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Cross-section optimization of tower crane lattice boom

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Abstract This paper discusses the problem of cross-section optimization of tower crane lattice boom. The trapezoid cross section has been analyzed. The triangular and rectangular cross sections have also been analyzed as special cases. Total mass of the construction has been selected as the objective function. Optimization parameters have been determined by Lagrange's multipliers method. Criterion of stress is used as the constraint function. Numerical examples are performed by usage of the obtained theoretical solutions. Those results were than compared with ones obtained by usage of finite element method based computer program. On the base of the analysis that was carried out, the recommendations are given regarding the application of trapezoid, triangular and rectangular cross sections of the lattice constructions.

Keywords Lattice boom · Lagrange multipliers · Design optimization · Stress · Tower crane

1 Introduction

The analysis of the cost structure of a metal construction, that was carried out by Farkas [6], is shown that the price is primarily influenced by the price of the material (30–73)%, while the other costs are lower: the manufacture (16–22)%, assembling (5–20)%, transportation (3–7)%, designing (2–3)%. The selection of optimum shape and optimum parameters of metal construction reduces the consumption of the material and its price.

The problem of construction optimization, using different methods, objective and constraint functions, was studied by several authors. Farkas [4–6] came to conclusion that the mathematical problems of the conditional extreme of the function of several variables can be applied onto structure cross-section in order to optimize it from the aspect of load and material consumption.

Jelicic and Atanackovic [8] determined optimal shape of elastic rod for axial load case. The optimization was achieved by implementation of Pontriyagin's maximum principle. The authors showed that the selection of the elastic rod optimal shape for certain cases of buckling can provide saving of the material up to 30%.

Atanackovic [1, 2] determined the optimal cross section of the rotating rod.

Banichuk, Ragnedda and Serra [3] determined the optimal dimensions of the cross sections shaped as regular polygons with "n" sides ($n \geq 3$). The optimization was carried out for adopted stress and deformation as the constraint functions, and minimal cross section area as the objective function.

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Selmic and Cvetkovic [9] considered the problem of determination of optimal parameters for rectangular cross section of lattice structures in case where the criterion of stress was the constraint function.

Selmic, Cvetkovic, Mijailovic and Kastratovic [10] determined optimal parameters of the triangular cross section of lattice structures, where both criteria of stress and deformation were used as constraint functions.

Selmic, Cvetkovic and Mijailovic [11], besides the determination of optimal dimensions of various cross sections for different supporters, analyzed theoretical bases of the optimization by application of Lagrange's multipliers method.

Gadus [7] preformed optimization of frameworks by finite element analysis, and achieved more than 35% savings of the material mass, keeping the required resistance and consistency parameters of construction.

What separates this paper from others? In this case, in model is included practical data that the tower crane booms are made as lattice structures with constant overall dimensions of cross section along boom. In order to satisfy requirement that maximum stress in any given cross section of the boom should be equal to allowable stress, here is suggested that the cross section area of flanges is variable along boom. However, it should be noted that in this paper only the flanges are considered, using allowable stress constraint. The buckling of compressed members (lattice) and the effect of bracing are neglected. This approximation should only be used for preliminary design purposes, and can be very useful to an engineer-designer in the first stage of the designing procedure, when he is faced with the problem of defining basic structure dimensions that would be close to optimum ones.

The analysis of real tower crane lattice booms has shown that they are made with triangular cross section. However, the analytical proof that the triangular cross section is indeed optimal can not be found in the literature. This is the reason why the optimization of trapezoid cross section was carried out, as follows. This optimization enables that with introduction of certain approximations, the optimal dimensions can be obtained. The necessary conditions, which determine whether the optimal cross section is triangular or rectangular, have also been obtained.

2 Defining of the optimization task

Lattice constructions have important applications in practice, especially in case of constructions with larger overall dimensions of which is required larger carrying capacity and minimal weight. The tower crane booms and truck-crane booms can be mentioned as the examples of those constructions, as well as constructions which are the integral part of halls and warehouses.

The cross section of lattice construction (Fig. 1) consists of flanges (positions 1–4) and lattice (positions 5–8). The flanges are usually made of pipes with circular, square or rectangular cross section. They can also be made of "L" or "U" profiles combination.

The analysis of the lattice structures is shown that the most used are those with triangular, rectangular and trapezoid cross section. The recommendations in available literature give basic dimensions of lattice construction cross section in function of its length. Still, they ignore many other influential factors, such as loads, material... On the other hand, there is no recommendation in the available literature regarding optimal cross section depending on exploitation conditions.

The mentioned reasons are the guidelines for the optimization of tower crane lattice boom. The optimization is performed (Fig. 2) in sense of:

- Determination of cross section optimal parameters and
- Selection of optimal cross section.

The determination of tower crane lattice boom trapezoid cross section optimal parameters in this case required determination of:

- c, d, h—dimensions of trapezoid cross section (Fig. 1),

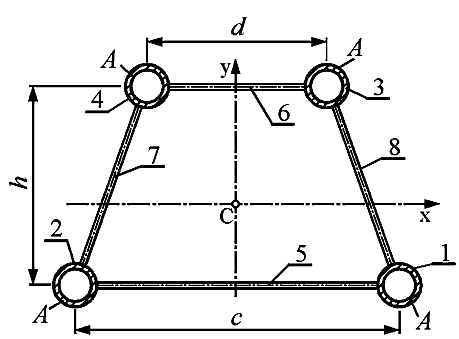

Fig. 1 Trapezoid cross section

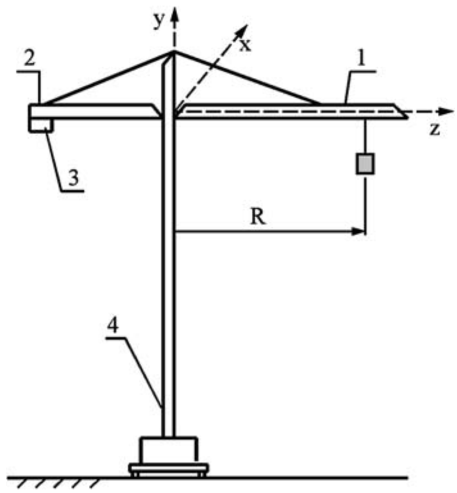

Fig. 2 Tower crane (*1*—boom, *2*—counter-boom, *3*—counterweight, *4*—tower)

– A–cross section area of one flange.

The dimensions of trapezoid cross section (c, d, h) are constant along the boom, while the cross section area of flange is variable.

