

MHD flow and heat transfer for the upper-convected Maxwell fluid over a stretching sheet

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Abstract In the present work, the effect of MHD flow and heat transfer within a boundary layer flow on an upper-convected Maxwell (UCM) fluid over a stretching sheet is examined. The governing boundary layer equations of motion and heat transfer are non-dimensionalized using suitable similarity variables and the resulting transformed, ordinary differential equations are then solved numerically by shooting technique with fourth order Runge–Kutta method. For a UCM fluid, a thinning of the boundary layer and a drop in wall skin friction coefficient is predicted to occur for higher the elastic number. The objective of the present work is to investigate the effect of Maxwell parameter β , magnetic parameter Mn and Prandtl number Pr on the temperature field above the sheet.

Keywords UCM fluid · Prandtl number · Magnetic parameter · Elastic parameter · Boundary layer · Stretching sheet

Nomenclature

b stretching rate [s^{-1}]

x horizontal coordinate [m]
 y vertical coordinate [m]
 u horizontal velocity component [$m s^{-1}$]
 v vertical velocity component [$m s^{-1}$]
 T temperature [K]
 t time [s]
 C_p specific heat [$J kg^{-1} K^{-1}$]
 f dimensionless stream function
 Pr Prandtl number, $\frac{\mu C_p}{k}$
 M^2 Magnetic parameter, $\frac{\sigma B_0^2}{\rho b}$
 q heat flux, $-k \frac{\partial T}{\partial y}$ [$J s^{-1} m^{-2}$]
 Nu_x local Nusselt number

Greek symbols

β Maxwell parameter
 η similarity variable, (4)
 θ dimensionless temperature
 k thermal diffusivity [$m^2 s^{-1}$]
 μ dynamic viscosity [$kg m^{-1} s^{-1}$]
 ν kinematic viscosity [$m^2 s^{-1}$]
 ρ density [$kg m^{-3}$]
 τ shear stress, $\mu \partial u / \partial y$ [$kg m^{-1} s^{-2}$]
 ψ stream function [$m^2 s^{-1}$]

Subscripts

x local value

Superscripts

' first derivative
" second derivative
''' third derivative

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1 Introduction

The studies of boundary layer flows of Newtonian and non-Newtonian fluids over a stretching surface have received much attention because of their extensive applications in the field of metallurgy and chemical engineering and particularly, in the extrusion of polymer sheet, from a die or in the drawing of plastic films. During the manufacture of these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. Such investigations of magneto-hydrodynamic (MHD) flow are very important industrially and have applications in different areas of research such as petroleum production and metallurgical processes. The properties of the end product depends greatly on the rate of cooling involved in these processes, the rate of cooling, and the desired properties of the end product can be controlled by the use of electrically conducting fluids and application of magnetic field [11]. The magnetic field has been used in the process of purification of molten metals from non-metallic inclusions.

The study of flow and heat transfer caused by a stretching surface is of great importance in many manufacturing processes such as in extrusion process, glass blowing, hot rolling, manufacturing of plastic and rubber sheets, crystal growing, continuous cooling and fibers spinning. Water is amongst the most widely used coolant liquid. In all these cases, a study of flow field and heat transfer can be of significant importance because the quality of the final product depends to a large extent on the skin friction coefficient and the surface heat transfer rate.

Sarpakaya [1] was the first researcher to study the MHD flow of a non-Newtonian fluid. Prandtl's boundary layer theory proved to be of great use in Newtonian fluids as Navier-Stokes equations can be converted into much simplified boundary layer equation which is easier to handle.

Crane [2] was the first among others to consider the steady two-dimensional flow of a Newtonian fluid driven by a stretching elastic flat sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point. Subsequently, various aspects of the flow and/or heat transfer problems for stretching surfaces moving in the finite fluid medium have been explored in many investigations, e.g. Refs. [3–9].

In a typical sheet production process the extrudate starts to solidify as soon as it exits from the die. The

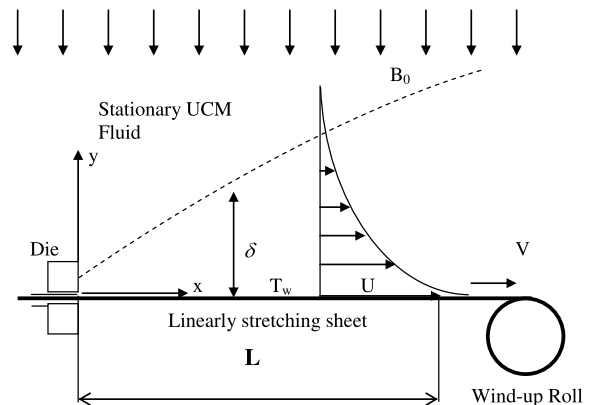


Fig. 1 Schematic showing flow above a stretching sheet

sheet is then brought into a required shape by a wind-up roll upon solidification (see Fig. 1). An important aspect of the flow is the extensibility of the sheet which can be employed effectively to improve its mechanical properties along the sheet. To further improve sheet mechanical properties, it is necessary to control its cooling rate. Physical properties of the cooling medium, e.g., its thermal conductivity, can play a decisive role in this regard [10]. The success of the whole operation can be argued to depend also on the rheological properties of the fluid above the sheet as it is the fluid viscosity which determines the (drag) force required to pull the sheet.

