Meccanica (2012) 47:1043–1048
DOI 10.1007/s11012-011-9461-x

BRIEF NOTES AND DISCUSSIONS

Similarity solution of MHD boundary layer flow with diffusion and chemical reaction over a porous flat plate with suction/blowing

Krishnendu Bhattacharyya · G.C. Layek

Received: 7 May 2010 / Accepted: 20 June 2011 / Published online: 13 July 2011

Abstract Similarity analysis of diffusion of chemically reactive solute distribution in MHD boundary layer flow of an electrically conducting incompressible fluid over a porous flat plate is presented. The reaction rate of the solute is considered inversely proportional along the plate. Adopting the similarity transformation technique the governing equations are converted into the self-similar ordinary differential equations which are solved by shooting procedure using Runge-Kutta method. For increase of the Schmidt number the solute boundary layer thickness is reduced. Most importantly, the effects of reaction rate and order of reaction on concentration field are of conflicting natures, due to increasing reaction rate parameter the concentration decreases, but for the increase in order of reaction it increases. In presence of chemical reaction, the concentration profiles attain negative value when Schmidt number is large.

Keywords Boundary layer flow · MHD · Chemically reactive solute · Porous flat plate · Suction/blowing

1 Introduction

The flow over a flat plate is a classical problem in fluid mechanics and the boundary layer structure over a flat plate occurs in different engineering processes. In the year 1908, Blasius [1] first considered the steady laminar boundary layer viscous flow over a flat plate. Using the similarity transformation a nonlinear third-order ordinary differential equation was obtained and finally a series solution was found. The numerical solution of above flow problem of Blasius is obtained by Howarth [2]. Rajagopal and Gupta [3] reported an exact solution for the flow of non-Newtonian fluid. In addition, some very important investigations in this direction were made in the articles [4–7].

The mass transfer analysis in boundary layer flow is of great importance in extending the theory of separation processes and chemical kinetics. The diffusion of a chemically reactive species in a laminar boundary layer flow over a flat plate was presented by Chambre and Young [8]. After that many researcher studied the heat and mass transfer with and without considering the chemical reaction in the boundary layer flow. Gebhart and Pera [9] studied the combined buoyancy effects of thermal and mass diffusion on vertical natural convection. Stan [10] discussed the boundary layer flow with chemical surface reaction. Cheng [11] showed the influence of lateral mass flux on free convection boundary layers in porous medium. The mass transfer effects on the flow past an impulsively started infinite vertical plate under several physical conditions

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K. Bhattacharyya (✉) · G.C. Layek
Department of Mathematics, The University of Burdwan,
Burdwan 713104, West Bengal, India
e-mail: krish.math@yahoo.com

K. Bhattacharyya (✉)
e-mail: krishnendu.math@gmail.com

G.C. Layek
e-mail: goralayek@yahoo.com

were analyzed by Soundalgekar [12], Soundalgekar et al. [13], Das et al. [14] and Muthucumaraswamy and Ganesan [15, 16]. The diffusion of a chemically reactive species from a stretching sheet was studied by Andersson et al. [17] and Mahmoud [18]. Fan et al. [19] obtained the similarity solution for diffusion of chemically reactive species in mixed convection flow over a horizontal moving plate. Anjalidavi and Kandasamy [20] studied the effects of chemical reaction, heat and mass transfer on laminar flow along a semi infinite horizontal plate. Mukhopadhyay and Layek [21] analyzed the radiation effects on forced convective flow and heat transfer over a porous plate in a porous medium. Recently, Chamkha et al. [22] explained the chemical reaction effect on unsteady MHD free convective flow of micropolar fluid in a vertical porous plate.

The study of magnetohydrodynamic (MHD) flow in the boundary layer is always interesting. This MHD flow of a viscous fluid over a plate has vast applications in many engineering problems such as plasma studies, MHD power generators, petroleum industries, the boundary layer control in the field of aerodynamics and many others. Riley [23] investigated the flow of an electrically conducting fluid on a vertical plate in presence of strong magnetic field normal to the flow. Watanabe and Pop [24] explained the hall effects on MHD boundary layer flow over a continuous moving flat plate. Anjalidavi and Kandasamy [25] analyzed the effects of chemical reaction on the flow past a semi infinite plate in presence of a magnetic field. Damseh et al. [26] obtained the similarity solution of forced convection flow with magnetic field and thermal radiation effects. Recently, Sharma and Singh [27] studied the MHD free convective flow over an inclined porous plate with variable thermal conductivity.

