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Influence of rotation and generalized magneto-thermoelastic on Rayleigh waves in a granular medium under effect of initial stress and gravity field

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Abstract Influence of rotation, relaxation times, magnetic field, initial stress and gravity field on attenuation coefficient (Imaginary part of frequency equation root) and Rayleigh waves velocity (the real part of frequency equation root) in an elastic half-space of granular medium is studied. The analytical solution is obtained by using Lame's potential techniques. The numerical calculations are carried out for the frequency equation of Rayleigh waves velocity. The results are displayed graphically. Some results of previous investigations are deduced as special cases from this study.

Keywords Granular medium · Rotation · Rayleigh waves · Initial stress · Gravity field · Magneto-thermoelastic · Relaxation times

Nomenclature

T	is temperature difference $(\theta - T_0)$
θ	is the ratio of the coefficients of heat transfer
t	is the time
$\overleftarrow{H}_o$	is the constant primary magnetic field vector
$\overleftarrow{E}$	is the electric intensity
s	is the specific heat per unit mass
ρ	is the density of the material
P	is the initial stress
F	is the coefficient of friction
F_j	are the components of Lorentz's body forces vector
τ_1, τ_2	are the mechanical relaxation times
g	is the earth gravity
$\vec{u}$	is the component of displacement vector
T_o	is reference temperature solid
$\overleftarrow{j}$	is the electric current density
ω_{ij}	is the rotation vector
μ_e	is the magnetic permeability
k	is the wave number
c	is speed of Rayleigh waves
λ, μ	are the Lame' constants
M	is the third elastic constant
κ	is thermal conductivity
K	is the thickness
$\overleftarrow{h}$	is the perturbed magnetic field over the constant primary magnetic field
σ_{ij}	is the stress tensor
ε	is coupling coefficient
(ξ, η, ζ)	are components of rotation vector of the grain about its center of gravity

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1 Introduction

The study of granular medium in recent times has been necessiated by its possible applications in soil mechanics, geophysical prospecting, mining engineering, earthquake science, etc. The dynamical problem of magneto-thermoelasticity has received much attention in the literature during the past decade. In recent years the theory of magneto-thermoelasticity which deals with the interactions among strain, temperature and electromagnetic fields has drawn the attention of many researchers because of its extensive uses in divers field, such as geophysics for understanding the effect of Earth's magnetic field on seismic waves, damping of acoustic waves in magnetic field, emissions of electromagnetic radiations from nuclear devices, development of highly sensitive superconduction magnetometer, electrical power engineering, optics etc. The dynamical problem in granular medium of generalized magneto-thermoelastic waves has been studied in recent times, necessitated by its possible applications in Soil mechanics, earthquakes Science, Geophysics, mining Engineering and Plasma physics, etc. In the motion of a deformable body, quantities at each point of it can't be recognized with out some uncertainty, and must be regarded as average values over a small region about the point. Abd-Alla and Mahmoud discussed magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylindrical under the hyperbolic heat conduction model [1]. Generalized magneto-thermoelastic Rayleigh waves in a granular medium under influence of gravity field and initial stress, also effect of the rotation on the radial vibrations in a non-homogeneous orthotropic hollow cylinder is discussed by Abd-Alla et al. [2, 4]. Mahmoud et al. [3, 5] discussed effect of the rotation on plane vibrations in a transversely isotropic infinite hollow cylinder, they solved effect of the rotation on wave motion through cylindrical bore in a micropolar porous cubic crystal.

The dynamical problem of a generalized thermoelastic granular infinite cylinder under initial stress has been illustrated by El-Naggar [6]. Abd-Alla et al. [7–12] studied wave propagation modeling in cylindrical human long wet bones with cavity, propagation of S-wave in a non-homogeneous anisotropic incompressible and initially stressed medium under influence of gravity field, effect of the rotation on a non-homogeneous infinite cylinder of orthotropic material, effect of magnetic field and non-homogeneity on the radial vibrations in hollow elastic cylinder, on Problem of Transient Coupled Thermoelasticity of an Annular Fin, effect of the rotation, and they studied magnetic field and initial stress on peristaltic motion of micropolar fluid, influences of rotation, magnetic field, initial stress and gravity on Rayleigh waves in a homogeneous orthotropic elastic half-space. Abd-Alla and Mahmoud [13–15] solved effect of the rotation on propagation of thermoelastic waves in a non-homogeneous infinite cylinder of isotropic material, analytical solution of wave propagation in non-homogeneous orthotropic rotating elastic media, magneto-thermo-viscoelastic interactions in an unbounded non-homogeneous body with a spherical gravity subjected to a periodic loading. Effect of the non-homogeneity on wave propagation on orthotropic elastic media, effect of rotation and magnetic field through porous medium on Peristaltic transport of a Jeffrey fluid in tube are investigated by Mahmoud [16, 17]. Rayleigh waves in a thermoelastic granular medium under initial stress has been explained by Ahmed [18]. Recently, Ahmed [19] discussed the influence of gravity on the propagation of waves in granular medium. Problem of Rayleigh waves propagation in an orthotropic thermoelastic medium under gravity and initial stress is discussed by Abd-Alla and Ahmed [20]. Abouelregal [21] presented Rayleigh waves in a thermoelastic solid half space using dual-phase-lag model. The medium under consideration is granular half-space overlying by a different granular layer and initial stress present in this medium have considerable effect in the propagation of Rayleigh waves, Ahmed [18].

This study focuses on the study of Rayleigh waves with models fitting with the earth. The granular medium under consideration is a discontinuous one and is composed of large or small grains. The effects of rotation, magnetic and gravity fields and initial stress on the propagation of Rayleigh waves in a granular media under incremental thermal stresses are investigated. The roots of this equation are in general complex, the real part measures the Rayleigh wave velocity and the imaginary part of an appropriate root measures the attenuation of the waves. When there is no coupling between the temperature and the strain field in the absence of the initial stress, the derived frequency equation reduces to an equation in the form of ninth-order determinant.

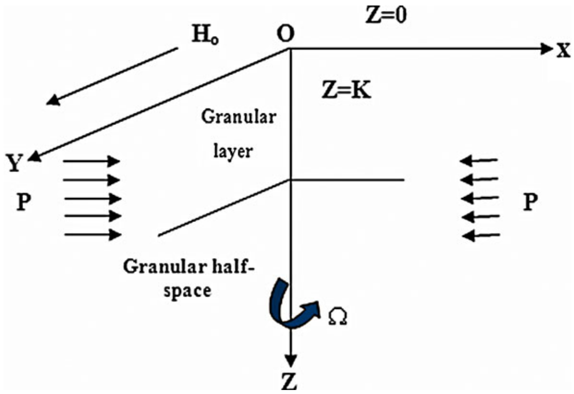

Fig. 1 Displaying of the problem

2 Formulation of the problem

Let us consider a system of orthogonal Cartesian axes $oxyz$, the interface and the free surface of the granular layer resting on the granular half-space of different material being the planes $z = K$, and $z = 0$, respectively. The origin o is any point on the free surface, z-axis is positive along the direction towards the exterior of the half-space, and the x-axis is positive along the direction of Rayleigh waves propagation. The both media are under initial compression stress P along x-direction and primary magnetic field $\overleftarrow{H}$ parallel to y-axis, as well as gravity field and incremental thermal stresses as shown in Fig. 1.

The state of deformation in the granular medium is described by the displacement vector $\vec{u}(u,0,w)$ and $\vec{\xi}(\xi,\eta,\zeta)$ are components of rotation vector of the grain about its center of gravity. The electromagnetic field is governed by Maxwell equations, under the consideration that the medium is a perfect electric conductor taking into account the absence of the displacement current.

The heat conduction equation in Lord and Shulman theory is given by

$$\kappa\nabla^2 T = \rho s\frac{\partial}{\partial t}\left[1+\tau_2\frac{\partial}{\partial t}\right]T + \gamma T_o\frac{\partial}{\partial t}\left[1+\tau_2\delta\frac{\partial}{\partial t}\right]\vec{\nabla}.\vec{u}. \quad (1)$$

The vector equation of motion is

$$\tau_{ij,i} + F_j = \rho[\ddot{u} + (\overleftarrow{\Omega}\times\overleftarrow{\Omega}\times\overleftarrow{u}) + (2\overleftarrow{\Omega}\times\overleftarrow{\dot{u}})]_j \quad i,j=1,2,3 \quad (2)$$

where $(\overleftarrow{\Omega}\times\overleftarrow{\Omega}\times\overleftarrow{u})$ is the centripetal acceleration due to the time varying motion only and $(2\overleftarrow{\Omega}\times\overleftarrow{u})$ is the Coriolis acceleration. Here $\overleftarrow{u}$ is the dynamic displacement vector measured from steady state deformed position and supposed to be small, F_j are the components of Lorentz's body forces vector.

Equation (2) has three Cartesian components:

$$\left.\begin{aligned}
&\frac{\partial\tau_{11}}{\partial x}+\frac{\partial\tau_{31}}{\partial z}+\frac{P}{2}\frac{\partial\omega_2}{\partial z}-\rho g\frac{\partial w}{\partial x}+F_x\\
&\quad=\rho\left(\frac{\partial^2 u}{\partial t^2}+2\Omega\frac{\partial w}{\partial t}-\Omega^2 u\right),\\
&\frac{\partial\tau_{21}}{\partial x}+\frac{\partial\tau_{23}}{\partial z}+F_y=0,\\
&\frac{\partial\tau_{13}}{\partial x}+\frac{\partial\tau_{33}}{\partial z}+\frac{P}{2}\frac{\partial\omega_2}{\partial x}+\rho g\frac{\partial u}{\partial x}+F_z\\
&\quad=\rho\left(\frac{\partial^2 w}{\partial t^2}-2\Omega\frac{\partial u}{\partial t}-\Omega^2 w\right),
\end{aligned}\right\} \quad (3)$$

where $\overleftarrow{F} = (-\mu_e H_o^2\nabla^2\Phi, 0, \mu_e H_o^2\nabla^2\Phi)$ and

$$\left.\begin{aligned}
&\tau_{32}-\tau_{23}+\frac{\partial M_{11}}{\partial x}+\frac{\partial M_{31}}{\partial z}=0,\\
&\tau_{31}-\tau_{13}+\frac{\partial M_{12}}{\partial x}+\frac{\partial M_{13}}{\partial z}=0,\\
&\tau_{12}-\tau_{21}+\frac{\partial M_{13}}{\partial x}+\frac{\partial M_{33}}{\partial z}=0.
\end{aligned}\right\} \quad (4)$$

The stress-strain-temperature relations have the forms

$$\left.\begin{aligned}
&\tau_{11}=(\lambda+2\mu+P)\frac{\partial u}{\partial x}+(\lambda+P)\frac{\partial w}{\partial z}\\
&\qquad-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T,\\
&\tau_{33}=\lambda\frac{\partial u}{\partial x}+(\lambda+2\mu)\frac{\partial w}{\partial z}-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T,\\
&\tau_{13}=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)+F\frac{\partial\eta}{\partial t},\\
&\tau_{12}=-F\frac{\partial\zeta}{\partial t},\\
&\tau_{32}=-F\frac{\partial\xi}{\partial t},
\end{aligned}\right\} \quad (5)$$

where η, ζ and ξ are the components of rotation vector of the grain about its center of gravity.