Generally it is observed that rheological properties of a material are specified by their constitutive equations. The simplest constitute equation for a fluid is a Newtonian one and the governing equation for such a fluid is the Navier-Stokes equation. But in many fields, such as food industry, drilling operations and bio-engineering, the fluids, rather synthetic or natural or mixtures of different stuffs such as water, particles, oils, red cells and other long chain of molecules. This combination imparts strong non-Newtonian characteristics to the resulting liquids. In these cases, the fluids have been treated as non-Newtonian fluids.

Although there is no doubt about the importance of the theoretical studies cited above, but they are not above reproach. For example, the viscoelastic fluid models used in these works are simple models such as second-order model and/or Watler's B model which are known to be good only for weakly elastic fluids subject to slow and/or slowly-varying flows [12]. To this should be added the fact that these two fluid models are known to violate certain rules of thermodynam-

ics [13]. A non-Newtonian second grade fluid does not give meaning full results for highly elastic fluids (polymer melts) which occur at high Deborah numbers [14, 15]. Therefore, the significance of the results reported in the above works are limited, at least as far as polymer industry is concerned. Obviously, for the theoretical results to become of any industrial significance, more realistic viscoelastic fluid models such as upper-convected Maxwell model or Oldroyd-B model should be invoked in the Analysis [24]. Indeed, these two fluid models have recently been used to study the flow of viscoelastic fluids above stretching and non-stretching sheets but with no heat transfer effects involved [16–18]. Hsiao [25, 26] studied the mixed convection of MHD viscoelastic fluid past a porous wedge and also analyzed the electromagnetic effect and non-uniform heat source effect on viscoelastic boundary layer flow. Ishak et al. [27] studied the boundary layer flow and heat transfer over an unsteady stretching vertical surface and Babaelahi et al. [28] investigated the effect of viscous and ohmic dissipations on viscoelastic MHD flow boundary layer over a stretching surface. Hayat et al. [29] studied the MHD stagnation-point flow of upper convected Maxwell fluid over stretching sheet.

The researcher [21] have done the work related to UCM fluid by using HAM-method and the researchers [14, 19, 20] have studied UCM fluid by using numerical methods with no heat transfer.

It is recognized that there are many other methods that could be considered in order to describe some reasonable solutions for this particular type of problem. But to the best of our knowledge, no numerical solution has previously been investigated for the combined effect of MHD flow and heat transfer of a UCM fluid above a stretching sheet. The focal point in the present work is to investigate same numerically.

2 Mathematical formulation

The equations governing the transfer of heat and momentum between a stretching sheet and the surrounding fluid (see Fig. 1) can be significantly simplified if it can be assumed that boundary layer approximations are applicable to both momentum and energy equations. Although this theory is incomplete for viscoelastic fluids, but has been recently discussed by Renardy [17], it is more plausible for Maxwell fluids as compared to other viscoelastic fluid models. For

MHD flow of an incompressible Maxwell fluids resting above a stretching sheet.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \lambda \left[u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right] = v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u, \tag{2}$$

where B_0 , is the strength of the magnetic field, ν is the kinematic viscosity of the fluid and λ is the relaxation time Parameter of the fluid. As to the boundary conditions, we are going to assume that the sheet is being stretched linearly. Therefore the appropriate boundary conditions on the flow are

$$u = Bx, \quad v = 0 \quad \text{at } y = 0, \tag{3}$$

$$u \rightarrow 0 \quad \text{as } y \rightarrow \infty,$$

where $B > 0$, is the stretching rate. Here x and y are, respectively, the directions along and perpendicular to the sheet, u and v are the velocity components along x and y directions. The flow is caused solely by the stretching of the sheet, the free stream velocity being zero. Equations (1) and (2) admit a self-similar solution of the form

$$u = Bx f'(\eta), \quad v = \sqrt{\nu B} f(\eta), \tag{4}$$

$$\eta = \left(\frac{B}{\nu} \right)^{\frac{1}{2}} y,$$

where superscript $'$ denotes the differentiation with respect to η . Clearly u and v satisfy (1) identically. Substituting these new variable in (2), we have

$$f''' - M^2 f' - (f')^2 + f f'' + \beta(2f f' f'' - f^2 f''') = 0. \tag{5}$$

Here $M^2 = \frac{\sigma B_0^2}{\rho B}$ and $\beta = \lambda B$ are magnetic and Maxwell parameters respectively.

