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Explicit analytical approximation to large-amplitude non-linear oscillations of a uniform cantilever beam carrying an intermediate lumped mass and rotary inertia

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Abstract This paper is concerned with analytical treatment of non-linear oscillations of planar, flexural large amplitude free vibrations of a slender, inextensible cantilever beam carrying a lumped mass with rotary inertia at an intermediate position along its span. An analytic approximate technique, namely Optimal Homotopy Asymptotic Method (OHAM) is employed for this purpose. It is proved that OHAM provide accurate solutions for large amplitudes and large modal constants in the considered nonlinear equations, when other classical methods fail. Our procedure provides us with a convenient way to optimally control the convergence of solution, such that the accuracy is always guaranteed. An excellent agreement of the approximate frequencies and periodic solutions with the numerical results and published results has been demonstrated. Two examples are given and the results reveal that this procedure is very effective, simple and accurate. This paper demonstrates the general validity and the great potential of the OHAM for solving strongly nonlinear problems.

Keywords Optimal Homotopy Asymptotic Method · Nonlinear oscillations · Cantilever beam

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1 Introduction

The problems of large-amplitude oscillations of nonlinear engineering structures have received considerable attention in the past years [1–6]. Engineering structures undergoing large-amplitude oscillations often involves discretizing the structure, when free vibration analysis is performed, and results in a temporal problem having inertia and static non-linearities. Such problems are not amenable to exact treatment because of their complexity and approximate techniques must be resorted to [7–9].

Concerning the problem of large-amplitude vibration of a uniform cantilever beam approached in this work, this subject is of practical interest because many engineering structures can be modelled as a slender, flexible cantilever beam carrying a lumped mass with rotary inertia at an intermediate point along its span hence they experience large-amplitude vibration. Approaching this problem, Hamdan and Dado [7], used the single-term harmonic balance method and two-terms harmonic balance method, but they obtained only the approximate solution to the period of oscillation. Qaisi and Al-Huniti [8] proposed, for the same problem, a power-series solution, but only for the vibration frequency. For the same problem, Wu et al. [9] combined the linearization of the governing equation with the method of harmonic balance, establishing approximate solutions to the non-linear oscillators for large amplitudes.

Often linearization techniques are employed in order to approximate such non-linear problems. One may question the accuracy of using such a linear mode method, which are a frequently used method in the analysis of non-linear continuous systems to approximate the large amplitude non-linear behaviour [7].

Another option to find approximate analytic solution for such non-linear oscillators could be employing perturbation methods, but since classical perturbation methods are, in principle, intended to solve problems involving a small parameter, an approximate solution using perturbation methods will not be adequate for the range of large amplitude vibrations to be considered in this work.

That is why new and reliable analytical techniques, which avoid linearization or small parameters, must be developed in order to achieve accurate analytical solutions for large-amplitude non-linear oscillations.

It is noted that several methods have been used to obtain approximate solutions for strongly non-linear oscillators. An interesting approach which combines the harmonic balance method and linearization of non-linear oscillation equation was presented in [10]. There also exists a wide range of literature dealing with approximate periodic solutions for nonlinear problems with large parameters by using a mixture of methodologies: the variational iteration method [11–14] some linearization methods [15, 16], the homotopy perturbation method (HPM) [17, 18], some modified Lindstedt-Poincare methods [19, 20] and so on.

In this paper, the optimal homotopy asymptotic method (OHAM) is proposed to find explicit analytical periodic solution to non-linear oscillations of planar, flexural large amplitude free vibrations of a slender, inextensible cantilever beam carrying a lumped mass with rotary inertia at an intermediate position along its span. The efficiency of our procedure is proved while an accurate solution is explicitly analytically obtained in an iterative way after only two iterations. The proposed method does not require a small parameter in the equation and provides a convenient and rigorous way to optimally control the convergence of the solution.