Noted that, the fundamental equations of the Lord and Shulman's theory can be obtained when $\delta = 1$ and

$\tau_1 = 0$. Green and Lindsay model (G-L) can be also obtained as a special case when $\delta = 0$. In the absence of the relaxation times τ_1 and τ_2, (1) reduces to the classical theory of thermoelasticity, where

$$\left.\begin{aligned} &M_{11} = M\frac{\partial \xi}{\partial x}, \qquad M_{13} = M\frac{\partial \xi}{\partial z}, \qquad M_{33} = M\frac{\partial \zeta}{\partial z}, \qquad M_{21} = M_{22} = M_{23} = 0, \\ &M_{12} = M\frac{\partial}{\partial x}(\omega_2 + \eta), \qquad M_{32} = M\frac{\partial}{\partial z}(\omega_2 + \eta), \qquad M_{13} = M\frac{\partial \zeta}{\partial z}. \end{aligned}\right\} \tag{6}$$

From (5) and (6), (3) and (4) tend to

$$\begin{aligned} &(\lambda + 2\mu + P + \mu_e H_o^2)\frac{\partial^2 u}{\partial x^2} + \left(\lambda + \mu + \frac{P}{2} + \mu_e H_o^2\right)\frac{\partial^2 w}{\partial x \partial z} + \left(\mu + \frac{P}{2}\right)\frac{\partial^2 u}{\partial z^2} \\ &\quad - \gamma\frac{\partial}{\partial x}\left(1 + \tau_1\frac{\partial}{\partial t}\right)T - F\frac{\partial^2 \eta}{\partial z \partial t} - \rho g\frac{\partial w}{\partial x} = \rho\left(\frac{\partial^2 u}{\partial t^2} + 2\Omega\frac{\partial w}{\partial t} - \Omega^2 u\right), \end{aligned} \tag{7}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \zeta}{\partial x} - \frac{\partial \xi}{\partial z}\right) = 0, \tag{8}$$

$$\begin{aligned} &\left(\lambda + \mu + \frac{P}{2} + \mu_e H_o^2\right)\frac{\partial^2 u}{\partial x \partial z} + \left(\mu - \frac{P}{2}\right)\frac{\partial^2 w}{\partial x^2} + (\lambda + 2\mu + \mu_e H_o^2)\frac{\partial^2 w}{\partial z^2} \\ &\quad - \gamma\frac{\partial}{\partial z}\left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} - F\frac{\partial^2 \eta}{\partial x \partial t} + \rho g\frac{\partial u}{\partial x} = \rho\left(\frac{\partial^2 w}{\partial t^2} + 2\Omega\frac{\partial u}{\partial t} - \Omega^2 w\right), \end{aligned} \tag{9}$$

$$\nabla^2 \xi - s_2\frac{\partial \xi}{\partial t} = 0, \tag{10}$$

$$\nabla^2(\eta + \omega_2) - s_2\frac{\partial \eta}{\partial t} = 0, \tag{11}$$

$$\nabla^2 \zeta - s_2\frac{\partial \zeta}{\partial t} = 0. \tag{12}$$

3 Solution of the problem

We assume that the displacements $\vec{u}$ are derivable from the displacement potentials Φ and Ψ by the relation

$$\vec{u} = \vec{\nabla}\Phi + \vec{\nabla} \times \overleftarrow{\Psi}, \qquad \overleftarrow{\Psi} = (0, \Psi, 0), \tag{13}$$

which reduces to

$$u = \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial z}, \qquad w = \frac{\partial \Phi}{\partial z} + \frac{\partial \Psi}{\partial x}. \tag{14}$$

Substituting from (14) into (7), (9) and (11), the wave equations tend to

$$\begin{aligned} &\alpha^2\nabla^2\Phi - \frac{\gamma}{\rho}\left(1 + \tau_1\frac{\partial}{\partial t}\right)T - g\frac{\partial \Psi}{\partial x} \\ &\quad - \frac{\partial^2 \Phi}{\partial t^2} - 2\Omega\frac{\partial \Psi}{\partial t} + \Omega^2\Phi = 0, \end{aligned} \tag{15}$$

$$\begin{aligned} &\beta^2\nabla^2\Psi - s_1\frac{\partial \eta}{\partial t} + g\frac{\partial \Phi}{\partial x} - \frac{\partial^2 \Psi}{\partial t^2} \\ &\quad - 2\Omega\frac{\partial \Phi}{\partial t} + \Omega^2\Psi = 0, \end{aligned} \tag{16}$$

$$\nabla^2\eta - s_2\frac{\partial \eta}{\partial t} + \nabla^4\Psi = 0, \tag{17}$$

where

$$\begin{aligned} &s_1 = \frac{F}{\rho}, \qquad s_2 = \frac{2F}{M}, \\ &\alpha^2 = \frac{\lambda + 2\mu + P + \mu_e H_o^2}{\rho}, \qquad \beta^2 = \frac{2\mu - P}{2\rho}. \end{aligned} \tag{18}$$

Using (14), the heat conduction equation (1) becomes:

$$\kappa \nabla^2 T = \rho s \frac{\partial}{\partial t}\left[1+\tau_2 \frac{\partial}{\partial t}\right] T + \gamma T_o \frac{\partial}{\partial t}\left[1+\tau_2 \delta \frac{\partial}{\partial t}\right]\nabla^2 \Phi. \tag{19}$$

From (15) and (19), we get

$$\left[\nabla^2 - \frac{1}{\grave{u}}\frac{\partial}{\partial t}\left(1+\tau_2\frac{\partial}{\partial t}\right)\right]\left[\alpha^2\nabla^2\Phi - \frac{\partial^2\Phi}{\partial t^2} - g\frac{\partial\Psi}{\partial x} - 2\Omega\frac{\partial\Psi}{\partial t} + \Omega^2\Phi\right] - \varepsilon\frac{\partial}{\partial t}\left(1+\tau_1\frac{\partial}{\partial t}\right)\left(1+\tau_2\delta\frac{\partial}{\partial t}\right)\nabla^2\Phi = 0. \tag{20}$$

Where the coupling coefficient ε appears in temperature equation only

$$\grave{u} = \frac{\kappa}{\rho s}, \qquad \varepsilon = \frac{T_o\gamma^2}{\rho\kappa}. \tag{21}$$

From (16) and (17), by eliminated η, we get

$$\left(\nabla^2 - s_2\frac{\partial}{\partial t}\right)\left(\beta^2\nabla^2\Psi - \frac{\partial^2\Psi}{\partial t^2} + \Omega^2\Psi + g\frac{\partial\Phi}{\partial x} - 2\Omega\frac{\partial\Phi}{\partial t}\right) + s_1\nabla^4\frac{\partial\Psi}{\partial t} = 0. \tag{22}$$

For a plane harmonic wave propagation in the x-direction, we assume

$$\Phi = \Phi_1(z)\exp(ik(x-ct)), \tag{23}$$

$$\Psi = \Psi_1(z)\exp(ik(x-ct)), \tag{24}$$

$$(\xi,\eta,\zeta) = (\xi_1,\eta_1,\zeta_1)(z)\exp(ik(x-ct)). \tag{25}$$

Substituting from (25) into (8), (10) and (12), one may obtain:

$$D\xi_1 - ik\zeta_1 = 0, \tag{26a}$$

$$D^2\xi_1 + q^2\xi_1 = 0, \tag{26b}$$

$$D^2\zeta_1 + q^2\zeta_1 = 0, \tag{26c}$$

where $q^2 = ikcs_2 - k^2$, $D = \frac{d}{dz}$.

Solutions of (26b) and (26c) are:

$$\xi_1 = A_1 e^{iqz} + A_2 e^{-iqz}, \quad \zeta_1 = B_1 e^{iqz} + B_2 e^{-iqz}, \tag{27}$$

where, A_1, A_2, B_1 and B_2 are arbitrary constants, from (26a) and (27) we obtain

$$q(A_1 e^{iqz} - A_2 e^{-iqz}) - k(B_1 e^{iqz} + B_2 e^{-iqz}) = 0, \tag{28}$$

then

$$A_j = \frac{(-1)^{j-1}k}{q}B_j, \quad j = 1,2. \tag{29}$$

Substituting from (23) and (24) into (20) and (22), we obtain:

$$\alpha^2 D^4 + \left(k^2(c^2-2\alpha^2) + \frac{ikc}{\grave{u}}(\alpha 4a^2\Gamma_2 + \grave{u}\varepsilon\Gamma_1\Gamma_3) + \Omega^2\right)D^2 + k^4(\alpha^2-c^2) + \left[\frac{ick^3\Gamma_2(c^2-\alpha^2)}{\grave{u}} - i\varepsilon ck^3\Gamma_1\Gamma_3 + \frac{ick\Gamma_2}{\grave{u}}\Omega^2\right]\Phi_1 - ikg\left[(D^2-k^2)\left(1-\frac{2\Omega c}{g}\right) + \frac{ikc\Gamma_2}{\grave{u}}\right]\Psi_1 = 0, \tag{30}$$

$$(\beta^2 - ikcs_1)D^4 + [k^2(c^2-2\beta^2) + ikc(\beta^2 s_2 - k^2 s_1)]D^2 + [k^4(\beta^2-c^2) + ik^3c\{s_2(c^2-\beta^2) - k^2 s_1\}]\Psi_1 + ikg\left[D^2 - k^2 + ikcs_2 - \frac{2\Omega c}{g}\right]\Phi_1 = 0, \tag{31}$$

where

$$\Gamma_1 = 1 - ikc\tau_1, \qquad \Gamma_2 = 1 - ikc\tau_2, \qquad \Gamma_3 = 1 - ikc\tau_2\delta.$$