The aim of the investigation is to find out complete similarity solutions for forced convective MHD boundary layer flow with diffusion of chemically reactive solute over a porous flat plate with suction/blowing. In the analysis, the order of chemical reaction is taken variable and the reaction rate is considered in such a way that it varies inversely along the plate. A complete set of self-similar equations are obtained and solved numerically using well known efficient shooting method for the solution of boundary value problem (BVP). Calculated numerical results are plotted and the different characteristics of the flow field and the solute diffusion are analyzed.

2 Analysis of the problem

Consider the steady two-dimensional MHD flow of viscous incompressible electrically conducting fluid and solute distribution with chemical reaction over a porous flat plate. Using boundary layer approximation, the equations for the flow and the concentration distribution may be written in usual notation as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+\frac{\sigma B^2}{\rho}(u_\infty-u) \quad \text{and} \tag{2}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-R(C-C_\infty)^n, \tag{3}$$

where u and v are velocity components in x- and y-directions respectively, $\upsilon(=\mu/\rho)$ is the kinematic fluid viscosity, ρ is the fluid density, μ is the coefficient of fluid viscosity, σ is the constant electrical conductivity of the fluid, u_∞ is the free stream velocity, C is the concentration, D is the diffusion coefficient, C_∞ is the value of the concentration in the free stream, n is the order of chemical reaction. $B(x)$ is the magnetic field in the y-direction and is given by $B(x)=B_0/(x)^{1/2}$, B_0 is constant and $R(x)$ is the variable reaction rate and is given by $R(x)=LR_0/x$, L is the reference length and R_0 is a constant.

The boundary conditions for the velocity components and the concentration are given by

$$u=0, \quad v=v_w \quad \text{at } y=0; \quad u\to u_\infty \quad \text{as } y\to\infty \tag{4}$$

and

$$C=C_w \quad \text{at } y=0; \quad C\to C_\infty \quad \text{as } y\to\infty, \tag{5}$$

where C_w is the plate concentration and v_w is prescribed suction/blowing at the porous plate and is given by $v_w=v_0/(x)^{1/2}$, v_0 is a constant with $v_0<0$ corresponds to suction and $v_0>0$ corresponds blowing.

We now introduce the following similarity transformations:

$$\begin{aligned}&\psi=\sqrt{u_\infty\upsilon x}\,f(\eta),\\ &C=C_\infty+(C_w-C_\infty)\phi(\eta) \quad \text{and}\\ &\eta=y\sqrt{u_\infty/\upsilon x},\end{aligned} \tag{6}$$

where ψ is the stream function defined in the usual notation as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ and η is the similarity variable.

In view of (6), equation (1) is identically satisfied and (2) and (3) reduce to the following self-similar equations

$$f''' + \frac{1}{2} f f'' + M(1 - f') = 0 \quad \text{and} \tag{7}$$

$$\phi + \frac{1}{2} Sc f \phi' - Sc\beta\phi^n = 0, \tag{8}$$

where $M = \sigma B_0^2/\rho u_\infty$ is the magnetic parameter, $Sc = \upsilon/D$ is the Schmidt number and $\beta = LR_0(C_w - C_\infty)^{n-1}/u_\infty$ is the reaction rate parameter.

The boundary conditions finally become

$$\begin{aligned} &f(\eta) = S, \quad f'(\eta) = 0 \quad \text{at } \eta = 0; \\ &f'(\eta) \to 1 \quad \text{as } \eta \to \infty \end{aligned} \tag{9}$$

and

$$\phi(\eta) = 1 \quad \text{at } \eta = 0; \quad \phi(\eta) \to 0 \quad \text{as } \eta \to \infty, \tag{10}$$

where $S = (-2v_w/u_\infty)(Re_x)^{1/2} = -2v_0/(u_\infty\upsilon)^{1/2}$, $Re_x = u_\infty x/\upsilon$ is the local Reynolds number; $S > 0$ (i.e. $v_0 < 0$) corresponds to suction and $S < 0$ (i.e. $v_0 > 0$) corresponds to blowing.

For the present problem, the physical parameter with engineering significance is the local Sherwood number which is proportional to the rate of solute transfer from the plate and is given by

$Sh_x = xM_w/(D(C_w - C_\infty))$ where

$M_w = D[\partial C/\partial y]_{y=0}$.

Using the relation (10) we finally get $Sh_x/Re_x = -\phi(0)$.

3 Numerical solution and discussions

The nonlinear coupled differential equations (7) and (8) along with the boundary conditions (9) and (10) form a boundary value problem and are solved using shooting method, by converting it into an initial value problem. In this method we have to choose a suitable finite value of $\eta \to \infty$, say η_∞. The method is very well described in the paper by Bhattacharyya et al. [28] and will not repeat here. We obtain the value of skin friction coefficient in absence of magnetic field and suction/blowing as $f''(0) = 0.332058$ which is well matched with the standard result $f''(0) = 0.33206$ obtained by Howarth [2].

