{-1}-\rho s\big)\bar{\Psi}=0, \tag{3.6}$$

where

$$L_{-1}=\frac{\partial^2}{\partial r^2}-\frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}.$$

The only non-vanishing component of the differential equation (3.3) in view of (3.4) gives

$$\kappa L_{-1}\bar{\Psi}+\left\{\gamma L_{-1}-2\kappa-\frac{2\gamma\rho s}{2\mu+\kappa}\right\}(r\sin\theta\,\bar{\nu})=0. \tag{3.7}$$

Eliminating the pressure p appearing in Eqs. (3.5) and (3.6), we arrive at

$$L_{-1}\big\{(\mu+\kappa)L_{-1}-\rho s\big\}\bar{\Psi}-\kappa L_{-1}(r\sin\theta\,\bar{\nu})=0. \tag{3.8}$$

The two equations (3.7) and (3.8) can be simplified to the following forms

$$L_{-1}\{L_{-1}-\alpha_1^2\}\{L_{-1}-\alpha_2^2\}\bar{\Psi}=0, \tag{3.9}$$

$$\{L_{-1}-\alpha_1^2\}\{L_{-1}-\alpha_2^2\}(r\sin\theta\bar{\nu})=0, \tag{3.10}$$

where

$$\alpha_1^2=\frac{2\rho s}{2\mu+\kappa},\qquad \alpha_2^2=\ell^2+\frac{\rho s}{\mu+\kappa},$$
$$\ell^2=\frac{\kappa(2\mu+\kappa)}{\gamma(\mu+\kappa)}.$$

The bounded solutions of the differential equations (3.9) and (3.10) are, respectively, found to be of the forms

$$\bar{\Psi}(r,\theta,s)=\left\{A\left(\alpha_1+\frac{1}{r}\right)e^{-\alpha_1 r}+B\left(\alpha_2+\frac{1}{r}\right)e^{-\alpha_2 r}+\frac{C}{r}+Dr^2\right\}\sin^2\theta, \tag{3.11}$$

and

$$\bar{\nu}(r,\theta,s)=\frac{(\mu+\kappa)}{\kappa}\left\{A\left(\alpha_1^2-\frac{\rho s}{\mu+\kappa}\right)\times\left(\alpha_1+\frac{1}{r}\right)\frac{1}{r}e^{-\alpha_1 r}+B\left(\alpha_2^2-\frac{\rho s}{\mu+\kappa}\right)\times\left(\alpha_2+\frac{1}{r}\right)\frac{1}{r}e^{-\alpha_2 r}\right\}\sin\theta, \tag{3.12}$$

where A, B, C and D are constants, depending only on the parameter s, to be determined from the imposed boundary conditions.

The boundary conditions (2.9) and (2.10), with the aid of (3.1) and (3.4), can be rewritten as

$$\frac{\partial\bar{\Psi}}{\partial\theta}=-\bar{U}r^2\sin\theta\cos\theta,\qquad \frac{\partial\bar{\Psi}}{\partial r}=-\bar{U}r\sin^2\theta,\qquad \bar{\nu}=0\quad\text{as } r\to\infty, \tag{3.13}$$

$$\frac{\partial\bar{\Psi}}{\partial\theta}=0,\qquad \frac{\partial\bar{\Psi}}{\partial r}=0,\qquad \bar{\nu}=0\quad\text{on } r=a. \tag{3.14}$$

Applying the boundary conditions (3.13) and (3.14) we obtain the values of the constants A, B, C and D in the following forms

$$A=\frac{-3a\bar{U}(s)}{2\Delta}\left\{\alpha_2^2-\frac{\rho s}{\mu+\kappa}\right\}\left(\alpha_2+\frac{1}{a}\right)e^{\alpha_1 a}, \tag{3.15}$$

$$B=\frac{3a\bar{U}(s)}{2\Delta}\left(\alpha_1^2-\frac{\rho s}{\mu+\kappa}\right)\left(\alpha_1+\frac{1}{a}\right)e^{\alpha_2 a}, \tag{3.16}$$

$$C=\frac{a^3\bar{U}(s)}{2}-\frac{3a^2\bar{U}(s)}{2\Delta}(\alpha_1^2-\alpha_2^2)\left(\alpha_1+\frac{1}{a}\right)\times\left(\alpha_2+\frac{1}{a}\right), \tag{3.17}$$

$$D=-\frac{\bar{U}(s)}{2}, \tag{3.18}$$

where

$$\Delta=\alpha_1^2\left(\alpha_2^2-\frac{\rho s}{\mu+\kappa}\right)\left(\alpha_2+\frac{1}{a}\right)-\alpha_2^2\left(\alpha_1^2-\frac{\rho s}{\mu+\kappa}\right)\left(\alpha_1+\frac{1}{a}\right).$$