The solutions of (30) and (31) take the form:

$$\Phi_1 = \sum_{j=1}^{4} C_j e^{ikN_j z} + D_j e^{-ikN_j z}, \tag{32a}$$

$$\Psi_1 = \sum_{j=1}^{4} E_j e^{ikN_j z} + F_j e^{-ikN_j z} \tag{32b}$$

Where the constants E_j and F_j are related with the constants C_j and D_j as the form

$$\begin{aligned} &E_j = m_j C_j, \qquad F_j = m_j D_j, \quad j = 1, 2, 3, 4, \\ &m_j = \frac{-i}{g(-k(N_j^2+1) + \frac{ic\Gamma_2}{\grave{u}})}\Bigg[\alpha^2 k^2 N_j^4 - \left(k^2(c^2 - 2\alpha^2) + \frac{ikc}{\grave{u}}(\alpha^2\Gamma_2 + \varepsilon\grave{u}\Gamma_1\Gamma_3) + \Omega^2\right)N_j^2 \\ &\quad + (\alpha^2 - c^2)\left(k^2 - \frac{ick\Gamma_2}{\grave{u}}\right) - ic\varepsilon k\Gamma_1\Gamma_3 + \frac{ic\Gamma_2}{\grave{u}k}\Omega^2\Bigg], \end{aligned} \tag{33}$$

where N_1, N_2, N_3 and N_4 are taken to be the complex roots of equation

$$N^8 + t_1 N^6 + t_2 N^4 + t_3 N^2 + t_4 = 0, \tag{34}$$

$$t_1 = -2k^2 + \frac{c^2k^2}{\alpha^2} + \frac{ick(\alpha^2\Gamma_2 + \varepsilon\grave{u}\Gamma_1\Gamma_3)}{\grave{u}\alpha^2} + \frac{\Omega^2}{\alpha^2} + \frac{1}{\beta^2 - ikcs_1}(k^2(c^2 - 2\beta^2) + ick(\beta^2 s_2 - k^2 s_1)), \tag{35a}$$

$$\begin{aligned} t_2 &= k^4 - \frac{c^2k^4}{\alpha^2} - \frac{ic\varepsilon k^3\Gamma_1\Gamma_3}{\alpha^2} + \frac{ick\Gamma_2\Omega^2}{\grave{u}\alpha^2} + \frac{k^4(\beta^2 - c^2)}{\beta^2 - ikcs_1} + \frac{ik^3c\{s_2(c^2-\beta^2) - k^2 s_1\}}{\beta^2 - ikcs_1} \\ &\quad + \frac{1}{\alpha^2(\beta^2 - ikcs_1)}[k^2(c^2 - 2\beta^2) + ick(\beta^2 s_2 - k^2 s_1)]\Bigg[k^2(c^2 - 2\alpha^2) \\ &\quad + \frac{ick(\alpha^2\Gamma_2 + \varepsilon\grave{u}\Gamma_1\Gamma_3)}{\grave{u}} + \Omega^2\Bigg] - \frac{ik^3c\Gamma_2(c^2 - \alpha^2)}{\grave{u}\alpha^2} - \frac{k^2g^2(1 - \frac{2\Omega c}{g})}{\alpha^2(\beta^2 - ikcs_1)}, \end{aligned} \tag{35b}$$

$$\begin{aligned} t_3 &= \frac{1}{\alpha^2(\beta^2 - ikcs_1)}\Bigg[(k^2(c^2 - 2\beta^2) + ick(\beta^2 s_2 - k^2 s_1))\Bigg(k^4(\alpha^2 - c^2) \\ &\quad + \frac{ick^3(\Gamma_2(c^2-\alpha^2) - \varepsilon\grave{u}\Gamma_1\Gamma_3 + \Gamma_2\Omega^2/k^2)}{\grave{u}}\Bigg) - g^2k^2\Bigg(\left(1 - \frac{2\Omega c}{g}\right)\left(icks_2 - \frac{2\Omega c}{g}\right) \\ &\quad + \frac{ick\Gamma_2}{\grave{u}} + -2k^2\left(1 - \frac{2\Omega c}{g}\right)\Bigg) + \Bigg(k^2(c^2 - 2\alpha^2) \\ &\quad + \frac{ick(\alpha^2\Gamma_2 + \varepsilon\grave{u}\Gamma_1\Gamma_3)}{\grave{u}} + \Omega^2\Bigg)(k^4(\beta^2 - c^2) + ik^3c(s_2(c^2 - \beta^2) - k^2 s_1))\Bigg], \end{aligned} \tag{35c}$$

$$\begin{aligned} t_4 &= \frac{1}{\alpha^2(\beta^2 - ikcs_1)}\Bigg[(k^4(\beta^2 - c^2) + ik^3c(s_2(c^2 - \beta^2) - k^2 s_1))\Bigg(k^4(\alpha^2 - c^2) \\ &\quad + \frac{ick^3(\Gamma_2(c^2-\alpha^2) - \varepsilon\grave{u}\Gamma_1\Gamma_3 + ick\Gamma_2\Omega^2)}{\grave{u}}\Bigg) - g^2k^4\left(k\left(1 - \frac{2\Omega c}{g}\right) - \frac{ic\Gamma_2}{\grave{u}}\right)\left(k - ics_2 + \frac{2\Omega c}{g}\right)\Bigg]. \end{aligned} \tag{35d}$$

From (16), (24), (25), (32a) and (32b), we obtain:

$$\eta_1 = \sum_{j=1}^{4}\left[\frac{1}{is_1c}\left(k(\beta^2(N_j^2+1) - c^2) - \frac{\Omega^2}{k}\right)m_j - ig - 2i\Omega c\right](C_j e^{ikN_j z} + D_j e^{-ikN_j z}). \tag{36}$$

Using (32a) and (32b), we obtain

$$T=\frac{\rho}{\gamma\Gamma_1}\sum_{j=1}^{4}[-\alpha^2k^2(N_j^2+1)+k^2c^2 - ik(g-2\Omega c)m_j+\Omega^2](C_je^{ikN_jz} + D_je^{-ikN_jz})e^{ik(x-ct)}. \tag{37}$$

With the lower medium, we use the symbols with dashes, for ξ_1, ζ_1, η_1, T, Φ_1, Ψ_1 and q, for $z>K$,

$$\xi_1'=-\frac{k}{q'}B_2'e^{-iq'z},\qquad \zeta_1'=B_2'e^{-iq'z}, \tag{38a}$$

$$\eta_1'=\sum_{j=1}^{4}\left[\frac{1}{is_1'c}\left(k(\beta'^2(N_j'^2+1)-c^2)-\frac{\Omega^2}{k}\right)m_j' - i(g-2\Omega c)\right]D_j'e^{-ikN_j'z}, \tag{38b}$$

$$T'=\frac{\rho'}{\gamma_1'\Gamma_1}\sum_{j=1}^{4}[-\alpha'^2k^2(N_j'^2+1)+k^2c^2 - ik(g-2\Omega c)m_j' + \Omega^2]D_j'e^{-ik(N_j'z-x+ct)}, \tag{38c}$$

$$\Phi_1'=\sum_{j=1}^{4}D_j'e^{-ikN_j'z},\qquad \Psi_1'=\sum_{j=1}^{4}F_j'e^{-ikN_j'z}. \tag{38d}$$

4 Boundary conditions and frequency equation

In this section, we obtain the frequency equation for the boundary conditions which specify on the interface $z=K$, i.e.,

$$\text{(i) } u=u',\qquad \text{(ii) } w=w',\qquad \text{(iii) } \xi=\xi',\qquad \text{(iv) } \eta=\eta',\qquad \text{(v) } \zeta=\zeta',\qquad \text{(vi) } M_{33}=M_{33}', \tag{39a}$$

$$\text{(vii) } M_{31}=M_{31}',\qquad \text{(viii) } M_{32}=M_{32}',\qquad \text{(ix) } \tau_{33}+\bar\tau_{33}=\tau_{33}'+\bar\tau_{33}',\qquad \text{(x) } \tau_{31}+\bar\tau_{31}=\tau_{31}'+\bar\tau_{31}', \tag{39b}$$

$$\text{(xi) } \tau_{32}+\bar\tau_{32}=\tau_{32}'+\bar\tau_{32}',\qquad \text{(xii) } T=T',\qquad \text{(xiii) } \frac{\partial T}{\partial z}+\theta T=\frac{\partial T'}{\partial z}+\theta T'. \tag{39c}$$

The boundary condition on the free surface $z=0$ are

$$\text{(xiv) } M_{33}=0,\qquad \text{(xv) } M_{31}=0,\qquad \text{(xvi) } M_{32}=0,\qquad \text{(xvii) } \tau_{33}+\bar\tau_{33}=0, \tag{40a}$$

$$\text{(xviii) } \tau_{31}+\bar\tau_{31}=0,\qquad \text{(xix) } \tau_{32}+\bar\tau_{32}=0,\qquad \text{(xx) } \frac{\partial T}{\partial z}+\theta T=0, \tag{40b}$$

from the conditions (iii), (v), (vi), (vii), we get

$$B_1e^{iqK}-B_2e^{-iqK}=-B_2'e^{-iq'K}, \tag{41a}$$

$$B_1e^{iqK}+B_2e^{-iqK}=B_2'e^{-iq'K}, \tag{41b}$$

$$M(B_1e^{iqK}-B_2e^{-iqK})=-M'B_2'e^{-iq'K}, \tag{41c}$$

$$M(B_1e^{iqK}+B_2e^{-iqK})=-M'B_2'e^{-iq'K}, \tag{41d}$$

hence

$$B_1=B_2=B_2'=0,\qquad \xi=\zeta=\xi'=\zeta'=0. \tag{42}$$

The other significant boundary conditions are responsible for the following relations

$$\text{(i) } \sum_{j=1}^{4}(1-N_jm_j)C_je^{ikN_jK}+(1+N_jm_j)D_je^{-ikN_jK}-(1-N_j'm_j')D_j'e^{-ikN_j'K}=0,$$

$$\text{(ii) } \sum_{j=1}^{4}(N_j+m_j)C_je^{ikN_jK}+(m_j-N_j)D_je^{-ikN_jK}-(m_j'-N_j')D_j'e^{-ikN_j'K}=0,$$

$$\text{(iv)}\quad \sum_{j=1}^{4}\left[\frac{1}{is_1c}\left(\beta^2k(N_j^2+1)-kc^2-\frac{\Omega^2}{k}\right)m_j-ig-2i\Omega c\right](C_je^{ikN_jK}+D_je^{-ikN_jK})$$

