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Optimization of a pentagonal cross section of the truck crane boom using Lagrange's multipliers and differential evolution algorithm

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Abstract The cross sections of truck crane booms are complex box-like cross sections which should provide continuous stress distribution. It is difficult to analytically determine the optimal relations among geometric parameters of such cross sections. The paper deals with the method for determination of relations among geometric parameters in order to reach the optimal shape of the cross section. The method is based on Lagrange's multipliers used for determination of extreme values. The optimization of geometric parameters has been also done by the algorithm of Differential Evolution (DE). The optimization of cross section is based on the strength criterion. The results of applied methods have been verified by means of numerical example for an existing solution. The comparative analysis of the results of both methods has been done, too.

Keywords Pentagonal cross section · Truck crane · Boom · DE algorithm · Lagrange's multipliers

1 Introduction

The world's truck crane manufacturers have been giving significance to design of the truck crane booms having box-like cross sections which increase bending and torsional rigidity and decrease the weight. Since the technology of box-like supports has been enhanced, the classic rectangular cross sections have been replaced with more complex polygonal ones [1–8].

The box-like supports have been made of sheet metal of various thicknesses because of optimization and material saving. The researches [1] and [9], which deal with the telescopic truck crane booms, have proved that there are some local peaks of stress at the points where the members are in contact, so the cross sections must be made of the thicker sheet metals at contact points. These stress peaks are rather noticeable when the boom segments are maximally drawn [9]. Both local stresses and the stresses at the polygonal cross sections are less at the point where external load is transmitted from one segment to another one [1–11].

The research of the optimal parameters of pentagonal cross section (Fig. 1) has been done by two methods. The results of the comparative analysis are also shown here.

The first method for cross section optimization is based on Lagrange's multipliers. This method provides the optimal values of geometric parameters of the cross section in the explicit form and their functional relations, too. The obtained relations closely determine the defining area of parameters which are significant for the minimal cross sectional area. The

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method is also suitable for forming the algorithms of the cross sectional area optimization.

The second method for cross section optimization is based on the algorithms of Differential Evolution (DE). DE algorithm is efficient in solving the optimization problems where the objective function need not be continual in an area and where the values of design parameters need not be close to the initial values.

Price and Storn [12] successfully applied DE algorithm during optimization of certain well-known nonlinear, non-differentiable and non-convex functions. The papers (cf. [13–17]) give a detailed description of DE algorithm as well as its application to various optimization problems.

This paper proves that DE algorithm can be successfully applied to optimize the cross sectional areas of the elements of supporting structures.

2 Mathematical formulation of optimization problem

The objective of the research is to define the geometric parameters of the cross section, as well as their relations, which will provide the minimum cross sectional area for defined load. The weight minimization corresponds to the volume minimization, i.e. to the cross sectional area minimization, and it is determined provided that the stress at the appropriate cross section is less than or equal to the permissible stress. The cross sectional area depends on the following: support height and weight, sheet metal thickness, and relations among the parameters. If there are a lot of optimization parameters and if the optimization of all parameters can not be done the dominant parameters need be chosen.

This is a general mathematical formulation of the above defined optimization problem:

$$\text{minimize } f(\boldsymbol{X}) \quad (1)$$

$$\text{subject to:} \quad g_j(\boldsymbol{X}) \le 0, \quad j = 1, \dots, m \quad (2)$$

where:

$f(\boldsymbol{X})$ objective function,

$g_j(\boldsymbol{X}) \le 0$ constraints defined by the search space,

m total number of constraints.

$\boldsymbol{X} = \{x_1, \dots, x_D\}^T$ is a design vector consisting of D design variables. The design variables are the values which should be determined during the optimization process. Each design variable is defined by its lower and upper boundaries.

3 Optimization done by Lagrange's multipliers

Generally, the truck crane boom is loaded by the longitudinal force N, the bending moments M_x, M_y, and the torsional moment T [1–9].

In order to determine the optimal values of geometric parameters by Lagrange's multipliers we start with the expression for cross sectional area which is the objective function $f(\boldsymbol{X})$.

The method of Lagrange's multipliers is applied to various deterministic and stochastic ways of the minimum search.

