Carlo Cercignani's interests for the foundations of physics

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Abstract Carlo Cercignani was known all over the world for his works on the Boltzmann equation and on kinetic theory. There was however another aspect of his scientific life, which is not much known. Namely, his interest for the foundations of physics, in particular for the possibility of understanding quantum mechanics through classical mechanics, which he shared with several people in Milan. A review of such researches is given here, together with some personal recollections of him.

1 Introduction

Carlo Cercignani was well known all over the world as one of the greatest experts in the problems related to the Boltzmann equation and more in general to kinetic theory, to which most of his scientific life was devoted. Very little known is instead his interest for the foundations of physics, especially the relations between classical and quantum mechanics. Two papers

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have a special interest in this connection; the first one [1] of the year 1972, by the title *"Zero-point energy in classical non-linear mechanics"*, in collaboration with Antonio Scotti and me, and the second one [2] of the year 1998, by the title *"On a nonquantum derivation of Planck's distribution law"*. His interest is also witnessed by another paper on the same subject, namely, *"Quantization à la Nernst for the plane rotator"* [3]. A further paper [4], with G. Benettin, A. Giorgilli and me (see also [5]), will also be mentioned. It reports on the discovery of the analog of the universal Feigenbaum number for Hamiltonian systems, and its hidden connection with the general problems discussed here will be indicated.

To say the core of the things in a few words, Carlo Cercignani shared with our group forty years ago, and continued to share along all of his life, the idea that the last word has not yet been said about the relations between classical and quantum mechanics. Perhaps new insights might come from the theory of classical dynamical systems, after the great impulse it received since the year 1954, both from the mathematical work of Kolmogorov (followed by those of Arnold, Moser and Nekhoroshev), and the physical work of Fermi, Pasta and Ulam.

A quick illustration of such works of Carlo Cercignani will be given here, after a recollection of the human and scientific atmosphere in which they were conceived. A short outline will also be given of the present state of the art in the field. Finally, some recollections
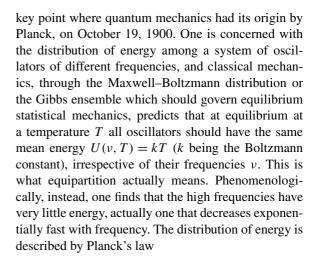
will be given concerning certain personal aspects of his life.

2 The Fermi Pasta Ulam problem and the foundations of physics in Milan in the years sixties

Things went as follows. Carlo had graduated in physics at Milan in the year 1961, and two years later also in mathematics, and had a great reputation among young people working around the Institute of Physics, at via Celoria 16. Actually such a reputation even went back to the times when he was a student and he was considered a kind of a genius. At the end of the years sixties he was working on some applications of hydrodynamics and kinetic theory, but from time to time he used to come to the Institute, where for instance he gave informal lectures on various sophisticated mathematical problems. There was a rather large group of young people working at the Institute or somehow associated with it, and the general atmosphere was very beautiful: one used to meet in the evenings, to go to the Alps in the week-ends, ... For what concerns research, in Milan there was a certain tradition in studies concerning foundational problems, going back to Caldirola and his pupils Loinger, Bocchieri, Prosperi, Scotti, Ghirardi, Lanz and others. Caldirola liked to mention that on several occasions he had the opportunity to take part in discussions within the Fermi group in Rome. Moreover Pauli had come several times to Milan, my tutor Montaldi had been several years in Munich with Heisenberg, Bruno Bertotti from Pavia had been with Schroedinger in Dublin, and Rosenfeld was considered as some kind of sponsor of the Milan group. For example, when Scotti and me first became aware of KAM theory (that in Italy absolutely no one knew in those years), I was requested to write down a short review that was sent to Rosenfeld. An enlarged version was later published, together with Antonio Scotti [6], for La Rivista del Nuovo Cimento.

One day Loinger discovered[1] the work of Fermi, Pasta and Ulam (FPU), in which the relevance of dynamics in connection with the equipartition principle was pointed out [7]. This, as everyone knows, is the key point where quantum mechanics had its origin by Planck, on October 19, 1900. One is concerned with the distribution of energy among a system of oscillators of different frequencies, and classical mechanics, through the Maxwell–Boltzmann distribution or the Gibbs ensemble which should govern equilibrium statistical mechanics, predicts that at equilibrium at a temperature T all oscillators should have the same mean energy $U(\nu, T) = kT$ (k being the Boltzmann constant), irrespective of their frequencies ν. This is what equipartition actually means. Phenomenologically, instead, one finds that the high frequencies have very little energy, actually one that decreases exponentially fast with frequency. The distribution of energy is described by Planck's law

$$U(\nu, T) = \frac{h\nu}{\exp(h\nu/kT) - 1},$$

which was just invented by a skillful fit to the data with the introduction of Planck's constant h, and quantum mechanics with its energy levels $E_n = nh\nu$ was just built above it.

Thus, the conflict between classical mechanics with its equipartition, and quantum mechanics which just by definition is Planck's law, is patent, and about say 99 percent of scientists decree that there is no problem: classical mechanics is wrong, being valid just in some limit situations, typically for low frequencies ν or high temperatures T, actually, in the limit $h\nu/kT \ll 1$, where Planck's law meets equipartition.