The boundary conditions (3) become

$$f'(0) = 1, \quad f(0) = 0 \quad \text{at } \eta = 0 \tag{6}$$

$$f'(\infty) \rightarrow 0, \quad f''(0) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty$$

3 Heat transfer analysis

By using usual boundary layer approximations, the equation of the energy for two-dimensional flow is given by

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2}, \tag{7}$$

where T , ρ , c_p and k are, respectively, the temperature, the density, specific heat at constant pressure and the thermal conductivity is assumed to vary linearly with temperature. We define the dimensionless temperature as

$$\left. \begin{aligned} \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty} && \text{(PST Case)} \\ g(\eta) &= \frac{T - T_\infty}{b\left(\frac{x}{l}\right)^2 \frac{1}{k} \sqrt{\frac{\nu}{b}}} && \text{(PHF Case)} \end{aligned} \right\}. \tag{8}$$

The thermal boundary conditions depend upon the type of the heating process being considered. Here, we are considering two general cases of heating namely, (1) Prescribed surface temperature and (2) prescribed wall heat flux, varying with the distance.

3.1 Prescribed surface temperature case (PST case)

For this heating process, the prescribed temperature is assumed to be a quadratic function of x is given by

$$\left. \begin{aligned} u &= Bx, & v &= 0, \\ T &= T_w(x) = T_\infty + A\left(\frac{x}{l}\right)^2 & \text{at } y = 0 \\ u &= 0, & T &\rightarrow T_\infty \text{ as } y \rightarrow \infty \end{aligned} \right\}, \tag{9}$$

where T is temperature of the fluid, T_w is surface temperature, T_∞ is the ambient temperature and l is the characteristic length. A is some constant and $B > 0$ is linear stretching constant.

Using (4), (7) and (10), the dimensionless temperature variable θ given by first equation of (8), satisfies

$$Pr[2f'\theta - \theta'f] = \theta'', \tag{10}$$

where $Pr = \frac{\mu c_p}{k}$ is the Prandtl number. The corresponding boundary conditions are

$$\left. \begin{aligned} \theta(\eta) &= 1 & \text{at } \eta = 0 \\ \theta(\eta) &= 0 & \text{as } \eta \rightarrow \infty. \end{aligned} \right\} \tag{11}$$

3.2 Prescribed heat flux case (PHF case)

The power law heat flux on the wall surface is considered to be a quadratic power of x in the form

$$u = Bx, \quad -k\left(\frac{\partial T}{\partial y}\right)_w = q_w = D\left(\frac{x}{l}\right)^2 \text{ at } y = 0 \tag{12}$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty \text{ as } y \rightarrow \infty.$$

Here D is constant. Using (4), (7) and (12), the dimensionless temperature variable g given by second condition of (8), satisfies

$$Pr'[2f'g - g'f] = g''. \tag{13}$$

The corresponding boundary conditions are

$$\left. \begin{aligned} g'(\eta) &= -1 & \text{at } \eta \rightarrow 0 \\ g(\eta) &= 0 & \text{at } \eta \rightarrow \infty. \end{aligned} \right\} \tag{14}$$

The rate of heat transfer between the surface and the fluid conventionally expressed in dimensionless form as a local Nusselt number and is given by

$$Nu_x \equiv -\frac{x}{T_w - T_\infty} \left(\frac{\partial T}{\partial y}\right)_{y=0} = -x\sqrt{Re} \theta'(0). \tag{15}$$

Similarly, momentum equation is simplified and exact analytic solutions can be derived for the skin-friction coefficient or frictional drag coefficient as

$$C_f \equiv \frac{(\mu \frac{\partial u}{\partial y})_{y=0}}{\rho(Bx)^2} = -f''(0) \frac{1}{\sqrt{Re_x}}, \tag{16}$$

where $Re_x = \frac{\rho Bx^2}{\mu}$ is known as local Reynolds number.

4 Numerical solution

We adopt the most effective shooting method (see Refs. [22, 23]) with fourth order Runge–Kutta integration scheme to solve boundary value problems in PST and PHF cases mentioned in the previous section. The non-linear equations (5) and (10) in the PST case are transformed into a system of five first order differential

equations as follows:

$$\begin{aligned} \frac{df_0}{d\eta} &= f_1, \\ \frac{df_1}{d\eta} &= f_2, \\ \frac{df_2}{d\eta} &= \frac{(f_1)^2 + M^2 f_1 - f_0 f_2 - 2\beta f_0 f_1 f_2}{1 - \beta f_0^2}, \\ \frac{d\theta_0}{d\eta} &= \theta_1, \\ \frac{d\theta_1}{d\eta} &= Pr[2f_1\theta_0 - \theta_1 f_0]. \end{aligned} \tag{17}$$