2 Formulation and solution approach

We consider a clamped beam at the base, free at the tip, which caries a lumped mass and rotary inertia at an arbitrary intermediate point along its span. The beam is considered to be uniform of constant length and mass per unit length and the thickness of this conservative beam is assumed to be small compared to the length so that the effects of rotary inertia and shearing deformation will be ignored. Moreover, the beam is assumed to be inextensible, which implies that the length of beam neutral axis remains constant during the motion. These assumptions are the same as those used in references [7–9]. In these conditions one can derive the discrete, single-mode, of order three nonlinearities, beam temporal problem [7]:

$$\frac{d^2u}{dt^2}+u+\alpha u^2\frac{d^2u}{dt^2}+\alpha u\left(\frac{du}{dt}\right)^2+\beta u^3=0 \tag{1}$$

subject to the initial conditions

$$u(0)=A; \qquad \frac{du}{dt}(0)=0 \tag{2}$$

This system, where α and β are modal constants which result from the discretization procedure [7], describes the large-amplitude free vibrations of the considered slender inextensible cantilever beam, which is assumed to be undergoing planar flexural vibrations. The third and fourth-terms in (1) represent inertia-type cubic non-linearity arising from the inextensibility assumption. The last term is a static-type cubic non-linearity associated with the potential energy stored in bending.

In order to analytically solve this problem, we introduce a new independent variable

$$\tau=\Omega t \tag{3}$$

then (1) and (2) become

$$u''+k^2u+\alpha u^2u''+\alpha uu'^2+\beta k^2u^3=0 \tag{4}$$

and

$$u(0)=A; \qquad u'(0)=0 \tag{5}$$

where the prime denotes differentiation with respect to τ and $k=\Omega^{-1}$. The new independent variable τ is chosen in such a way that the solution of (4), which satisfies the assigned initial conditions in (5) is a periodic function of τ, of period 2π. The period of the corresponding non-linear oscillation is given by $T = 2\pi/\Omega$. Here, both periodic solution $u(\tau)$ and frequency Ω depend on A.

Under the transformation

$$u(\tau) = Ax(\tau) \tag{6}$$

Equation (4) becomes

$$x'' + k^2x + ax^2x'' + axx'^2 + bk^2x^3 = 0 \tag{7}$$

where $a = \alpha A^2$, $b = \beta A^2$ and the initial conditions become

$$x(0) = 1; \qquad x'(0) = 0 \tag{8}$$

Equation (7) has the general form

$$x'' + k^2x + f(\tau, x, x', x'') = 0 \tag{9}$$

By the homotopy technique, we construct a homotopy in a more general form [21–26]:

$$\begin{aligned} &H(\phi(\tau,p), h(\tau,p)) \\ &\quad = (1-p)L(\phi(\tau,p)) - h(\tau,p)N(\phi(\tau,p), K(p)) \\ &\quad = 0 \end{aligned} \tag{10}$$

where L is a linear operator:

$$L(\phi(\tau,p)) = \frac{\partial^2\phi(\tau,p)}{\partial\tau^2} + \phi(\tau,p) \tag{11}$$

while N is a nonlinear operator

$$\begin{aligned} &N(\phi(\tau,p), K(p)) \\ &\quad = \frac{\partial^2\phi(\tau,p)}{\partial\tau^2} + K^2(p)\phi(\tau,p) \\ &\qquad + f\left(\tau, \phi(\tau,p), \frac{\partial\phi(\tau,p)}{\partial\tau}, \frac{\partial^2\phi(\tau,p)}{\partial\tau^2}\right) \end{aligned} \tag{12}$$

where $p \in [0, 1]$ is the embedding parameter, $h(\tau, p)$ is an auxiliary function so that $h(\tau, 0) = 0$ and $h(\tau, 0) \neq 0$ for $p \neq 0$. From (8), we obtain the initial conditions:

$$\phi(0,p) = 1, \qquad \left.\frac{\partial\phi(\tau,p)}{\partial\tau}\right|_{\tau=0} = 0 \tag{13}$$

Obviously when $p = 0$ and $p = 1$ it holds

$$\begin{aligned} &\phi(\tau,0) = x_0(\tau), \qquad \phi(\tau,1) = x(\tau) \\ &K(0) = k_0, \qquad K(1) = k \end{aligned} \tag{14}$$

where $x_0(\tau)$ is an initial approximation for $x(\tau)$. Therefore, as the embedding parameter p increases from 0 to 1, $\phi(\tau, p)$ varies from the initial approximation $x_0(\tau)$ to the solution $x(\tau)$, so does $K(p)$ from the initial approximation k_0 to the exact value k.