However, there are a few scientists who feel that things are subtler, and there still is a delicate dynamical aspect of the problem that should be settled. This actually is the point that Fermi (with Pasta and Ulam) made in what probably was the last work of his life. Notice that he had addressed the problem already in the year 1923, by elaborating on a theorem of Poincaré about the number of integrals of motion of a generic Hamiltonian system, which is usually interpreted as supporting the common opinion that classical mechanics implies equipartition also in a dynamical sense. But Fermi was not content with this, and when for the first time he had available a computer that might allow him to check conjectures by "numerical experiments", he made a dynamical check of equipartition. He considered the system of oscillators which correspond (as normal modes, in the familiar way) to a system of point particles on a line with nearest neighbour slightly nonlinear forces. For initial data with energy given just to one low-frequency oscillator, or to a

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[1] I do not know in which way. Perhaps, through the work of Izrailev and Chirikov (1966), although I came to know of such a paper only much later.

few low-frequency ones, energy was expected to flow up, equally distributing itself among all oscillators, as required by the equipartition principle as equilibrium is attained. Instead, the paradoxical result was found that, up to the available computational time, a state of apparent equilibrium was attained, in which energy was shared only among a small packet of low-frequency oscillators, while the high-frequency ones essentially remained deprived of any energy. Such a "final" state appeared to be a stationary one, which did not move at all. According to Fermi, this was some new, relevant discovery. For the figures, see the original paper, or for example the review [8].

So, the FPU "paradox" saw the light, and it made patent the deep role played by dynamics in statistical mechanics. By the way, just in the same year 1954 Kolmogorov had proven his celebrated theorem on invariant tori, on which Moser and Arnold came back in the years 1961, 1962. Thus some kind of a new era in the mathematical aspects of perturbation theory had started, and at the same time a new popular acquaintance with the coexistence of ordered and chaotic motions took place, especially through a diffuse availability of computers....

Now, which had been the reaction of the scientific community to the FPU paradox? In the year 1965 [9] Kruskal, Zaburksy and their collaborators had taken up a certain mathematical side of the problem, paving the road to solitons and integrable partial differential equations, but had said nothing on the physical side of the FPU problem. However, such works impressed Carlo very much, and he studied them carefully. What came out is a review by the title *"Solitons. Theory and application"* [10] that he wrote for La Rivista del Nuovo Cimento.

The physical relevance was instead immediately proclaimed one year later by Izrailev and Chirikov [11], who completely understood that a fundamental problem had been opened. They also were so clever as to indicate a possible way out. Let us ponder on this. First of all they made a crucial discovery. Namely, if one repeats the FPU "experiment" for initial data exactly as FPU, but at a high enough energy, then the FPU paradox disappears, because equipartition is attained very quickly. In other words, there exists a kind of critical energy E_c, and the paradox only occurs for an energy $E < E_c$. Then they also imagined a way in which the paradox could be completely eliminated for macroscopic systems, i.e., for systems with a very

large number N of constituents (formally, in the so-called thermodynamic limit $N \to \infty$, for fixed values of the specific energy E/N and of the specific "volume" L/N, if L is the size of the FPU system). The very simple idea is that the critical specific energy $\epsilon_c = E_c/N$ vanishes in the thermodynamic limit. Then, for any macroscopic system, which by definition has specific energy $\epsilon = E/N > 0$, one would have $\epsilon > \epsilon_c$, and the "ordered" FPU-like states violating equipartition would not exist at all. With such a conjecture the problem of eliminating the FPU paradox then became a mathematical one, namely, to prove or indicate in any possible way, that the energy threshold E_c does not grow at all or grows less than N for increasing N.

This was the situation when Loinger discovered the FPU problem. Now, at the end of the years sixties Scotti had come back to Milan from California where, working in high energy physics, he had become acquainted with the use of computers. So Loinger involved him and Bocchieri, and they repeated the computations for a FPU-type model with realistic intermolecular forces, given by the so-called Lennard-Jones potential

$$V(r) = 4V_0\big[(\sigma/r)^{12} - (\sigma/r)^6\big],$$

which involves two parameters V_0 and σ. The only other parameter entering the model was the mass m of the particles. They too confirmed the existence of an energy threshold E_c, but their new contribution was an indication, based on their numerical results, that the specific threshold $\epsilon_c = E_c/N$ may not vanish with increasing N [12]. This result was perhaps even expected because, when shortly later Loinger gave a talk on the general problem of the relations between classical and quantum mechanics, he explicitly said he liked that result. Most people in the audience judged him as crazy. I, instead, was fascinated.

At that moment I entered the game. Since I was already working with Scotti on equilibrium statistical mechanics, and really had begun to share my life with him, I started helping him in some further computations he was performing on the FPU problem. We were impressed by the fact that the "final" distribution of energy among the oscillators was decreasing exponentially fast with frequency, and just had the crazy idea to fit it to a Planck-like law. The fit turned out to be rather good, but what really struck us as something incredible and almost absurd, was what occurred with the free

parameter introduced in the fit. Indeed we had chosen a Planck-like distribution in which the only free parameter was an action A taking the place of Planck's constant h, and the fit gave for A a value rather near to h. It took us perhaps one month to really understand what had occurred. If in a classical mechanical model one determines an action A, then this has to be a pure number, say α, times some "natural" action which is defined in terms of the parameters entering the model. In our case they were the parameters V_0, σ and m previously mentioned, and so standard dimensional analysis gives

$$A = \alpha \sqrt{m V_0}\, \sigma.$$

Now, from the relevant textbooks on molecular physics one finds that for the rare gases one has

$$\sqrt{m V_0}\, \sigma = 2 Z \hbar$$

where Z is the atomic number and $\hbar = h/2\pi$. So h had been introduced into the classical FPU model from outside, through the molecular parameters of the Lennard-Jones potential. Moreover we had actually worked, as previously Bocchieri Scotti and Loinger, with the parameters of Argon, and by chance the value of the pure number α was such that, combined with the values of the molecular parameters of Argon, it produces $A \simeq \hbar$.