We are going now to evaluate the resultant drag force exerted by the fluid on the sphere by using the well known formula

$$\bar{F}_z(s)=2\pi a^2\int_0^\pi\{\bar{t}_{rr}(a,\theta,s)\cos\theta-\bar{t}_{r\theta}(a,\theta,s)\sin\theta\}\sin\theta d\theta. \tag{3.19}$$

Using the stress formula (2.5) and after some straight forward manipulations the formula (3.19) can be simplified to the form

$$\bar{F}_z(s)=2\pi a\left\{\frac{1}{3}\rho a^2\big(s\bar{U}(s)\big)+\frac{3\kappa_1\kappa_2}{\kappa_3}\bar{U}(s)(\ell a+1)+\frac{3a\kappa_1}{\pi}\big(s\bar{U}(s)\bar{\Phi}(s)\big)\right\}, \tag{3.20}$$

where

$$\bar{\Phi}(s)=\frac{\pi\rho s}{\Delta_1}\{\big(a\kappa_3(\alpha_1\kappa_2+2\alpha_2\kappa_1)+\kappa_2^2+4\ell a\kappa_1^2\big)+\kappa_1\kappa_2\big(\ell\kappa(a\alpha_1\alpha_2+\alpha_1+\alpha_2-\ell)+\kappa_3\alpha_1(\alpha_2+\ell^2 a)\big)\}, \tag{3.21}$$

and

$$\Delta_1=\kappa_1\kappa_3 s\{2a\kappa_1\alpha_2^2+\kappa_2(a\alpha_1\alpha_2+\alpha_1+\alpha_2)\},$$
$$\kappa_1=(\mu+\kappa),\qquad \kappa_2=(2\mu+\kappa),$$
$$\kappa_3=\kappa_2+2\ell a\kappa_1.$$

4 Inverse Laplace transform

Taking the inverse Laplace transform to Eq. (3.20) and using the convolution theorem, we obtain the following drag formula in the physical domain

$$F_z(t)=2\pi a\left\{\frac{1}{3}\rho a^2\frac{dU(t)}{dt}+\frac{3\kappa_1\kappa_2}{\kappa_3}U(t)(\ell a+1)+\frac{3a\kappa_1}{\pi}\int_0^t\frac{dU(\tau)}{d\tau}\Phi(t-\tau)d\tau\right\}, \tag{4.1}$$

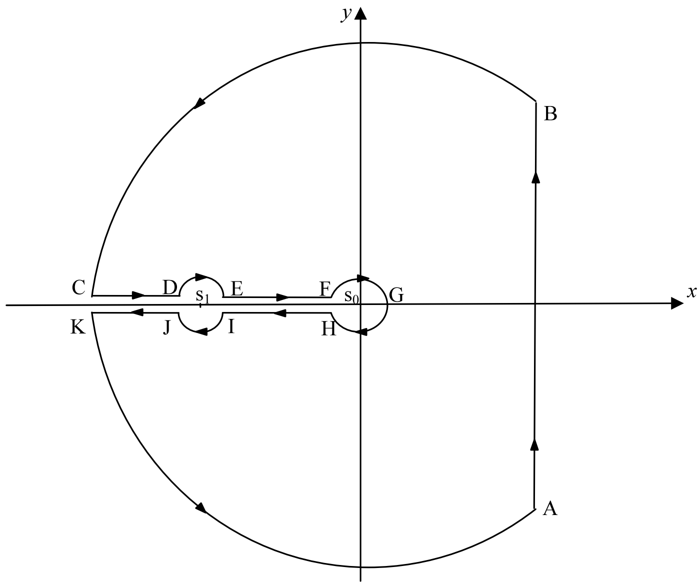

Fig. 2 The modified Bromwich contour Γ

where the time dependent function $\Phi(t)$ represents the inverse Laplace transform of the function $\bar{\Phi}(s)$ defined by the relation (3.21). In order to obtain the inverse Laplace transform of $\bar{\Phi}(s)$, we shall use the complex inversion formula of the Laplace transform defined by [20]

$$\Phi(t) = \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} e^{st}\bar{\Phi}(s)ds, \tag{4.2}$$

where the constant σ is assumed to be greater than all the real parts of the singularities of $\bar{\Phi}(s)$ [20, 21]. The function $\bar{\Phi}(s)$ defined by Eq. (3.21) has only two real branch points at $s_o = 0$, $s_1 = -\eta$, where $\eta = \kappa_1 \ell^2/\rho$, and a simple pole at $s_o = 0$.