Expanding $\phi(\tau, p)$ and $K(p)$ in series with respect to the parameter p, one has respectively

$$\phi(\tau,p) = x_0(\tau) + px_1(\tau) + p^2x_2(\tau) + \cdots \tag{15}$$

$$K(p) = k_0 + pk_1 + p^2k_2 + \cdots \tag{16}$$

If the initial approximation $x_0(\tau)$ and the auxiliary function $h(\tau, p)$ are properly chosen so that the above series converges as $p = 1$, one has:

$$x(\tau) = x_0(\tau) + x_1(\tau) + x_2(\tau) + \cdots \tag{17}$$

$$k = k_0 + k_1 + k_2 + \cdots \tag{18}$$

Notice that the series (15) and (16) contain the auxiliary function $h(\tau, p)$ which determines their convergence regions.

The results of the m th-order approximations are given by

$$\bar{x}(\tau) \approx x_0(\tau) + x_1(\tau) + \cdots + x_m(\tau) \tag{19}$$

$$\bar{k} \approx k_0 + k_1 + \cdots + k_{m-1} \tag{20}$$

We propose the auxiliary function $h(\tau, p)$ be of the form:

$$\begin{aligned} h(\tau,p) = {} & pC_1 + p^2C_2 + \cdots + p^{m-1}C_{m-1} \\ & + p^m g_m(\tau) \end{aligned} \tag{21}$$

where $C_1, C_2, \ldots, C_{m-1}$ can be constants and the last term $g_m(\tau)$ can be a function depending on the variable τ.

Substituting (15) and (16) into (12) yields:

$$\begin{aligned} N(\phi) = {} & N_0(x_0, k_0) + pN_1(x_0, x_1, k_0, k_1) \\ & + p^2N_2(x_0, x_1, x_2, k_0, k_1, k_2) + \cdots \end{aligned} \tag{22}$$

If we substitute (22) and (21) into (10) and equating the coefficients of like powers of p to zero, we obtain the following linear equations

$$L(x_0)=0, \qquad x_0(0)=1, \qquad x_0'(0)=0 \tag{23}$$

$$\begin{aligned} &L(x_i)-L(x_{i-1}) \\ &\quad -\sum_{j=1}^{i} C_j N_{i-j}(x_0,x_1,\dots,x_{i-j},k_0,k_1,\dots,k_{i-j}) \\ &\quad =0 \\ &x_i(0)=x_i'(0)=0, \quad i=1,2,\dots,m-1 \end{aligned} \tag{24}$$

$$\begin{aligned} &-\sum_{j=1}^{m-1} C_j N_{m-j} - g_m(\tau)N_0 = 0 \\ &x_m(0)=x_m'(0)=0 \end{aligned} \tag{25}$$

Note that k_i can be determined avoiding the presence of secular terms in (24). At this moment, the m th-order approximation given by (19) depends on the parameters $C_1, C_2, \dots, C_{m-1}$, as well as of the function $g_m(\tau)$. The constants $C_1, C_2, \dots, C_{m-1}$ and those constants which appear in the expression of $g_m(\tau)$, can be identified via various methods, for example the least square method, the Galerkin method, the collocation method and so on.

It is to be noted that our procedure has a very important feature in that it contains the auxiliary function $h(\tau,p)$, which provides us with a simple and rigorous way to adjust and control the convergence of solution. In this procedure it is very important to properly choose the last function $g_m(\tau)$ which appears in the m th-order approximation (19). More details are presented in [26].