In any case the FPU paradox was thus enhanced, and Planck's law appeared to enter classical physics, at those low temperatures for which equipartition is not attained dynamically. This remark was published in the Physical Review Letters [13]. Actually Bocchieri and Loinger did not dare to sign the paper, and Scotti and me were left as the only two authors. The reactions in the scientific community were in general not very good.

3 Enters Carlo. The 1972 CGS work

Here, Carlo enters the game. One day I was explaining these things on a blackboard to Maria Marinaro, a physicist from Naples whom I had met in London a little before, at a conference where I had presented our results. While I was discussing with her, Carlo showed up at the door of the room. All doors were left opened in those years, and everyone passing through could freely join a discussion. I still remember Carlo on that

occasion, elegant as usual in his blue suit, leaning casually against the door jamb, and listening with great attention. After perhaps ten minutes he left. But a few minutes later he came back, and told me that he too had happened to think of the relations between classical and quantum mechanics, although he never had heard of the FPU problem. So we arranged that we should talk about it, together with Scotti, and so we did in a few days.

What he had in mind is the role of zero-point energy. This, as Planck himself says in the preface to his book on the quantum theory of radiation, is the quintessence itself of quantum mechanics. As is well known, with his "second theory" of the year 1911 Planck had suggested that each oscillator should possess, in addition to the thermal energy $U(\nu, T)$ given by the standard Planck's formula, also an energy $h\nu/2$. So the total energy should be $E(\nu, T) = U(\nu, T) + h\nu/2$, i.e.,

$$E(\nu, T) = \frac{h\nu}{\exp(h\nu/kT) - 1} + \frac{1}{2} h\nu.$$

At zero temperature the "thermal part" U vanishes, and only the "zero-point" energy (*nullspunkt* means zero temperature in German) remains. This energy of a non thermal nature is known to produce mechanical effects, such as the so-called Casimir effect. So one has on the one hand an energy of disordered or chaotic type, and on the other one an energy of ordered type, as was particularly stressed by Nernst in the year 1916. But at that time no one of us was aware of these features concerning such two types of energy, and the main idea of Carlo was precisely to introduce such notions in the FPU frame. In such a way he was led to conceive that one should go beyond the conception of a global energy threshold of the system, introduced by Bocchieri Scotti and Loinger. He instead insisted on the fact that each frequency should present a separate energy threshold, which should be identified with Planck's zero-point energy.

Actually, the idea that zero-point energy should play the role of some threshold between ordered and chaotic motions can even be found in the very original paper of Planck of the year 1911, although Carlo only knew what Planck says in his book. Indeed, in Planck's paper the conception is advanced that, when an oscillator absorbs energy starting from a state of no energy, the process is "regular" until the oscilla-

tor reaches the energy[2] $h\nu$. At that moment, Planck imagines, the process becomes dynamically so complicated and unstable that a purely dynamical description becomes impossible, and some probabilistic considerations become necessary. So the oscillator has a certain probability of having its energy increased, and the complementary probability to lose energy falling back to the state of no energy.

In order to check the reliability of the conjecture advanced by Carlo, that the threshold of the FPU model could correspond to zero-point energy, what we did was just checking that the total zero-point energy $(1/2)\sum_i h\nu_i$ turns out to be of the same order of magnitude of the total critical energy E_c discovered by Bocchieri Loinger and Scotti, and this actually turned out to be the case. Some days later I wrote a paper in a few hours, Carlo and Tonino Scotti added a few remarks, and the paper was sent and published in Physics Letters [1].

One might mention that no one of us even knew of the paper of Izrailev and Chirikov, which we came to know much later. It actually turns out that the idea that there should exist an energy threshold for each frequency, leading from ordered to "chaotic motions", actually is the main point of the Russian authors. But Carlo's intention was exactly the opposite one. Indeed Izrailev and Chirikov wanted to use such a feature as a means to eliminate the physical relevance of the FPU problem, just by trying to prove that the thresholds vanish in the thermodynamic limit. Instead, Carlo's idea was that such thresholds should be the mathematical counterpart of a physical phenomenon which reflects a "reality", namely, the coexistence of ordered and chaotic motions, or the coexistence of a thermal and a non thermal energy. And this can only obtain if the thresholds do not vanish for macroscopic systems.

In conclusion, Carlo invented by himself the idea that

1. a threshold for each frequency should exist,
2. it should persist in the thermodynamic limit, and
3. it should play the role of the door through which quantum mechanics may be "explained" by classical mechanics.

Carlo, who is known everywhere for his great technical abilities, his mental powers (for example, he could recite all of Dante's Divine Comedy)[3] and his great skill, was in that case able to produce an incredible free invention, just based on pure intuition or phantasy, and even dared to involve himself in a game that most in the scientific community consider with great skepticism, or suspicion, to say the least.