Subsequently the boundary conditions in (6) and (11) take the form,

$$\begin{aligned} f_0(0) = 0, \quad f_1(0) = 1, \quad f_1(\infty) = 0, \\ \theta_0(0) = 0, \quad \theta_0(\infty) = 0. \end{aligned} \tag{18}$$

Here $f_0 = f(\eta)$ and $\theta_0 = \theta(\eta)$. Aforementioned boundary value problem is first converted into an initial value problem by appropriately guessing the missing slopes $f_2(0)$ and $\theta_1(0)$. The resulting IVP is solved by shooting method for a set of parameters appearing in the governing equations with a known value of $f_2(0)$ and $\theta_1(0)$. The convergence criterion largely depends on fairly good guesses of the initial conditions in the shooting technique. The iterative process is terminated until the relative difference between the current iterative values of $f_2(0)$ matches with the previous iterative value of $f_2(0)$ up to a tolerance of 10^{-6} . Once the convergence is achieved we integrate the resultant ordinary differential equations using standard fourth order Runge–Kutta method with the given set of parameters to obtain the required solution.

5 Results and discussion

The exact solution do not seem feasible for a complete set of equations (5) and (10) because of the non linear form of the momentum and thermal boundary layer equations. This fact forces one to obtain the solution of the problem numerically. Appropriate similarity transformation is adopted to transform the governing partial differential equations of flow and heat transfer into a system of non-linear ordinary differential equations. The resultant boundary value problem

Table 1 Values of $f''(0)$ for various parametric values of Maxwell parameter β

β	$f''(0)$		
	Sadeghy et al. [19]	Present results	Present results
	$M = 0.0$		$M = 0.2$
0.0	-1.0000	-0.999962	-1.095445
0.2	-1.0549	-1.051948	-1.188270
0.4	-1.10084	-1.101850	-1.275878
0.6	-1.0015016	-1.150163	-1.358733
0.8	-1.19872	-1.196692	-1.437369
1.2	-	-1.285257	-1.512280
1.6	-	-1.368641	-1.095445
2.0	-	-1.447617	-1.188270

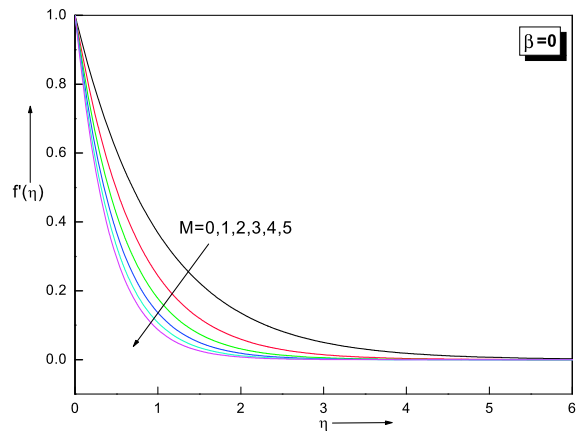


Fig. 2 The effect of MHD parameter M on u -velocity component f' at $\beta = 0$

is solved by the efficient shooting method. Present results are compared with some of the earlier published results in some limiting cases are shown in Table 1. The effect of several parameters controlling the velocity and temperature profiles are shown graphically and discussed briefly.

Figures 2 and 3 show the effect of magnetic parameter, M , in the absence of Maxwell parameter (at $\beta = 0$) on the velocity profile above the sheet. An increase in the magnetic parameter leads in decrease of both u and v velocity components at any given point above the sheet. This is due to the fact that applied transverse magnetic field produces a drag in the form of Lorentz force thereby decreasing the magnitude of velocity. The drop in horizontal velocity as a consequence of increase in the strength of magnetic field is