3 Mathematical model

Vector of given parameters is:

$$\vec{x} = \big(N, M_x, M_y, [\sigma], \ldots\big) \tag{1}$$

where

– N is the axial force, supposed to act on the cross-section center,
– M_x, M_y are the moments of flexion for axes x and y, respectively,
– $[\sigma]$ is the limiting (allowable) stress,

Vector of variables is given by

$$\vec{y} = (c, d, h, \ldots). \tag{2}$$

Constrain function is presented as:

$$\varphi_1 = \varphi_1(c, d, h, \ldots), \qquad \varphi_2 = \varphi_2(c, d, h, \ldots), \ldots \tag{3}$$

The determination of optimum parameters was performed by the method of Lagrange multipliers for extremes of the function of several variables. Therefore, for the function $F = F(c, d, h, \ldots)$ to have relative maximum or minimum at a certain point, it is necessary that constrain function (3) be satisfied, as well as the following equations:

$$\frac{\partial \Phi}{\partial a} = 0, \qquad \frac{\partial \Phi}{\partial b} = 0, \qquad \frac{\partial \Phi}{\partial h} = 0, \quad \ldots \tag{4}$$

where Lagrange's function is:

$$\Phi(c, d, h, \ldots) = F(c, d, h, \ldots) + \lambda_1 \cdot \varphi_1(c, d, h, \ldots) + \lambda_2 \cdot \varphi_2(c, d, h, \ldots) + \cdots \tag{5}$$

and λ—unknown Lagrange multipliers. So, the system of equations (4) can be expressed as:

$$\begin{aligned} &\frac{\partial F}{\partial a} + \lambda_1 \frac{\partial \varphi_1}{\partial a} + \lambda_2 \frac{\partial \varphi_2}{\partial a} + \cdots = 0, \\ &\frac{\partial F}{\partial b} + \lambda_1 \frac{\partial \varphi_1}{\partial b} + \lambda_2 \frac{\partial \varphi_2}{\partial b} + \cdots = 0, \\ &\frac{\partial F}{\partial h} + \lambda_1 \frac{\partial \varphi_1}{\partial h} + \lambda_2 \frac{\partial \varphi_2}{\partial h} + \cdots = 0, \quad \ldots \end{aligned} \tag{6}$$

The system of equations (3) and (6) defines unknown optimum parameters $(c_O, d_O, h_O, \ldots)$ and unknown Lagrange multipliers λ.

4 Constraint function

The stress criterion was used as constraint function. According to this criterion, the constraint function is based on the condition that for all values of coordinate (z) maximum stress which occurs in one of the flanges of the boom should be equal to allowable stress:

$$\varphi(z) = \left| \frac{N(z)}{4A} + \frac{M_x(z)}{I_x} \cdot y + \frac{M_y(z)}{I_y} \cdot x \right| - [\sigma] = 0. \tag{7}$$

Moments of inertia of trapezoid cross section are given by expressions:

$$I_x = A \cdot h^2, \tag{8}$$

$$I_y = \frac{A}{2}(c^2 + d^2), \tag{9}$$

Substituting the expressions (8) and (9) in (7) gives the expression which defines constraint function based

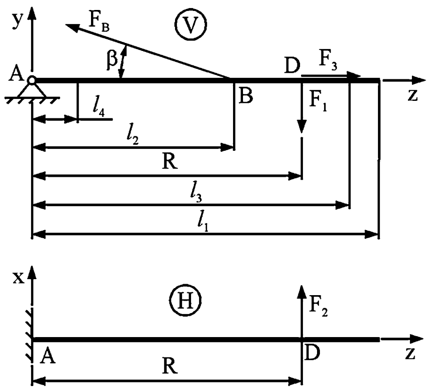

Fig. 3 Static model of tower crane boom

on stress criterion:

$$\varphi(z) = \frac{1}{A}\left|\frac{N(z)}{4} + \frac{M_x(z)}{h^2}\cdot y + \frac{2\cdot M_y(z)}{c^2+d^2}\cdot x\right| - [\sigma] = 0, \quad (10)$$

The model of a tower crane boom was analyzed as a static model in two different planes: in vertical plane it is pin connected beam at A (joint with the tower of the crane), supported by cable at B; in horizontal plane it is modeled as fixed supported beam (Fig. 3) [10, 11], where:

- F_1—is the resultant force acting in vertical plane at crane hook radius R due to the dead load, crab and pulley weight, and inertial force that emerges due to the load lifting,
- F_2—is the force acting in horizontal plane at the crane (hook) radius R, a tangential force due to load rotation,
- F_3—is the force acting in the direction of lengthwise axis at the crane (hook) radius R, a centrifugal force due to the load rotation,
- F_B—is force in the cable,
- R—is the crane (hook) radius, which determines the load position in respect of the tower of the crane ($l_4 \le R \le l_3$),
- l_1—overall boom length,
- l_2—the distance between joint position of boom and tower of the crane and joint position of boom and cable.

Carrying capacity was approximated based on the catalogue data as following function [10]:

$$Q = \sqrt{428.861 - 41.106\sqrt{R}}, \quad [N], \quad (11)$$

where crane (hook) radius is in meters.

The variations of axial force and bending moments can be determined, based on static model of the boom, as functions of crane (hook) radius (R) and lengthwise coordinate (z).

By the application of expression (10), a correspondence between flange cross section area, load, allowable stress and dimensions of trapezoid cross section, can be written as a next function:

$$A = \frac{1}{[\sigma]}\left|\frac{N(z)}{4} + \frac{M_x(z)}{h^2}\cdot y + \frac{2\cdot M_y(z)}{c^2+d^2}\cdot x\right|. \quad (12)$$

As it is assumed that dimensions of trapezoid cross section (c, d, h) have constant values along the boom, it can be concluded that, based on the analysis of the expression (12), the cross section area of flanges are dependent of the axial force and bending moment's variations along the boom. Static model of boom (Fig. 3) is subjected to concentrated forces (F_1, F_2, F_3). This causes linear change of bending moments with lengthwise coordinate "z". The configuration of this linear function changes in position of constrains and loads. Based on this, it follows that cross section area of flanges vary linearly with lengthwise coordinate "z", i.e., (12) can be written as:

$$A(z) = \begin{cases} a_{10} + a_{11}\cdot z & \text{for } 0 \le z \le l_2 \wedge l_2 \le R \le l_3 \\ a_{20} + a_{21}\cdot z & \text{for } l_2 < z \le R \wedge l_2 \le R \le l_3 \\ b_{10} + b_{11}\cdot z & \text{for } 0 \le z \le R \wedge 0 \le R < l_2 \\ b_{20} + b_{21}\cdot z & \text{for } R < z \le l_2 \wedge 0 \le R < l_2 \\ 0 & \text{for } R < z \le l_1 \wedge l_2 \le R \le l_3 \\ & \vee l_2 < z \le l_1 \wedge 0 \le R < l_2. \end{cases} \quad (13)$$

In correspondence with load position, in following text the optimal parameters of trapezoid cross section of tower crane lattice boom are determined:

- c, d, h—dimensions of trapezoid cross section,
- parameters which determine cross section area of flanges
  - $a_{10}, a_{11}, a_{20}, a_{21}$ for $l_2 \le R \le l_3$,
  - $b_{10}, b_{11}, b_{20}, b_{21}$ for $0 \le R < l_2$.