$$-\sum_{j=1}^{4}\left[\frac{1}{s_1'c}\left(\beta'^2k(N_j'^2+1)-kc^2-\frac{\Omega^2}{k}\right)m_j'-ig-2i\Omega c\right]D_j'e^{-ikN_j'K}=0,$$

$$\text{(viii)}\quad \sum_{j=1}^{4}MN_j\left(\frac{1}{s_1c}(\beta^2k(N_j^2+1)-kc^2)m_j-ig\right)(C_je^{ikN_jK}-D_je^{-ikN_jK})$$

$$+k^2m_j(N_j^2+1)(C_je^{ikN_jK}+D_je^{-ikN_jK})+\left(M'N_j'\left(\frac{1}{s_1'c}(\beta'^2k(N_j'^2+1)-kc^2)m_j'-ig\right)\right.$$

$$\left.-k^2m_j'(N_j'^2+1)\right)D_j'e^{-ikN_j'K}=0,$$

$$\text{(ix)}\quad \sum_{j=1}^{4}[-k^2\{\lambda(1-N_jm_j)+N_j(\lambda+2\mu)(m_j+N_j)\}]C_je^{ikN_jK}$$

$$+[-k^2\{\lambda(1+N_jm_j)-N_j(\lambda+2\mu)(m_j-N_j)\}]D_je^{-ikN_jK}$$

$$+[k^2(N_j^2+1)(\rho\gamma\alpha^2-\mu_eH_o^2)-k\rho\gamma(kc^2-igm_j)](C_je^{ikN_jK}+D_je^{-ikN_jK})$$

$$-[-k^2\{\lambda'(1+N_j'm_j')-N_j'(\lambda'+2\mu')(m_j'-N_j')\}]D_j'e^{-ikN_j'K}$$

$$-[k^2(N_j'^2+1)(\rho'\gamma'\alpha'^2-\mu_e'H_o^2)-k\rho'\gamma'(kc^2-igm_j')]D_j'e^{-ikN_j'K}=0,$$

$$\text{(x)}\quad \sum_{j=1}^{4}\left[\mu k^2(m_j(N_j^2-1))+\frac{iFk}{s_1}((\beta^2k(N_j^2+1)-kc^2)m_j-ig)\right]$$

$$\times(C_je^{ikN_jK}+D_je^{-ikN_jK})+2\mu k^2N_j(-C_je^{ikN_jK}+D_je^{-ikN_jK})$$

$$-\left[\mu'k^2(m_j'(N_j'^2-1)+2N_j')+\frac{iFk}{s_1'}((\beta'^2k(N_j'^2+1)-kc^2)m_j'-ig)\right]D_j'e^{-ikN_j'K}=0,$$

$$-\frac{\rho'}{\gamma'}[-\alpha'^2k^2(N_j'^2+1)+k^2c^2-ik(g-2\Omega c)m_j'+\Omega^2]D_j'e^{-ikN_j'K}=0,$$

$$\text{(xiii)}\quad \sum_{j=1}^{4}\frac{\rho}{\gamma}[-\alpha^2k^2(N_j^2+1)+k^2c^2-ik(g-2\Omega c)m_j+\Omega^2][(\theta+ikN_j)C_je^{ikN_jK}$$

$$+(\theta-ikN_j)D_je^{-ikN_jK}]-\frac{\rho'}{\gamma'}[-\alpha'^2k^2(N_j'^2+1)+k^2c^2-ik(g-2\Omega c)m_j'+\Omega^2]$$

$$\times(\theta-ikN_j')D_j'e^{-ikN_j'K}=0,$$

$$\text{(xvi)}\quad \sum_{j=1}^{4}\left(\frac{1}{s_1c}(\beta^2k(N_j^2+1)-kc^2)m_j-ig\right)(MN_j(C_j-D_j))+k^2m_j(N_j^2+1)(C_j+D_j)=0,$$

$$\text{(xvii)}\quad \sum_{j=1}^{4}-\lambda k^2[(1-N_jm_j)C_j+(1+N_jm_j)D_j]+(\lambda+2\mu)[-k^2N_j\{(m_j+N_j)C_j$$

$$+(m_j-N_j)D_j\}]+[k^2(N_j^2+1)(\rho\gamma\alpha^2+\mu_eH_o^2)-k\rho\gamma(kc^2-igm_j)](C_j+D_j)=0,$$

(xviii) $\sum_{j=1}^{4} -2k^2N_j(C_j - D_j) + k^2m_j(N_j^2 - 1) + \frac{iFk}{s_1}[(\beta^2k(N_j^2+1) - kc^2)m_j - ig](C_j + D_j) = 0,$

(xx) $\sum_{j=1}^{4} [-\alpha^2k^2(N_j^2+1) + k^2c^2 - ik(g - 2\Omega c)m_j + \Omega^2][(\theta + ikN_j)C_j + (\theta - ikN_j)D_j] = 0.$

By eliminating C_j, D_j and D'_j from the relevant results, we will get a determinant of a twelfth-order form which determines the wave velocity and attenuation coefficient.

5 Special cases and discussion

5.1 In presence of rotation and absence of the gravity field

In this case, we put the gravity field $g = 0$, and the rotation $\Omega \neq 0$, (30) and (31) take the form:

$$\alpha^2D^4 + \left(k^2(c^2 - 2\alpha^2) + \frac{ikc}{\grave{u}}(\alpha_2^2\Gamma_2 + \grave{u}\varepsilon\Gamma_1\Gamma_3) + \Omega^2\right)D^2 + \left[k^4(\alpha^2 - c^2) + \frac{ick^3[\Gamma_2(c^2 - \alpha^2 + \frac{\Omega^2}{k^2}) - \varepsilon\grave{u}\Gamma_1\Gamma_3]+}{\grave{u}}\right]\Phi_1 + 2ick\Omega\Psi_1 = 0, \tag{43}$$

$$(\beta^2 - ikcs_1)D^4 + 2[k^2(c^2 - 2\beta^2) + ikc(s_2\beta^2 - s_1k^2)]D^2 + 2k^4(2\beta^2 - c^2) + [2ics_2k^3(c^2 - \beta^2) - ics_1k^5]\Psi_1 - 2ick\Omega\Phi_1 = 0, \tag{44}$$

which take the solutions

$$\Phi_1 = \sum_{j=1}^{4} L_je^{ikN_jz} + Q_je^{-ikN_jz,}, \qquad \Psi_1 = \sum_{j=1}^{4} n_je^{ikN_jz} + p_je^{-ikN_jz}, \tag{45}$$

where

$$n_j = I_jL_j, \qquad p_j = I_jQ_j, \tag{46}$$

where

$$I_j = \frac{-2ick\Omega}{[\alpha^2k^2N_j^4 - (k^2(c^2 - 2\alpha^2) + \frac{ikc}{\grave{u}}(\alpha_2^2\Gamma_2 + \varepsilon\grave{u}\Gamma_1\Gamma_3) + \Omega^2)N_j^2 + (\alpha^2 - c^2)(k^2 - \frac{ick\Gamma_2}{\grave{u}}) - ic\varepsilon k\Gamma_1\Gamma_3 + \frac{ic\Gamma_2}{\grave{u}k}\Omega^2]}, \tag{47}$$

$$R^8 + \Lambda_1R^6 + \Lambda_2R^4 + \Lambda_3R^2 + \Lambda_4 = 0, \tag{48}$$

$$\Lambda_1 = -\frac{2(c^2 - 2\beta^2)}{(\beta^2 - ikcs_1)} - \frac{ic(\beta^2s_2 - k^2s_1)}{k(\beta^2 - ikcs_1)} - \frac{(c^2 - 2\alpha^2) + \varepsilon\grave{u}\Gamma_1\Gamma_3 + \Omega^2}{k^2\alpha^2} - \frac{ic}{\grave{u}k}\Gamma_2, \tag{49a}$$

$$\Lambda_2 = \left\{2\left[\frac{(c^2 - 2\alpha^2)}{k^2} + \frac{ic\alpha^2}{k^3}\Gamma_2 + \frac{\varepsilon\acute{u}\Gamma_1\Gamma_3 + \Omega^2}{k^4}\right] \times [k^2(c^2 - 2\beta^2) + ick(\beta^2s_2 - k^2s_1)] + \left[1 - \frac{c^2}{\alpha^2} + \frac{ic}{\grave{u}k\alpha^2}\left[\Gamma_2\left(c^2 - \alpha^2 + \frac{\Omega^2}{k^2}\right) - \frac{\varepsilon\acute{u}\Gamma_1\Gamma_3}{k^4}\right]\right] + \frac{2(\beta^2 - c^2)}{(\beta^2 - ikcs_1)} + \frac{2ics_2(c^2 - \beta^2) - ick^2s_1}{k(\beta^2 - ikcs_1)}\right\}, \tag{49b}$$

$$\Lambda_3 = \frac{1}{\alpha^2(\beta^2 - ikcs_1)k^2}\left[2(\alpha^2 - c^2) + \frac{ic}{\grave{u}k}\left(\Gamma_2\left(c^2 - \alpha^2 + \frac{\Omega^2}{k^2}\right) - \frac{\varepsilon\acute{u}\Gamma_1\Gamma_3}{k^4}\right)\right.$$
$$\left.\times [k^2(c^2 - 2\beta^2) + ick(\beta^2 s_2 - k^2 s_1)]\right], \tag{49c}$$

$$\Lambda_4 = \frac{1}{\alpha^2(\beta^2 - ikcs_1)k^2}\left\{\left[\frac{(\alpha^2 - c^2)}{k^4} + \frac{ic}{\grave{u}k^5}\left(\Gamma_2\left(c^2 - \alpha^2 + \frac{\Omega^2}{k^2}\right) - \frac{\varepsilon\acute{u}\Gamma_1\Gamma_3}{k^8}\right)\right.\right.$$
$$\left.\left.\times [2k^4(2\beta^2 - c^2) + 2ick^3 s_2(c^2 - \beta^2) - ick^5 s_1] - \frac{2ic\Omega}{k^7}\right]\right\}. \tag{49d}$$

