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Radiation effects on the thermal boundary layer flow over a moving plate with convective boundary condition

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Abstract The steady laminar boundary layer flow over a moving plate in a moving fluid with convective surface boundary condition and in the presence of thermal radiation is investigated in this paper. Under certain conditions, the present problem reduces to the classical Blasius and Sakiadis problems. The effects of radiation and convective parameters on the thermal field are thoroughly examined and discussed. Dual solutions are found to exist when the plate and the fluid move in the opposite directions.

Keywords Boundary layer · Thermal radiation · Convective boundary condition · Dual solutions · Mechanics of fluid

List of symbols

a Convective parameter
c Constant
C_f Skin friction coefficient
c_p Specific heat at constant pressure
f Dimensionless stream function
k Thermal conductivity
k^* Mean absorption coefficient
N Radiation parameter
Pr Prandtl number
q_r Radiative heat flux
T Fluid temperature
T_f Hot fluid temperature
T_w Plate temperature
T_∞ Ambient temperature
u, v Velocity components along the x and y directions, respectively
U Composite velocity
U_w Plate velocity
U_∞ Free stream velocity
x, y Cartesian coordinates along the plate and normal to it, respectively

Greek symbols

α Thermal diffusivity
ε Velocity ratio parameter
η Similarity variable
θ Dimensionless temperature
ν Kinematic viscosity
ρ Fluid density
σ^* Stefan-Boltzmann constant
τ_w Wall shear stress
ψ Stream function

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Subscripts

w At the wall

∞ In the free stream

Superscript

$'$ Differentiation with respect to η

1 Introduction

The effects of thermal radiation on the classical Blasius flow have been investigated by Bataller [1], and the same author was then extended it to the Sakiadis flow [2]. For Blasius flow, the development of the velocity boundary layer is caused solely on the moving fluid over a stationary flat surface, while for Sakiadis flow, it is due to the moving flat surface in a quiescent fluid. The Blasius and Sakiadis flows are two different problems and cannot be mathematical transformed into one another [3]. It is well known that the skin friction coefficient along a continuous moving flat surface in a quiescent fluid (Sakiadis problem) is about 30% higher than those along a static flat plate in a moving fluid (Blasius problem). This fact indicates that we cannot regard the velocity difference $|U_w - U_\infty|$, where U_w and U_∞ are the plate velocity and the free stream velocity respectively, as a relative velocity in the sense of Galilei. The solution depends not only on the velocity difference $|U_w - U_\infty|$ but also on the velocity ratio U_w/U_∞.

The Blasius flow with convective surface boundary condition has been investigated by Aziz [4], and very recently by Magyari [5]. Aziz found that similarity solution for the energy equation exists if the heat transfer coefficient h_f is proportional to $x^{-1/2}$, where x is the distance from the leading edge of the plate. This problem was then extended by Bataller [6] to include the effect of radiation, and also to the Sakiadis flow. Bataller considered two sets of boundary conditions separately, for Blasius and Sakiadis flows.

The objective of the present paper is to combine these two problems, by using the composite velocity $U = U_w + U_\infty$ introduced by Afzal et al. [7]. By using this composite velocity, we are able to investigate the flow characteristics when both the plate and the fluid are in moving conditions. The problems considered by Bataller [6] are two special cases of the present study, i.e. when $U_w = 0$ and $U_\infty = 0$.

2 Mathematical formulation

Consider a steady, two-dimensional laminar flow of a viscous fluid passing through a moving flat plate with constant velocity U_w, in the same or opposite direction to the free stream U_∞. The x-axis extends parallel to the surface, while the y-axis extends upwards, normal to the surface. The simplify governing equations are [1, 2, 6]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{3}$$

where u and v are the velocity components along the x and y axes, respectively, T is the fluid temperature in the boundary layer, q_r is the radiative heat flux, and ν, ρ, c_p and k are the kinematic viscosity, fluid density, specific heat at a constant pressure and the thermal conductivity, respectively. The boundary conditions for the flow field are [7, 8]

$$u = U_w, \qquad v = 0 \quad \text{at } y = 0,$$
$$u \to U_\infty \quad \text{as } y \to \infty. \tag{4}$$

It is assumed that the bottom surface of the plate is heated by convection from a hot fluid of uniform temperature T_f which provides a heat transfer coefficient h_f. Under this assumption, the boundary conditions for the thermal field may be written as [4, 6]

$$-k\frac{\partial T}{\partial y} = h_f(T_f - T_w) \quad \text{at } y = 0,$$
$$T \to T_\infty \quad \text{as } y \to \infty, \tag{5}$$

with k being the thermal conductivity, T_w the uniform temperature on the top surface of the plate and T_∞ is the temperature of the ambient cold fluid. Here we have $T_f > T_w > T_\infty$.