Lagrange's function is defined in the following manner:

$$\Phi(\boldsymbol{X}) = f(\boldsymbol{X}) + \lambda \cdot g(\boldsymbol{X}) \quad (3)$$

where:

λ-Lagrange's multiplier,
$g(\boldsymbol{X})$-constraint function.

The following conditions need be met so that the objective function has its minimum:

$$\frac{\partial \Phi(\boldsymbol{X})}{\partial x_i} = 0, \quad \text{where } i = 1, \dots, D. \quad (4)$$

Equation (4) can be also written as:

$$\frac{\partial f(\boldsymbol{X})}{\partial x_i} + \lambda \cdot \frac{\partial g(\boldsymbol{X})}{\partial x_i} = 0, \quad i = 1, \dots, D \quad (5)$$

When the multiplier λ is eliminated we get the equations which define the optimal values of parameters.

3.1 Objective function and constraints

The objective function is pentagonal cross section area. The optimization of three parameters (H, B, h) is done for this objective function (Fig. 1). Other geometric parameters such as wall thicknesses t_1, t_2 and t_3 are not treated in this method as optimization parameters. Due to the method itself and due to the fact that parameters t_1, t_2 and t_3 are more precisely determined by contacting stresses, i.e. by local stability [5, 7, 9], they are considered to be defined values in this method. The

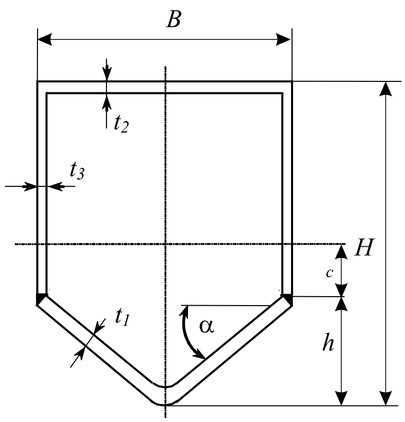

Fig. 1 Pentagonal cross section

local stress is important for the analysis at the point where segments are in contact, but it is not so important when we define the basic geometric parameters of the cross section [2, 5, 7, 9].

These are defined parameters:

N-axial force acting on the centre of the cross section,
M_x and M_y-bending moments for x and y axes,
T-torsional moment,
$[\sigma]$-permissible stress of the basic boom material.

The geometric parameters of the truck crane boom are:

W_x and W_y-resisting bending moments for x and y axes,
W_t-polar moment of resistance.

The design parameters can be expressed in the form of design vector:

$$\boldsymbol{X} = (k, d, H)$$

where:

$$d = \frac{h}{H}, \qquad k = \frac{B}{H} \tag{6}$$

The constraints corresponds to the sum of normal and tangential stresses at the cross section [2]:

$$g_1(k, d, H) = \left(\frac{N}{A} + \frac{M_x}{W_y} + \frac{M_y}{W_x}\right)^2 + 4 \cdot \left(\frac{T}{W_t}\right)^2 - [\sigma]^2 = 0 \tag{7}$$

If we introduce the following relations:

$$\delta_1 = \frac{t_1}{H}, \qquad \delta_2 = \frac{t_2}{H} \quad \text{and} \quad \delta_3 = \frac{t_3}{H} \tag{8}$$

we get the following objective function:

$$f(k, d, H) = H^2 \cdot [k \cdot \delta_2 + 2 \cdot (1 - d) \cdot \delta_3 + \delta_1 \cdot \sqrt{4 \cdot d^2 + k^2}] \tag{9}$$

Since we define the geometric parameters of truck crane boom, the member $4 \cdot (\frac{T}{W_t})^2$ of the constraints (7) can be ignored [3–9]:

$$g_1(k, d, H) = \left(\frac{N}{A} + \frac{M_x}{W_y} + \frac{M_y}{W_x}\right)^2 - [\sigma]^2 = [\sigma_e]^2 - [\sigma]^2 = 0 \tag{10}$$

On the basis of the autocrane manufacturers' recommendation [18], and references [5, 7, 9], as well, the parameter values (8) of the pentagonal cross section (Fig. 1) are within the following constraints

$$\delta_1 = 0.02\text{–}0.03; \qquad \delta_2 = 0.02\text{–}0.027; \qquad \delta_3 = 0.015\text{–}0.02 \tag{11}$$