Also,

$$\eta_1 = \sum_{j=1}^{4}\left[\frac{1}{s_1 c}k\left(\beta^2(R_j^2 + 1) - c^2 - \frac{\Omega^2}{k}\right)I_j - 2i\Omega c\right](L_j e^{ikR_j z} + Q_j e^{-ikR_j z}), \tag{50}$$

$$T = \frac{\rho}{\gamma\Gamma_1}\sum_{j=1}^{4}[-\alpha^2 k^2(R_j^2 + 1) + k^2c^2 + ik(2\Omega c)I_j + \Omega^2](L_j e^{ikR_j z} + Q_j e^{-ikR_j z})e^{ik(x-ct)}. \tag{51}$$

With the lower medium, we use the symbols with dashes, for ξ_1, ζ_1, η_1, T, Φ_1, Ψ_1 and q, for $z > K$,

$$\xi_1' = -\frac{k}{q'}B_2' e^{-iq'z}, \qquad \zeta_1' = B_2' e^{-iq'z}, \tag{52a}$$

$$\eta_1' = \sum_{j=1}^{4}\left[\frac{1}{s_1' c}\left(k(\beta'^2(R_j'^2 + 1) - c^2) - \frac{\Omega^2}{k}\right)I_j' - 2i\Omega c\right]Q_j' e^{-ikR_j' z}, \tag{52b}$$

$$T' = \frac{\rho'}{\gamma_1'\Gamma_1}\sum_{j=1}^{4}[-\alpha'^2 k^2(R_j'^2 + 1) + k^2c^2 + 2i\Omega ck I_j' + \Omega^2]Q_j' e^{-ik(R_j' z - x + ct)}, \tag{52c}$$

$$\Phi_1' = \sum_{j=1}^{4} Q_j' e^{-ikR_j' z}, \qquad \Psi_1' = \sum_{j=1}^{4} p_j' e^{-ikR_j' z}. \tag{53}$$

Using the boundary conditions, we get:

$$d_{1j} = (1 - R_j I_j)L_j e^{ikR_j K} + (1 + R_j I_j)Q_j e^{-ikR_j K}, \qquad d_{1j}' = (1 - R_j' I_j')Q_j' e^{-ikR_j' K},$$

$$d_{2j} = (R_j + I_j)L_j e^{ikR_j K} + (I_j - R_j)Q_j e^{-ikR_j K}, \qquad d_{2j}' = (I_j' - R_j')Q_j' e^{-ikR_j' K},$$

$$d_{3j} = \left[\frac{1}{s_1 c}\left(-ik\beta^2(R_j^2 + 1) + ikc^2 - \frac{\Omega^2}{ik}\right)I_j + (-2\Omega c)\right](L_j e^{ikR_j K} + Q_j e^{-ikR_j K}),$$

$$d_{3j}' = \left[\frac{1}{s_1' c}\left(-ik\beta'^2(R_j'^2 + 1) + ikc^2 - \frac{\Omega^2}{ik}\right)I_j' + (-2\Omega c)\right]Q_j' e^{-ikR_l' K},$$

$$d_{4j} = MV_j\left(\frac{1}{s_1 c}\left(-ik\beta^2(R_j^2 + 1) + ikc^2 - \frac{\Omega^2}{ik}\right)I_j + (-2\Omega c)\right)(L_j e^{ikR_j K} - Q_j e^{-ikR_j K})$$
$$+ k^2 I_j(R_j^2 + 1)(L_j e^{ikR_j K} + Q_j e^{-ikR_j K}),$$

$$d_{4j}' = \left(\frac{1}{s_1' c}\left(-ik\beta'^2(R_j'^2 + 1) + ikc^2 - \frac{\Omega^2}{ik}\right)I_j' + (-2\Omega c)\right)MR_j' Q_j' e^{-ikR_j' K} + k^2 I_j'(R_j'^2 + 1)Q_j' e^{-ikR_j' K},$$

$$
\begin{aligned}
d_{5j} &= [-k^2\{\lambda(1-R_jI_j)+R_j(\lambda+2\mu)(I_j+R_j)\}]L_je^{ikR_jK} \\
&\quad +[-k^2\{\lambda(1+R_jI_j)-R_j(\lambda+2\mu)(I_j-R_j)\}]Q_je^{-ikR_jK} \\
&\quad +[k^2(R_j^2+1)(\rho\gamma\alpha^2-\mu_eH_o^2)-k\rho\gamma(kc^2)](L_je^{ikR_jK}+Q_je^{-ikR_jK}), \\
d'_{5j} &= [-k^2\{\lambda'(1+R'_jm'_j)-R'_j(\lambda'+2\mu')(I'_j-R'_j)\}]Q'_je^{-ikR'_jK} \\
&\quad -[k^2(R'^2_j+1)(\rho'\gamma'\alpha'^2-\mu'_eH_o^2)-k\rho'\gamma'(kc^2)]Q'_je^{-ikR'_jK,} \\
d_{6j} &= \left[\mu k^2(I_j(R_j^2-1))+\frac{iFk}{s_1}((\beta^2k(R_j^2+1)-kc^2)I_j)\right] \\
&\quad \times(L_je^{ikR_jK}+Q_je^{-ikR_jK})+2\mu k^2R_j(-L_je^{ikR_jK}+Q_je^{-ikR_jK}), \\
d'_{6j} &= \left[\mu'k^2((I'_j(R'^2_j-1))+2R'_j)+\frac{iFk}{s'_1}((\beta'^2k(R'^2_j+1)-kc^2)I'_j)\right]Q'_je^{-ikR'_jK}, \\
d_{7j} &= \frac{\rho}{\gamma}[-\alpha^2k^2(R_j^2+1)+k^2c^2-ik(-2\Omega c)I_j+\Omega^2](L_je^{ikR_jK}+Q_je^{-ikR_jK}), \\
d'_{7j} &= \frac{\rho'}{\gamma'}[-\alpha'^2k^2(R'^2_j+1)+k^2c^2-ik(-2\Omega c)m'_j+\Omega^2]Q'_je^{-ikV_jK}, \\
d_{8j} &= \frac{\rho}{\gamma}[-\alpha^2k^2(R_j^2+1)+k^2c^2-ik(-2\Omega c)I_j+\Omega^2][(\theta+ikR_j)L_je^{ikR_jK}+(\theta-ikV_j)Q_je^{-ikR_jK}], \\
d'_{8j} &= \frac{\rho'}{\gamma'}[-\alpha'^2k^2(R'^2_j+1)+k^2c^2-ik(-2\Omega c)I'_j+\Omega^2](\theta'-ikR'_j)Q'_je^{-ikR'_jK}, \\
d_{9j} &= \left(\frac{1}{s_1c}\left(-ik\beta^2\left(R_j^2+1+ikc^2-\frac{\Omega^2}{ik}\right)I_j+(-2\Omega c)\right)\right)(MR_j(L_j-Q_j))+k^2I_j(R_j^2+1)(L_j+Q_j), \\
d_{10j} &= -\lambda k^2[(1-R_jm_j)C_j+(1+R_jm_j)D_j]+(\lambda+2\mu)\{-k^2R_j[(I_j+R_j)L_j \\
&\quad +(I_j-R_j)D_j]\}-[k^2(R_j^2+1)(\rho\gamma\alpha^2+\mu_eH_o^2)-k\rho\gamma(kc^2)](L_j+Q_j), \\
d_{11j} &= -2k^2R_j(L_j-Q_j)+k^2I_j(R_j^2-1)+\frac{iFk}{s_1}[(\beta^2k(R_j^2+1)-kc^2)I_j], \\
d_{12j} &= [-\alpha^2k^2(R_j^2+1)+k^2c^2-ik(-2\Omega c)I_j+\Omega^2][(\theta+ikR_j)L_j+(\theta-ikR_j)Q_j], \\
d'_{9j} &= d'_{10j}=d'_{11j}=d'_{12j}=0, \quad j=3,4,5,6.
\end{aligned}
\tag{54}
$$

Equation (54) has complex roots, the real part gives Rayleigh wave velocity and the imaginary part gives the attenuation coefficient due to the friction of the granular nature of the medium. Analytically, one may observed that the Rayleigh wave velocity and attenuation coefficient depend on the rotation, magnetic field, initial stress, gravity field, granular rotation and thermal relaxation times. It's shown from (34) that due to effect of the thermal field and gravity field change from fourth-order to eight-order which involves four positive roots. The transcendental equation (54) in the determinant form, represents the required wave velocity equation of wave propagated in generalized magneto-thermoelastic granular body under the influence of rotation, gravity field and initial stress.

5.2 In presence of rotation, relaxation times, granularity and there is no coupling ($\varepsilon = 0$) between the temperature and strain field

In this case, presence of rotation ($\Omega \neq 0$), no gravity field ($g = 0$), no initial stress ($P = 0$), no magnetic field ($H_o = 0$), coupled parameter ($\varepsilon = 0$) and the ratio of the coefficients of heat transfer ($\theta = 0$), we obtain:

$$\lim_{\varepsilon\to 0}\Lambda_1 = -\frac{2(c^2-2\beta^2)}{(\beta^2-ikcs_1)} - \frac{ic(\beta^2 s_2 - k^2 s_1)}{k(\beta^2-ikcs_1)} - \frac{(c^2-2\alpha^2)+\Omega^2}{k^2\alpha^2} - \frac{ic}{\grave{u}k}\Gamma_2, \tag{55}$$

$$\begin{aligned}\lim_{\varepsilon\to 0}\Lambda_2 = \Bigg\{&2\left[\frac{(c^2-2\alpha^2)}{k^2} + \frac{ic\alpha^2}{k^3}\,\Gamma_2 + \frac{\Omega^2}{k^4}\right]\times[k^2(c^2-2\beta^2)+ick(\beta^2 s_2 - k^2 s_1)]\\ &+\left[1-\frac{c^2}{\alpha^2}+\frac{ic}{\grave{u}k\alpha^2}\left[\Gamma_2\left(c^2-\alpha^2+\frac{\Omega^2}{k^2}\right)\right]\right] + \frac{2(\beta^2-c^2)}{(\beta^2-ikcs_1)} + \frac{2ics_2(c^2-\beta^2)-ick^2 s_1}{k(\beta^2-ikcs_1)}\Bigg\},\end{aligned} \tag{56}$$

$$\begin{aligned}\lim_{\varepsilon\to 0}\Lambda_3 = \frac{1}{\alpha^2(\beta^2-ikcs_1)k^2}\Bigg\{&\Bigg[2(\alpha^2-c^2)+\frac{ic}{\grave{u}k}\left(\Gamma_2\left(c^2-\alpha^2+\frac{\Omega^2}{k^2}\right)\right)\\ &\times[k^2(c^2-2\beta^2)+ick(\beta^2 s_2-k^2 s_1)]\Bigg]\Bigg\},\end{aligned} \tag{57}$$

$$\begin{aligned}\lim_{\varepsilon\to 0}\Lambda_4 = \frac{1}{\alpha^2(\beta^2-ikcs_1)k^2}\Bigg\{&\Bigg[\frac{(\alpha^2-c^2)}{k^4}+\frac{ic}{\grave{u}k^5}\left(\Gamma_2\left(c^2-\alpha^2+\frac{\Omega^2}{k^2}\right)\right)\\ &\times[2k^4(2\beta^2-c^2)+2ick^3 s_2(c^2-\beta^2)-ick^5 s_1]-\frac{2ic\Omega}{k^7}\Bigg]\Bigg\}.\end{aligned} \tag{58}$$