3 Explicit analytical approximation

For the motion of a uniform cantilever beam governed by (7), the function $f(\tau,x,x',x'')$ is given by

$$f(\tau,x,x',x'')=ax^2x''+axx'^2+bk^2x^3$$

From (23), (24) and (25) ($m=2$), we obtain the following equations:

$$x_0''+x_0=0, \qquad x_0(0)=1, \qquad x_0'(0)=0 \tag{26}$$

$$\begin{aligned} &x_1''+x_1-(x_0''+x_0) \\ &\quad -C_1[x_0''+k_0^2x_0+ax_0^2x_0''+ax_0x_0'^2+bk_0^2x_0^3]=0 \\ &x_1(0)=x_1'(0)=0 \end{aligned} \tag{27}$$

$$\begin{aligned} &x_2''+x_2-(x_1''+x_1) \\ &\quad -C_1[x_1''+k_0^2x_1+2k_0k_1x_0 \\ &\quad +a(2x_0x_1x_0''+x_0^2x_1''+2x_0x_0'x_1'+x_0'^2x_1) \\ &\quad +b(2k_0k_1x_0^3+3k_0^2x_0^2x_1)] \\ &\quad -g_2(\tau)[x_0''+k_0^2x_0+a(x_0^2x_0''+x_0x_0'^2) \\ &\quad +bk_0^2x_0^3]=0 \\ &x_2(0)=x_2'(0)=0 \end{aligned} \tag{28}$$

Equation (26) has the solution

$$x_0(\tau)=\cos\tau \tag{29}$$

If this result is substituted into (27) and assuming C_1 to be constant, we obtain the following equation

$$\begin{aligned} &x_1''+x_1+C_1\left(k_0^2-1+\frac{3bk_0^2-2a}{4}\right)\cos\tau \\ &\quad +\frac{C_1(2a+ab-b)}{3b+4}\cos 3\tau=0 \\ &x_1(0)=x_1'(0)=0 \end{aligned} \tag{30}$$

where C_1 is an unknown constant at this moment. Avoiding the presence of a secular term needs:

$$k_0^2=\frac{2a+4}{3b+4} \tag{31}$$

with this requirement, the solution of (30) is

$$x_1(\tau)=\frac{C_1(2a+ab-b)}{8(3b+4)}(\cos 3\tau-\cos\tau) \tag{32}$$

If we substitute (29), (31) and (32) into (28), we obtain the equation in x_2:

$$\begin{aligned} &x_2''+x_2+\frac{C_1(2a+ab-b)}{3b+4}\cos 3\tau \\ &\quad +C_1\left[\frac{-C_1(2a+ab-b)(2a+3ab+3b)}{8(3b+4)^2}\right. \\ &\quad \left.+\frac{k_0k_1}{2}(3b+4)\right]\cos\tau \\ &\quad +C_1\left[\frac{bk_0k_1}{2}-\frac{C_1(2a+ab-b)(3a+8)}{8(3b+4)}\right]\cos 3\tau \end{aligned}$$

$$+\left[\frac{3C_1^2(2a+ab-b)(b-4ab-6a)}{8(3b+4)^2}\right]\cos 5\tau$$
$$+g_2(\tau)\left[\frac{b-2a-ab}{3b+4}\right]\cos 3\tau$$
$$x_2(0)=x_2'(0)=0 \tag{33}$$

No secular terms in $x_2(\tau)$ requires that

$$k_1=\frac{C_1(2a+ab-b)(2a+3ab+3b)}{4k_0(3b+4)^3} \tag{34}$$

From (18), (31) and (34) we obtain

$$k=\sqrt{\frac{2a+4}{3b+4}}+\frac{C_1(2a+ab-b)(2a+3ab+3b)}{4(3b+4)^3}\sqrt{\frac{3b+4}{2a+4}} \tag{35}$$

and therefore

$$\Omega=k^{-1}=\left(\sqrt{\frac{2a+4}{3b+4}}+\frac{C_1(2a+ab-b)(2a+3ab+3b)}{4(3b+4)^3}\times\sqrt{\frac{3b+4}{2a+4}}\right)^{-1} \tag{36}$$

Substituting (34) into (33) and considering $g_2(\tau)=C_2+2C_3\cos 2\tau+2C_4\cos 4\tau$ with C_2, C_3 and C_4 unknown constants, we obtain $C_4=-C_3$ and

$$x_2(\tau)=\left[\frac{(C_1+C_2)(b-2a-ab)}{8(3b+4)}-\frac{C_1^2(2a+ab-b)(3a+8)}{64(3b+4)}+\frac{bC_1^2(2a+ab-b)(2a+3ab+3b)}{64(3b+4)^3}\right]\times(\cos\tau-\cos 3\tau)+\left[\frac{C_1^2(2a+ab-b)(b-4ab-6a)}{64(3b+4)^2}+\frac{C_3(b-2a-ab)}{24(3b+4)}\right](\cos\tau-\cos 5\tau)-\frac{C_3(b-ab-2a)}{48(3b+4)}(\cos\tau-\cos 7\tau) \tag{37}$$

The second order approximate solution is

$$\bar{x}(\tau)=x_0(\tau)+x_1(\tau)+x_2(\tau)$$

where x_0, x_1 and x_2 are given by (29), (32) and (37) respectively.