These values (11) refer to autocranes, while are different for other types of cranes. This statement does not decrease the significance of appliance of Lagrange's method, because it allows optimization of these three parameters. Having this in mind, the adopted values of parameters are:

$$\delta_1 = 0.0273; \qquad \delta_2 = 0.0221; \qquad \delta_3 = 0.0175 \tag{12}$$

so the values of their relations are:

$$\frac{\delta_1}{\delta_3} = 1.56; \qquad \frac{\delta_2}{\delta_3} = 1.26 \tag{13}$$

The accepted limits of defined parameters do not lower the generality of the optimization of the parameters (H, B, h).

The relation between the bending moments is defined in practice and researches [2–9] as:

$$M_y = \psi \cdot M_x \tag{14}$$

where the value of ψ coefficients is within the boundaries [5, 7, 9]: $\psi = 0.4\text{–}0.75$.

The relation (14) can be expressed as:

$$M_y = \frac{M_x}{M} \tag{15}$$

so the value of M coefficient is within the following boundaries: (1.3–2.5). The value of the permissible stress is also defined as $[\sigma] = 10(\frac{\text{kN}}{\text{cm}^2})$, with assumption that the material properties are not significantly in the elastic zone [19].

3.2 Optimization of geometric parameters

In order to simplify the objective function (9) it is transformed into the following form:

$$f(k,d,H) = H^2 \cdot \left[k \cdot \delta_2 + 2 \cdot (1-d) \cdot \delta_3 + \delta_1 \cdot \sqrt{4 \cdot d^2 + k^2}\right] = H^2 \cdot [S1], \tag{16}$$

where:

$$[S1] = \left[k \cdot \delta_2 + 2 \cdot (1-d) \cdot \delta_3 + \delta_1 \cdot \sqrt{4 \cdot d^2 + k^2}\right] \tag{17}$$

The values of the moments of resistance for appropriate axes are:

$$W_x = \frac{H^3 \cdot \delta_2 \cdot k \cdot \left[3 - d \cdot \left(2 \cdot d^2 + 3 \cdot d - 3\right)\right]}{6 \cdot (2-d)} + \frac{H^3 \cdot \delta_3 \cdot (1-d) \cdot (2-d)}{2 \cdot (3-d)^2} \cdot \left\{(d-1)^2 + \frac{f^2 \cdot (3-d)^2}{3 \cdot (2-d)^2}\right\} + \frac{H^3 \cdot k_s \cdot \delta_1 \cdot (2-d)}{2 \cdot (3-d^2)} \cdot \left\{d^2 + \frac{(d^2 - 6 \cdot d + 6)^2}{3 \cdot (2-d)^2}\right\} \tag{18}$$

$$W_y = H^3 \cdot \left[\frac{\delta_2 \cdot k^2}{6} + \delta_3 \cdot (1-d) \cdot k + \frac{\delta_1 \cdot k}{6} \cdot \sqrt{4 \cdot d^2 + k^2}\right] \tag{19}$$

The expressions for moments of resistance (18) are not suitable for the application of Lagrange's multipliers. Thus, their approximation has been done provided that their accuracy is not reduced (the error at this approximation does not exceed the value of 5%). The simplified expression for the moment of resistance for x axis is:

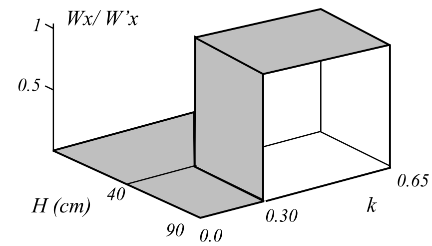

Fig. 2 Relation between the moments of resistance for x axis

$$W_x'(k,d,H) = A \cdot \frac{H}{2} \cdot \alpha \tag{20}$$

The graphic interpretation of the approximation can be seen in Fig. 2. There is no significant deviation of the values of the moments of resistance stated in the equations (18) and (20), where the approximation coefficient is $\alpha = 0{,}33$.