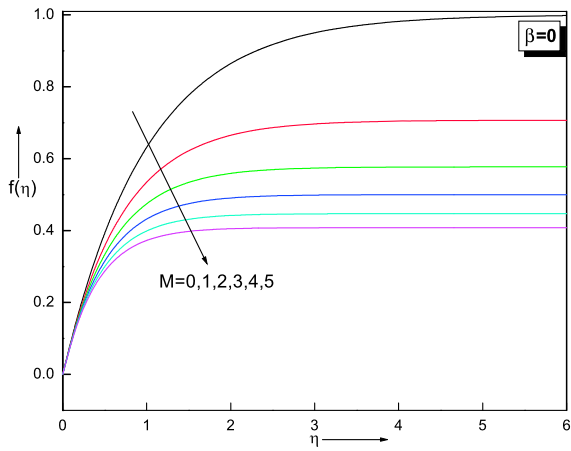


Fig. 3 The effect of MHD parameter M on v -velocity component f' at $\beta = 0$

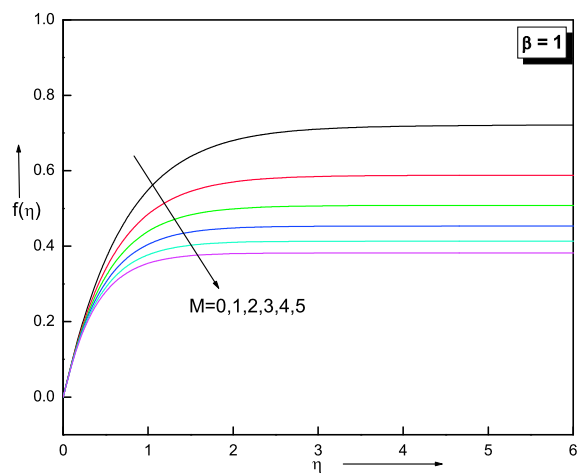


Fig. 5 The effect of MHD parameter M on v -velocity component f' at $\beta = 1$

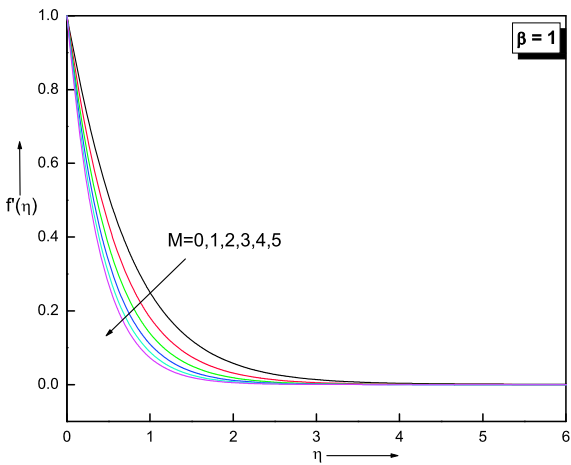


Fig. 4 The effect of MHD parameter M on u -velocity component f' at $\beta = 1$

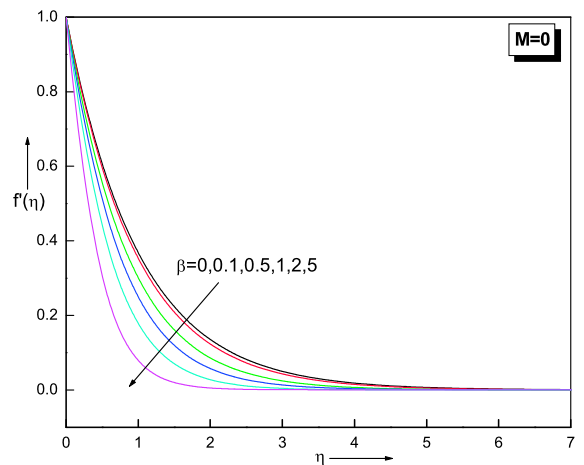


Fig. 6 The effect of elastic parameter β on u -velocity component f' at $M = 0$

observed. Figures 4 and 5 show the same effect as said above, but, in the presence of Maxwell parameter (at $\beta = 1$). That is, an increase in the magnetic parameter leads in decrease of fluid velocity at any given point above the sheet.

Figures 6 and 7 show the effect of Maxwell parameter β , in the absence of magnetic number (at $M = 0$) on the velocity profile above the sheet. An increase in the Maxwell parameter is noticed to decrease both u - and v -velocity components at any given point above the sheet.

Figures 8 and 9 show the effect of Maxwell parameter on the temperature profiles above the sheet for both PST and PHF cases. An increase in the Maxwell pa-

rameter is seen to decrease the fluid temperature $\theta(\eta)$ and $g(\eta)$ above the sheet. That is, the thermal boundary layer becomes thicker for larger the magnetic parameter.

Figures 10 and 11 show the effect of magnetic parameter on the temperature profiles above the sheet for both PST and PHF cases. An increase in the magnetic parameter is seen to increase the fluid temperature $\theta(\eta)$ above the sheet. That is, the thermal boundary layer becomes thicker for larger the magnetic parameter.

Figures 12 and 13 show the effect of Prandtl number on the temperature profiles above the sheet for

