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Thermal effect on vibration of non-homogenous visco-elastic rectangular plate of linearly varying thickness

A.K. Gupta · Lalit Kumar

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Abstract An analysis for vibration of non-homogenous visco-elastic rectangular plate of linearly varying thickness subjected to thermal gradient has been discussed in the present investigation. For visco-elastic, the basic elastic and viscous elements are combined. We have taken Kelvin model for visco-elasticity that is the combination of the elastic and viscous elements in parallel. Here the elastic element means the spring and the viscous element means the dashpot. The governing differential equation of motion has been solved by Galerkin's technique. Deflection, time period and logarithmic decrement at different points for the first two modes of vibration are calculated for various values of thermal gradients, non homogeneity constant, taper constant and aspect ratio for non-homogenous visco-elastic rectangular plate which is clamped on two parallel edges and simply supported on remaining two edges. Comparison studies have been carried out with homogeneous visco-elastic rectangular plate to establish the accuracy and versatility.

A.K. Gupta (✉)
Department of Mathematics, M.S. College, Saharanpur, UP, India
e-mail: gupta_arunnitin@yahoo.co.in

L. Kumar
Department of Mathematics, Kisan (PG) College, Simbhaoli, Ghaziabad, UP, India
e-mail: abbutyagi@yahoo.com

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Abbreviations

x, y	Coordinate in the plane of plate
M_x, M_y	Bending moments
M_{yx}	Twisting moments
E	Young's modulus
G	Shear modulus
ν	Poisson's ratio
h	Thickness of plate
ρ	Mass density per unit length of plate material
D_1	Flexural rigidity
$\tilde{D}$	Visco elastic operator
t	Time
η	Visco elastic constant
$w(x, y, t)$	Transverse deflection of plate at point
a, b	Length and breath of the plate
$\alpha, \alpha_1, \alpha_2$	Temperature constants
β	Taper constant
α_3	Non-homogeneity constant
τ	Temperature excess above a given reference
Λ	Logarithmic decrement
K	Time period

1 Introduction

Thermal effect of vibration of non-homogenous visco-elastic plates are of great interest in the field Engineering such as for better designing of gas turbines, jet engine, space craft and nuclear power projects. During heating up periods the structure are exposed to high intensity, high fluxes and the material properties thus undergo significant changes, in particular the thermal effect on the modules of elasticity of the material can not be taken as negligible. Several authors [1–5] have studies the thermal effect on vibration of homogenous plates of variable thickness but none of the authors has so far considered thermal effect on vibration of non-homogenous rectangular plates of linearly varying thickness. Pronsato et al. [9] have discussed on transverse vibration of rectangular membrane with discontinuously varying density. It is well known [8] that in the presence of a thermal gradient, the elastic coefficient of homogenous materials becomes function of the space variables. Bambill et al. [6] discussed the transverse vibration of an orthotropic rectangular plate of linearly varying thickness and with a free hole edge. Li and Zhou [7] have studied non-linear vibration and thermal bucking of heated orthotropic circular plate by shooting method. Recently, Gupta and Khanna [11] studied vibration of visco-elastic rectangular plate with linearly thickness variations in both directions. Pradeep and Ganesan [12] solved the problem of buckling & vibration of rectangular composite viscoelastic sandwich plates under thermal load. Li et al. [13] discussed the vibration of thermally post-buckled orthotropic circular plate.

Visco-elastic, as its name implies, is a generalization of elasticity and viscosity. The ideal linear elastic element is the spring. When a tensile force is applied to it, the increase in the distance between its two ends is proportional to the force. The ideal linear viscous element is the dashpot.

The main purpose of the present investigation is to study the effect of non-homogeneity on thermally induced vibration on visco-elastic rectangular plate of linearly varying thickness. To determine the frequency equation, Galerkin's technique has been applied. It is considered that the visco-elastic properties of the plate are of Kelvin type.