The expression for the moment of resistance for y axis can be also transformed as:

$$W_y(k,d,H) = A \cdot \frac{k \cdot H}{2} \cdot \beta \tag{21}$$

In the same manner as for the coefficient α, by using the graphic representation of the equality of equations (19) and (21), we get the coefficient value as follows (approximation error less than 5%):

$$\beta = 0.64 \tag{22}$$

According to the above approximation, the constraints (10) is:

$$g_1 = \frac{N}{H^2 \cdot [S1]} + \frac{2 \cdot M_x}{\alpha \cdot H^3 \cdot [S1]} + \frac{2 \cdot M_y}{\beta \cdot H^3 \cdot k \cdot [S1]} - [\sigma] = 0 \tag{23}$$

In order to apply Lagrange's condition, it is necessary to differentiate the constraints (23) and the objective function (16) with respect to the stated parameters, and then to find the following ratios:

$$\frac{\frac{\partial g_1}{\partial H}}{\frac{\partial f}{\partial H}} = \frac{-N}{H^4 \cdot [S1]^2} - \frac{3 \cdot M_x}{\alpha \cdot H^5 \cdot [S1]^2} - \frac{3 \cdot M_y}{\beta \cdot k \cdot H^5 \cdot [S1]^2}$$

$$\frac{\frac{\partial g_1}{\partial d}}{\frac{\partial f}{\partial d}} = -\frac{N}{H^4 \cdot [S1]^2} - \frac{2 \cdot M_x}{\alpha \cdot H^5 \cdot [S1]^2} - \frac{2 \cdot M_y}{\beta \cdot H^5 \cdot k \cdot [S1]^2} \tag{24}$$

$$\frac{\frac{\partial g_1}{\partial k}}{\frac{\partial f}{\partial k}} = -\frac{N}{H^4 \cdot [S1]^2} - \frac{2 \cdot M_x}{\alpha \cdot H^5 \cdot [S1]^2} - \frac{2 \cdot M_y \cdot \{(\delta_2 + \delta_1 \cdot \frac{k}{\sqrt{4 \cdot f^2 + k^2}}) \cdot k + [S1]\}}{H^5 \cdot k^2 \cdot [S1]^2 \cdot \beta \cdot [\delta_2 + \frac{\delta_1 \cdot k}{\sqrt{4 \cdot f^2 + k^2}}]}$$

Setting the first and second expression (24) equal to each other, we have:

$$k = \frac{\alpha}{M \cdot \beta} \tag{25}$$

For the given values of parameter M (1.3–2.5), the value of parameters $k = 0.2$–0.4 is obtained from equation (25). As, in the real world, the value of parameter k ranges within [5, 7, 9] $k = 0.4$–0.65, it can be concluded that only parameter value of

$$k = 0.4 \tag{26}$$

fulfills all given constraints.

If we equalize the first, second and third expression (24), by means of certain transformations, we reach the following expression:

$$k = \frac{-2}{(1.26 - 2 \cdot t) + 1.56 \cdot \sqrt{4 \cdot t^2 + 1}} \tag{27}$$

where: $\frac{d}{k} = t$. Transformation of expression (27) results in the expression of the required parameter d:

$$d = k \cdot \left[\sqrt{14 - 7.3 \cdot k} - 2.85\right] \tag{28}$$

Mutual dependence of optimised parameters d and k, within defined boundaries, is presented in Fig. 3.

The pentagonal height can be obtained from the limit function (23) if we ignore the members of a very

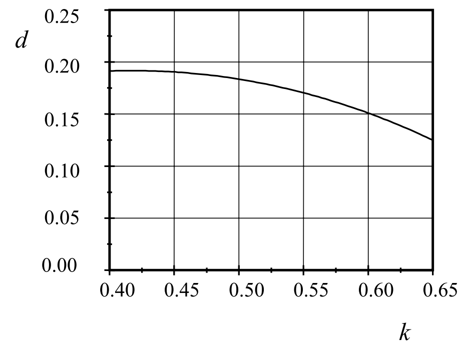

Fig. 3 Dependence of optimised parameters d and k

small value:

$$H = \sqrt[3]{\frac{M_x \cdot d^2 \cdot \{2 \cdot d \cdot [S] + M \cdot (1+k) \cdot [S_1]\}}{\alpha \cdot M \cdot (1+k) \cdot [S_1] \cdot [S] \cdot [\sigma]}} \tag{29}$$

4 Optimization by DE method

4.1 Brief description of DE algorithm

The DE algorithm is briefly described here, and the control parameters of the algorithm are also dealt with. A detailed description of the DE algorithm can be seen in references [12–17].