5 Objective function

As it was mentioned before, the price of the material, hence, material consumption has the main influence in price of metal construction overall. Hence, total mass of the construction has been selected as the objective function:

$$F \equiv m = \rho \cdot V, \tag{14}$$

where

- ρ—is density of the material,
- V—volume of the lattice construction.

Volume of the lattice construction is equal to the sum of flanges volume (V_p), volume of lattice (V_r) and volume of other elements (V_o):

$$V = V_p + V_r + V_o. \tag{15}$$

The volume of the flanges is determined by next expressions:

$$V_p = 4 \cdot \int_0^{L_1} A dz = \begin{cases} 4 \cdot \int_0^{L_2} a_{10} + a_{11} \cdot z dz + 4 \cdot \int_{L_2}^{R} a_{20} + a_{21} \cdot z dz \\ \quad \text{for } l_2 \le R \le l_3 \\ 4 \cdot \int_0^{R} b_{10} + b_{11} \cdot z dz + 4 \cdot \int_{R}^{L_2} b_{20} + b_{21} \cdot z dz \\ \quad \text{for } 0 \le R < l_2, \end{cases} \tag{16}$$

After integration expression (16) becomes

$$V_p = \begin{cases} 4 \cdot a_{10} \cdot L_2 + 2 \cdot a_{11} \cdot L_2^2 \\ \quad + 4 \cdot a_{20} \cdot (R - L_2) \\ \quad + 2 \cdot a_{21} \cdot (R^2 - L_2^2) \\ \quad \text{for } l_2 \le R \le l_3 \\ 4 \cdot b_{10} \cdot R + 2 \cdot b_{11} \cdot R^2 + 4 \cdot b_{20} \\ \quad \cdot (L_2 - R) + 2 \cdot b_{21} \cdot (L_2^2 - R^2) \\ \quad \text{for } 0 \le R < l_2. \end{cases} \tag{17}$$

Volume of lattice is determined by:

$$V_r = k_1(c + d) + k_2\sqrt{4h^2 + (c - d)^2}, \tag{18}$$

while volume of other elements is determined by following expression

$$V_o = k_3(V_p + V_r). \tag{19}$$

The variation of coefficients values k_1, k_2, k_3 is minimal in observed cases, so they can be assumed to be constant [4–6, 10, 11].

Substituting the expression (19) in (15) provide expression which determine volume of lattice construction

$$V = (1 + k_3) \cdot (V_p + V_r). \tag{20}$$

6 Optimal parameters

Lagrange's function (5) is given as:

$$\Phi = V + \lambda_1 \cdot \varphi(z_1) + \lambda_2 \cdot \varphi(z_2) + \lambda_3 \cdot \varphi(z_3) + \lambda_4 \cdot \varphi(z_4), \tag{21}$$

where $\varphi(z_i)$ represent constraint function based on criterion of stress written for the following values of lengthwise coordinates

- $0 \le z_1 \le l_2, 0 \le z_2 \le l_2, l_2 \le z_3 \le R, l_2 \le z_4 \le R$ for $l_2 \le R \le l_3$,
- $0 \le z_1 \le R, 0 \le z_2 \le R, R \le z_3 \le l_2, R \le z_4 \le l_2$ for $0 \le R < l_2$.

After substituting expressions (10), (20), (21) in expression (4) the systems of equations are obtained. Solutions of these systems of equations determine unknown parameters as functions of load position and position of the flange in which maximal stress occurs. There are four possible combinations of load position and position of the flange in which maximal stress occurs:

- $l_2 \le R \le l_3$, maximal actual stress occurs in one of the lower flanges (Fig. 3—position 1 and 2)

$$\begin{aligned} & k_1 + k_2 \cdot \frac{c - d}{\sqrt{4 \cdot h^2 + (c - d)^2}} = \frac{c^2 - d^2}{(c^2 + d^2)^2} \cdot \frac{2 \cdot F_2 \cdot R^2}{[\sigma]}, \\ & \frac{h^3}{\sqrt{4 \cdot h^2 + (c - d)^2}} = \frac{F_1 \cdot R \cdot (R - l_2)}{4 \cdot k_2 \cdot [\sigma]}, \\ & 2 \cdot c \cdot d \cdot \left(k_1 + k_2 \cdot \frac{c - d}{\sqrt{4 \cdot h^2 + (c - d)^2}}\right) \\ & \quad = \left(c^2 - d^2\right) \cdot \left(k_1 - k_2 \cdot \frac{c - d}{\sqrt{4 \cdot h^2 + (c - d)^2}}\right), \\ & a_{10} = \frac{1}{[\sigma]}\left[\frac{1}{4} \cdot (F_1 \frac{R}{l_2} \text{ctg}\beta - F_3) + \frac{F_2 \cdot R \cdot c}{c^2 + d^2}\right], \\ & a_{11} = \frac{1}{[\sigma]}\left[\frac{F_1 \cdot (R - l_2)}{2 \cdot h \cdot l_2} - \frac{F_2 \cdot c}{c^2 + d^2}\right], \\ & a_{20} = \frac{1}{[\sigma]}\left[\frac{-F_3}{4} + \frac{F_1 \cdot R}{2 \cdot h} + \frac{F_2 \cdot R \cdot c}{c^2 + d^2}\right], \\ & a_{21} = \frac{1}{[\sigma]}\left[-\frac{F_1}{2 \cdot h} - \frac{F_2 \cdot c}{c^2 + d^2}\right]; \end{aligned} \tag{22}$$

- $0 \le R < l_2$, maximal actual stress occurs in one of the lower flanges (Fig. 3—position 1 and 2)

$$
\begin{aligned}
&k_1 + k_2 \cdot \frac{c-d}{\sqrt{4\cdot h^2 + (c-d)^2}} \\
&\quad = \frac{c^2-d^2}{(c^2+d^2)^2}\cdot\frac{2\cdot F_2\cdot R^2}{[\sigma]},\\
&\frac{h^3}{\sqrt{4\cdot h^2+(c-d)^2}} = \frac{F_1\cdot R\cdot(l_2-R)}{4\cdot k_2\cdot[\sigma]},\\
&2\cdot c\cdot d\cdot\left(k_1 + k_2\cdot\frac{c-d}{\sqrt{4\cdot h^2+(c-d)^2}}\right)\\
&\quad = (c^2-d^2)\cdot\left(k_1 - k_2\cdot\frac{c-d}{\sqrt{4\cdot h^2+(c-d)^2}}\right), \quad (23)\\
&b_{10} = \frac{1}{[\sigma]}\left[\frac{-1}{4}\cdot\left(F_1\frac{R}{l_2}\text{ctg}\beta - F_3\right) + \frac{F_2\cdot R\cdot c}{c^2+d^2}\right],\\
&b_{11} = \frac{1}{[\sigma]}\left[\frac{F_1\cdot(l_2-R)}{2\cdot h\cdot l_2} - \frac{F_2\cdot c}{c^2+d^2}\right],\\
&b_{20} = \frac{1}{[\sigma]}\left[-F_1\frac{R}{4\cdot l_2}\text{ctg}\beta + \frac{F_1\cdot R}{2\cdot h}\right],\\
&b_{21} = \frac{1}{[\sigma]}\left[-\frac{F_1\cdot\eta}{2\cdot h\cdot l_2}\right].
\end{aligned}
$$