To use the complex inversion formula of the Laplace transform, we have to integrate the function $\bar{\Phi}(s)$ along the modified Bromwich contour Γ illustrated in Fig. 2. The contour consists of three small circular arcs DE, IJ and FGH each of radius ε, say. It is formed also of two arcs BC and KA of large radii R, say. The contour Γ also contains four straight lines connecting the circular arcs as shown in Fig. 2 and another vertical line AB along which s takes the value $\sigma + iy$. When taking the limits as $\varepsilon \to 0$ and $R \to \infty$, the integral along AB matches the integral (4.2). Since the considered function has no singular points inside Γ, then the integral along Γ vanishes.

From the above discussion we have

$$\oint_\Gamma e^{st}\bar{\Phi}(s)ds = 0, \tag{4.3}$$

$$\operatorname*{Lim}_{R\to\infty}\int_{AB} e^{st}\bar{\Phi}(s)ds = 2\pi i\Phi(t), \tag{4.4}$$

$$\int_{FGH} e^{st}\bar{\Phi}(s)ds = O\big(\varepsilon^{\frac{1}{2}}\big) \to 0 \quad \text{as } \varepsilon \to 0, \tag{4.5}$$

$$\int_{DE} e^{st}\bar{\Phi}(s)ds + \int_{IJ} e^{st}\bar{\Phi}(s)ds = O(\varepsilon) \to 0 \quad \text{as } \varepsilon \to 0, \tag{4.6}$$

$$\int_{BC} e^{st}\bar{\Phi}(s)ds + \int_{KA} e^{st}\bar{\Phi}(s)ds \to 0 \quad \text{as } R \to \infty. \tag{4.7}$$

The integrals along the straight lines (CD, JK) and (EF, HI) are evaluated and are found to be of the forms

$$\begin{aligned}
&\operatorname*{Lim}_{\substack{\varepsilon\to 0\\ R\to\infty}}\left\{\int_{CD} e^{st}\bar{\Phi}(s)ds + \int_{JK} e^{st}\bar{\Phi}(s)ds\right\} \\
&= -2\pi i\int_\eta^\infty \frac{e^{-xt}}{\Delta_2}\Big\{\kappa_1\ell^2\big(2\kappa_2 b_1\big\{\ell^2 a^2\kappa_1^2 - \mu \\
&\quad - 3\rho x a^2\big\} + b_2\big\{\kappa_2\kappa - 2\rho x a^2(2\kappa_1 + \kappa_2)\big\}\big) \\
&\quad + \rho x\big(\kappa_2 + 2\rho x a^2\big) \\
&\quad \times \big(2b_1\kappa_2 + b_2(2\kappa_1 + \kappa_2)\big)\Big\}dx,
\end{aligned} \tag{4.8}$$

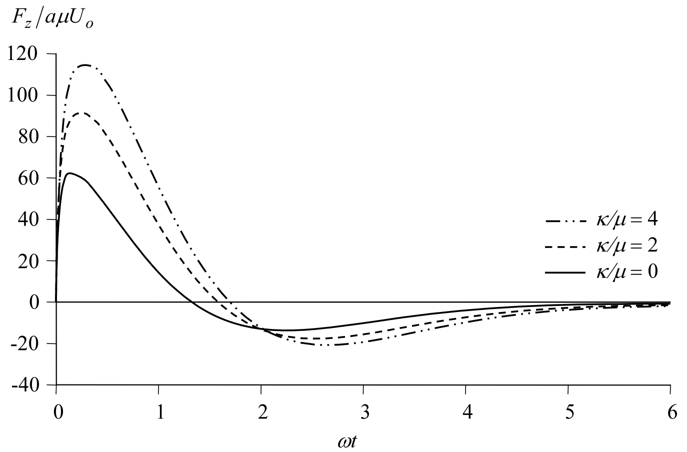

Fig. 3 Drag force versus time for case (1)

