Meccanica (2008) 43: 37–46
DOI 10.1007/s11012-007-9092-4

Unsteady conjugate problem of a dissipative fluid in a horizontal channel with a periodic variation temperature

Joaquín Zueco

Received: 7 June 2007 / Accepted: 24 September 2007 / Published online: 28 November 2007

Abstract The unsteady two-dimensional transient heat transfer problem referring to a fully laminar flow developing in a parallel-plane channel exposed to a periodic variation surface temperature with distance is numerically studied. The effects of channel thickness, Péclet number, wall-to-fluid conductivity ratio, thermal diffusivity ratio, angular frequency and the viscous dissipation parameter are determined in the solutions. The non-linear equations are discretized by means an implicit finite difference scheme and the electric analogy to the resulting system is applied to convert these equations into a network-electrical model that was solved using a computer code (electric circuits simulator). In this scheme, only spatial discretization is necessary, while time remains as a real continuous variable, and its programming does not require manipulation of the sophisticated mathematical software that is inherent in other numerical methods. The network simulation method, which satisfies the conservation law for the heat flux variable and the uniqueness law for temperature, also permits the direct visualization of the local and/or integrated transport variables at any point or section of the medium.

J. Zueco (✉)
Department of Engineering Thermal and Fluids,
University Politechnique of Cartagena,
Campus Muralla del Mar, 30203 Cartagena, Spain
e-mail: joaquin.zueco@upct.es

Keywords Transient heat conduction · Conjugate problem · Viscous dissipation · Network method · Heat transfer

Abbreviations

A	Ratio of diffusivities, α_s/α_f
B	Dimensionless angular frequency
Br	Brinkman number
c_e	Specific heat
C	Capacitor
k	Thermal conductivity
G	Control-voltage current-source
L_0	Half distance between the channel walls
L_1	Thickness of the pipe
N	Number of cells
Nu	Nusselt number
Pe	Péclet number
q	Heat flux
R	Resistor
t	Time
T	Temperature
u	Velocity
U	Dimensionless velocity
x	Axial co-ordinate
y	Vertical co-ordinate

Greek symbols

α	Diffusivity

β	Angular frequency
ΔT	Oscillation amplitude temperature
ΔX	Axial thickness of the cell
ΔY	Vertical thickness of the cell
$\varGamma$	Dimensionless geometric parameter
θ	Dimensionless temperature
μ	Dynamic viscosity
ρ	Density
τ	Dimensionless time

Subscripts

f	Associated to fluid
i, j	Associated with i, j nodal point
$i, i-\Delta, i+\Delta$	Associated to the centre, left and right position on the cell
m	Medium value
mea	Associated to measurement
s	Associated to solid
w	Solid-fluid interface

1 Introduction

In recent years, numerous authors have published papers on the stationary conjugate (conduction-convection) heat transfer problem in laminar flow in pipes under different sets of boundary conditions. Pagliarini [1], Vick and Özisik [2] studied Graetz's problem with axial conduction in the pipe and fluid, and determined the range of Péclet numbers for which axial conduction is negligible. For the same problem, Wijeysundera [3] and Guedes et al. [4] obtained solutions for a convective boundary condition. Barletta et al. [5] studied analytically and numerically the problem of the conjugate forced convection heat transfer in a plane channel with longitudinally periodic regime. For transient problems, Cotta et al. [6] and Weigong and Kakac [7] obtained solutions for a periodic variation of the input temperature in ducts of constant thickness, which had previously been studied by Olek et al. [8] and Yan [9], the latter including a convective boundary condition. Yan and Lee [10] solved the unsteady conjugated mixed convection in a vertical channel, Lee and [11] studied the unsteady conjugated mixed convection inside ducts with convection from the ambient. Zueco et al. [12] studied the laminar forced convection with network simulation in the thermal entrance region of ducts. Zueco et al. [13] studied the effect of viscous dissipation on a vertical channel for a laminar flow. Barletta and Magyari [14] solved the problem of forced convection with viscous dissipation in the thermal entrance region of a circular duct with prescribed wall.

The behavior of fluid motion and temperature during the transient regime is useful for understanding the physical situation in many engineering applications. In addition, the effect of the viscous dissipation must be taken into account to obtain real solutions to engineering problems. In this paper, we study the two-dimensional unsteady heat transfer related to a fully development laminar forced convection flow in a rectangular channel submitted to a periodic variation surface temperature with constant thermal properties, taking into account viscous dissipation. The axial conduction in fluid is not considered (because the Péclet number is large), and the influence of the Brinkman number, the pipe thickness, ratio of diffusivities and the ratio thermal conductivities on the heat transfer in the horizontal channel is analyzed.

A numerical technique based on electrical analogy, the Network Simulation Method (NSM hereafter), is used to obtain the numerical solution of the problem. First a spatial discretization is applied to the transient boundary-layer equations, and a set of ordinary differential equations is obtained, one for each control volume. The main advantage of the method is that time derivatives are not replaced by finite differences (similar to the method of lines [15]), but only require finite-difference schemes for the spatial variable. In this way, the time remains as a continuous variable, which results in greater accuracy and no time interval needs to be established by the programmer. The simulation is carried out in a PC using suitable software, PSPICE in this work [16].