Using the Rosseland approximation for radiation [9, 10], the radiative heat flux is simplified as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{6}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. This approximation was also used by Bataller [11, 12], Pal [13], Pal and Mondal [14], Mukhopadhyay and Layek [15], and very recently by Ishak [16]. Further, it is assumed that the temperature differences within the flow such that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms we obtain

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{7}$$

Using (6) and (7), (3) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\alpha + \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*}\right)\frac{\partial^2 T}{\partial y^2}, \tag{8}$$

where $\alpha = k/(\rho c_p)$ is the thermal diffusivity. If we take $N = kk^*/(4\sigma^* T_\infty^3)$ as the radiation parameter, (8) becomes

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\alpha}{k_0}\frac{\partial^2 T}{\partial y^2}, \tag{9}$$

with $k_0 = 3N/(3N+4)$.

The continuity equation (1) is satisfied by introducing a stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}. \tag{10}$$

The momentum and energy equations can be transformed into the corresponding ordinary differential equations by the following transformation [4]:

$$\eta = \left(\frac{U}{\nu x}\right)^{1/2} y, \qquad f(\eta) = \frac{\psi}{(\nu x U)^{1/2}}, \qquad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \tag{11}$$

where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature and $U = U_w + U_\infty$ (see Afzal et al. [7]). The transformed nonlinear ordinary differential equations are:

$$f''' + \frac{1}{2} f f'' = 0, \tag{12}$$

$$\frac{1}{Pr}\theta'' + \frac{1}{2} k_0 f \theta' = 0, \tag{13}$$

where primes denote differentiation with respect to η and $Pr = \nu/\alpha$ is the Prandtl number. In order that similarity solutions of (1)–(5) exist, we take [4]

$$h_f = c x^{-1/2}, \tag{14}$$

where c is a constant.

The transformed boundary conditions are

$$\begin{aligned} &f(0) = 0, \qquad f'(0) = \varepsilon, \\ &\theta'(0) = -a[1 - \theta(0)], \\ &f'(\eta) \to 1 - \varepsilon, \qquad \theta(\eta) \to 0 \quad \text{as } \eta \to \infty, \end{aligned} \tag{15}$$

where $\varepsilon = U_w/U$ is the velocity ratio parameter, and $a = \frac{c}{k}\sqrt{\nu/U}$ is the convective parameter.

The assumption (14) is necessary for a in the boundary conditions (15) to be independent of x. Without this assumption, the solutions generated are the local similarity solutions (see Ishak et al. [17]). We note that the present problem reduces to those of Bataller [6] when $\varepsilon = 0$ (also when $\varepsilon = 1$), while it reduces to those of Aziz [4] when $\varepsilon = 0$ and $k_0 = 1$. The case $0 < \varepsilon < 1$ is when the plate and the fluid move in the same direction, while they move in the opposite directions when $\varepsilon < 0$, and when $\varepsilon > 1$. If $\varepsilon < 0$, the free stream is directed towards the positive x-direction, while the plate moves towards the negative x-direction. If $\varepsilon > 1$, the free stream is directed towards the negative x-direction, while the plate moves towards the positive x-direction. However, in this paper we consider only the case $\varepsilon \le 1$, i.e. the direction of the free stream is fixed (towards the positive x-direction). Similar problem, but without the effects of thermal radiation ($k_0 = 1$) as well as convective boundary condition ($a \to \infty$) has been considered by Afzal et al. [7]. Without energy equation, the present problem reduces to those of Blasius [18] when $\varepsilon = 0$ and those of Sakiadis [19] when $\varepsilon = 1$.