All the material constants, which are used in numerical calculations, have been taken for the alloy 'DURALIUM' which is mainly used in modern technology. Deflection, time period and logarithmic decrement at different points for the first two mode of vibration are calculated for various values of thermal gradients, non-homogeneity parameter, taper constant and aspect ratio for non-homogenous visco-elastic rectangular plate which is clamped on two parallel edges and simply supported on the remaining two edges.

2 Analysis and equation of motion

The governing differential equation of transverse motion of a visco-elastic plate of variable thickness in Cartesian co-ordinates, as by Bhatnagar and Gupta [10], is

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \rho h \frac{\partial^2 w}{\partial t^2}. \tag{1}$$

The expression for M_x, M_y, M_{yx} are given by

$$\left.\begin{aligned} M_x &= -\tilde{D} D_1\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) \\ M_y &= -\tilde{D} D_1\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) \\ M_{yx} &= -\tilde{D} D_1(1-\nu)\frac{\partial^2 w}{\partial x \partial y} \end{aligned}\right\}. \tag{2}$$

On substitution the values M_x, M_y and M_{yx} from (2) in (1), one has

$$\begin{aligned} &\tilde{D}\Bigg[D_1\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) \\ &\quad + 2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right) \\ &\quad + 2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x^2 \partial y}\right) \\ &\quad + \frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) \\ &\quad + \frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) + 2(1-\nu)\frac{\partial^2 D_1}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y}\Bigg] \\ &\quad + \rho h \frac{\partial^2 w}{\partial t^2} = 0. \end{aligned} \tag{3}$$

The solution of (3) can be sought in the form of a product of two functions as follows:

$$w = w(x, y, t) = W(x, y) T(t) \tag{4}$$

where $W(x, y)$ is a function of the coordinates x, y and $T(t)$ is a time function.

Using (4) in (3) and simplifying, one has

$$\begin{aligned}
&\left[D_1\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)\right.\\
&\quad+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+\frac{\partial^3 W}{\partial x\partial y^2}\right)\\
&\quad+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+\frac{\partial^3 W}{\partial x^2\partial y}\right)\\
&\quad+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right)\\
&\quad+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\nu\frac{\partial^2 W}{\partial x^2}\right)\\
&\quad\left.+2(1-\nu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}\right]\Big/\rho h W=-\frac{1}{\tilde{D}T}\frac{d^2T}{dt^2}. \qquad (5)
\end{aligned}$$

Taking both sides of (5) are equal to a constant p^2, we have

$$\begin{aligned}
&D_1\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)\\
&\quad+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+\frac{\partial^3 W}{\partial x\partial y^2}\right)\\
&\quad+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+\frac{\partial^3 W}{\partial x^2\partial y}\right)\\
&\quad+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right)\\
&\quad+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\nu\frac{\partial^2 W}{\partial x^2}\right)\\
&\quad+2(1-\nu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}-\rho h p^2 W=0 \qquad (6)
\end{aligned}$$

and

$$\frac{d^2T}{dt^2}+p^2\tilde{D}T=0. \qquad (7)$$

Equation (6) is differential equation of transverse motion and (7) is a differential equation of time function of vibration of non-homogeneous visco-elastic plate of variable thickness.

It is assumed that the non-homogeneous visco-elastic rectangular plate is subjected to a steady one dimensional temperature distribution along the length, i.e. x-direction, as

$$\tau=\tau_0\left(1-\frac{x}{a}\right) \qquad (8)$$

where τ denotes the temperature excess above the reference temperature at any point at distance $\frac{x}{a}$ and τ_0 denotes the temperature excess above reference temperature at the end i.e. $x=a$.

The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed in this form,

$$E=E_0(1-\gamma\tau) \qquad (9)$$

where E_0 is the value of the Young's modulus at some reference temperature i.e. $\tau=0$ and γ is the slope of the variation of E with τ. The modulus variations, in view of expressions (8) and (9) become

$$E(x)=E_0\left(1-\alpha\left(1-\frac{x}{a}\right)\right) \qquad (10)$$

where $\alpha=\gamma\tau_0$ $(0\le\alpha<1)$, a parameter known as temperature gradient.