After tanking $\gamma \to 0$, boundary conditions become:

$$\begin{aligned}
d'_{5j} &= [-k^2\{\lambda'(1+R'_j I'_j) - R'_j(\lambda'+2\mu')(I'_j - R'_j)\}]Q'_j e^{-ikR'_j K},\\
d_{6j} &= \left[\mu k^2(I_j(R_j^2-1)) + \frac{iFk}{s_1}((\beta^2 k(R_j^2+1)-kc^2)I_j)\right]\\
&\quad\times(L_j e^{ikrR_jK} + Q_j e^{-ikR_jK}) + 2\mu k^2 R_j(-L_j e^{ikR_jK} + Q_j e^{-ikR_jK}),\\
d'_{6j} &= \left[(\mu' k^2(I'_j(R'^2_j-1)) + 2R'_j) + \frac{iFk}{s'_1}((\beta'^2 k(R'^2_j+1)-kc^2)I'_j)\right]Q'_j e^{-ikR'_jK},\\
d_{7j} &= \left(\frac{1}{s_1 c}\left(-ik\beta^2\left(R_j^2+1+ikc^2-\frac{\Omega^2}{ik}\right)I_j + (-2\Omega c)\right)(MR_j(L_j-Q_j))\right.\\
&\quad\left.+k^2 I_j(R_j^2+1)(L_j+Q_j)\right),\\
d_{8j} &= -\lambda k^2[(1-R_jI_j)L_j + (1+R_jI_j)Q_j] + (\lambda+2\mu)\{-k^2R_j[(I_j+R_j)L_j + (I_j-R_j)Q_j]\},\\
d_{9j} &= -2k^2R_j(L_j-Q_j) + k^2 I_j(R_j^2-1) + \frac{iFk}{s_1}[(\beta^2 k(R_j^2+1)-kc^2)I_j],\\
d_{10j} &= [-\alpha^2k^2(R_j^2+1) + k^2c^2 - ik(-2\Omega c)I_j + \Omega^2][(ikR_j)L_j - (ikR_j)Q_j],\\
d'_{7j} &= d'_{8j} = d'_{9j} = d'_{10j} = 0, \quad j = 3,4,5,6.
\end{aligned}$$

$$\det(d_{ij}) = 0. \tag{59}$$

Equation (59) has complex roots, the real part gives Rayleigh wave velocity and the imaginary part gives the attenuation coefficient. The frequency equation (54) which determines the wave velocity equation for the Rayleigh waves in a granular medium, when the rotation and initial stress are absent, we have

$$\alpha^2 = \frac{\lambda + 2\mu}{\rho}, \qquad \beta^2 = \frac{\mu}{\rho}. \tag{60}$$

Finally, if there is (i) no rotation, magnetic field, thermal relaxation times, gravity field, and granular rotation is vanishes and (ii) absence of the magnetic field, thermal relaxation times, gravity field, there is uncoupling between temperature and strain field, and the granular rotation is vanishes, the results obtained by Ahmed [18, 19] are deduced as special case from this study with slight changes in symbols with additional to the graphs that not included in the last work.

For a computation work by half-interval method program, we use Sand Stone as a granular medium and Nephiline as a granular layer taking into consideration that the friction coefficient $F = 0.5$ and third elastic constants $M_1 = 0.4$, $M_2 = 0.6$.

5.3 In presence of rotation, there are no coupling ($\varepsilon = 0$) between the temperature and strain field, there are no relaxation times and granularity

In this case presence of rotation ($\Omega \neq 0$), no relaxation times ($\tau_1 = \tau_2 = 0$) no gravity field ($g = 0$), no initial stress ($P = 0$), no magnetic field ($H_o = 0$), coupled parameter ($\varepsilon = 0$) and the ratio of the coefficients of heat transfer ($\theta = 0$), and the friction coefficient $F = 0$, we obtain:

$$\begin{aligned}
\lim_{\varepsilon\to 0} \Lambda_1 &= -\frac{2(c^2 - 2\beta^2)}{\beta^2} - \frac{(c^2 - 2\alpha^2) + \Omega^2}{k^2\alpha^2} - \frac{ic}{\grave{u}k}, \\
\lim_{\varepsilon\to 0} \Lambda_2 &= 2\left[\frac{(c^2 - 2\alpha^2)}{k^2} + \frac{ic\alpha^2}{k^3} + \frac{\Omega^2}{k^4}\right] \times [k^2(c^2 - 2\beta^2)] \\
&\quad + \left[1 - \frac{c^2}{\alpha^2} + \frac{ic}{\grave{u}k\alpha^2}\left[\left(c^2 - \alpha^2 + \frac{\Omega^2}{k^2}\right)\right]\right] + \frac{2(\beta^2 - c^2)}{\beta^2}, \\
\lim_{\varepsilon\to 0} \Lambda_3 &= \frac{1}{\alpha^2\beta^2k^2}\left\{\left[2(\alpha^2 - c^2) + \frac{ic}{\grave{u}k}\left(\left(c^2 - \alpha^2 + \frac{\Omega^2}{k^2}\right)\right)[k^2(c^2 - 2\beta^2)]\right]\right\}, \\
\lim_{\varepsilon\to 0} \Lambda_4 &= \frac{1}{\alpha^2\beta^2k^2}\left\{\left[\frac{(\alpha^2 - c^2)}{k^4} + \frac{ic}{\grave{u}k^5}\left(\left(c^2 - \alpha^2 + \frac{\Omega^2}{k^2}\right)\right)[2k^4(2\beta^2 - c^2)] - \frac{2ic\Omega}{k^7}\right]\right\},
\end{aligned}$$

$$\begin{aligned}
d_{1j} &= (1 - R_jI_j)L_je^{ikR_jK} + (1 + R_jI_j)Q_je^{-ikR_jK}, \\
d'_{1j} &= (1 - R'_jI'_j)Q'_je^{-ikR'_jK}, \\
d_{2j} &= (R_j + I_j)L_je^{ikR_jK} + (I_j - R_j)Q_je^{-ikR_jK}, \\
d'_{2j} &= (I'_j - R'_j)Q'_je^{-ikR'_jK}, \\
d_{3j} &= (-k^2\{\lambda(1 - R_jI_j) + R_j(\lambda + 2\mu)(I_j + R_j)\}]L_je^{ikR_jK} + [-k^2\{\lambda(1 + R_jI_j) \\
&\quad - R_j(\lambda + 2\mu)(I_j - R_j)\}]Q_je^{-ikR_jK}, \\
d'_{3j} &= [k^2\{\lambda'(1 + R'_jI'_j) + R'_j(\lambda' + 2\mu')(I'_j - R'_j)\}]Q'_je^{-ikR'_jK}, \\
d_{4j} &= -\lambda k^2[(1 - R_jI_j)L_j + (1 + R_jI_j)Q_j] + (\lambda + 2\mu)\{-k^2R_j[(I_j + R_j)L_j + (I_j - R_j)Q_j]\}, \\
d_{5j} &= -2k^2R_j(L_j - Q_j) + k^2I_j(R_j^2 - 1)(L_j + Q_j), \\
d_{6j} &= [-\alpha^2k^2(R_j^2 + 1) + k^2c^2 + 2ik\Omega cI_j + \Omega^2][(ikR_j)L_j - (ikR_j)Q_j], \\
d'_{4j} &= d'_{5j} = d'_{6j} = 0, \quad j = 1, 2.
\end{aligned}$$

So, (59) tends to

$$\det(d_{ij}) = 0. \tag{61}$$

Equation (61) has complex roots, the real part gives Rayleigh wave velocity and the imaginary part gives the attenuation coefficient, where

$$d_{11} = (1 - R_1 I_1)e^{ikR_1K}, \quad d_{12} = (1 + R_1 I_1)e^{-ikR_1K}, \quad d_{13} = (1 - R_2 I_2)e^{ikR_2K},$$
$$d_{14} = (1 + R_2 I_2)e^{-ikR_2K}, \quad d_{15} = (1 - R_1 I_1')e^{-ikR_1'K}, \quad d_{16} = (1 - R_2' I_2')e^{-ikR_2'K},$$
$$d_{21} = (1 - R_1 I_1)e^{ikR_1K}, \quad d_{22} = (I_1 - R_1)e^{-ikR_1K}, \quad d_{23} = (R_2 + I_2)e^{ikR_2K},$$
$$d_{24} = (I_2 - R_2)e^{-ikR_2K}, \quad d_{25} = (I_1' - R_1')e^{-ikR_1'K}, \quad d_{26} = (I_2' - R_2')e^{-ikR_2'K},$$
$$d_{31} = -k^2\{\lambda(1 - R_1 I_1) + R_1(\lambda + 2\mu)(I_1 + R_1)\}e^{ikR_1K},$$
$$d_{32} = -k^2\{\lambda(1 + R_1 I_1) - R_1(\lambda + 2\mu)(I_1 - R_1)\}e^{-ikR_1K},$$
$$d_{33} = -k^2\{\lambda(1 - R_2 I_2) + R_2(\lambda + 2\mu)(I_2 + R_2)\}e^{ikR_2K},$$
$$d_{34} = -k^2\{\lambda(1 + R_2 I_2) - R_2(\lambda + 2\mu)(I_2 - R_2)\}e^{-ikR_2K},$$
$$d_{35} = k^2\{\lambda'(1 + R_1' I_1') + R_1'(\lambda' + 2\mu')(I_1' - R_1')\}e^{-ikR_1'K},$$
$$d_{36} = k^2\{\lambda'(1 + R_2' I_2') + R_2'(\lambda' + 2\mu')(I_2' - R_2')\}e^{-ikR_2'K},$$
$$d_{41} = -\lambda k^2(1 - R_1 I_1) - k^2 R_1(\lambda + 2\mu)(I_1 + R_1),$$
$$d_{42} = -\lambda k^2(1 + R_1 I_1) - k^2 R_1(\lambda + 2\mu)(I_1 - R_1),$$
$$d_{43} = -\lambda k^2(1 - R_2 I_2) - k^2 R_2(\lambda + 2\mu)(I_2 + R_2),$$
$$d_{44} = -\lambda k^2(1 + R_2 I_2) - k^2 R_2(\lambda + 2\mu)(I_2 - R_2),$$
$$d_{45} = k^2\lambda'(1 + R_1' I_1') + R_1'(\lambda' + 2\mu')(I_1' - R_1'),$$
$$d_{46} = k^2\lambda'(1 + R_2' I_2') + R_2'(\lambda' + 2\mu')(I_2' - R_2'),$$
$$d_{51} = -2k^2 R_j + k^2 I_j(R_j^2 - 1), \quad d_{52} = 2k^2 R_j + k^2 I_j(R_j^2 - 1),$$
$$d_{53} = -2k^2 R_j + k^2 I_j(R_j^2 - 1),$$
$$d_{54} = 2k^2 R_j + k^2 I_j(R_j^2 - 1), \quad d_{55} = 0, \quad d_{56} = 0,$$
$$d_{61} = (ikR_1)[-\alpha^2 k^2(R_1^2 + 1) + k^2c^2 + 2ik\Omega c I_1 + \Omega^2],$$
$$d_{62} = (-ikR_2)[-\alpha^2 k^2(R_2^2 + 1) + k^2c^2 + 2ik\Omega c I_2 + \Omega^2],$$
$$d_{63} = (ikR_1)[-\alpha^2 k^2(R_1^2 + 1) + k^2c^2 + 2ik\Omega c I_1 + \Omega^2],$$
$$d_{64} = (-ikR_2)[-\alpha^2 k^2(R_2^2 + 1) + k^2c^2 + 2ik\Omega c I_2 + \Omega^2],$$
$$d_{65} = 0, \quad d_{66} = 0.$$