Using the transformations (3) and (6), the second order approximate solution of (4) becomes:

$$u(t)=B\cos\Omega t+C\cos 3\Omega t+D\cos 5\Omega t+E\cos 7\Omega t \tag{38}$$

where B, C, D, E and Ω are given respectively by:

$$\begin{aligned}
B&=A+\frac{(12C_1+6C_2+C_3)(\beta A^3-\alpha A^3-\alpha\beta A^5)}{48(3\beta A^2+4)}\\
&\quad-\frac{C_1^2(2\alpha A^3+\alpha\beta A^5-\beta A^3)(18\alpha\beta A^5+33\beta^2A^4+52\alpha\beta A^4+36\alpha A^2+94\beta A^2+64)}{32(3\beta A^2+4)^3}\\
C&=\frac{(2C_1+C_2)(2\alpha A^3+\alpha\beta A^5-\beta A^3)}{8(3\beta A^2+4)}\\
&\quad+\frac{C_1^2(2\alpha A^3+\alpha\beta A^5-\beta A^3)(24\alpha\beta^2A^5+70\alpha\beta A^4+69\beta^2A^4+48\alpha A^2+192\beta A^2+128)}{64(3\beta A^2+4)^3}\\
D&=\frac{C_1^2(2\alpha A^3+\alpha\beta A^5-\beta A^3)(6\alpha A^2+4\alpha\beta A^4-\beta A^2)}{64(3\beta A^2+4)^2}+\frac{C_3(2\alpha A^2+\alpha\beta A^4-\beta A^2)}{24(3\beta A^2+4)}\\
E&=\frac{C_3(\beta A^2-2\alpha A^2-\alpha\beta A^4)}{48(3bA^2+4)}
\end{aligned} \tag{39}$$

$$\Omega = \left(\sqrt{\frac{2\alpha A^2 + 4}{3\beta A^2 + 4}} + \frac{C_1(2\alpha A^2 + \alpha\beta A^4 - \beta A^2)(2\alpha A^2 + 3\alpha\beta A^4 + 3\beta A^2)}{4(3\beta A^2 + 4)^3} \sqrt{\frac{3\beta A^2 + 4}{2\alpha A^2 + 4}} \right)^{-1}$$

The constants C_i, $i = 1, 2, 3, 4$ will be obtained using the least square method.

4 Results and discussion

In this analysis, periodic solutions are analyzed for the cantilever beam under study. Beside the role of the large amplitude A, a special role is played by the modal constants α and β, which depends on the inertia parameters of the attached inertia element with mass M and rotary inertia J. We do not approach the simplest cases when the modal constants α, β are small (0.1 or 0.2), because in these cases it is easy to achieve accurate periodic solutions even for large amplitudes using known procedures. Difficulties appear when these modal constants become larger [9] and the oscillator experiences large amplitudes. Here, the meaning of "large" implies the fact that the peak amplitude reach a value where the non-linear terms are of an order comparable to that of the linear ones. More specific, the amplitude may be of the order of beam length.

We illustrate the accuracy of our procedure for large modal constants and large amplitudes comparing the obtained approximate analytical solutions with the numerical integration results obtained using a fourth-order Runge-Kutta method. We will also compare these results with published results [9].

We further consider larger values for the modal constants α and β (1 or 2) and we also consider large values of the initial amplitude A (5 or 10). In order to prove the accuracy of the obtained results, two examples are analysed.