It is assumed that thickness and non-homogeneity varies in the x-direction only, consequently, the thickness, density and flexural rigidity of the plate become functions of x only. Let the two opposite edges $y=0$ and $y=b$ of the plate be simply supported. So that the plate when undergoing free transverse vibrations with frequency p may have levy-type solution as

$$W(x,y)=W_1(x)\sin\left(\frac{\pi y}{b}\right). \qquad (11)$$

Substitution of (11) in (6) comes out as

$$\begin{aligned}
&D_1\left[\frac{\partial^4 W_1}{\partial x^4}-2\left(\frac{\pi}{b}\right)^2\frac{\partial^2 W_1}{\partial x^2}+\left(\frac{\pi}{b}\right)^4 W_1\right]\\
&\quad+2\frac{\partial D_1}{\partial x}\left[\frac{\partial^3 W_1}{\partial x^3}-\left(\frac{\pi}{b}\right)^2\frac{\partial W_1}{\partial x}\right]\\
&\quad+\frac{\partial^2 D_1}{\partial x^2}\left[\frac{\partial^2 W_1}{\partial x^2}-\nu\left(\frac{\pi}{b}\right)^2 W_1\right]\\
&\quad-\rho p^2 h W_1=0. \qquad (12)
\end{aligned}$$

Thus (12) reduces to a form independent of y and upon introducing the non-dimensional variables

$$\bar{H}=\frac{h}{a},\qquad \bar{\rho}=\frac{\rho}{a},\qquad \bar{D}=\frac{D_1}{a^3},\qquad \bar{W}=\frac{W_1}{a},\qquad X=\frac{x}{a} \tag{13}$$

(12) becomes, in non-dimensional form, as

$$\bar{D}\left(\frac{\partial^4\bar{W}}{\partial X^4}-2r^2\frac{\partial^2\bar{W}}{\partial X^2}+r^4\bar{W}\right)+2\frac{\partial\bar{D}}{\partial X}\left(\frac{\partial^3\bar{W}}{\partial X^3}-r^2\frac{\partial\bar{W}}{\partial X}\right)+\frac{\partial^2\bar{D}}{\partial X^2}\left(\frac{\partial^2\bar{W}}{\partial X^2}-\nu r^2\bar{W}\right)-a^3\bar{\rho}p^2\bar{h}\bar{W}=0 \tag{14}$$

where

$$r=\frac{\pi a}{b}. \tag{15}$$

In view of the previous assumption, the thickness and non-homogeneity vary linearly in the x-direction only, one assumes,

$$\bar{H}(X)=H_0(1-\beta X) \tag{16}$$

where β is the taper constant and $H_0=\bar{H}|_{x=0}$ and

$$\bar{\rho}=\rho_0(1-\alpha_3 X) \tag{17}$$

where α_3 is the non-homogeneity constant and $\rho_0=\bar{\rho}|_{x=0}$.

The rigidity given by equation

$$\bar{D}=\frac{E_0H_0^3(1-\beta X)^3[1-\alpha(1-X)]}{12(1-\nu^2)}. \tag{18}$$

Using (16), (17) and (18) in (14), one obtain

$$A_1\left(\frac{\partial^4\bar{W}}{\partial X^4}-2r^2\frac{\partial^2\bar{W}}{\partial X^2}+r^4\bar{W}\right)+A_2\left(\frac{\partial^3\bar{W}}{\partial X^3}-r^2\frac{\partial\bar{W}}{\partial X}\right)+A_3\left(\frac{\partial^2\bar{W}}{\partial X^2}-\nu r^2\bar{W}\right)-p^2(1-\alpha_3X)\ell\bar{W}=0. \tag{19}$$

Here

$$A_1=(1-\alpha+\alpha X)(1-\beta X)^2,$$

$$A_2=2(1-\beta X)(3\alpha\beta-3\beta+\alpha-4\alpha\beta X),$$

$$A_3=6\beta(\beta-\alpha-\alpha\beta+2\alpha\beta X),$$

$$\ell=\frac{12(1-\nu^2)\rho_0a^3}{E_0H_0^2}$$

and p^2 is the frequency parameter.