4 Present state of the FPU problem

After such a proposal concerning the FPU problem and quantum mechanics, that involved Carlo, Tonino Scotti and me, in a sense the community of scientists working on the foundational aspects of the FPU problem, turned out to be divided into two groups; those that would like to prove that the thresholds disappear in the thermodynamic limit, and those that would like to prove the contrary. There are here two extremely difficult problems. The first one is to invent a concrete mathematical position of the problem which suits the physical interpretation. The second one is of an analytic nature, and consists in the corresponding technical proof of an actual theorem, in the strict mathematical sense.

One might ask whether the situation has presently been clarified, after so many years. The answer is still negative. Some substantial progress has however been made, and I will try now to mention a few steps that appear particularly relevant to me. More details on certain aspects of the problem can be found in a review that I wrote with Giancarlo Benettin, Andrea Carati and Antonio Giorgilli [8] on the occasion of a conference on the FPU problem organized by Giovanni Gallavotti in Rome. Other points of view illustrated at that conference may be found in the book [14].

The first progress was a paper written in the year 1982 by a group of persons around Giorgio Parisi [15], where it was proposed that the "final" FPU state actually is an apparent equilibrium state, i.e., a metaequilibrium one, which on a second extremely long time scale might approach the final standard equilibrium. Moreover, a very beautiful mechanism was conceived to explain analytically how a quick approach is made to the intermediate metaequilibrium state. A visual

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[2]Both Planck and Nernst propose that the value of the zero-point energy should be $h\nu$ rather than $h\nu/2$.

[3]I once was joking with him about this, telling him that according to some people he could do it both forward and backward. And he replied: "saying backward, do you mean verse by verse, or word by word?"
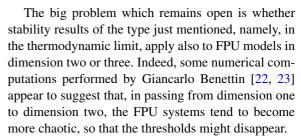
numerical exhibition of such a passage from metaequilibrium to equilibrium was then given by Antonio Giorgilli, Luisa Berchialla and me [16]. Furthermore the mechanism for the quick approach to metaequilibrium proposed by the group around Parisi was investigated by Dario Bambusi and Antonio Ponno [17] in the spirit of the old works of Kruskal and his collaborators, and a new path was thus opened to the use of PDE's as normal forms for FPU-like systems.

The second progress concerns the problem of how should one formulate an analog of the FPU problem if one considers initial data of "generic type", rather that of the special FPU type in which only low-frequency oscillators or just some few oscillators are initially excited. Indeed in the latter case the attainment of equipartition is a mark of having reached equilibrium. Instead, for generic initial data, one is in presence of equipartition already from the start, and so one has to invent some new way of establishing whether some analog of the FPU phenomenon even exists in that case. The answer that was proposed [18, 19] is that one should look at the problem from the point of view of the Fluctuation Dissipation Theorem, which concerns the exchange of energy between two interacting systems, for example a FPU system in contact with a heat reservoir. In such a case, equilibrium is attained when the time autocorrelation of the FPU system's energy has decayed to zero, and an analog of the FPU phenomenon occurs if the autocorrelation decays to some nonvanishing value, and moreover remains stabilized at that value. Physically, this corresponds to the fact that the measured specific heat of the FPU system is not the classical canonical one, but a smaller one, as qualitatively occurs in quantum mechanics.

The third progress consists of an extension of classical perturbation theory to the thermodynamic limit (N tending to infinity with nonvanishing specific energy E/N and specific volume L/N). Classical methods do not work in such a case. The great advance was performed by Andrea Carati [20], who showed how results independent of N are obtained if one weakens the requirements, and just formulates the problem in a probabilistic frame by introducing L_2 norms with respect to Gibbs measure, in place of the standard sup norm. In such a way he has obtained quite recently, together with his pupil Alberto Maiocchi, a Nekhoroshev-like theorem in the thermodynamic limit for the so-called ϕ^4 model (a simple variant of the FPU one) [21].

The big problem which remains open is whether stability results of the type just mentioned, namely, in the thermodynamic limit, apply also to FPU models in dimension two or three. Indeed, some numerical computations performed by Giancarlo Benettin [22, 23] appear to suggest that, in passing from dimension one to dimension two, the FPU systems tend to become more chaotic, so that the thresholds might disappear.

So, we are now, in the year 2011, more or less in the same situation as in the year 1972, when the proposals of Izrailev and Chirikov and of Cercignani–Galgani–Scotti stood one against the other. The situation did however change because, with the analytic progress recently accomplished in perturbation theory in the thermodynamic limit one may be confident that the problem will be finally settled analytically, in favour of the one thesis or the other. I obviously hope (and I'm sure Carlo too would) that the thresholds will be proven to persist in the thermodynamic limit, even for dimensions two and three. If not, it will in any case be true that our idea proved to be helpful in stimulating deep analytical results, and all of us will content ourselves with applying the old Italian adagio *"Se non è vera, è ben trovata."*[4]

5 A strange work on universal numbers

Let us now come to the work [4]. This deals with the well known discovery of Feigenbaum concerning universal numbers in dynamical systems. By studying some mappings depending on a parameter (say μ) he found that a sequence of bifurcations occur, of "period doubling" type: at μ_1, say, a fixed point passes from stable to unstable and a periodic orbit of period two shows up, then at μ_2 that orbit becomes unstable and an orbit of period four shows up, and so on. The sequence μ_n has a limit, and he observed that the ratio $(\mu_{n+1} - \mu_n)/(\mu_n - \mu_{n-1})$ too tends to a limit which is universal, i.e., does not depend on the mapping. The latter limit is the universal Feigenbaum number 4.669... In our paper we were the first in the world to point out that for conservative mappings (i.e., for the Hamiltonian equations of motion of mechanics) the corresponding number does not coincide with the Feigenbaum one, still being universal for the subclass of Hamiltonian systems. Such a number is 8.721...