– $l_2 \le R \le l_3$, maximal actual stress occurs in one of the upper flanges (Fig. 3—position 3 and 4)

$$
\begin{aligned}
&k_1 + k_2 \cdot \frac{c-d}{\sqrt{4\cdot h^2 + (c-d)^2}} = \frac{4\cdot c\cdot d\cdot F_2\cdot R^2}{(c^2+d^2)^2\cdot[\sigma]},\\
&\frac{h^3}{\sqrt{4\cdot h^2+(c-d)^2}} = \frac{F_1\cdot R\cdot(R-l_2)}{4\cdot k_2\cdot[\sigma]},\\
&2\cdot c\cdot d\cdot\left(k_1 - k_2\cdot\frac{c-d}{\sqrt{4\cdot h^2+(c-d)^2}}\right)\\
&\quad = (d^2-c^2)\cdot\left(k_1 + k_2\cdot\frac{c-d}{\sqrt{4\cdot h^2+(c-d)^2}}\right), \quad (24)\\
&a_{10} = \frac{1}{[\sigma]}\left[\frac{-1}{4}\cdot\left(F_1\frac{R}{l_2}\text{ctg}\beta - F_3\right) + \frac{F_2\cdot R\cdot d}{c^2+d^2}\right],\\
&a_{11} = \frac{1}{[\sigma]}\left[\frac{F_1\cdot(R-l_2)}{2\cdot h\cdot l_2} - \frac{F_2\cdot d}{c^2+d^2}\right]\\
&a_{20} = \frac{1}{[\sigma]}\left[\frac{F_3}{4} + \frac{F_1\cdot R}{2\cdot h} + \frac{F_2\cdot R\cdot d}{c^2+d^2}\right],\\
&a_{21} = \frac{1}{[\sigma]}\left[-\frac{F_1}{2\cdot h} - \frac{F_2\cdot d}{c^2+d^2}\right];
\end{aligned}
$$

– $0 \le R < l_2$, maximal actual stress occurs in one of the upper flanges (Fig. 3—position 3 and 4)

$$
\begin{aligned}
&k_1 + k_2 \cdot \frac{c-d}{\sqrt{4\cdot h^2 + (c-d)^2}} = \frac{4\cdot c\cdot d\cdot F_2\cdot R^2}{(c^2+d^2)^2\cdot[\sigma]},\\
&\frac{h^3}{\sqrt{4\cdot h^2+(c-d)^2}} = \frac{F_1\cdot R\cdot(l_2-R)}{4\cdot k_2\cdot[\sigma]},\\
&2\cdot c\cdot d\cdot\left(k_1 - k_2\cdot\frac{c-d}{\sqrt{4\cdot h^2+(c-d)^2}}\right)\\
&\quad = (d^2-c^2)\cdot\left(k_1 + k_2\cdot\frac{c-d}{\sqrt{4\cdot h^2+(c-d)^2}}\right), \quad (25)\\
&b_{10} = \frac{1}{[\sigma]}\left[\frac{1}{4}\cdot\left(F_1\frac{R}{l_2}\text{ctg}\beta - F_3\right) + \frac{F_2\cdot R\cdot d}{c^2+d^2}\right],\\
&b_{11} = \frac{1}{[\sigma]}\left[\frac{F_1\cdot(l_2-R)}{2\cdot h\cdot l_2} - \frac{F_2\cdot d}{c^2+d^2}\right],\\
&b_{20} = \frac{1}{[\sigma]}\left[F_1\frac{R}{4\cdot l_2}\text{ctg}\beta + \frac{F_1\cdot R}{2\cdot h}\right],\\
&b_{21} = \frac{1}{[\sigma]}\left[-\frac{F_1\cdot\eta}{2\cdot h\cdot l_2}\right].
\end{aligned}
$$

The conditions that determine in which flange will occur maximal stress for any given lengthwise coordinate "z", are defined with expression (10). Therefore, the maximal stress will occur in one of the lower flanges (Fig. 3—position 1 and 2), corresponding the load positions, if the following conditions are met:

– $l_2 \le R \le l_3$
  – $0 \le z \le l_2$

$$
\begin{aligned}
&\left|\frac{1}{4}\cdot\left(F_1\cdot\frac{R}{l_2}\cdot\text{ctg}\beta - F_3\right) + \frac{F_1\cdot(R-l_2)\cdot z}{2\cdot h\cdot l_2} + \frac{F_2\cdot(R-z)\cdot c}{c^2+d^2}\right|\\
&\quad > \left|\frac{-1}{4}\cdot\left(F_1\cdot\frac{R}{l_2}\cdot\text{ctg}\beta - F_3\right) + \frac{F_1\cdot(R-l_2)\cdot z}{2\cdot h\cdot l_2} + \frac{F_2\cdot(R-z)\cdot d}{c^2+d^2}\right|, \quad (26)
\end{aligned}
$$

  – $l_2 < z \le R$

$$
\begin{aligned}
&\left|\frac{-F_3}{4} + \frac{F_1\cdot(R-z)}{2\cdot h} + \frac{F_2\cdot(R-z)\cdot c}{c^2+d^2}\right|\\
&\quad > \left|\frac{F_3}{4} + \frac{F_1\cdot(R-z)}{2\cdot h} + \frac{F_2\cdot(R-z)\cdot d}{c^2+d^2}\right|, \quad (27)
\end{aligned}
$$

– $0 \le R < l_2$
  – $0 \le z \le R$

$$
\left|\frac{-1}{4}\cdot\left(F_1\cdot\frac{R}{l_2}\cdot\text{ctg}\beta - F_3\right)\right.
$$

$$+\frac{F_1\cdot(l_2-R)\cdot z}{2\cdot h\cdot l_2}+\frac{F_2\cdot(R-z)\cdot c}{c^2+d^2}\Bigg|$$
$$>\left|\frac{1}{4}\cdot\left(F_1\cdot\frac{R}{l_2}\cdot\text{ctg}\beta-F_3\right)+\frac{F_1\cdot(l_2-R)\cdot z}{2\cdot h\cdot l_2}\right.$$
$$\left.+\frac{F_2\cdot(R-z)\cdot d}{c^2+d^2}\right|, \tag{28}$$

– $R<z\le l_2$

$$\left|\frac{-1}{4}\cdot F_1\cdot\frac{R}{l_2}\cdot\text{ctg}\beta+\frac{F_1\cdot R\cdot(l_2-z)}{2\cdot h\cdot l_2}\right|$$
$$>\left|\frac{1}{4}\cdot F_1\cdot\frac{R}{l_2}\cdot\text{ctg}\beta+\frac{F_1\cdot R\cdot(l_2-z)}{2\cdot h\cdot l_2}\right|. \tag{29}$$

7 Selection of optimal boom cross section

As it was mentioned before, the analysis of real tower crane lattice booms has shown that they are made with triangular cross section. However, the analytical proof that the triangular cross section is indeed optimal one, can not be found in the literature. Regarding the selection of optimal boom cross section, it is significant to define the conditions for which triangular and rectangular boom cross sections will be optimal. The previous analysis enables determination of the necessary conditions, which specify whether the optimal cross section of lattice boom is triangular ($d=0$ or $c=0$) or rectangular ($c=d$).