$$\begin{aligned}&\lim_{\substack{\varepsilon\to 0\\ R\to\infty}}\left\{\int_{EF} e^{st}\bar{\Phi}(s)ds+\int_{HI} e^{st}\bar{\Phi}(s)ds\right\}\\ &\quad=4\pi i\ell^2\kappa_1^2\kappa_2\int_0^{\eta}\frac{e^{-xt}}{\Delta_3}b_1\\ &\qquad\times\left\{a^2b_3^2-2ab_3-1\right\}dx, \end{aligned}\tag{4.9}$$

where

$$\begin{aligned}\Delta_2(x)&=x\big\{4\ell^4a^2\kappa_1^3-\ell^2\kappa_1\big(\kappa_2^2+2\rho xa^2(4\kappa_1+\kappa_2)\\ &\quad+4b_1b_2a^2\kappa_1\kappa_2\big)+\big(\rho x(2\kappa_1+\kappa_2)\\ &\quad+2b_1b_2\kappa_1\kappa_2\big(2\rho xa^2+\kappa_2\big)\big)\big\},\\ \Delta_3(x)&=x\big\{4\ell^4a^2\kappa_1^3+\ell^2\kappa_1\big(\kappa_2^2-2\rho xa^2(2\kappa_1+\kappa)\\ &\quad+4ab_3\kappa_1\kappa_2\big)+\rho x\kappa\big(2\rho xa^2+\kappa_2\big)\big\},\end{aligned}$$

and

$$b_1=\sqrt{\frac{2\rho x}{\kappa_2}},\qquad b_2=\sqrt{\frac{\rho x}{\kappa_1}-\ell^2},$$
$$b_3=\sqrt{\ell^2-\frac{\rho x}{\kappa_1}}.$$

Now we substitute the values of the integrals (4.4)–(4.9) into (4.3) to obtain the desired function $\Phi(t)$ in the physical domain as follows

$$\begin{aligned}\Phi(t)&=-2\ell^2\kappa_1^2\kappa_2\int_0^{\eta}\frac{e^{-xt}}{\Delta_3}b_1\left\{a^2b_3^2-2ab_3-1\right\}dx\\ &\quad+\int_{\eta}^{\infty}\frac{e^{-xt}}{\Delta_2}\big\{\kappa_1\ell^2\big(2\kappa_2b_1\big[\ell^2a^2\kappa_1^2-\mu\\ &\quad-3\rho xa^2\big]+b_2\big[\kappa_2\kappa-2\rho xa^2(2\kappa_1+\kappa_2)\big]\big)\\ &\quad+\rho x\big(\kappa_2+2\rho xa^2\big)\\ &\quad\times\big(2b_1\kappa_2+b_2(2\kappa_1+\kappa_2)\big)\big\}dx.\end{aligned}\tag{4.10}$$

From the above equation and Eq. (4.1) we obtain a general formula to calculate the drag force exerted by the fluid on the surface of a sphere placed in an unsteady micropolar fluid flow in the following simple form

$$\begin{aligned}F_z(t)&=2\pi a\bigg[\frac{1}{3}\rho a^2\frac{dU(t)}{dt}+\frac{3\kappa_1\kappa_2}{\kappa_3}(\ell a+1)U(t)\\ &\quad+\frac{3a\kappa_1}{\pi}\int_0^t\frac{dU(\tau)}{d\tau}\Phi(t-\tau)d\tau\bigg].\end{aligned}\tag{4.11}$$

The classical case of viscous fluid flow is recovered as a special case of this work when the micropolarity constant κ tends to zero. In this case the relation (4.11) simply reduces to

$$\begin{aligned}F_z(t)&=2\pi a\bigg[\frac{1}{3}\rho a^2\frac{dU(t)}{dt}+3\mu U(t)\\ &\quad+3a\sqrt{\frac{\rho\mu}{\pi}}\int_0^t\frac{dU(\tau)}{d\tau}\frac{1}{\sqrt{(t-\tau)}}d\tau\bigg].\end{aligned}\tag{4.12}$$

The formula (4.12) is in agreement with that of Basset (see Basset [22] and Landau and Lifshitz [23]).