6 Numerical results and discussion

In order to show theoretical results and present some numerical results obtained. We investigate the variation of attenuation coefficient (imaginary part of frequency equation root) and Rayleigh wave velocity in a perfectly conducting granular medium under effect of rotation Ω, magnetic field, initial stress P and gravity field g, for computational work. From Fig. 3a shows the effects of rotation Ω on variation of attenuation coefficient with respect to initial stress with the various values of rotation Ω, $k = 10^{-5}$, $\tau_1 = 10^{-2}$, $K = 10^{-3}$, $H_o = 4 \times 10^3$ which it decreases with increasing of initial stress and it increases with increasing of rotation Ω.

Figure 2(a,b) shows variation of the attenuation coefficient (imaginary part of frequency equation root) and velocity of Rayleigh waves (real part of frequency equation root), with respect to initial stress, for the various values of gravity field; we found that

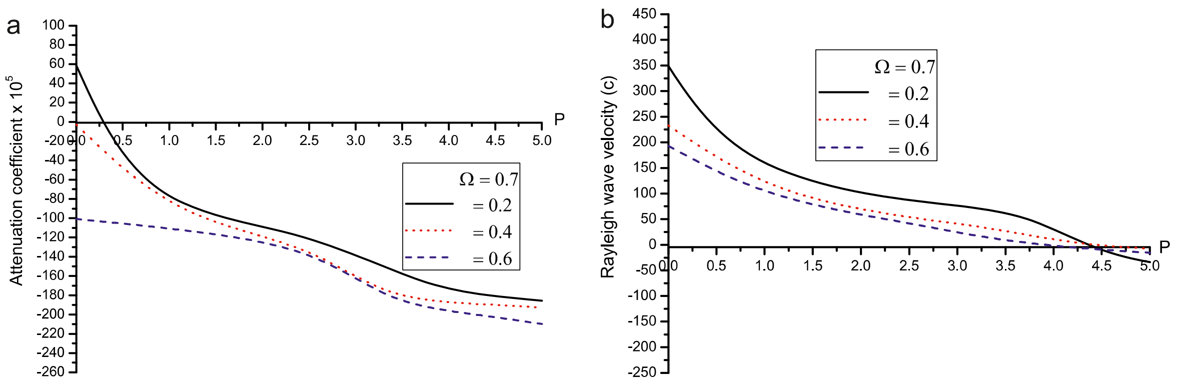

Fig. 2 (**a**) Variation of attenuation coefficient with respect to initial stress, for the various values of gravity field. (**b**) Variation of Rayleigh wave coefficient with respect to initial stress, for the various values of gravity field

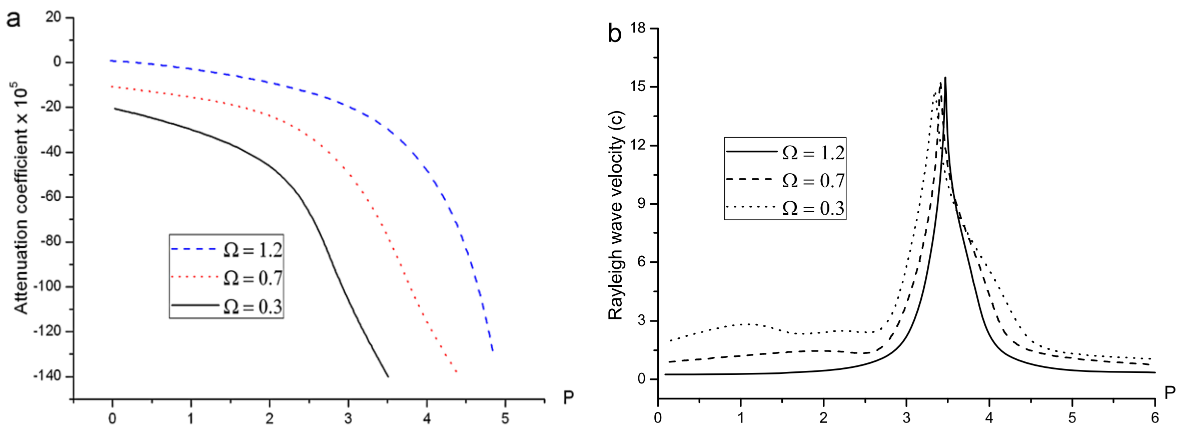

Fig. 3 (**a**) Variation of attenuation coefficient with respect to initial stress, for the various values of rotation Ω. (**b**) Variation of Rayleigh wave coefficient with respect to initial stress, for the various values of rotation Ω

the attenuation coefficient and velocity of Rayleigh waves decreased with increasing values of initial stress P and gravity field g. From Fig. 3b, it's noticed that the various values of Rayleigh wave velocity are fixed for the small values of initial stress P, jump to its maximum value that help an engineers in their applications, and run to its fixed value.

Figure 4(a,b) shows the effects of rotation Ω and magnetic field H_o on variation of attenuation coefficient and Rayleigh wave velocity with respect to initial stress with the various values of magnetic field H_o, $k = 10^{-5}$, $\tau_1 = 10^{-2}$, $K = 10^{-3}$, which it decreases with increasing of initial stress and it decreases with increasing of magnetic field H_o. Figure 5(a,b) shows the effects of rotation Ω and wave number k on variation of attenuation coefficient and Rayleigh wave velocity with respect to initial stress with the various values of wave number k, $\tau_1 = 10^{-2}$, $K = 10^{-3}$, $H_o = 4 \times 10^3$, which it decreases with increasing of initial stress and it decreases with increasing of wave number k. Figure 6(a,b) shows the effects of rotation Ω and relaxation time τ_1 on variation of attenuation coefficient and Rayleigh wave velocity with respect to initial stress with the various values of relaxation time, $k = 10^{-5}$, $K = 10^{-3}$, $H_o = 4 \times 10^3$, which it decreases with increasing of initial stress and it increases with increasing of relaxation time τ_1.

Figure 7(a,b) shows the effects of rotation Ω on attenuation coefficient and Rayleigh wave velocity c with respect to thickness K with the various values of rotation, $k = 10^{-5}$, $\tau_1 = 10^{-2}$, $K = 10^{-3}$, $H_o = 4 \times 10^3$, which it increases with increasing of thickness K and it increases with decreasing of rotation Ω. Figure 8 shows the effects of rotation Ω on

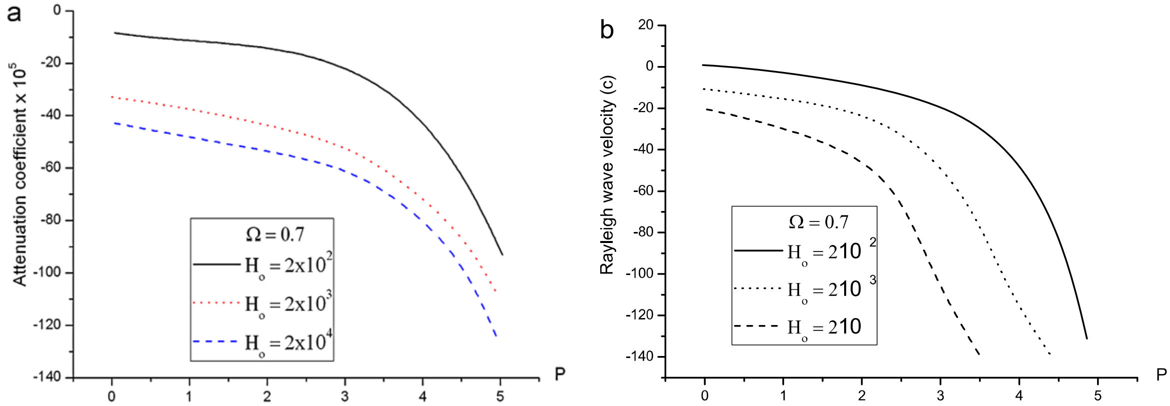

Fig. 4 (**a**) Variation of attenuation coefficient with respect to initial stress, for the various values of magnetic field H_0. (**b**) Variation of Rayleigh wave coefficient with respect to initial stress, for the various values of magnetic field H_0