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[4]If it is not true, it is at least a beautiful invention.

But how did we happen to be interested in this problem, and which are the relations with the general problems discussed here? Things went as follows. In the summer 1980 I was attending a conference in Pojana Brasov (Romania), and an afternoon I decided to go for a walk. But Herbert Spohn met me, and insisted that I should listen to a talk by Eckmann, which he expected to be very interesting. So I went. The talk was very interesting indeed, and was a general review about the Feigenbaum matter, of which I knew nothing. As everything was concerned with dissipative systems, I asked the speaker which was the situation concerning Hamiltonian systems, and he told me that a work had been published containing the answer, namely, that the same phenomenon had been observed in Hamiltonian systems too, and with the same Feigenbaum number.

The reason why I was interested in universal numbers is again the relation between quantum and classical mechanics, in connection with the celebrated pure number 137. This had been introduced by Sommerfeld in connection with the fine structure constant, and essentially goes back again to Planck's constant $\hbar$. Indeed $\hbar$ is an action, and if one looks for a classical analog of it in the frame of electrodynamics one meets with the action e^2/c where e is the electron charge and c the speed of light. So $\hbar$ is a multiple of e^2/c and the factor is just 137.035999679... In other words there exists the "fine structure constant" $\alpha = e^2/\hbar c$, a pure number, and one has $\alpha \simeq 1/137$.

This is a point on which I happened to have infinitely many conversations with Carlo and Tonino Scotti, and many times we indulged in imagining how that magic number 137 might be extracted from some hidden dynamical property of Hamiltonian systems. All this came to my mind during the talk of Eckmann in Pojana Brasov. So when I came back to Milan I discussed this new perspective with Carlo, and we decided to make an attempt. Actually the work was done by my two jewels, Giancarlo Benettin and Antonio Giorgilli who, independently in Padua and in Milan, conceived some numerical method to look for fixed points, especially suited for conservative, as opposed to dissipative, systems. Actually both of them found out that the published results were incorrect, that Hamiltonian systems indeed have a Feigenbaum-type number, the value of which however is 8.721.., different from the Feigenbaum one. But also different from 137 or 1/137. We were disappointed, but in any case

a new original result in Hamiltonian systems had been discovered (see also [5]). This is indeed the moral of all the story. We are looking for something which is considered crazy, and discover different, but in any case new, things. Somehow as Colombo, who wanted to go to India, and discovered America.

6 Carlo comes back to Planck's law: the Foundations of Physics 1998 paper

For a long time Carlo was mostly involved in his main research themes on kinetic theory, while in my group we were continuing studying the mathematical aspects of perturbation theory and of the FPU problem. We also had started a deep study, which still continues in the present days, on the dynamical aspects of electron theory when radiation reaction and retardation are taken into account (see for example [24, 25] and [26]).

For what concerns a possible deduction of Planck's law in a classical frame I had done a consistent progress (see the papers [27, 28] and the paper [29] with Giancarlo Benettin). This passed through a deep understanding (which took about ten years) of the paper of Nernst of the year 1916, by the title *"On an attempt to return from quantum mechanics to the assumption of continuous variations of energy"* [30], the main point of which was the conception that Planck's law may be consistent with equipartition. This is another way—possibly the first one, historically—of dealing with the idea, already mentioned in this paper, of the Fluctuation Dissipation Theorem. Namely, that Gibbs' law, with the corresponding equipartition, just plays the role of a measure for the initial data, whereas the actually exchanged energy depends on dynamics, in particular on the existence of an energy threshold. Nernst was indeed speaking in terms of a threshold between ordered (*geordnete*) and disorderer (*ungeordnete*) motions. So I had given some interpretation of Nernst's deduction of Planck's law, which makes an explicit reference to the energy thresholds, the role of which had been particularly stressed by Carlo. In this connection, the paper [3] too was written.

Much time passed, and one day Carlo showed me an article he had written on that subject, and after some adjustments he published it in the Foundations of Physics Letters [2]. I actually never fully understood the new point he was making, and this was perhaps a little disappointing for him. The characteristic feature of Carlo's new approach was an essential

elimination of dynamics. Actually he was considering just the electromagnetic field, and his main point was that, even if it be dealt with classically, the electromagnetic field has to contain implicitly the concept of the photon, an essential point being that photons necessarily have to be considered as indistinguishable particles. Then, he shows that the standard statistical concepts used by Boltzmann, with indistinguishability taken into account, leads to Planck's law (the general Wien's law too, considered as a property of thermodynamics, should be taken into account).