7.1 Triangular cross section

Triangular cross section (Fig. 4) can be considered as a special case of trapezoid cross section. Then its optimal parameters can be obtained when in expressions (22)–(29) is included one of following conditions $d=0$ (Fig. 4a) or $c=0$ (Fig. 4b).

7.1.1 Triangular cross section

In case when the load is positioned on a boom gantry ($l_2\le R\le l_3$), and maximal actual stress occurs in one of the lower flanges, optimal parameters of triangular boom cross section are obtained by the expressions (22), from which emanate:

$$c=\sqrt{\frac{F_2\cdot R^2}{k_1\cdot[\sigma]}},$$

$$h=\frac{c}{2}\cdot\sqrt{\left(\frac{k_2}{k_1}\right)^2-1},$$

$$a_{10}=\frac{1}{[\sigma]}\left[\frac{1}{4}\cdot\left(F_1\frac{R}{l_2}\text{ctg}\beta-F_3\right)+\frac{F_2\cdot R}{c}\right], \tag{30}$$

$$a_{11}=\frac{1}{[\sigma]}\left[\frac{F_1\cdot(R-l_2)}{2\cdot h\cdot l_2}-\frac{F_2}{c}\right]$$

$$a_{20}=\frac{1}{[\sigma]}\left[\frac{-F_3}{4}+\frac{F_1\cdot R}{2\cdot h}+\frac{F_2\cdot R}{c}\right],$$

$$a_{21}=\frac{1}{[\sigma]}\left[-\frac{F_1}{2\cdot h}-\frac{F_2}{c}\right],$$

The triangular boom cross section will be optimal if the following conditions are met:

$$\frac{F_2}{F_1}=2\cdot\left(1-\frac{l_2}{R}\right)\cdot\left[\left(\frac{k_2}{k_1}\right)^2-1\right]^{-\frac{3}{2}},\quad k_2\ge k_1. \tag{31}$$

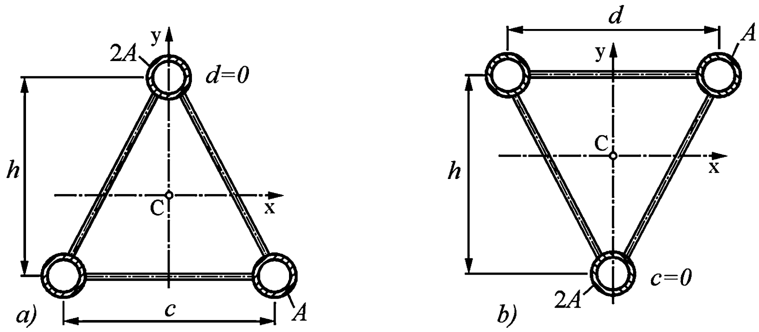

Fig. 4 Triangular boom cross section

In the case when the load is not on the boom gentry ($0 \le R < l_2$), and maximal actual stress occurs in one of the lower flanges, optimal parameters of triangular boom cross section are obtained by the expressions (23) and can be expressed as:

$$\begin{aligned} c &= \sqrt{\frac{F_2 \cdot R^2}{k_1 \cdot [\sigma]}}, \\ h &= \frac{c}{2} \cdot \sqrt{\left(\frac{k_2}{k_1}\right)^2 - 1}, \\ b_{10} &= \frac{1}{[\sigma]}\left[\frac{-1}{4} \cdot \left(F_1 \frac{R}{l_2} \mathrm{ctg}\beta - F_3\right) + \frac{F_2 \cdot R}{c}\right], \\ b_{11} &= \frac{1}{[\sigma]}\left[\frac{F_1 \cdot (l_2 - R)}{2 \cdot h \cdot l_2} - \frac{F_2}{c}\right], \\ b_{20} &= \frac{1}{[\sigma]}\left[-F_1 \frac{R}{4 \cdot l_2} \mathrm{ctg}\beta + \frac{F_1 \cdot R}{2 \cdot h}\right], \\ b_{21} &= \frac{1}{[\sigma]}\left[-\frac{F_1 \cdot \eta}{2 \cdot h \cdot l_2}\right], \end{aligned} \tag{32}$$

In this case, the triangular boom cross section will be optimal if the following conditions are met:

$$\frac{F_2}{F_1} = 2 \cdot \left(\frac{l_2}{R} - 1\right) \cdot \left[\left(\frac{k_2}{k_1}\right)^2 - 1\right]^{-\frac{3}{2}}, \quad k_2 \ge k_1. \tag{33}$$

In cases when the maximum actual stress occurs in upper flange, the analysis of the expressions (24) and (25) shows that, regardless of the positions of the load, the triangular boom cross section in not an optimal one.

7.1.2 *Triangular cross section $c = 0$*

The analysis of the optimal parameters of the triangular boom section ($c = 0$) shows that, if the maximum actual stress occurs in lower flange, the triangular boom cross section ($c = 0$) in not an optimal one, regardless of the positions of the load.

In case when the load is positioned on a boom gantry ($l_2 \le R \le l_3$), and maximum actual stress occurs in one of the upper flanges, the optimal parameters of triangular boom cross section can be obtained by the expression a (22), from which emanate:

$$\begin{aligned} h &= \sqrt[4]{\frac{F_1^2 \cdot R^2 \cdot (R - L_2)^2}{4 \cdot k_1 \cdot k_2 \cdot [\sigma] \cdot [(\frac{k_2}{k_1})^2 - 1]}}, \\ d &= 2 \cdot h \cdot \left[\left(\frac{k_2}{k_1}\right)^2 - 1\right]^{-\frac{1}{2}}, \\ a_{10} &= \frac{1}{[\sigma]}\left[\frac{-1}{4} \cdot \left(F_1 \frac{R}{l_2} \mathrm{ctg}\beta - F_3\right) + \frac{F_2 \cdot R}{d}\right], \\ a_{11} &= \frac{1}{[\sigma]}\left[\frac{F_1 \cdot (R - l_2)}{2 \cdot h \cdot l_2} - \frac{F_2}{d}\right] \\ a_{20} &= \frac{1}{[\sigma]}\left[\frac{F_3}{4} + \frac{F_1 \cdot R}{2 \cdot h} + \frac{F_2 \cdot R}{d}\right], \\ a_{21} &= \frac{1}{[\sigma]}\left[-\frac{F_1}{2 \cdot h} - \frac{F_2}{d}\right]; \end{aligned} \tag{34}$$