3 Results and discussion

The nonlinear ordinary differential equations (12) and (13) subject to the boundary conditions (15) were solved numerically using Runge-Kutta-Fehlberg method with shooting technique. To validate the numerical results obtained, these equations were also solved numerically using the Keller-box method, which is very familiar to the present authors (cf. [20–25]), for certain values of parameters. Table 1 shows the comparison for the values of $\theta(0)$ with those reported by Aziz [4] and Bataller [6], and they are found to be in excellent agreement. The temperature profiles for certain values of parameters are presented

Table 1 Values of $\theta(0)$ for various values of a and Pr when $k_0 = 1$ (without thermal radiation) and $\varepsilon = 0$ (fixed plate)

a	Aziz [4]		Bataller [6]		Present results	
	$Pr = 0.72$	$Pr = 10$	$Pr = 0.72$	$Pr = 10$	$Pr = 0.72$	$Pr = 10$
0.05	0.1447	0.0643	0.144661678	0.064256124	0.144661	0.064256
0.10	0.2528	0.1208			0.252758	0.120752
0.20	0.4035	0.2155	0.403527127	0.215485674	0.403523	0.215484
0.40	0.5750	0.3546			0.575014	0.354565
0.60	0.6699	0.4518	0.669916801	0.451761720	0.669916	0.451759
0.80	0.7302	0.5235			0.730170	0.523512
1	0.7718	0.5787	0.771822220	0.578660018	0.771822	0.578656
5	0.9441	0.8729			0.944174	0.872883
10	0.9713	0.9321	0.971285852	0.932127987	0.971285	0.932128

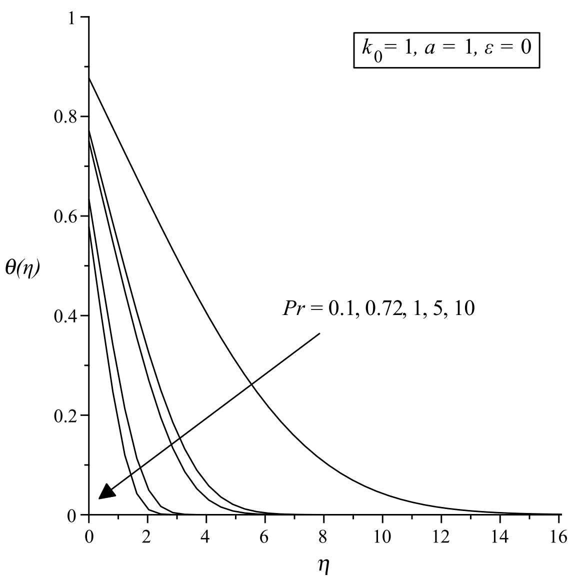

Fig. 1 Temperature profiles $\theta(\eta)$ for various values of Pr when $k_0 = 1$, $a = 1$ and $\varepsilon = 0$

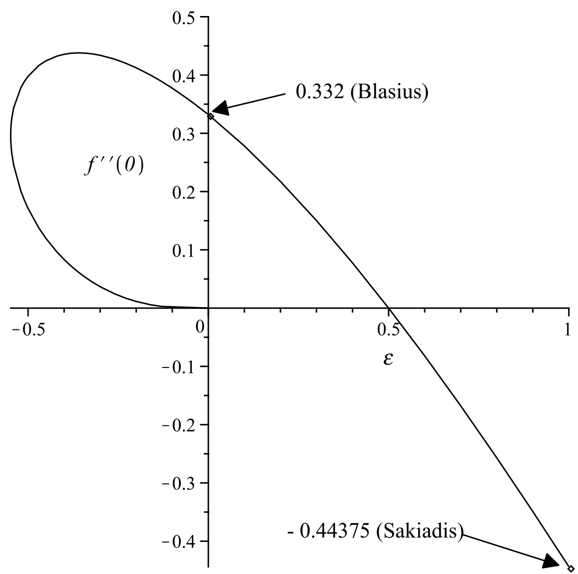

Fig. 2 Variation of the skin friction coefficient $f''(0)$ with ε

in Fig. 1. We note that the values of $-\theta'(0)$ as reported in [4] can be obtained by using the relation $\theta'(0) = -a[1 - \theta(0)]$ from (15).

The variation of the skin friction coefficient $f''(0)$ with ε is shown in Fig. 2. We note that the radiation parameter N, the convective parameter a and the Prandtl number Pr have no influence to the flow field, which is clear from (12) and (15). Further discussion on the results presented in Fig. 2 can be found in [7] and [8].