The deflection function $\bar{W}(X)$, of the plate, is assumed to be a finite sum of characteristic functions $\bar{W}_k(X)$ as

$$\bar{W}(X)=\sum_{k=1}^{n}C_k\bar{W}_k(X) \tag{20}$$

where C_k's are undetermined coefficients and $\bar{W}_k$ are the characteristic functions chosen to satisfy the boundary conditions of plate. For a rectangular plate clamped at both the edges $X=0$ and $X=1$ (and simply supported at the remaining two edges) boundary conditions are

$$\bar{W}|_{X=0}=\left.\frac{\partial\bar{W}}{\partial X}\right|_{X=0}=0,\qquad \bar{W}|_{X=1}=\left.\frac{\partial\bar{W}}{\partial X}\right|_{X=1}=0. \tag{21}$$

Using Galerkin's technique, one has

$$\int L[\bar{W}(X)]\bar{W}(X)dX=0 \tag{22}$$

where $L[\bar{W}(X)]$ is left hand side of (19). Taking the first two terms of the sum (20), for the function $\bar{W}(X)$ as a solution of (19), as

$$\bar{W}(X)=C_1X^2(1-X)^2+C_2X^3(1-X)^3. \tag{23}$$

Substituting (23) into (22) and then eliminating C_1 and C_2 gives the frequency equation as

$$\begin{vmatrix}2(F_1+B_1p^2) & (F_2+B_2p^2)\\ (F_2+B_2p^2) & 2(F_3+B_3p^2)\end{vmatrix}=0 \tag{24}$$

where

$$F_1 = \frac{4}{5} - \frac{2}{5}\alpha - \frac{4}{5}\beta + \frac{6}{35}\alpha\beta + \frac{2}{7}\beta^2 - \frac{1}{35}\alpha\beta^2 + r^2\left(\frac{4}{105} - \frac{2}{105}\alpha - \frac{4}{105}\beta + \frac{1}{90}\alpha\beta + \frac{1}{70}\beta^2 - \frac{1}{315}\alpha\beta^2\right) + r^4\left(\frac{1}{630} - \frac{1}{1260}\alpha - \frac{1}{630}\beta + \frac{1}{1386}\alpha\beta + \frac{1}{2310}\beta^2 - \frac{1}{5544}\alpha\beta^2\right) + \nu r^2\left(\frac{1}{105}\alpha\beta - \frac{1}{105}\beta^2\right),$$

$$B_1 = \ell\left(-\frac{1}{630} + \frac{1}{1260}\alpha_3\right),$$

$$F_2 = \frac{12}{35} - \frac{6}{35}\alpha - \frac{12}{35}\beta + \frac{11}{105}\alpha\beta + \frac{11}{105}\beta^2 - \frac{2}{105}\alpha\beta^2 + r^2\left(\frac{2}{105} - \frac{1}{105}\alpha - \frac{2}{105}\beta + \frac{43}{4930}\alpha\beta + \frac{47}{4930}\beta^2 - \frac{2}{1155}\alpha\beta^2\right) + r^4\left(\frac{1}{1386} - \frac{1}{2772}\alpha - \frac{1}{1386}\beta + \frac{1}{3003}\alpha\beta + \frac{1}{5148}\beta^2 - \frac{1}{12012}\alpha\beta^2\right) + \nu r^2\left(\frac{1}{231}\alpha\beta - \frac{1}{231}\beta^2\right),$$