Now, every normal person would be tempted to say that assuming the existence of photons is just a direct way of assuming quantum mechanics itself, and from this point of view Carlo's approach might be considered to be just some kind of revisitation of the Bose–Einstein approach. Everything depends on the consideration one gives to the way in which Carlo introduces the photons; whether this is just some verbal rephrasing of known things, or the actual pointing out of something extremely deep, which I'm still unable to fully appreciate. The extremely high consideration I have of Carlo suggests to me that this is probably the case, and I'm looking forward to have an illumination on this point. In the meantime, just after having written down a first draft of this paper, I had the opportunity to pass an afternoon, sitting in a beautiful garden and discussing this problem with Andrea Carati and Tonino Scotti, and Andrea has almost convinced us he is on the way of understanding the actual point made by Carlo. If we will be able to find some clear statement in this connection, this will be a beautiful occasion to pay homage to Carlo, and explain it to everybody.

If we will be able to do this, a key element will be the following one. I remember Carlo particularly pointing it out to me. The point concerns the way one should conceive frequency. We are all accustomed, following Rayleigh, to think of the electromagnetic field in terms of its normal modes, and thus to attach to a packet of normal modes (or to the corresponding photons) the label ω of frequency as a parameter, in the same way as the mass m is attached as a label to a particle. However, a particle has also a variable velocity v, and a corresponding kinetic energy $K_m(v) = mv^2/2$. Now, the point Carlo was making to me is that, along the lines of Bose, ω should be thought of as the analog of v, rather than of m. Indeed, first of all ω depends on the reference system, as does the velocity of a particle, and even changes within a given reference system

when light impinges on a moving wall (this actually is a key point in the deduction of Wien's displacement law). So the energy ϵ of a particle of light has to be a function of ω, i.e., one should have $\epsilon = \epsilon(\omega)$, in the same way in which the kinetic energy of a particle is a function of velocity.

So if one day, by arguments as those concerning solitons, it will be understood that particles of light can be conceived within classical physics, then they will have an energy $\epsilon = \epsilon(\omega)$. Taking indistinguishability into account (think again of the interaction with a wall), one then immediately deduces a Planck-like law involving the function $\epsilon = \epsilon(\omega)$ of a still undetermined form. Finally, Wien's displacement law gives $\epsilon = \hbar\omega$. Finally, he adds the zero-point energy as related to ordered, or nonthermal, motions, and gives an argument (time invariance) to prove that it has the factor $1/2$ in front.

In any case, whether Carlo's approach will prove to be the good solution or not, this paper of him is a further witness of the fact that, even in the last part of his life, he was deeply involved in the problem of the relations between quantum and classical mechanics, with the actual aim of deducing quantum mechanics from the classical one.

7 Some personal recollections about Carlo

Carlo Cercignani was an extremely reserved person, with whom it was not easy to have conversations about deep problems concerning life. But he thought very much about that, and his choice was to commit his thinkings to some writings. The first thing he wrote down, that I really liked very much, is a novel, by the title *"Morte di un professore"* or *"Muerte a Pastru-fazio"* (he never made a decision between such two possible titles). The book is a joke, as many of the things he wrote. But Chap. 6 was entirely devoted to the verbal transcription of an ideal talk given by a scientist (by the name Veroviro, which mimics Truesdell, but the true speaker was Carlo himself), in which a long discussion is given of the problem whether liberty of a human being is possible in a deterministic world.

Then he wrote several poems, which were collected in a booklet [31] by the title *"Scherzi in versi"*. Among them, the one I like most is the poem by the title 1921

(written in the year 1987), which starts with *"Il pendolo semplice e il verso degli angoli, coppie e momenti, e l'orientazion dei segmenti .."*. I recited this at his funeral, having made the effort of learning it by heart. He also made many very beautiful translations from several poets, as Shakespeare, Queneau, Borges. He even translated both the Iliad and the Odyssey, in some verses that he especially chose with the aim of better fitting the original rhythm of Homer. In the final years he also wrote one more novel that, according to his wife Silvana, is the most beautiful thing he ever wrote. The title is *"La creazione secondo (according to) Michele"*.

Just a few months after his passing away, putting some order to my own bookshelves, I found out a poem that he had given me perhaps one year before, and somehow had escaped my attention. This poem seems now particularly interesting to me, as it gives an indication of how he lived his last times, when he was almost completely paralyzed. He actually continued to deal with people in his usual way, somehow as joking at first, then perhaps reciting from memory with his beautiful voice a poem suited to the moment,[5] then reentering into himself and starting thinking about something.

The poem I found has the title *"Beethoven in cielo"* (i.e., *in heaven*), and is essentially a meditation on pain (*dolore*). Carlo describes how after death he very joyfully goes to heaven. But strangely enough, the chorus of the angels he hears is a little monotonous, and Carlo cannot refrain from telling them. From the style, he recognizes it as a piece of Beethoven, but one unknown to him. The angels confirm it, saying Beethoven composed it in heaven. So he asks to meet the composer, what he is allowed to do. Requested by Beethoven of his opinion about his music, Carlo at first refrains to speak, but then, as Beethoven insists, with great humility admits he would have expected some more sublime music. And Beethoven says he completely agrees. *"Tutto in cielo mi vien male, che iattura"* "Everything in heaven comes out badly to me. I'm even refraining from composing anymore. You know why? I'm lacking the creative spark, the note

that most shines; this note is pain … Only the one who cries and groans by pain will have humanity, divine gift …. Did you ever cry together with your wife? The one who doesn't do it is unable to capture true love… God too, when was seen among us, was he a king, or wanted he to be rich? He was a man's son, full of pain…". So Carlo looks at Beethoven, as terrified. "How strange—he thinks—is the flowing of the world. A few hours ago I was asking that death should save pain to my heart. Now, here, in this high and blessed (*beato*) world, I regret pain. Oh human heart, truly unfathomable and indeed strange".