Example 1 For the modal constants $\alpha = 1$, $\beta = 1$, and the initial amplitude $A = 5$, following the procedure described above it is obtained the convergence-control constants:

$$C_1 = -0.102968962; \qquad C_2 = -0.005613042$$
$$C_3 = 0.038438926; \qquad C_4 = -0.038438926$$

and consequently the approximate periodic solution becomes:

$$\begin{aligned} u(t) &= 5.317402609 \cos \Omega t - 0.576807191 \cos 3\Omega t \\ &\quad + 0.292349339 \cos 5\Omega t \\ &\quad - 0.032944754 \cos 7\Omega t \end{aligned} \tag{40}$$

where $\Omega = 1.342143172$, while for the same modal constants α and β, when the initial amplitude A raises to $A = 10$, we obtain the constants:

$$C_1 = -0.029698817; \qquad C_2 = -0.002408906$$
$$C_3 = 0.001571857; \qquad C_4 = -0.001571857$$

and the approximate periodic solution in this case will be:

$$\begin{aligned} u(t) &= 10.686783311 \cos \Omega t - 1.307660795 \cos 3\Omega t \\ &\quad + 0.631757266 \cos 5\Omega t \\ &\quad - 0.010879766 \cos 7\Omega t \end{aligned} \tag{41}$$

where $\Omega = 1.382367422$.

Example 2 For the modal constants $\alpha = 2$, $\beta = 2$, and the initial amplitude $A = 5$, following the same procedure we obtain the convergence-control constants:

$$C_1 = -0.056226544; \qquad C_2 = -0.003304042$$
$$C_3 = -0.073429731; \qquad C_4 = 0.073429731$$

and consequently the approximate periodic solution is in this case:

$$\begin{aligned} u(t) &= 5.47307199 \cos \Omega t - 0.618622196 \cos 3\Omega t \\ &\quad + 0.018895842 \cos 5\Omega t \\ &\quad + 0.126654363 \cos 7\Omega t \end{aligned} \tag{42}$$

where $\Omega = 1.374317516$, while for the same modal constants α and β, but for a larger initial amplitude $A = 10$, we obtain the constants:

$$C_1 = -0.015351906; \qquad C_2 = -0.001411507$$
$$C_3 = -0.000041392; \qquad C_4 = 0.000041392$$

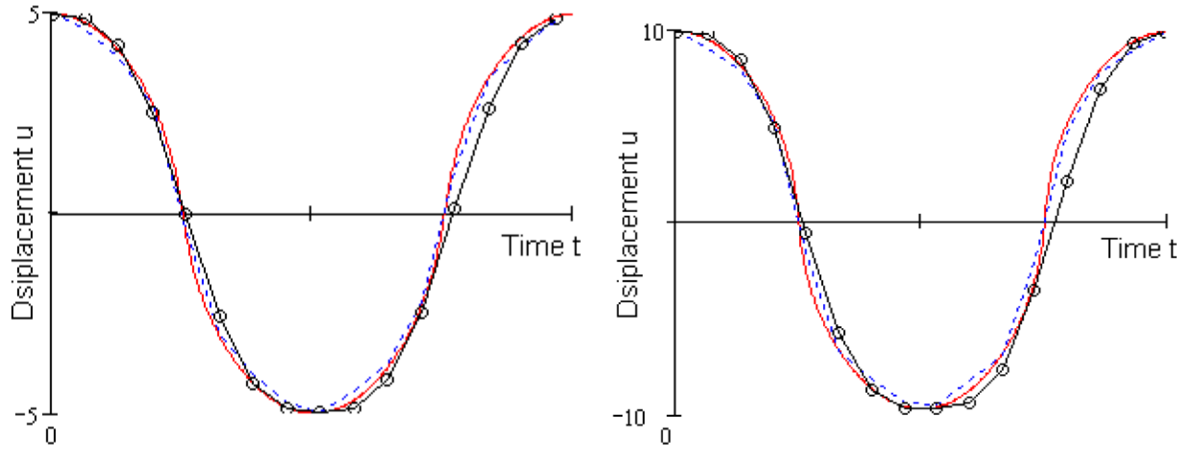

Fig. 1 **a** and **b** present a comparison of the obtained analytical solutions (40) and (41) with numerical ones and also with known results [9] for the modal constants $\alpha = \beta = 1$ and initial amplitudes $A = 5$ and $A = 10$, respectively, for a period of the motion (—— numerical results; – – – – OHAM results; –o–o–o– results from [9])