$$B_2 = \ell\left(-\frac{1}{1386} + \frac{1}{2772}\alpha_3\right),$$

$$F_3 = \frac{2}{35} - \frac{1}{35}\alpha - \frac{2}{35}\beta + \frac{3}{154}\alpha\beta + \frac{13}{770}\beta^2 - \frac{3}{770}\alpha\beta^2 + r^2\left(\frac{1}{385} - \frac{1}{770}\alpha - \frac{1}{385}\beta + \frac{1}{1092}\alpha\beta + \frac{53}{60060}\beta^2 - \frac{1}{4004}\alpha\beta^2\right) + r^4\left(\frac{1}{12012} - \frac{1}{24024}\alpha - \frac{1}{12012}\beta + \frac{1}{25740}\alpha\beta + \frac{1}{45045}\beta^2 - \frac{1}{102960}\alpha\beta^2\right) + \nu r^2\left(\frac{1}{2002}\alpha\beta - \frac{1}{2002}\beta^2\right),$$

$$B_3 = \ell\left(-\frac{1}{12012} + \frac{1}{24024}\alpha_3\right).$$

The frequency (24) is a quadratic equation in p^2 from which the two values of p^2 can be found.

Choosing $C_1 = 1$, one obtains $C_2 = -\frac{F_4}{F_5}$ where $F_4 = 2(F_1 + p^2 B_1)$, $F_5 = 2(F_2 + p^2 B_2)$.

Therefore

$$\bar{W} = X^2(1-X)^2 - \frac{F_4}{F_5}X^3(1-X)^3. \tag{25}$$

3 Time functions of vibrations of non-homogeneous visco-elastic plates

Time function of vibrations of visco-elastic plates is defined by the general ordinary differential (7). Their form depends on the visco-elastic operator $\tilde{D}$. For Kelvin's model, one has

$$\tilde{D} \equiv \left(1 + \frac{\eta}{G}\frac{d}{dt}\right). \tag{26}$$

Taking temperature dependence of shear modulus G and visco-elastic constants η as [8]:

$$G(\tau) = G_0(1 - \gamma_1\tau), \qquad \eta(\tau) = \eta_0(1 - \gamma_2\tau) \tag{27}$$

where G_0 is shear modulus and η_0 is visco-elastic constant at some reference temperature i.e. at $\tau = 0$, γ_1 and γ_2 are slope variation of τ with G and η respectively. Using (8) in (27), one has

$$G(X) = G_0[1 - \alpha_1(1-X)],$$
$$\eta(X) = \eta_0[1 - \alpha_2(1-X)] \tag{28}$$

where

$$\alpha_1 = \gamma_1\tau_0, \quad 0 \le \alpha_1 < 1, \quad \text{and}$$
$$\alpha_2 = \gamma_2\tau_0, \quad 0 \le \alpha_2 < 1.$$

Using (28) in (26), we get

$$\tilde{D} \equiv \left(1 + q\frac{d}{dt}\right) \tag{29}$$

where

$$q = \frac{\eta_0[1 - \alpha_2(1-X)]}{G_0[1 - \alpha_1(1-X)]}. \tag{30}$$

Using (29) in (7), one obtains

$$\frac{d^2T}{dt^2} + p^2 q\frac{dT}{dt} + p^2 T = 0 \tag{31}$$

and its solution comes out as

$$T(t)=e^{-\frac{p^2qt}{2}}(e_1\cos st+e_2\sin st) \tag{32}$$

where $s^2=p^2\frac{1}{4}p^4\,q^2$, and e_1 and e_2 are constants of integration.

Let us assume that the initial conditions are $T=1$ and $\frac{dT}{dt}=0$ at $t=0$, so (32) becomes

$$T(t)=e^{-\frac{p^2qt}{2}}\left(\cos st+\frac{p^2q}{2s}\sin st\right). \tag{33}$$

Thus, deflection $w(x,y,t)$ may be expressed from (4), (11), (25) and (33), as

$$w(x,y,t) = \bar{W}(X)e^{-\frac{p^2qt}{2}}\left(\cos st+\frac{p^2q}{2s}\sin st\right)\sin\frac{\pi y}{b}. \tag{34}$$

Time period of the vibration of the plate is given by

$$K=\frac{2\pi}{p} \tag{35}$$

where p is frequency given by (24).