The numerical computations of above self-similar equations with proper boundary conditions are performed for some sets of values of parameters involved in the equations viz. the suction/blowing parameter S, the Schmidt number Sc, the reaction rate parameter β and the order of chemical reaction n. In the analysis the magnetic parameter is kept fixed, $M = 0.3$. The computed results are depicted in some figures and physical reasons for the behavior are also rendered.

The effects of externally applied suction/blowing through the porous plate on the concentration profiles are presented in Fig. 1. For the increase of suction at the plate, the concentration at a point decreases. On

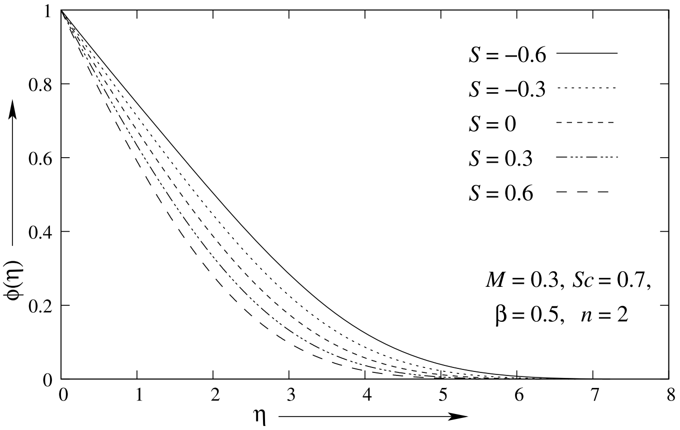

Fig. 1 Dimensionless reactive concentration profiles $\phi(\eta)$ for several values of S

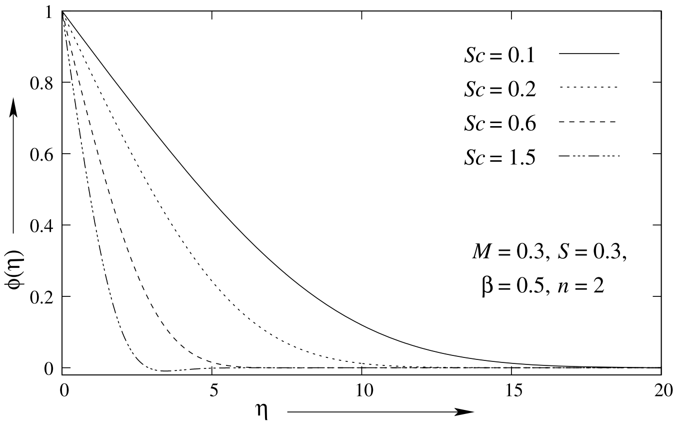

Fig. 2 Dimensionless reactive concentration profiles $\phi(\eta)$ for several values of *Sc*

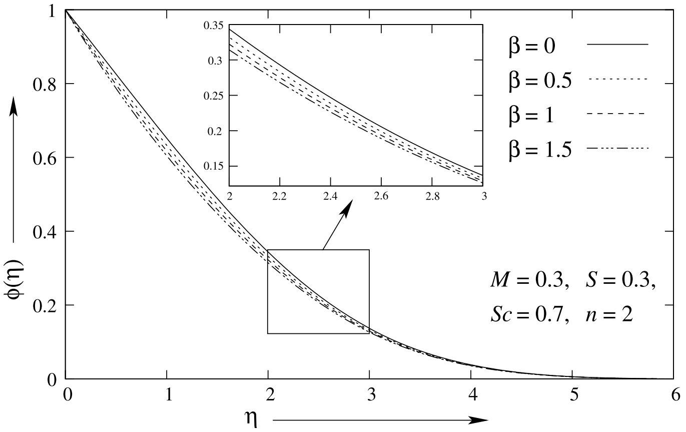

Fig. 3 Dimensionless reactive concentration profiles $\phi(\eta)$ for several values of β

the other hand, reverse effects are noticed for applied blowing i.e. with increasing blowing, the concentration increases. The above effects are physically significant and the reasons behind this are that the thickness of solute boundary layers decreases with suction and increases with blowing.

In Fig. 2, the effect of the Schmidt number on the concentration distribution is exhibited. The concentration $\phi(\eta)$ at a fixed η quickly decreases with increasing values of *Sc*, because the Schmidt number acts to reduce the solute boundary layer thickness. Moreover, after certain increment of *Sc* the value of solute profiles exhibit negative value for some η and this can be seen for the curve $Sc = 1.5$ in Fig. 2. The physical reason behind this negative nature of concentration profiles at large Schmidt number is the chemical reaction effect.