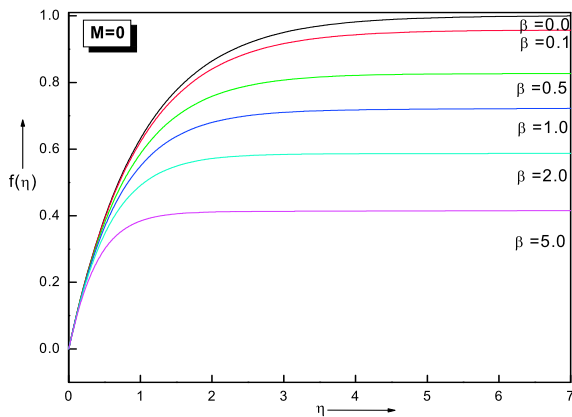


Fig. 7 The effect of elastic parameter β on v -velocity component f' at $M = 0$

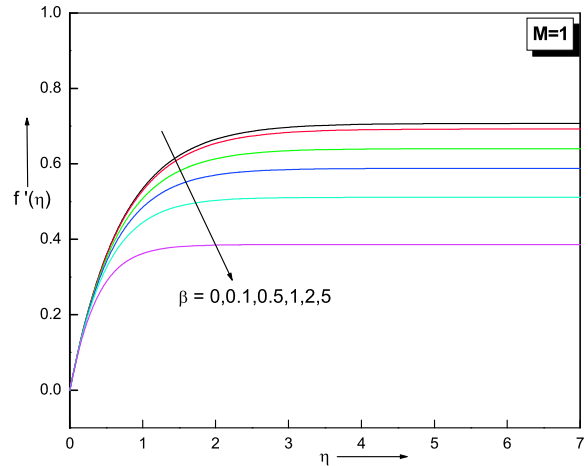


Fig. 9 The effect of elastic parameter β on v -velocity component f' at $M = 1$

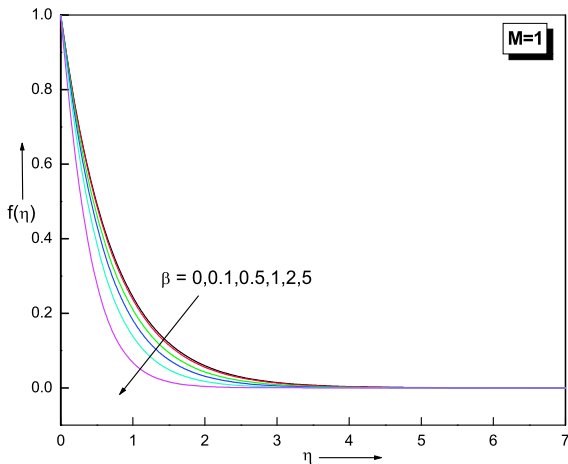


Fig. 8 The effect of elastic parameter β on u -velocity component f at $M = 1$

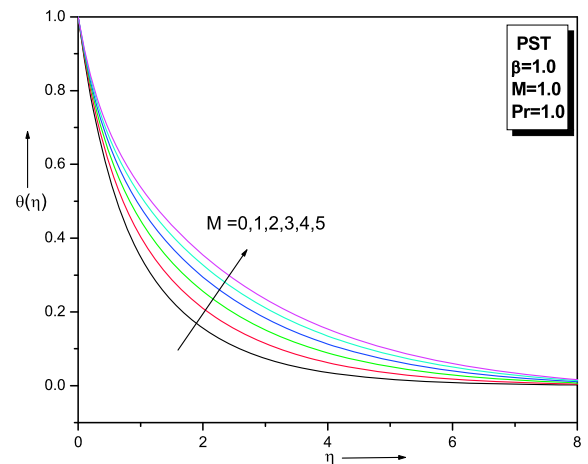


Fig. 10 The effect of MHD parameter M on temperature profiles $\theta(\eta)$

both PST and PHF cases. An increase in the Prandtl number is seen to decrease the fluid temperature $\theta(\eta)$ above the sheet. That is not surprising realizing the fact that the thermal boundary becomes thinner for, larger the Prandtl number. Therefore, with an increase in the Prandtl number the rate of thermal diffusion drops. This scenario is valid for both PST and PHF cases. For the PST case the dimensionless wall temperature is unity for all parameter values. However, it may be other than unity for the PHF case because of its differing thermal boundary conditions.

A drop in skin friction as investigated in this paper has an important implication that in free coating operations, elastic properties of the coating formulations may be beneficial for the whole process. Which means

that less force may be needed to pull a moving sheet at a given withdrawal velocity or equivalently higher withdrawal speeds can be achieved for a given driving force resulting in, increase in the rate of production [24].

The accuracy of the results have been validated by comparing the results of skin friction with the reported results of Sadeghy et al. [19], these results agree very well as seen in Table 1 and results of heat transfer rate is also tabulated in Table 2, we can see that, as increase in parametric values of Pr wall temperature gradient is decreasing but as increase in parametric values of Maxwell parameter the wall temperature is increasing.