DE is a simple, but still strong evolutionary algorithm used for realization of the global minimum in numerous real optimization problems. The DE algorithm has the following control parameters: the population size *NP*, the crossover constant *CR* and the mutation constant F. Coding of chromosomes with real numbers, i.e. presentation of chromosomes as vectors of real values, is used in numerical application of DE in optimization processes.

Generation of the initial population is performed stochastically. The population size *NP* is commonly ten times bigger than the number of design variables. At the beginning, each design variable is a random value which is found within the defined upper and lower limits. While defining the limits, attention should be paid that the values of design variables are not out of range which is really acceptable.

The mutation constant in DE is a real parameter, which controls the increase of difference between two individuals in the search space. The difference between two randomly chosen vectors defines the magnitude and direction of mutation. When the difference is added to a randomly chosen vector, it becomes a mutant vector. The basic idea of DE is that mutation is self-adaptive in the search space and the current population. At the beginning of the optimization process, the magnitude of mutation is large because the vectors in the population are far away from the search space. When the process starts to converge, the magnitude of mutation starts to decrease. The self-adaptive mutation in DE leads the solution of the optimization process toward the global minimum [15].

There are some basic rules, which are defined in [15], for taking the best values for *CR*. High values are effective for all problems, but they are not always the fastest ones. The problems with heavy interaction between design variables generally require a high *CR*. But, if interaction between design variables is lower, a lower *CR* can be used, which results in obtaining a satisfactory solution with a smaller number of iterations (faster solution). According to reference [15] the values of control parameters are presented in Table 2.

4.2 Optimization done by DE algorithm

Design parameters can be expressed in the form of design vector:

$$\boldsymbol{X} = (k, d, \delta_1, \delta_2, \delta_3, H)$$

In addition to the variables k, d, H which have been optimized by Lagrange's multipliers, the variables δ_1, δ_2 and δ_3 are also optimized.

The objective function is:

$$f(k, d, \delta_1, \delta_2, \delta_3, H) = A = H^2 \cdot \left[\frac{k}{d} \cdot \delta_2 + \frac{\delta_1}{d} + \delta_3 \cdot \sqrt{4 + \left(\frac{1-k}{d} \right)^2} \right] \tag{30}$$

with the following constraints:

$$g_1(k, d, \delta_1, \delta_2, \delta_3, H) = \left(\frac{N}{A} + \frac{M_x}{W_y} + \frac{M_y}{W_x} \right) - 10 \le 0,$$

$$\begin{aligned} h_1(k, d, \delta_1, \delta_2, \delta_3, H) &= \delta_1 - 1.56 \cdot \delta_3 = 0, \\ h_2(k, d, \delta_1, \delta_2, \delta_3, H) &= \delta_2 - 1.26 \cdot \delta_3 = 0 \end{aligned} \tag{31}$$

The constraints $g_1(k, d, \delta_1, \delta_2, \delta_3, H)$ results from (10) while the constraints $h_1(k, d, \delta_1, \delta_2, \delta_3, H)$ and $h_2(k, d, \delta_1, \delta_2, \delta_3, H)$ result from (13).

The boundaries of design variables δ_1, δ_2 and δ_3 are defined by equation (11) and the boundaries of k, d, H are accepted according to references [5, 7, 9]. Their upper and lower boundaries are shown in Table 1. The boundaries which enable the design variables to be within defined boundaries are directly used into DE algorithm [14].

The parameters related to DE algorithm are shown in Table 2.

In Table 2 there are some final design variables for various accepted values M and M_x. On the basis of the final design variables, the values B, b, σ_e have been calculated as well as the numerical value of the objective function, i.e. the minimal trapezoidal cross sectional area of the truck crane boom has been obtained.

5 Analysis of results

By using (9) and (29), dependences of the area and height of a polygonal cross section can be obtained in the function of external load and parameter M. The values of load corresponds to loads of field auto cranes TD-16 manufactured by [18]: IMK 14 oktobar—Krusevac and ADH-16 ILR—Belgrade.