The variations of the local Nusselt number $-\theta'(0)$, which represents the heat transfer rate at the surface, with ε for different values of N and a when the other parameters are fixed to unity are presented in Figs. 3 and 4, respectively. These figures show that it is possible to obtain dual solutions of the similarity equations (12) and (13) subjected to boundary conditions (15) when the plate and the fluid move in the opposite directions. For negative values of ε, there is a critical value ε_c, with two solution branches for $\varepsilon_c < \varepsilon < 0$, unique solution for $\varepsilon \geq 0$, a saddle-node bifurcation at $\varepsilon = \varepsilon_c$ and no solution for $\varepsilon < \varepsilon_c$. The boundary layer approximations breakdown at $\varepsilon = \varepsilon_c$, and thus no solution is obtained for $\varepsilon < \varepsilon_c$. Based on our computations, $\varepsilon_c = -0.5482$ for all values of N and a considered. This value of ε_c is in agreement with those

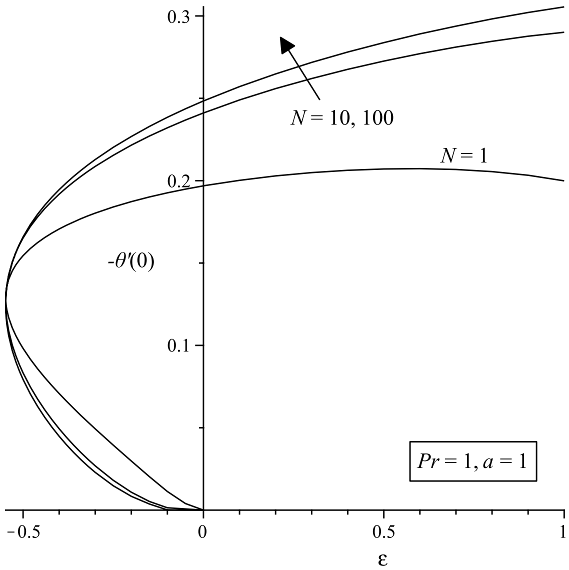

Fig. 3 Variation of the local Nusselt number $-\theta'(0)$ with ε and N when $Pr = 1$ and $a = 1$

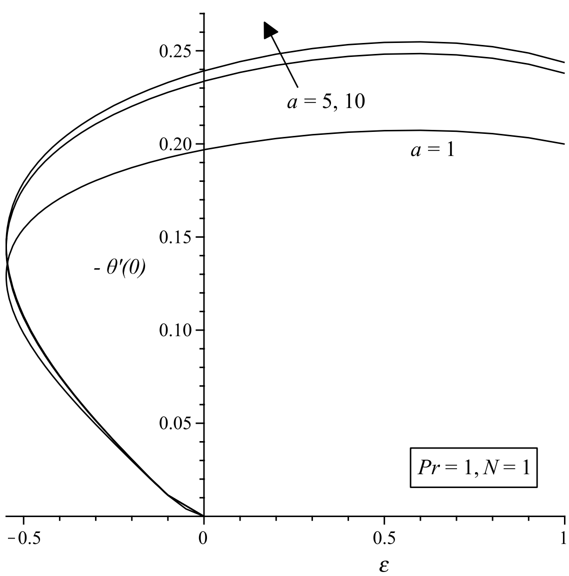

Fig. 4 Variation of the local Nusselt number $-\theta'(0)$ with ε and a when $Pr = 1$ and $N = 1$

reported by Afzal et al. [7], Klemp and Acrivos [26] and Merkin [27].

Between the two solutions, which solution is physically relevant depends essentially on the stability of the solutions. The full stability analysis is beyond the scope of the present study, however we expect that the upper branch solutions are physically stable and occur

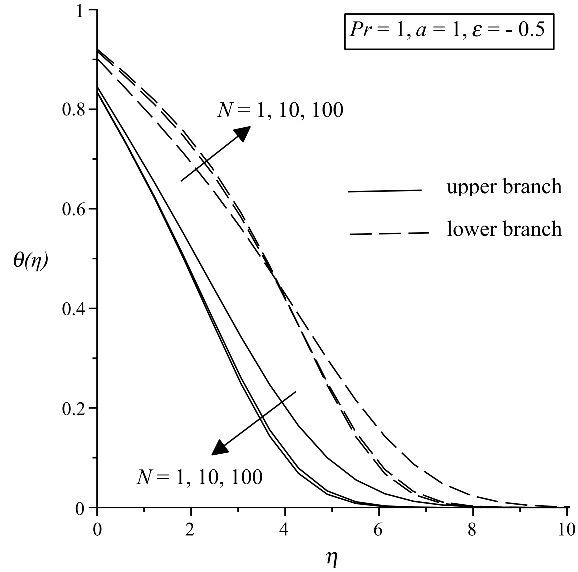

Fig. 5 Temperature profiles for different values of N when $Pr = 1$, $a = 1$ and $\varepsilon = -0.5$