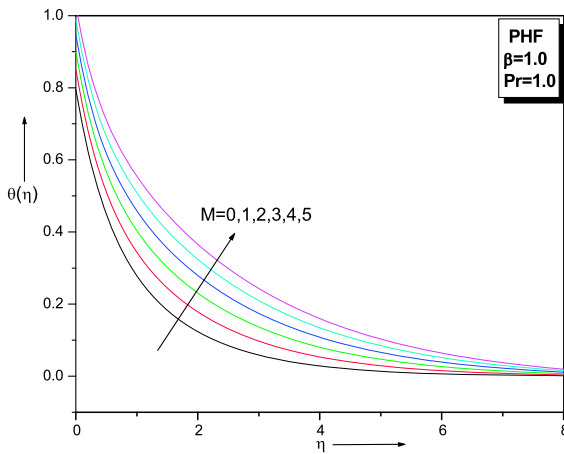


Fig. 11 The effect of MHD parameter M on temperature profiles $\theta(\eta)$

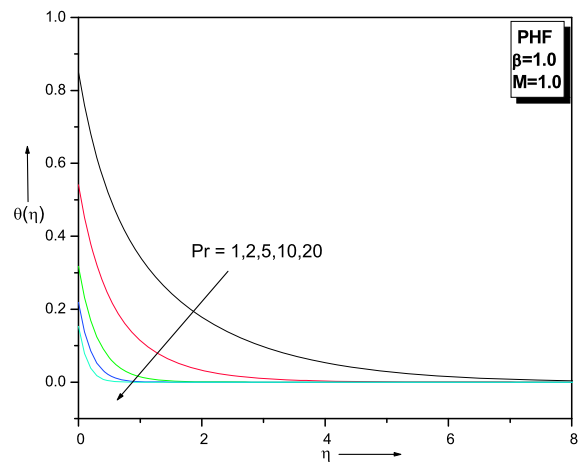


Fig. 13 The effect of Prandtl number Pr on temperature profiles $\theta(\eta)$

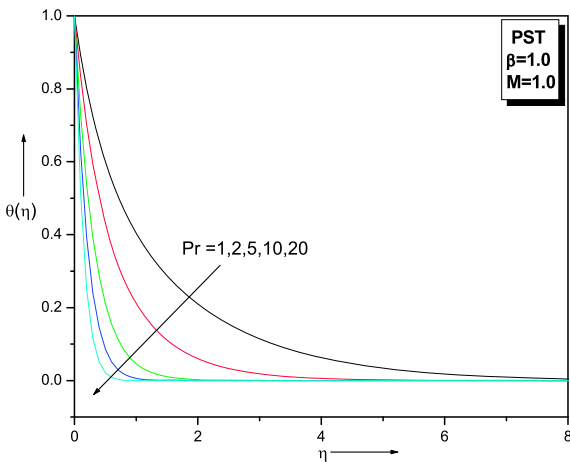


Fig. 12 The effect of Prandtl number Pr on temperature profiles $\theta(\eta)$

Table 2 Wall temperature gradient $\theta'(0)$ values for different physical parameters

Pr	β	$-\theta'(0)$ (PST case)
1.0		1.000174
2.0		1.523090
3.0	0.0	1.923609
4.0		2.260770
5.0		2.557560
1.0		0.9800923
2.0		1.504233
3.0	0.2	1.905729
4.0		2.243542
5.0		2.540800
1.0	0.0	1.000174
	0.2	0.980092
	0.4	0.960788
	0.6	0.942318
	0.8	0.924698

6 Conclusions

The present work analyses, the MHD flow and heat transfer within a boundary layer of UCM fluid above a stretching sheet. Numerical results are presented to illustrate the details of the flow and heat transfer characteristics and their dependence on the various parameters.

We observe that, when the magnetic parameter increases the velocity decreases, also, for increase in Maxwell parameter, there is decreases in velocity. The effect of magnetic field and Maxwell parameter on the UCM fluid above the stretching sheet is to suppress the

velocity field, which in turn causes the enhancement of the temperature field.

Also it is observed that, an increase of Prandtl number results in decreasing thermal boundary layer thickness and more uniform temperature distribution across the boundary layer in both the PST and PHF cases. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat

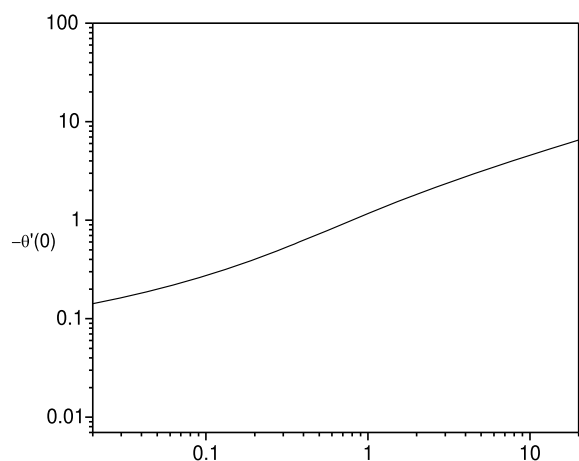


Fig. 14 Dimensionless heat flux $-\theta'(0)$ at the sheet vs Prandtl number

is able to diffuse away from the heated surface more rapidly than for higher values of Pr .

The dimensionless wall temperature gradient $-\theta'(0)$ takes a higher value at large Prandtl number Pr (see Fig. 14).