The triangular boom cross section will be optimal if the following conditions are met:

$$k_2 \ge k_1. \tag{35}$$

In the case when the load is on the gantry ($0 \le R < l_2$), and the maximal actual stress occurs in on of the upper flanges, the optimal parameters of triangular boom cross section can be obtained by the expression (23) and they are:

$$\begin{aligned} h &= \sqrt[4]{\frac{F_1^2 \cdot R^2 \cdot (L_2 - R)^2}{4 \cdot k_1 \cdot k_2 \cdot [\sigma] \cdot [(\frac{k_2}{k_1})^2 - 1]}}, \\ d &= 2 \cdot h \cdot \left[\left(\frac{k_2}{k_1}\right)^2 - 1\right]^{-\frac{1}{2}} \\ b_{10} &= \frac{1}{[\sigma]}\left[\frac{1}{4} \cdot \left(F_1 \frac{R}{l_2} \mathrm{ctg}\beta - F_3\right) + \frac{F_2 \cdot R}{d}\right], \\ b_{11} &= \frac{1}{[\sigma]}\left[\frac{F_1 \cdot (l_2 - R)}{2 \cdot h \cdot l_2} - \frac{F_2}{d}\right], \end{aligned} \tag{36}$$

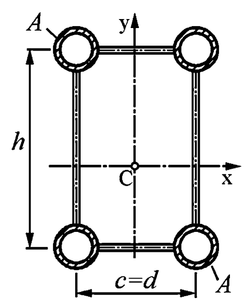

Fig. 5 Rectangular boom cross section

$$b_{20} = \frac{1}{[\sigma]}\left[F_1 \frac{R}{4 \cdot l_2} \mathrm{ctg}\beta + \frac{F_1 \cdot R}{2 \cdot h}\right],$$

$$b_{21} = \frac{1}{[\sigma]}\left[-\frac{F_1 \cdot \eta}{2 \cdot h \cdot l_2}\right].$$

The triangular boom cross section will be an optimal one, if the condition (35) is satisfied.

Still, the triangular boom cross section ($c = 0$) is not the optimal one, because when compared its overall volume with the corresponding volume of trapezoid boom cross section (20), it can be concluded that regardless of load position, volume of triangular cross section lattice boom is always larger then the volume of trapezoid cross section lattice boom.

7.2 Rectangular boom cross section

Rectangular boom cross section (Fig. 5) can also be considered as the special case of the trapezoid boom cross section, whereby its optimal parameters can be obtained by the (22)–(29) which include the condition $c = d$. The analysis of the expressions (22)–(25) with mentioned condition included showed that rectangular boom cross section is not the optimal one, regardless of load position and the position and the position of the flange in which maximal actual stress occurs.

8 Numerical example and analysis of results

The verification of the theoretical results has been performed by the numerical example of the rotation working operation of tower crane lattice boom, with data given in [10]:

- limiting stress $[\sigma] = 240$ MPa,
- $k_1 = 0.078$, $k_2 = 0.062$, $k_3 = 0.05$,
- crab weight—3.2 kN,
- pulley weight—5.8 kN,
- coefficient of the drive class 1.05,
- dynamic coefficient 1.15,
- angular speed of the boom 0.084 s^{-1},
- angular acceleration of the boom 0.017 s^{-2},
- angle between the boom and the rope $\beta = 10°$,
- geometrical parameters of the boom $l_1 = 50$ m, $l_2 = 24$ m, $l_3 = 44.8$ m, $l_4 = 7.5$ m.

The centrifugal force due to load rotation (F_3) is axial force acting on a boom gantry. Numerical analysis of actual exploitation conditions showed that this force is less then 0.4% of axial force acting on a boom between tower and cable. Therefore, the centrifugal force due to load rotation (F_3) is disregarded in this numerical example.

The analysis of the variation of optimal dimensions of boom cross section along the crane (hook) radius (Fig. 6), it can be concluded that trapezoid cross section is the optimal one. The different relations between opposite sides of trapezoid (c and d), regarding the load position. So, if the load is on boom gantry ($l_2 \le R \le l_3$) the actual maximal stress (pressure stress) would occur in one of the lower flanges, depending of the direction of boom rotation. In that case the trapezoid boom cross section will be placed so that $c > d$. on the other hand, if the load is between the tower and the cable ($0 \le R < l_2$), the actual maximal stress (pressure stress) would occur in one of the upper flanges, and in that case is $c > d$.

The analysis of the variation of optimal values of cross section flanges area with the lengthwise coordinate (Fig. 7), it can be concluded that maximal values of those areas occurs in range of 30% (the load position $0 \le R \le l_2$ m excluded). On the joint position of

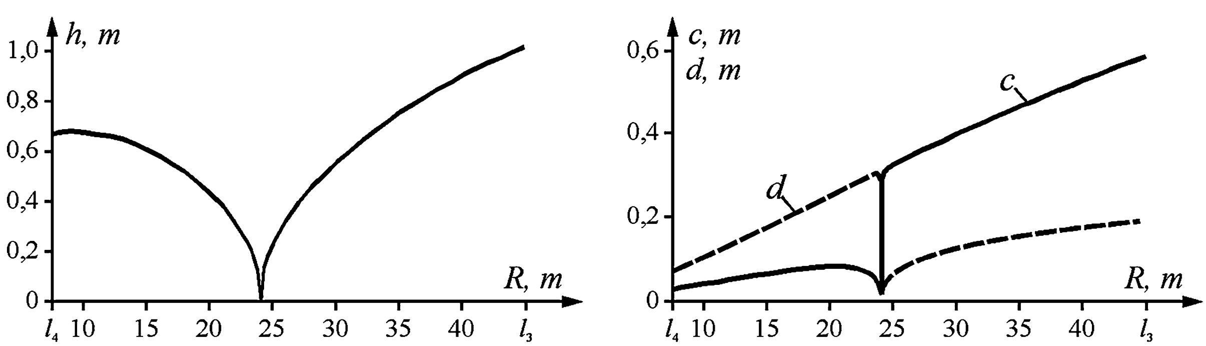

Fig. 6 Variation of optimal dimensions of trapezoid cross section (c, d, h) with the crane (hook) radius (R)

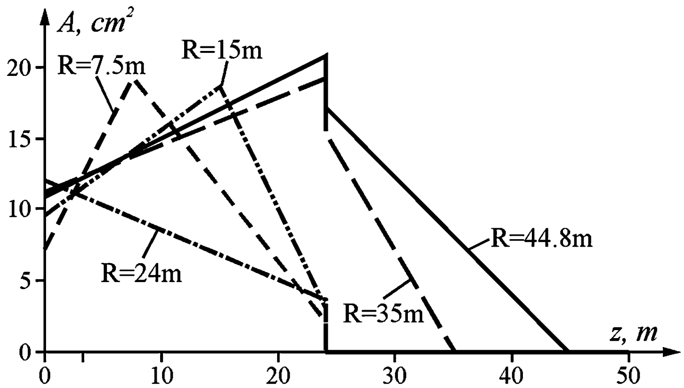

Fig. 7 Variation of optimal values of cross section flange area with the lengthwise coordinate (z) for different values of crane (hook) radius (R)

boom and cable, the variation of cross section flanges area can be explained with the fact that the axial force acting on position ($0 \leq z \leq R$) is considerably grater then the force acting on position $R < z \leq l_2$.