By using the data from Table 2, as well as the objective functions (9), a comparative analysis of obtained

Table 1 Initial values of design variables

	k	d	δ_1	δ_2	δ_3	H [cm]
Lower boundary	0.4	1.2	0.02	0.02	0.015	60
Upper boundary	0.8	2.2	0.03	0.027	0.02	90

Table 2 Parameters of DE algorithm

NP (initial population)	*D* (number of design variables)	*CR* (crossover constant)	*F* (mutation constant)	*Itermax* (maximum number of iterations)
60	6	0.5	0.5	1000

Table 2 Accepted, final and optimised values of design variables

		Accepted values					
		$M = 2.5$	$M = 2.0$	$M = 1.7$	$M = 2.0$	$M = 2.5$	$M = 1.33$
		Mx = 40000 [kNcm]	Mx = 20000 [kNcm]	Mx = 20000 [kNcm]	Mx = 25 000 [kNcm]	Mx = 35000 [kNcm]	Mx = 50000 [kNcm]
Final values of design variables	κ	0.409106	0.405120	0.496425	0.496096	0.601418	0.598885
	d	0.181811	0.243516	0.204915	0.158380	0.138936	0.187421
	δ_1	0.027801	0.026257	0.025507	0.026503	0.026941	0.026831
	δ_2	0.021614	0.022227	0.022242	0.021943	0.021792	0.021177
	δ_3	0.017526	0.017364	0.017259	0.017511	0.017360	0.017037
	H [cm]	88.73709	72.992371	71.083419	74.664797	78.158764	94.737182
	Number of iterations	27	22	13	34	27	39
Calculated values	B [cm]	36.3029	29.5707	35.2876	37.0409	47.0061	56.7367
	b [cm]	16.1334	17.7748	14.5661	11.8254	10.8591	17.7557
	A_{min} [cm^2]	415.2759	276.5679	277.4342	311.9744	371.7298	532.4742
	σ_e [kN/cm^2]	9.895207	9.825457	9.901661	9.884895	9.804667	9.895787

values for the cross section area can be presented for both methods (Fig. 4).

By analysing the obtained results (Fig. 4), it can be stated that there is considerable congruence, which is especially noticed for the value of parameter $k = 0.6$; $M \in 1.33$–2.5 (Fig. 4c). In the other two cases, $k = 0.4$ and $k = 0.5$, the results obtained by the Lagrange multiplier method give better solutions, i.e. smaller of cross-sectional areas, and, at that, stresses do not exceed the allowed value.

6 Conclusion

Both optimisation methods can be successfully used for determination of dependence of geometrical parameters of a pentagonal cross section of the support structure of an auto crane boom.

The Lagrange multiplier method is advantageous in defining the objective function in analytical form, which is very suitable for practical application. The obtained expressions can be very useful for engineers-designers, particularly in the first phase of design when they cope with the problem of defining dimensions of the initial structure which is close to the optimum one.

The DE algorithm gives dependence of the objective function in analytical form, but it enables introduction of a larger number of boundaries, a broader scope of initial values of design variables and a larger number of solutions satisfying the given boundaries. The DE algorithm provides discrete values of load of certain parameters as well as the minimum value of the objective function for the given values of load. In addition, the goal of applying the DE algorithm was to make comparison with the first method at certain points (given values of load).

By comparing the results obtained, it can be stated that there is overlapping among them to a large extent. By observing the whole scope of analysis with the application of the Lagrange multiplier method, better solutions are obtained (Fig. 4). It can be explained by the fact that the Lagrange multiplier method explicitly defines optimum values of parameters while the DE method provides more solutions that satisfy the given boundaries, so that minimum values of the objective function should be found among them.

More exact solutions obtained by applying the Lagrange multiplier method are especially noticed for the given values of parameters $k = 0.4$ and $k = 0.5$, (Figs. 4a and 4b), while significant overlapping of values of the obtained solutions are noticeable for the value of parameter $k = 0.6$ (Fig. 4c).

This fact confirms that the Lagrange multiplier method can also be successfully applied for optimization of structures, especially in the initial phase of design.