Logarithmic decrement of the vibration is given by

$$\Lambda=\log_e\frac{w_2}{w_1} \tag{36}$$

where w_1 is the deflection at any point of the plate at a time period $K=K_1$ and w_2 is the deflection at the same point at the time period succeeding K_1.

4 Results and discussion

Time period, deflection and logarithmic decrement corresponding to the first two modes of vibration at different points for C-S-C-S non-homogeneous visco-elastic rectangular plate has been computed for different combinations of non-homogeneity parameter, taper constant, aspect ratio and thermal constants. Results are plotting in Figs. 1–8. For numerical computation, the following materials parameters are used [10]:

$E_0=7.08\times10^{10}$ N/m^2,

$G_0=2.682\times10^{10}$ N/m^2,

$\eta_0=1.4612\times10^{6}$ N s/m^2,

$\rho_0=2.80\times10^{3}$ kg/m^3,

$\nu=0.345$,

$H_0=0.01$ M.

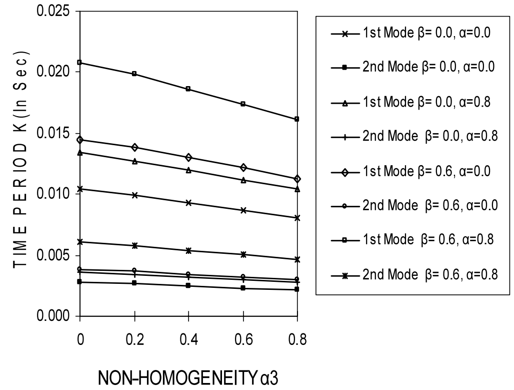

Fig. 1 Variation of time period with non homogeneity constant of visco-elastic non homogeneous rectangular plate of linearly varying thickness

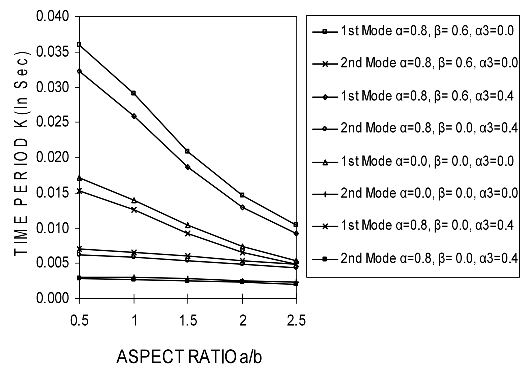

Fig. 2 Variation of time period with aspect ratio of visco-elastic non homogeneous rectangular plate of linearly varying thickness

Figures 1 and 2 show that the time period (K) for first two modes of vibration decrease with an increase of non-homogeneity parameter (α_3) and aspect ratio $(\frac{a}{b})$ respectively. Whenever taper constant (β) and thermal constant (α) increase then time period increases for first two modes of vibration. Figures 3 and 4 show that the deflection (w) for fixed aspect ratio $(\frac{a}{b}=1.5)$ starts from zero to increase then decrease to zero for first mode of vibration but for the second mode of vibration value starts zero to increase then decease then increase and finally become to zero for fixed Y and increasing value of X for time 0 K and 5 K respectively. Figures 5 and 6 show that the deflection (w) for first two modes of vibration decreases with an increase in aspect ratio $(\frac{a}{b})$.

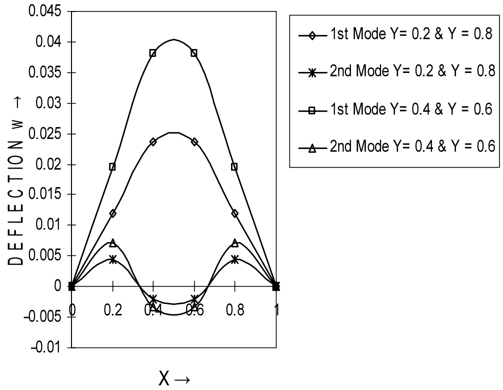

Fig. 3 Transverse deflection w vs X of visco-elastic non homogeneous rectangular plate of linearly varying thickness at initial time 0.K. $\beta = 0.6$, $\alpha = 0.8$ and $a/b = 1.5$ for all α_1, α_2, α_3