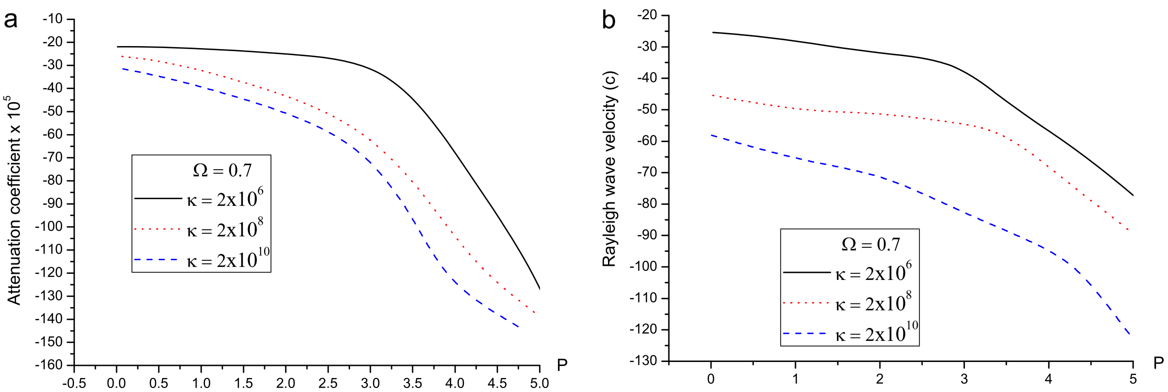

Fig. 5 (**a**) Variation of attenuation coefficient with respect to initial stress, for the various values of wave number. (**b**) Variation of Rayleigh wave coefficient with respect to initial stress, for the various values of wave number

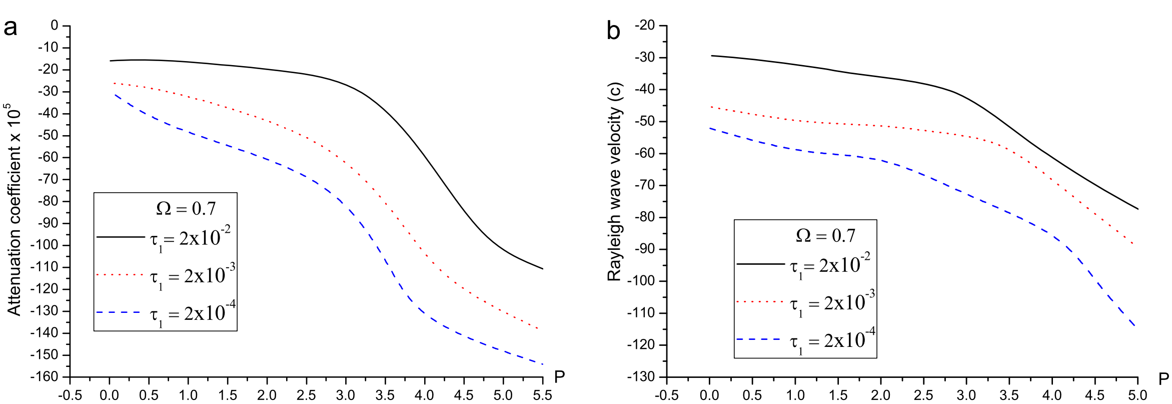

Fig. 6 (**a**) Variation of attenuation coefficient with respect to initial stress, for the various values of relaxation time. (**b**) Variation of Rayleigh wave velocity with respect to initial stress, for the various values of relaxation time

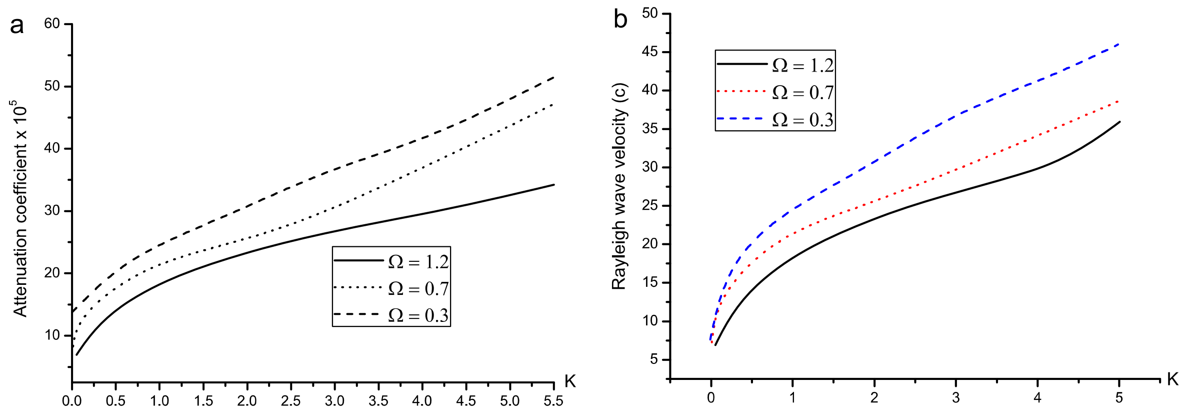

Fig. 7 (**a**) Variation of attenuation coefficient with respect to thickness, for the various values of rotation. (**b**) Variation of Rayleigh wave velocity with respect to thickness, for the various values of rotation Ω

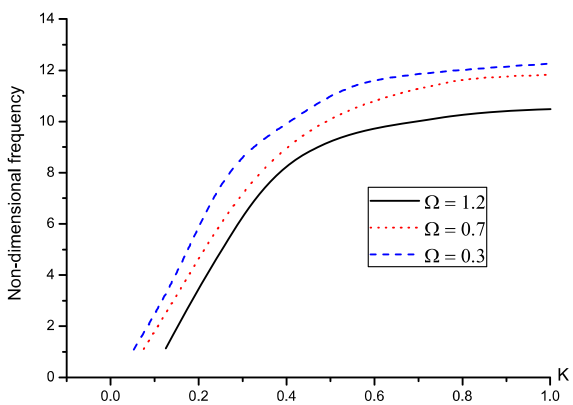

Fig. 8 Variation of non-dimensional frequency with respect to thickness, for the various values of rotation Ω

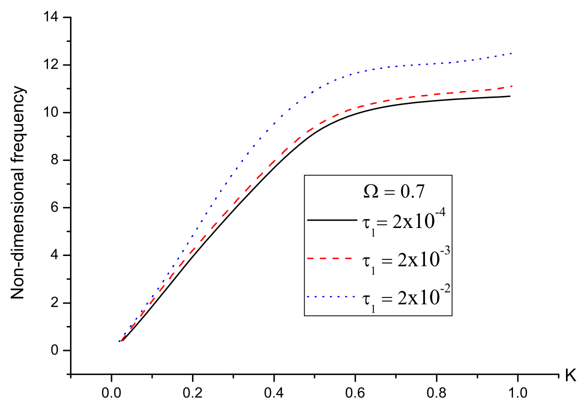

Fig. 10 Variation of non-dimensional frequency with respect to thickness, for the various values of relaxation times

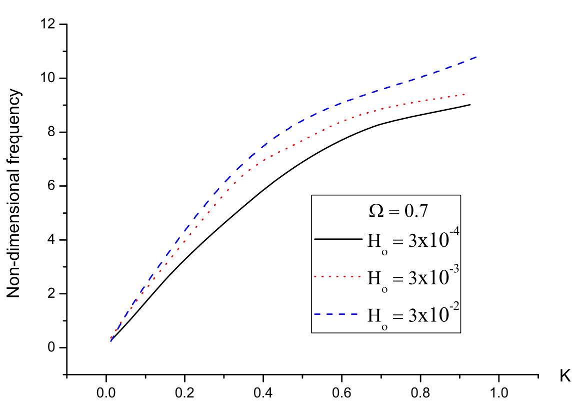

Fig. 9 Variation of non-dimensional frequency with respect to thickness, for the various values of magnetic field H_o

non-dimensional frequency, which it increases with increasing of thickness K and it decreases with increasing the rotation. Figure 9 shows the effects of magnetic field H_o on non-dimensional frequency, which it increases with increasing of thickness K and it increases with increasing the magnetic field. Figure 10 shows the effects of relaxation time on non-dimensional frequency, which it increases with increasing of thickness K and it increases with increasing the relaxation time. Figure 11 shows the effects of wave number on non-dimensional frequency, which it increases with increasing of thickness K and it increases with increasing the wave number. Figure 12 shows the effects of gravity field on non-dimensional frequency, which it has no effect for small values of thickness K and it has Trivial effect for higher values, non-dimensional fre-

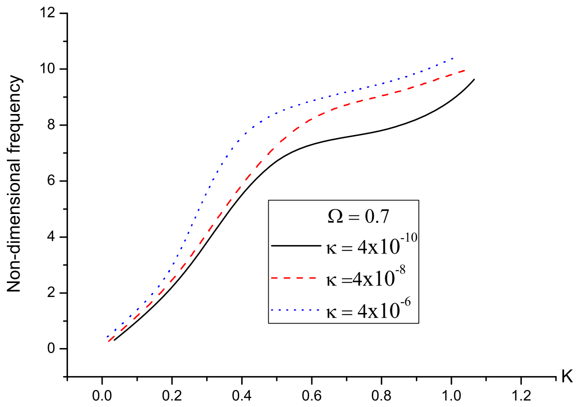

Fig. 11 Variation of non-dimensional frequency with respect to thickness, for the various values of wave number

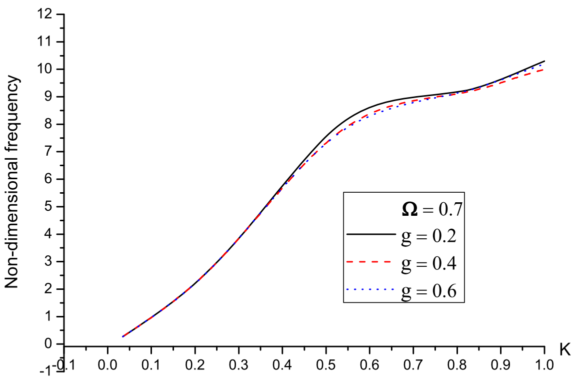

Fig. 12 Variation of non-dimensional frequency with respect to thickness, for the various values of gravity field

quency increases with increasing of thickness K and it increases with increasing the gravity field.

Finally, from the previous results obtained in general case and special cases, it is shown that the rotation, magnetic field, initial stress, gravity field, thermal relaxation times, and granular rotations have utilitarian aspects on Rayleigh wave velocity and attenuation coefficient.

7 Conclusion

The effects of rotation, relaxation times, magnetic field, gravity field, initial stress and wave number are very pronounced on attenuation coefficient, Rayleigh wave velocity and non-dimensional frequency. When the medium is an orthotropic and the effect of rotation is neglected, the frequency equation reduces to previous work. Special cases are investigated. The results indicate that the effect of rotation is very sensitive.

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