9 Optimization by computer program based on finite element analysis

The optimization of previous example was run by computer program based on finite element analysis with built in routines for design optimization. Those routines were used for optimization of analyzed model in order to additionally verify application of obtained theoretical results in actual exploitation (Fig. 8).

This kind of optimization implied modeling and analysis of the basic model of tower crane lattice boom. In order to create finite element model of tower crane lattice boom, the computer code is written within the used computer program, which enabled automatic execution of iteration procedure for obtaining required results. This code enabled geometrical modeling, specification of material properties, application of loads, generation of finite element mesh, as well as solving and obtaining corresponding output data. In this case the output data are maximal stresses in corresponding cross sections.

Subsequently, the postprocessor analysis was carried out. In that analysis, first the independent variables were specified (dimensions of the boom cross section c, d and h and the flanges cross section areas A_i). Then the dependent variables (constraint function-stress criterion: maximal allowable stress in corresponding boom cross sections) and the objective function (minimal weight, i.e. minimal volume of analyzed model) were specified as well. Finally, the methods and tools of optimization execution were assigned. The used computer program offered two optimization methods: the first order method and sub-problem optimization method. Both methods were used in this case.

The maximum number of independent variables is 60, but 20 was recommended, which was limitation of used computer program to which analyzed model had to be adapted. Because of this, the tower crane lattice boom was segmented into 17 sections. The cross section areas A_i, $i = 1 \ldots 17$ of those sections were different, and their length was 2 m and 4 m, except for the two last sections. Their length was, respectively, 2.8 m and 5.2 m. The maximal number of dependent variables is 100, but in this case it is equal to the number of different cross section areas along the boom, hence 17.

For every variable, regardless if it was independent or not, the range of variation had to be specified. The range of variation specification independent variables was in the direct connection with the value of the maximal allowable stress, which in this case was 240 MPa.

The range of variation specification of independent variables was more complicated. Too wide a range results in extend of optimization analysis, whereas too narrow a range may exclude optimal solutions. Subproblem optimization method was used to establish this range. This is an advanced zero-order method which uses approximations (curve fitting) to all dependent variables and the objective function. It is a general

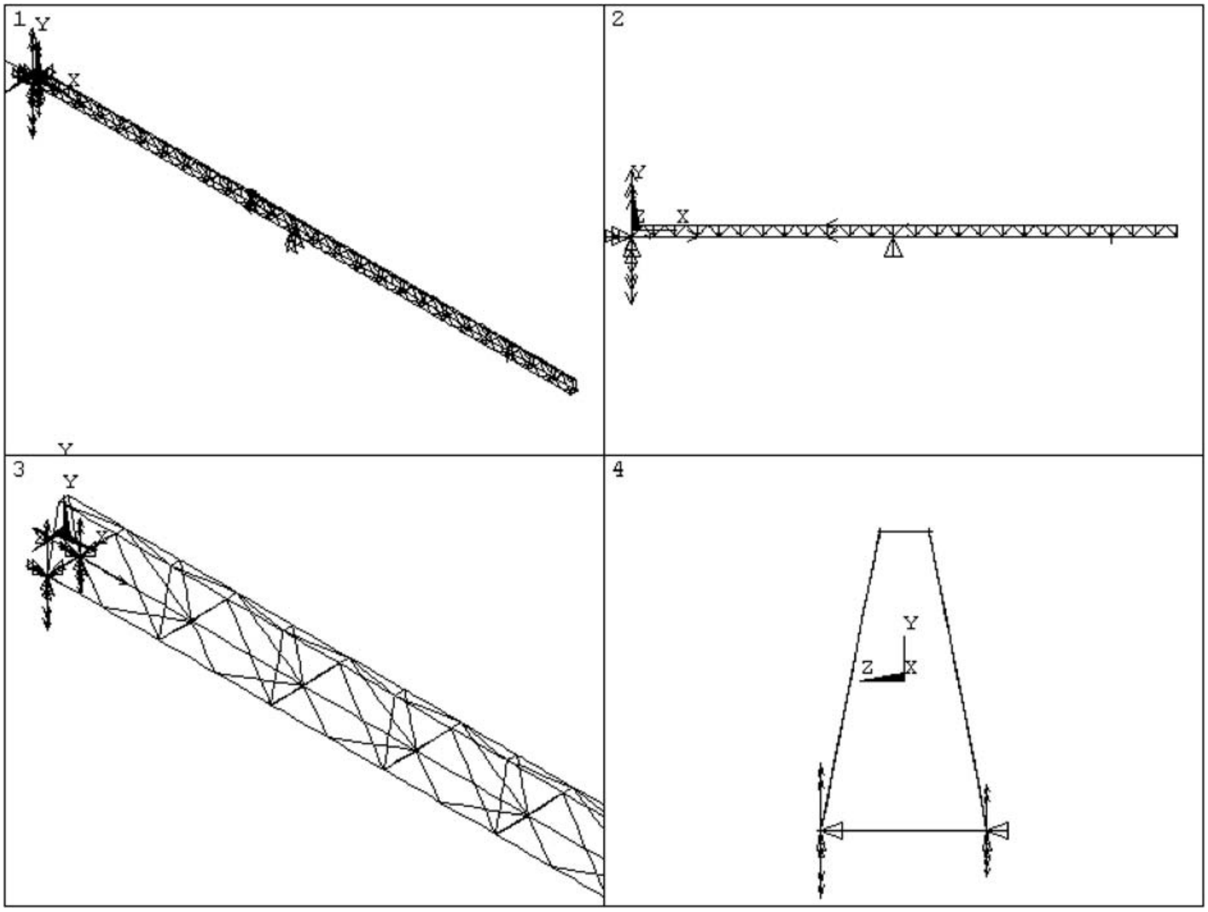

Fig. 8 Analysis of the tower crane boom by means of the computer code based on the method of finite elements

method that can be applied efficiently to a wide range of engineering problems.

The first order method was used for optimization in this case. This method uses derivative information, that is, gradients of the dependent variables with respect to the design variables. It is highly accurate and works well for problems having dependent variables that vary widely over a large range of design space. However, this method can be computationally intense.