Next, we shall discuss the variation of reactive solute distribution for different values of the reaction rate parameter β. In Fig. 3, the dimensionless concentration profiles $\phi(\eta)$ for increment of β are represented and from this figure it reveals that the value of the concentration decreases with increasing values of β. But for large values of η, the effect dies out and so the concentration boundary layer thickness remains almost the same.

Now, we represent the effect of the increasing order of chemical reaction n on the solute distribution and the solute profiles $\phi(\eta)$ for several value of n are shown in Fig. 4. An observation is made from the figure that the value of the dimensionless concentration at a point increases in response to higher value of reaction order n. Hence the effect of order of the reaction is opposite to that of the rate of reaction.

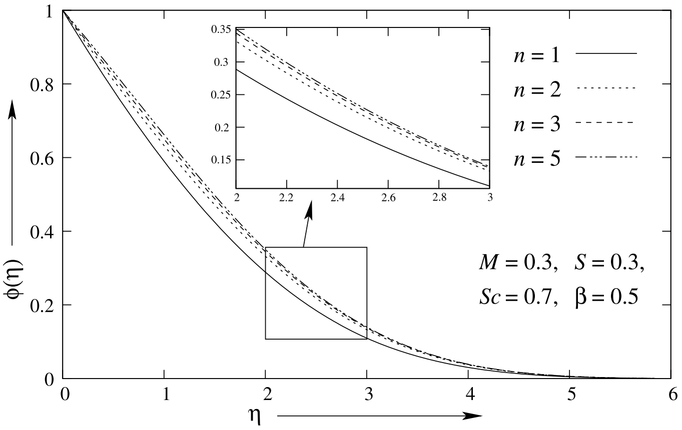

Fig. 4 Dimensionless reactive concentration profiles $\phi(\eta)$ for several values of n

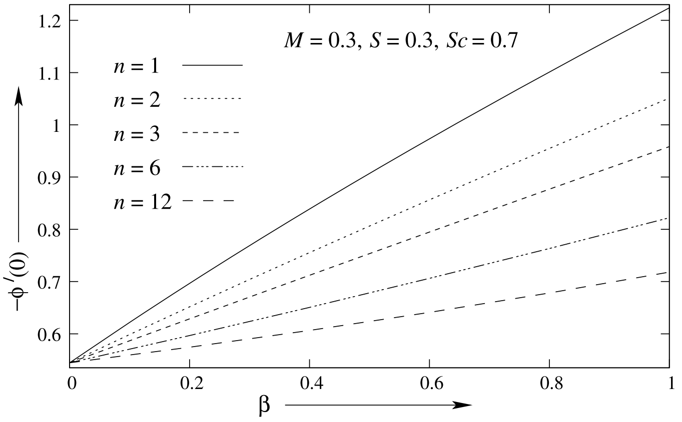

Fig. 5 The value of $-\phi'(0)$ against β for several value of n

Finally, a clear view of the deviation of value of $Sh_x/Re_x[= -\phi(0)]$ which is proportional to the rate of solute transfer with increment of the reaction rate parameter β and the order of the reaction n can be observed from Fig. 5. For the increment of β the value of $-\phi(0)$ increases, while $-\phi(0)$ decreases when n increases and it is also noticed that the rate of increment with β is reduced when n increases. Thus the rate of solute transfer depends much on the order of the chemical reaction.

4 Concluding remarks

In this investigation, our objective is to obtain the similarity solution of forced convective MHD boundary layer flow of an electrically conducting fluid with diffusion of chemically reactive species with variable reaction order over a porous flat plate. The reaction rate of the reactive species is taken inversely proportional along the plate. Using similarity transformation, the governing equations are transformed into self-similar equations. The numerical solutions of the self-similar equations are obtained using shooting technique with the help of forth order Runge-Kutta method. The following remarks can be concluded from the analysis:

(a) Due to the increase of suction at the plate the solute boundary layer thicknesses reduces. On the other hand, boundary layer thickness becomes thicker for the blowing.
(b) The reactive concentration profile and the concentration boundary layer thickness decreases with the increasing Schmidt number.

(c) Interestingly, in presence of chemical reaction, for large value of Schmidt number, the concentration attains negative values in certain range.

(d) Due to increase in the order of reaction the mass transfer from the plate reduces, whereas increase in the rate of reaction causes to enhance the mass transfer rate.

Acknowledgements Authors are very much thankful to the reviewers for their contractive suggestions. One of the authors (K. Bhattacharyya) gratefully acknowledges the financial support of National Board for Higher Mathematics (NBHM), DAE, Mumbai, India for pursuing this work.

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