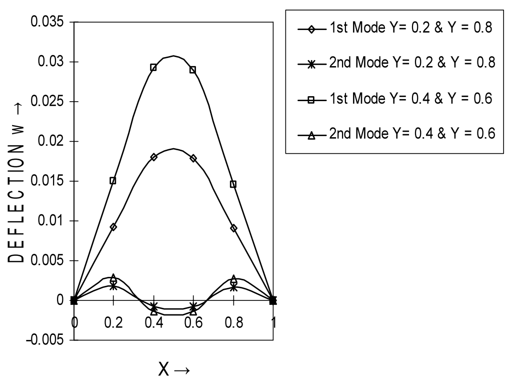

Fig. 4 Transverse deflection w vs X of visco-elastic non homogeneous rectangular plate of linearly varying thickness at time 5.K. $\beta = 0.6$, $\alpha = 0.8$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\alpha_3 = 0.4$ and $a/b = 1.5$

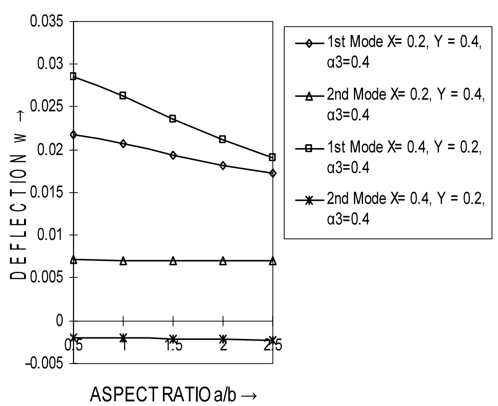

Fig. 5 Transverse deflection w vs aspect ratio a/b of visco-elastic non homogeneous rectangular plate of linearly varying thickness at initial time 0.K. $\beta = 0.6$, $\alpha = 0.8$, $\alpha_1 = 0.2$ and $\alpha_2 = 0.3$

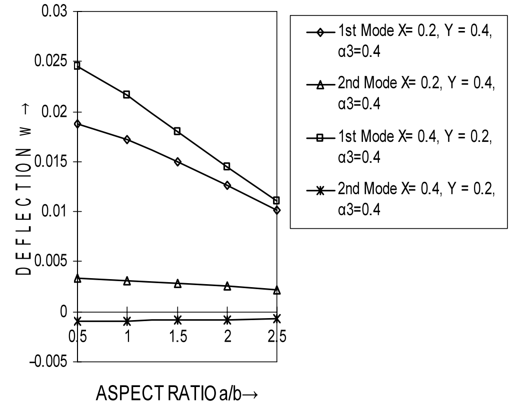

Fig. 6 Transverse deflection w vs aspect ratio a/b of visco-elastic non homogeneous rectangular plate of linearly varying thickness at time 5.K. $\beta = 0.6$, $\alpha = 0.8$, $\alpha_1 = 0.2$ and $\alpha_2 = 0.3$

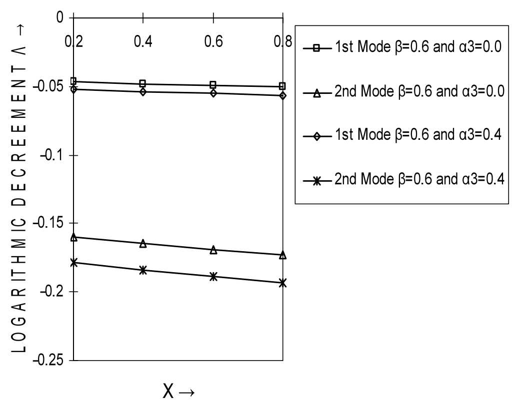

Fig. 7 Logarithmic decrement Λ vs X of visco-elastic non homogeneous rectangular plate of linearly varying thickness. $\alpha = 0.8$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $a/b = 1.5$ and for all Y