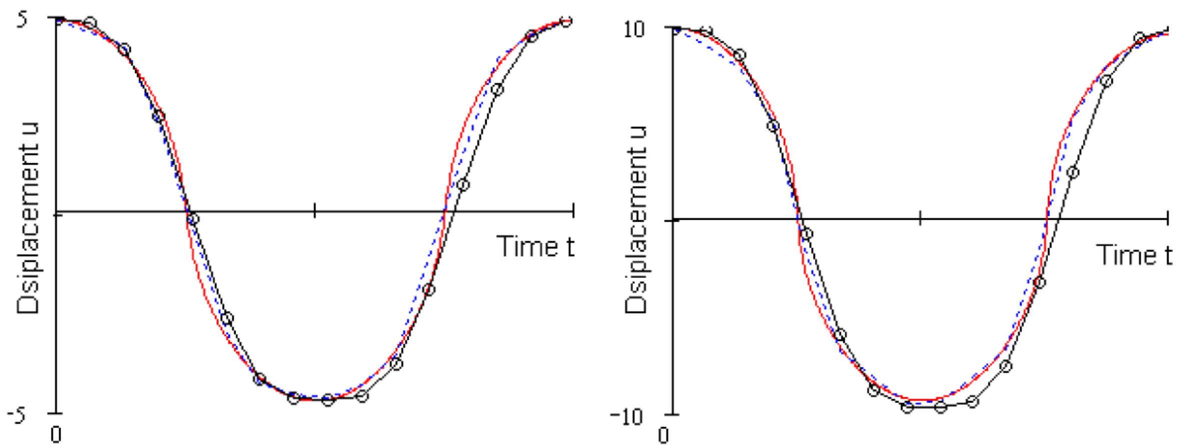

Fig. 2 **a** and **b** present a comparison of the obtained analytical solutions (42) and (43) with numerical ones and also with known results [9] for the modal constants $\alpha = \beta = 2$ and initial amplitudes $A = 5$ and $A = 10$, respectively, for the first period of the motion (—— numerical results; – – – – OHAM results; –o–o–o– results from [9])

and the approximate periodic solution in this case:

$$\begin{aligned} u(t) &= 10.69151238 \cos \Omega t - 1.344253519 \cos 3\Omega t \\ &\quad + 0.652167205 \cos 5\Omega t \\ &\quad + 0.000014221 \cos 7\Omega t \end{aligned} \tag{43}$$

where $\Omega = 1.400493712$.

It can be seen from the above Figures that the solutions obtained by the proposed procedure is nearly identical with the numerical solutions obtained using a fourth-order Runge-Kutta method. Moreover, the analytical solutions obtained by our procedure proved to be more accurate than other known results obtained by combining the linearization of the governing equation with the method of harmonic balance [9], especially for large values of the modal constants α and β. In the case of large modal constants $\alpha = \beta = 2$ and very large amplitude $A = 10$, Fig. 2b indicates an evident discrepancy of numerical solutions and solutions from [9], while the analytic solution obtained in this paper is still valid.

5 Conclusions

In this paper, the Optimal Homotopy Asymptotic Method (OHAM) is employed to propose a new explicit analytic approximate solution for non-linear oscillations of planar, flexural large amplitude free vibrations of a slender, inextensible cantilever beam carrying a lumped mass with rotary inertia at an intermediate position along its span. This procedure is optimized to control the convergence of solutions through some convergence-control functions such that accuracy is always guaranteed even if the nonlinear equation does not contain any small or large parameters.

The obtained results demonstrate that the methods is applicable to oscillations with large parameters and unlike other known methods, the present analytical approximate procedure yield periodic solutions which are valid even for large modal constants α and β and large amplitude of oscillation. Excellent agreement of the analytical approximate periodic solutions with numerical solution has been demonstrated and discussed on several examples.

This new approach proves to be very rapid, effective and accurate and this is proved by comparing the solutions obtained through the proposed method with the solutions obtained via numerical integration as well as with published results in the literature.

This paper shows one step in the attempt to develop a new nonlinear analytical technique valid for large parameters and the proposed procedure can easily be used to find analytical approximate solutions to other strongly nonlinear oscillators.

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