This poem is reproduced here in Appendix,[6] together with the one I like most, by the title 1921 (1987), that starts with *"Il pendolo semplice e il verso degli angoli, coppie e momenti, e l'orientazion dei segmenti … "*, and with another, very beautiful, short one by the title *"Leggendo le georgiche (1994)"*, that starts with *"Che dir delle stelle, del cielo d'autunno, dell'ansia che prende"*.

Appendix

Beethoven in cielo

L'anima mia dal corpo si è staccata
con una lotta che sarà obliata.
Ma dopo tanta angoscia e sofferenza,
gioia è librarsi come pura essenza!
Il brulichio del mondo è come un velo
e salgo verso un'alta meta: il cielo!
Lunga e svelta qual freccia è la mia via.
Sento lungi una splendida armonia.
D'angeli un coro dolcemente aspetta
per accogliere chi lassù si affretta.
Tra poco sarò giunto; quale incanto!
Che monotono sembri però il canto
a nascondere agli angeli non riesco.
Ridon felici: – E' un animo tedesco!
La musica da voi sale fin qua!
Dio glorifica, allor, *l'eternità*
cantiamo; e veda che ce ne intendiamo!
Ma di andare all'unìsono cerchiamo! –
E cantano un corale grande; e presto

---

penso: – E' lo stile di Beethoven questo;
quel pezzo lì però mi è sconosciuto. –
Chiedo allora: – Cos'è? – Ordine ha avuto
di scriverlo – mi dicon – dal Signore
l'anima di Beethoven. Con fervore
lo eseguiamo ogni volta che c'è festa;
musica qui non c'è miglior di questa!–
–Lo credo! Ma vorrei che a me mostrata
fosse l'anima sua. Non sarà stata
vana così la gita.– Divertiti,
mi conducon per bei prati fioriti
e m'indican lo spirito divino
che solitario va, in lento cammino,
sotto le palme. – Da lui, benché indegno,
da lui or voglio andare in questo regno,
colui che onora più l'ingegno umano! –
Mi vede allora e mi porge la mano:
– Benvenuto, o terreno ospite, prono
al poter della musica e al suo suono!
Per te fu il coro angelico eseguito,
da me composto in cielo, ed ho gradito
che gli angeli lo affrontin con impegno:
del mio corale, pure in questo regno,
son le quarte eccedenti assai temute!
Ma le mie note, di', ti son piaciute? –
Confuso non rispondo. Ed egli lesto
e cortese prosegue: – Animo onesto,
tu sei sincero! E' giusta l'opinione:
fuggivi in terra pur l'adulazione. –
Allora del suo dir colgo il vantaggio;
dico: – O mio eroe – , facendomi coraggio,
– o mio maestro! Ora ho ascoltato il canto
con entusiasmo. Devo dir soltanto
che, tra gli angeli, io, su queste cime,
musica mi aspettavo più sublime! –
Egli risponde sorridendo: – Senti,
la penso come te se ti lamenti.
Tutto in ciel mi vien male, che iattura!
Ho smesso di comporre addirittura.
Solo per il giudizio universale
ho dovuto impegnarmi, bene o male,
per non imbarazzare assai il buon Dio,

di scriver per gli ottoni un pezzo mio:
lo devo far, ma non ne sono lieto.
Ma sai perché, altrimenti, ormai mi vieto
di comporre? Mi manca la scintilla
più creativa, la nota che più brilla:
questa nota è il dolore! Sì, il dolore
che ti afferra e ti fa stringere il cuore;
come un metallo forte suona e vibra
e ti fa risuonare in ogni fibra.
E' un vero amico e ti fa superiore:
solo chi piange e geme dal dolore
avrà l'umanità, dono divino.
Cosa lega alla madre ogni bambino?
Le grandi pene della notte in cui
c'è Dio soltanto in veglia con lei e lui.
Non hai mai pianto insieme con tua moglie?
Chi non lo fa l'amor vero non coglie,
un dolore profondo e condiviso:
il suo ricordo è come un paradiso.
Sopporta il santo pena ed afflizione:
brilla in lui il raggio della perfezione.
Fama di eroe ottener sol ti è concesso,
se fermamente domini te stesso.
Tremi il tuo cuore nella sofferenza!
Vivrai nel canto della discendenza.
Dio stesso quando qui fra noi si scorse,
fu forse un re, si volle ricco forse?
Fu figlio d'uomo, pieno del dolore,
che ognor s'incontra in qualsisia maggiore
cosa; è la nota mia fondamentale.
Ma qui tutto è beato e senza male;
la cetra mi è caduta allor di mano. –
Lo guardo ora atterrito: – Come è strano
lo scorrere del mondo. Poche ore
fa, chiedevo alla morte che il dolore
al cuore mio venisse risparmiato.
Ora qui, in questo mondo alto e beato,
si rimpiange il dolore! Oh, cuore umano,
veramente insondabile e ben strano! –

Carlo Cercignani

1921 (1987)