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References

- Sarpakaya T (1961) Flow of non-Newtonian fluids in a magnetic field. *AIChE J* 7:324–328
- Crane LJ (1970) Flow past a stretching plate. *Z Angew Math Phys* 21:645–647
- Grubka LG, Bobba KM (1985) Heat transfer characteristics of a continuous stretching surface with variable temperature. *J Heat Transf* 107:248–250
- Dutta BK, Gupta AS (1987) Cooling of a stretching sheet in a various flow. *Ind Eng Chem Res* 26:333–336
- Jeng DR, Chang TCA, Dewitt KJ (1986) Momentum and heat transfer on a continuous surface. *J Heat Transf* 108:532–539
- Chakrabarti A, Gupta AS (1979) Hydromagnetic flow and heat transfer over a stretching sheet. *Q Appl Math* 37:73–78
- Andersson HI, Bech KH, Dandapat BS (1992) Magneto-hydrodynamic flow of a power-law fluid over a stretching sheet. *Int J Non-Linear Mech* 27:929–936
- Afzal N (1993) Heat transfer from a stretching surface. *Int J Heat Mass Transf* 36:1128–1131
- Prasad KV, Abel S, Datti PS (2003) Diffusion of chemically reactive species of a non-Newtonian fluid immersed in a porous medium over a stretching sheet. *Int J Non-Linear Mech* 38:651–657
- Agassant JF, Avens P, Sergent J, Carreau PJ (1991) *Polymer processing: principles and modelling*. Hanser Publishers, Munich
- Schulz DN, Glass JE (1991) *Polymers as rheology modifiers*. ACS symposium series, vol 462. American Chemical Society, Washington
- Bird RB, Armstrong RC, Hassager O (1987) *Dynamics of polymeric liquids*, vol 1. Wiley, New York
- Fosdick RL, Rajgopal KR (1979) Anomalous features in the model of second-order fluids. *Arch Ration Mech Anal* 70:145
- Hayat T, Abbas Z, Sajid M (2006) Series solution for the upper-convected Maxwell fluid over a porous stretching plate. *Phys Lett A* 358:396–403
- Sadeghy K, Najafi AH, Saffaripour M (2005) Sakiadis flow of an upper convected Maxwell fluid. *Int J Non-Linear Mech* 40:1220–1228
- Alizadeh-Pahlavan A, Aliakbar V, Vakili-Farahani F, Sadeghy K (2009) MHD flows of UCM fluids above porous stretching sheets using two-axillary-parameter homotopy analysis method. *Commun Nonlinear Sci Numer Simul* 14:473–488
- Renardy M (1997) High Weissenberg number boundary layers for the upper convected Maxwell fluid. *J Non-Newton Fluid Mech* 68:125
- Rao IJ, Rajgopal KR (2007) On a new interpretation of the classical Maxwell model. *Mech Res Commun* 34:509–514
- Sadeghy K, Hajibeygi H, Taghavi S-M (2006) Stagnation point flow of upper-convected Maxwell fluids. *Int J Non-Linear Mech* 41:1242–1247
- Aliakbar V, Alizadeh-Pahlavan A, Sadeghy K (2009) The influence of thermal radiation on MHD flow of Maxwellian fluids above stretching sheets. *Commun Nonlinear Sci Numer Simul* 14(3):779–794
- Alizadeh-Pahlavan A, Sadeghy K (2009) On the use of homotopy analysis method for solving unsteady MHD flow of Maxwellian fluids above impulsively stretching sheets. *Commun Nonlinear Sci Numer Simul* 14(4):1355–1365
- Conte SD, de Boor C (1972) *Elementary numerical analysis*. McGraw-Hill, New York
- Cebeci T, Bradshaw P (1984) *Physical and computational aspects of convective heat transfer*. Springer, New York
- Rajagopal KR In: Montieivo Marques MDP, Rodrigues JF (eds) *Boundary layers in non-Newtonian fluids*
- Hsiao K-L (2011) MHD mixed convection for viscoelastic fluid past a porous wedge. *Int J Non-Linear Mech* 46:1–8
- Hsiao K-L (2010) Viscoelastic fluid over a stretching sheet with electromagnetic effects and non-uniform heat source/sink. *Math Probl Eng* 2010:740943, 14 pages
- Ishak A, Nazar R, Pop I (2009) Boundary layer flow and heat transfer over an unsteady stretching vertical surface. *Meccanica* 44:369–375
- Babaelahi M, Domairry G, Joneidi AA (2010) Viscoelastic MHD flow boundary layer over a stretching surface with viscous and ohmic dissipations. *Meccanica* 45:817–827
- Hayat T, Abbas Z, Sajid M (2009) MHD stagnation point flow of an upper convected Maxwell fluid over a stretching sheet. *Chaos Solitons Fractals* 39:840–848