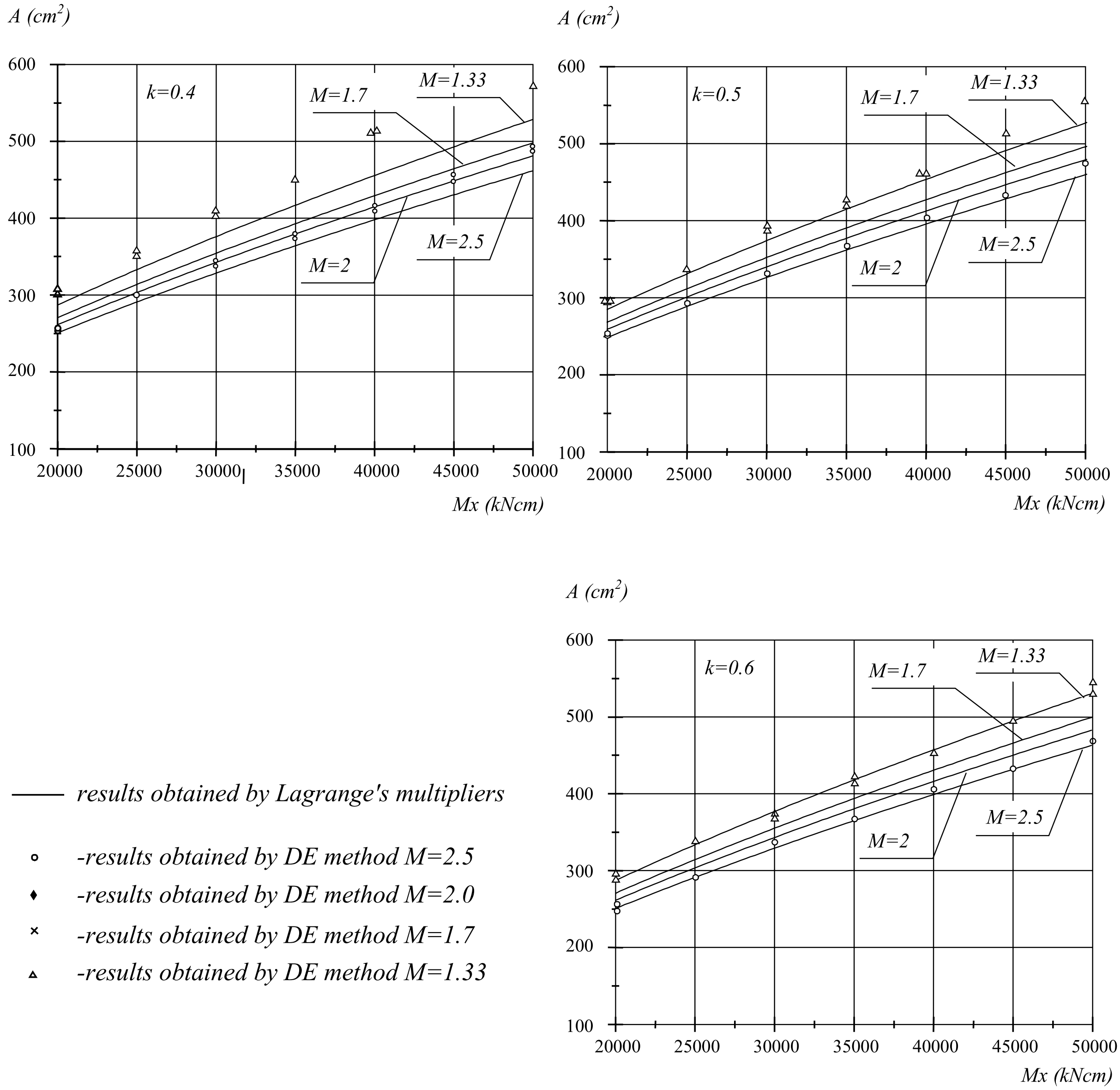

Fig. 4 Comparative illustration of the results of both methods

On the basis of research, it is concluded that neither method requires use of huge mathematical apparatus, software and hardware, and they have an important role in defining basic geometrical parameters for cross sections of support structures in general.

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