Il pendolo semplice e il verso   degli angoli, coppie e momenti,
e l'orientazion dei segmenti,   che sembrano avere un po' perso

quel ruolo di bei caposaldi,   che noi studiavamo convinti
sui tomi, a caratteri stinti,   del buon Levi Civita–Amaldi;
i solidi rigidi e i fili   poggiati o sospesi per aria,
disposti per far catenaria,   e, ancor, coniugati, i profili;
le formule dell'ellissoide   d'inerzia, le tre rotazioni,
il calcolo delle reazioni,   il grafico della cicloide;
e dopo? anche il moto centrale   e la geometria delle masse,
e come se ciò non bastasse   le formule del potenziale;
la velocità areolare,   insieme col noto rapporto
tra assi e periodi e uno storto   poligono funicolare;
l'epicicloide ordinaria,   le verghe, i vettori, i versori.
la legge d'inerzia, i cursori,   l'odografo, la legge oraria;
e quei giroscopi, che ognuno   ricorda, che sembrano armille;
rinasco, rinasco nel mille,   eh sì, novecentoventuno!
Il Finzi è un ragazzo aitante,   che svolge, sicuro e veloce,
il calcolo, pure il più atroce,   e sgomina ... un determinante;
e ancor la Pastori, serafica,   in certe serate d'inverno,
risolve, su un lindo quaderno,   problemi di statica grafica.
Sul tavolo la lucernetta   fa un orbe di luce conchiuso,
nel quale discutere è d'uso   problemi di base e rulletta.
Max Abraham, presso alla morte,   ancor polemizza ma è stanco;
la teppa, rizzata sul banco,   ne aveva deciso la sorte
da anni: doveva lontano   andare la *Kultur* germanica
che quel professor di meccanica   voleva illustrare a Milano.
I giorni si fanni più foschi;   che importa se poi il suo tensore
potrà risultare migliore   di quello che ha dato Minkowski?
Ma è giunta notizia che oggi   la vecchia teoria newtoniana,
ed è una notizia ben strana,   su basi sicure non poggi.
Oltr'alpe, se pur tra i fragori   di guerra, son stati trovati
dei nuovi concetti, e, applicati,   dei vecchi si trovan migliori.
Qualcuno scuotendo la testa,   davanti ai propositi empi,
ripete: – Che tempi! Che tempi!   Dovevo sentire anche questa! –
Tal altro si sente già certo   di quella scoperta recente
e dice: – Che mente! Che mente   quel tipo, quell'Einstein ... Alberto! –
Oh sere, passare a studiare   le formule dell'avvenire,
passate a cercar di capire,   sapendo pur sol balbettare
ja, bitte, Forelle e Kartoffel   quei fogli di stampa ancor fresca,
riempiti di lingua tedesca,   e i simboli di E.B.Christoffel!
Ci sono certuni che stiman   che sia una pura follia
studiar quella nuova teoria   con dentro il tensore di Riemann:
che cuore partire all'assalto   di pagine, quindici a quindici,
ripiene di formule e indici   che stanno un po' in basso e un po' in alto!
E Weyl, che negli ultimi mesi,   seguendo i precetti di Mie,
produce ancor nuove teorie,   chiamate di *gauge* dagli inglesi!
Misteri grandissimi ancor   ha in serbo la quantizzazione,
descritta con la condizione   che fu escogitata da Bohr.
Quel mondo si è rotto ed ormai   i giovani studiano a caso
le cose più strane e col vaso   Pandòra ancor semina guai!
Sì certi equilibri son rotti   e circolan libri un po' strani
che srivono, qui, il Cercignani,   e, un po' più in là, il Gallavotti.

Perfino ai congressi si sente　parlar di cose un po' strane:
scompaiono le lagrangiane,　emerge il fibrato tangente.
O tempo vicino e lontano,　sei sempre presente nel cuore!
quand'era un versore un versore　e non un simbolo strano,
ch'è simile a una derivata　parziale, che invece non è;
nessuno mi spiega il perché　di questa Babele sfrenata.
Eh sì, non lo spiega nessuno　ed io vorrei che tornasse
quel tempo, che ci si trovasse　nel milnovecentoventuno.
Tornare nel tempo che fu,　poter imparare i tensori
col Finzi, amar la Pastori!　quei giorni non tornano più!

Carlo Cercignani

Leggendo le georgiche (1994)

Che dir delle stelle, del cielo　d'autunno, dell'ansia che prende
se il sole ogni giorno discende　più presto e si copre d'un velo;
o se primavera finisce　piovosa e nei campi matura
la messe di spighe e Natura　le steli d'umori arricchisce?
E quando nei campi dorati　falciare vuoi il fragile orzo
e i venti, rigonfi di sforzo,　tu vedi scontrarsi adirati
e, come guerrieri nemici,　avvolti di nuvole scure,
strappare le spighe mature,　svellendo perfin le radici,
scagliarle nell'aria; e le nere　tempeste avvolgere in spire,
facendoli in alto salire,　gli steli e le stoppie leggere.
Il cielo non è più celeste　e senti continua scrosciare
la pioggia; salita dal mare,　la nube si addensa in tempeste
oscure d'orribile pioggia;　il cielo precipita in terra
e scende feroce a far guerra.　Si gonfiano il fiume e la roggia,
con strepito il mare ribolle;　si allagano i campi ridenti
dell'opra di giorni pazienti　e sul seminato e sul colle.
Furente con noi Giove Pluvio,　brandendo la folgore iroso
ci manda col buio nuvoloso,　o sembra, un suo nuovo Diluvio...

Carlo Cercignani

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