If the fluid flow is assumed to move steadily, i.e. when $U(t)=U_o$, where U_o is a constant, the drag formula (4.11) becomes

$$F_z(t)=\frac{6\pi\kappa_1\kappa_2}{\kappa_3}aU_o(\ell a+1),\tag{4.13}$$

which is coincident with that obtained by Ramkissoon and Majumdar [9].

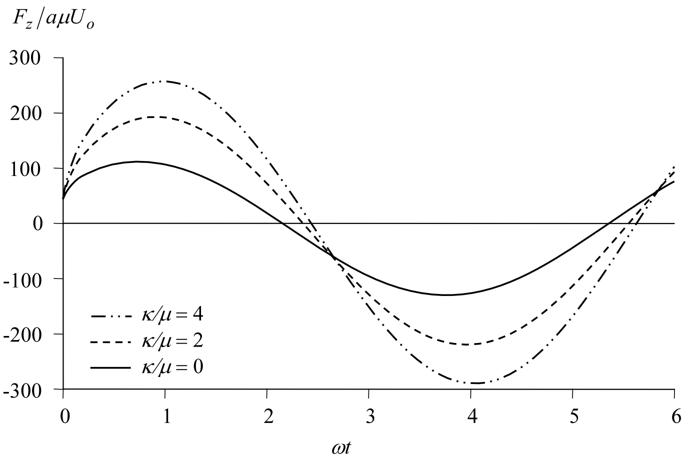

Fig. 4 Drag force versus time for case (2)

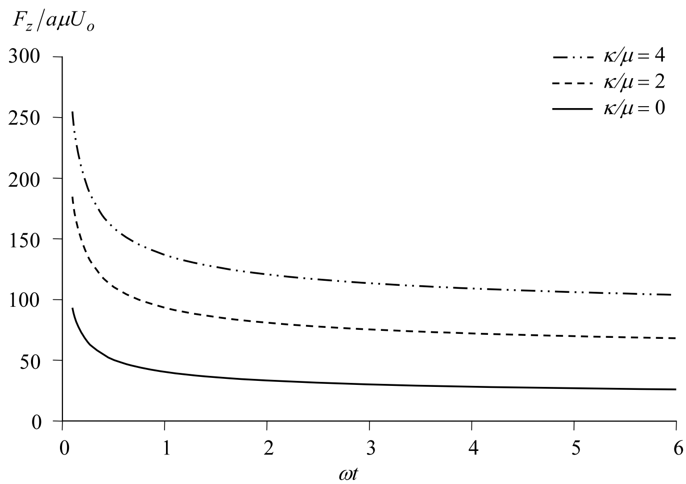

Fig. 5 Drag force versus time for case (3)

5 Drag of some flows

In this section we employ the general drag formula (4.11) to evaluate the resultant drag force on the surface of a sphere translating in a micropolar fluid with some given speeds.

Case (1) The case of damping oscillation, with frequency ω, is considered here by assuming that $U(t) = U_o e^{-\omega t}\sin(\omega t)$, therefore

$$F_z(t) = 2\pi a U_o\bigg[\frac{1}{3}\rho a^2 \omega e^{-\omega t}\big(\cos(\omega t) - \sin(\omega t)\big) + \frac{3\kappa_1\kappa_2}{\kappa_3}(\ell a + 1)e^{-\omega t}\sin(\omega t) + \frac{3a\kappa_1\omega}{\pi}\int_0^t e^{-\omega\tau}\big(\cos(\omega\tau) - \sin(\omega\tau)\big) \times \Phi(t-\tau)\,d\tau\bigg]. \tag{5.1}$$

Case (2) In this case we consider the oscillatory flow given by applying the velocity $U(t) = U_o\sin(\omega t)$ to get

$$F_z(t) = 2\pi a U_o\bigg[\frac{1}{3}\rho a^2\omega\cos(\omega t) + \frac{3\kappa_1\kappa_2}{\kappa_3}(\ell a + 1)\sin(\omega t) + \frac{3a\kappa_1\omega}{\pi}\int_0^t \cos(\omega\tau)\Phi(t-\tau)\,d\tau\bigg]. \tag{5.2}$$