The created finite element model was adapted to the maximal opportunities that used computer program had to offer. It should be mentioned that calculation of optimal parameters required a lot of time. For example, just one optimization iteration could take up to 90 minutes, and 3 to 4 optimization iterations were necessary for only one load position.

The optimization was executed for different load positions along the tower crane lattice boom and in the accordance with the obtained analytical expressions. This enabled comparison of the results.

10 Comparison of the results

The optimization results obtained by the computer program based on finite element analysis are represented on following graphs (Fig. 9), together with the results obtained by application of analytical expressions.

Based on the comparison of the optimal dimensions of boom cross section obtained in these two manners, it can be concluded that the error does not exceed 10%, which is acceptable from an engineering point of view. However, grater error occurs in case

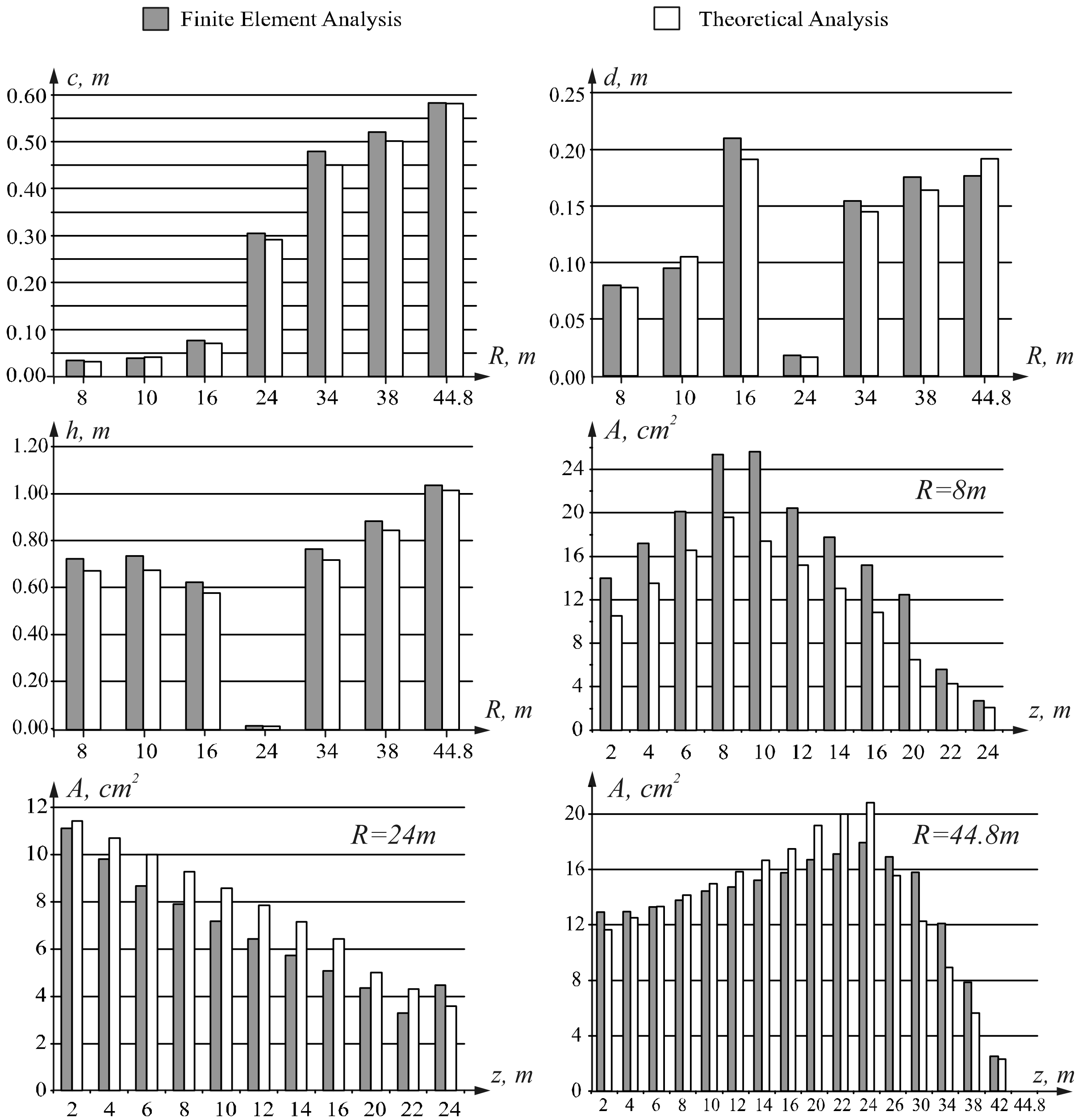

Fig. 9 Comparison of optimal dimensions of boom cross section (c, d, h) and comparison of the optimal values of flanges cross section areas for different values of crane (hook) radius (R)

of flanges cross section areas comparison. This error is caused by the size of the segments on which the boom was segmented, for the purposes of finite element analysis. Still, it can be observed that in places where the segments are smaller and in which proximity there are no characteristic cross sections (cross section with constrains or applied load), the error is lesser. This draws to conclusion that the increase of segments number and reduction of their size, especially in the proximity of characteristic cross sections, would result in significant decrease of error. However, the results obtained by the computer program based on finite element analysis are more then useful. They can be considered as starting results for further optimization of analyzed model and at the same time they emphasizes even more the significance of the obtained theoretical results.

11 Conclusion

The optimization of the cross section of tower crane lattice boom was performed in this paper. The analysis was carried out based on stress criterion. Equations (22)–(29) can be useful to an engineer-designer in the first stage of the designing procedure, when the problem of defining basic structure dimensions that would be close to optimal ones arise**.** Obtained results are compared with ones obtained by the computer program based on finite element analysis, whereby good agreement of the results was achieved, regarding the limitations of used computer program.

The established model included practical data that the overall dimensions of boom cross section (c, d, h) are constant along the boom. Based on this paper it can be concluded that it is necessary that flanges cross section area vary linearly with boom lengthwise coordinate (z), which is determined by expression (13), in order to achieve maximal material exploitation.

The analysis of the optimal parameters of the rectangular boom section showed that the rectangular boom section is not the optimal one, while the triangular boom cross section $(d = 0)$ is optimal, but only in special cases which are defined in this paper (30)–(35).

The proposed model of tower crane lattice boom can be used for the design purposes, but the designers should consider the buckling of compressed members (lattice) and the effect of bracing, too. Another problem is that a crane boom is a very flexible structure; therefore, it needs limitations for vertical and horizontal deflection as well.

So, the directions of further research in this area are in the multicriteria analysis, which include additional constraint functions, like deformation criterion, dynamic stability criterion... From the exploration point, it would be convenient to determine optimal values of carrying capacity, regarding dimensions of boom cross section and dynamic exploitation parameters.

The proposed model of tower crane lattice boom can be applied on other lattice constructions as well, with minor modifications. The truck crane lattice boom is the best example for this, because its model is the closest one to the statical model of tower crane lattice boom.

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