in practice, since the upper branch is the only solution for $0 \leq \varepsilon \leq 1$. Figure 3 shows that for the upper branch solutions (which we expect to be physically relevant), the values of $-\theta'(0)$ are higher for larger values of N. Thus, the thermal radiation reduces the heat transfer rate at the surface. This observation is supported by Fig. 5, which shows that the temperature gradient at the surface is higher for larger values of N. Figure 3 also shows that the values of $-\theta'(0)$ become indistinguishable for very large values of N. Similar behaviors are observed for the variations of $-\theta'(0)$ with a. For $N \to \infty$ (without thermal radiation) and $a \to \infty$ (without convective boundary condition), the solutions for the present problem were given by Afzal et al. [7], for Prandtl number $Pr = 0.72$.

The velocity and temperature profiles presented in Figs. 5, 6, 7 show that the boundary layer thickness is lower for the upper branch solutions compared to the lower branch solutions. Both upper and lower branch profiles satisfy the far field boundary conditions (15) asymptotically, and thus supporting the validity of the numerical results obtained.

4 Conclusions

In this paper, we studied theoretically the problem of steady laminar boundary layer flow and heat transfer over a moving flat surface in a parallel stream

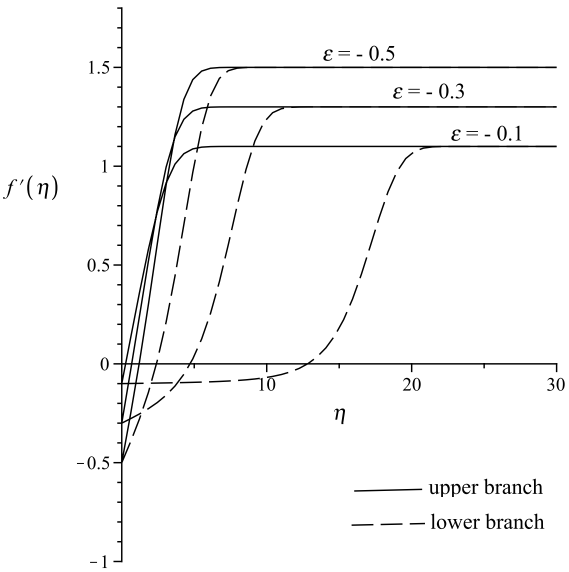

Fig. 6 Velocity profiles for different values of ε

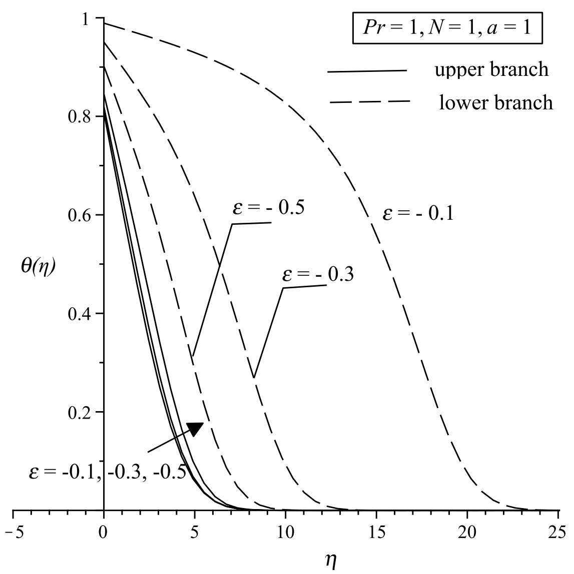

Fig. 7 Temperature profiles for different values of ε when $Pr = 1$, $N = 1$ and $a = 1$

with convective boundary condition. Similarity solution for the thermal field is possible when the convective heat transfer from the lower surface varies like $x^{-1/2}$, where x is the distance from the slot where the plate is issued. We found that the heat transfer rate at the surface decreases in the presence of thermal radiation and convective boundary condition. Further, dual solutions are found to exist when the plate and the fluid move in the opposite directions.

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