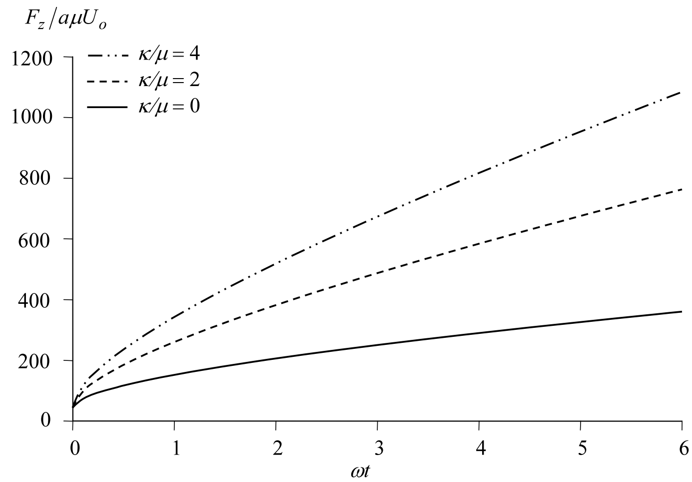

Fig. 6 Drag force versus time for case (4)

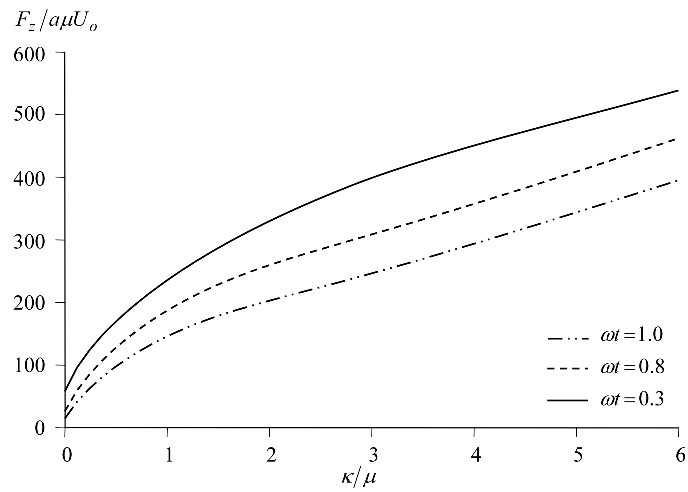

Fig. 7 Drag force versus micropolarity coefficient for case (1)

Case (3) The sudden motion is considered here by taking $U(t) = U_o H(t)$, where $H(t)$ is the Heaviside unit step function defined by

$$H(t) = \begin{cases} 1 & \text{if } t > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{5.3}$$

In this case the drag formula (4.11) reduces to the form

$$F_z(t) = \frac{3a\kappa_1 U_o}{\kappa_3}\{2\pi\kappa_2(\ell a + 1)H(t) + a\kappa_3 \Phi(t)\}. \tag{5.4}$$

This latter relation yields the correct behavior of the steady motion when the time t becomes infinite.

Case (4) Here we consider the case of accelerating velocity, i.e. $U(t) = tU_o$, then we have

$$F_z(t) = 2\pi a U_o\left[\frac{1}{3}\rho a^2 + \frac{3\kappa_1\kappa_2}{\kappa_3}(\ell a + 1)t + \frac{3a\kappa_1}{\pi}\int_0^t \Phi(t-\tau)d\tau\right]. \tag{5.5}$$

6 Numerical results and conclusion

To illustrate our results graphically, formula (4.11) is employed for different cases of the speed $U(t)$. In view of (2.4), the material parameters γ, ρ and

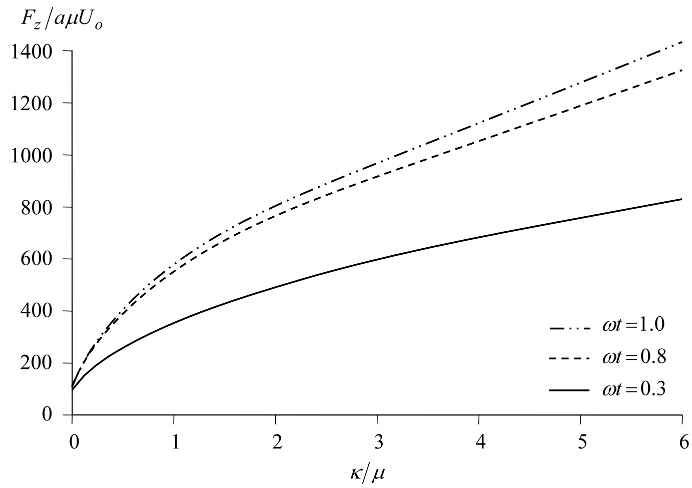

Fig. 8 Drag force versus micropolarity coefficient for case (2)