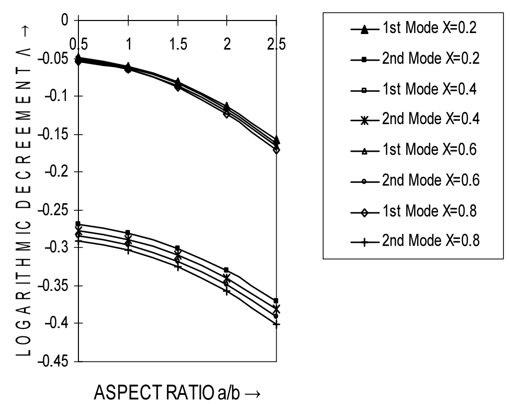

Fig. 8 Logarithmic decrement Λ vs aspect ratio a/b of visco-elastic non homogeneous rectangular plate of linearly varying thickness. $\alpha = 0.8$, $\beta = 0.0$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\alpha_3 = 0.4$ and for all Y

Figures 7 and 8 show that logarithmic decrement (Λ) for first two modes of vibration decreases with an increase in X and in the aspect ratio ($\frac{a}{b}$) respectively. It can be seen from Fig. 7 that as non-homogeneity parameter (α_3) increases logarithmic decrement (Λ) decreases for both modes of vibration.

5 Conclusion

The results for homogeneous isotropic visco-elastic rectangular plate are compared to the published paper [11] and found to be in close agreement. After comparing authors conclude that as nonhomogeneity created time period and logarithmic decrement decrease while deflection increase in comparison to homogeneous visco-elastic rectangular plate. Therefore engineers can see and develop the plates in the manner so that they can fulfill the requirements.

References

1. Tomar JS, Gupta AK (1983) Thermal effect on frequencies of an orthotropic rectangular plate of linearly varying thickness. J Sound Vib 90(3):325–331
2. Fanconneau G, Marangoni RD (1970) Effect of thermal gradient on the natural frequencies of a rectangular plate. Int J Mech Sci 12(2):113–122
3. Rao CK, Satyanarayana B (1975) Effect of thermal gradient of frequencies of tapered rectangular plates. AIAA J 131(8):1123–1126
4. Tomar JS, Gupta AK (1984) Vibration of an orthotropic elliptic plate of non-uniform thickness and temperature. J Sound Vibr 96(1):29–35
5. Tomar JS, Gupta AK (1985) Thermal effect on axisymmetric vibrations of an orthotropic circular plate of parabolically varying thickness. Ind J Pure Appl Math 16(5):537–545
6. Bambill DV, Rossit CA, Laura PAA, Rossi RE (2000) Transverse vibrations of an orthotropic rectangular plate of linearly varying thickness and with a free edge. J Sound Vib 235(3):530–538
7. Li SR, Zhou YH (2001) Shooting method for non linear vibration and thermal buckling of heated orthotropic circular plates. J Sound Vib 248(2):379–386
8. Hoff NJ (1958) High temperature effect in aircraft structures. Pergamon, New York
9. Pronsato ME, Laura PAA, Juan A (1999) Transverse vibrations of rectangular membrane with discontinuously varying density. J Sound Vib 222(2):341–344
10. Bhatnagar NS, Gupta AK (1988) Vibration analysis of viscoelastic circular plate subjected to thermal gradient. Model Simul Control B 15(1):17–31
11. Gupta AK, Khanna A (2007) Vibration of visco-elastic rectangular plate with linearly thickness variations in both directions. J Sound Vib 301(3–5):450–457
12. Pradeep V, Ganesan N (2007) Buckling & vibration of rectangular composite visco-elastic sandwich plates under thermal loads. Compos Struct 77(4):419–429
13. Li S, Batra RC, Ma L (2007) Vibration of thermally post-buckled orthotropic circular plate. J Therm Stresses 30(1):43–57