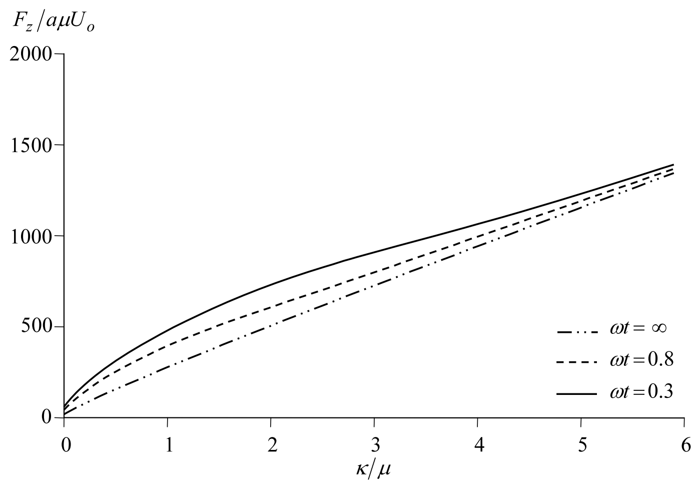

Fig. 9 Drag force versus micropolarity coefficient for case (3)

μ have been assigned the following values during numerical calculations; the parameter γ is taken equal to 1.3 $\mathrm{g\,cm\,s^{-1}}$, the density ρ is assumed to be 1.05 $\mathrm{g\,cm^{-3}}$ and the viscosity coefficient μ is assigned the value 0.05 $\mathrm{g\,cm^{-1}\,s^{-1}}$. The last two values represents the mean density and mean viscosity of animal blood which can be modeled as micropolar fluids. The drag force is calculated and represented graphically against the time for different values of κ/μ in Figs. 3, 4, 5 and 6 and against the micropolarity coefficient ratio κ/μ for different values of time in Figs. 7, 8, 9 and 10. From Figs. 3, 4, 5 and 6 it can be noticed that the increase of the micropolarity factor κ/μ increases the values of the drag force. Also, from Fig. 3 we observe that the drag vanishes after a short time; of course the decay of the drag force occurring in this case is expected since we consider here the case of damping oscillation. Figures 7 and 9 show that the values of the drag force decrease with the increase of the time; this behavior is in accord with damping oscillation and sudden motion. In Fig. 10 we find that the increase of the time increases the values of the drag force which is also expected because the cases of sine oscillation and accelerating speed are considered and they both proportional to the time.

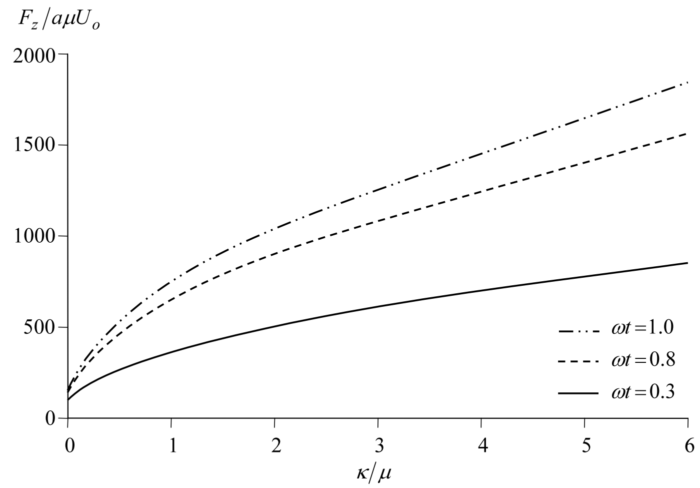

Fig. 10 Drag force versus micropolarity coefficient for case (4)

The well known drag formula (4.12) obtained by Basset (see Basset [22] and Landau and Lifshitz [23]) in the case of viscous fluid flow is recovered as a special case of the present work when the micropolarity parameter κ becomes zero.

Also, when the speed $U(t)$ is assumed to be constant the drag formula (4.11) reduces to that of Ramkissoon and Majumdar [9] in the case of steady state micropolar fluids. This behavior is also seen in Fig. 9 when the time tends to infinity.

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