2 Physical and mathematical model

Figure 1 illustrates an infinite rectangular channel whose outer surface temperature is oscillating. The fluid flow is considered to be the hydro-dynamically developed forced convection of a Newtonian fluid with constant thermal properties of the fluid and pipe; the longitudinal heat conduction effect can be neglected, but the effect of the viscous dissipation is taken into account. Under these conditions, the set of governing equations or mathematical model can be formulated as

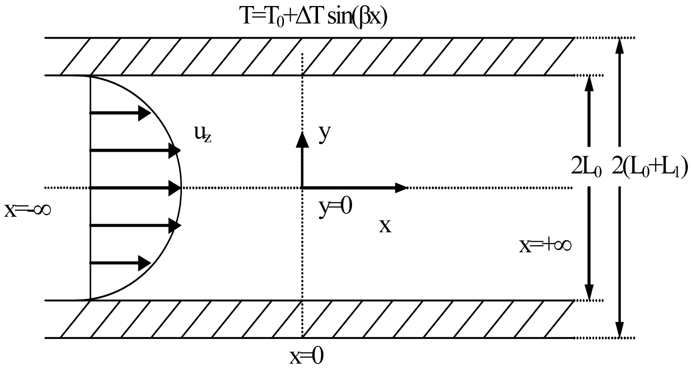

Fig. 1 Geometry of the problem

follows:

$$k_s \partial^2 T_s/\partial x^2 + k_s \partial^2 T_s/\partial y^2 = (\rho c_e)_s \partial T_s/\partial t, \quad (1)$$

$$k_f \partial^2 T_f/\partial y^2 + \mu(\partial u/\partial y)^2 = u_x(\rho c_e)_f \partial T_f/\partial x + (\rho c_e)_f \partial T_f/\partial t \quad (2)$$

where u_x is the Poiseuille velocity profile,

$$u_x(y) = 1.5u_m[1-(y/L_0)^2] \quad (3)$$

with the initially and boundary conditions,

$$T_f = T_s = 0 \quad \forall x, y,\ t = 0, \quad (4a)$$

$$\partial T_f/\partial y = 0 \quad \text{at } y = 0, \quad (4b)$$

$$T_s = T_f, \qquad k_s \partial T_s/\partial y = k_f \partial T_f/\partial y \quad \text{at } y = \pm L_0, \quad (4c)$$

$$T_s = T_0 + \Delta T \sin(\beta x) \quad \text{at } y = \pm L_1. \quad (4d)$$

Equations (1) and (2) refer to the solid and liquid regions, respectively. In order to generalize the solution, the above equations and the initial and boundary conditions may be written in dimensionless form, resulting in

$$\partial^2\theta_s/\partial X^2 + \partial^2\theta_s/\partial Y^2 = 1/A\partial\theta_s/\partial\tau, \quad \text{for } 1 < Y < \Gamma, \quad (5)$$

$$\partial^2\theta_f/\partial Y^2 + 9BrY^2 = U_x Pe\partial\theta_f/\partial X + \partial\theta_f/\partial\tau, \quad \text{for } 0 < Y < 1 \quad (6)$$

with the initially and boundary conditions,

$$\theta_f = \theta_s = 0 \quad \forall X, Y, \tau = 0, \quad (7a)$$

$$\partial\theta_f/\partial Y = 0 \quad \text{at } Y = 0, \quad (7b)$$

$$\theta_s = \theta_f,\ k_{sf}\partial\theta_s/\partial Y = \partial\theta_f/\partial Y \quad \text{at } Y = 1, \quad (7c)$$

$$\theta_s = \sin(BX/Pe) \quad \text{at } Y = \Gamma \quad (7d)$$

where

$$\begin{aligned} &\theta = (T-T_0)/\Delta T; \quad X = x/L_0; \quad Y = y/L_0; \\ &\Gamma = L_1/L_0, \quad \tau = t\alpha_f/L_0^2, \\ &U_x = 1.5[1-Y^2], \\ &Br = \mu u_m^2/k_f\Delta T, \quad B = Pe\beta L_0, \\ &Pe = u_m L_0/\alpha_f, \\ &k_{sf} = k_s/k_f, \quad A = \alpha_s/\alpha_f \end{aligned} \quad (8)$$

where α is the diffusivity, $\alpha = k(\rho c_e)^{-1}$, k_{sf} the ratio thermal conductivities and Br the Brinkman number. For most practical applications, the dimensionless heat flux at the solid-fluid interface ($q_{w,x}$), the bulk temperature (θ_b) and local Nusselt number (Nu_x) are required. These values may be computed from

$$q_{w,x} = \partial\theta/\partial Y|_{Y=0}, \quad (9)$$

$$\theta_b(X) = 1.5\int_0^1 \theta(X,Y)(1-Y^2)dY, \quad (10)$$

$$Nu_x = \frac{-q_{w,x}}{\theta_w(X,1)-\theta_b(X)}. \quad (11)$$

3 Numerical solution

A second-order central difference scheme has been used to discretize the energy equation in both solid and fluid domain. The numerical method is based on the existing analogy between electric circuit theory and heat conduction theory, which, it should be emphasized, has nothing to do with the classical thermo-electrical analogy. This method uses discrete intervals for the spatial variable (the time variable being a continuous function), a development that is also adopted by the mathematically oriented Method of Lines [15]. For time-domain circuit simulation, Pspice uses a numerical implicit integration formula (for example, trapezoidal integration, with a truncation-error time-step, is employed in the Spice2 program) to form companion models for capacitors and inductors at each time-point, and applies the Newton-Raphson method to linearize non-linear devices. The circuit is simulated at each time-point by iteratively solving a system of linearized equations in the form of $Ax = b$, where A is typically the so-called modified nodal analysis circuit matrix, or Quasi-Newton iteration matrix (see Nagel [17]).

The total time interval that is specified by the user is divided into discrete time-points and the program determines the circuit solution at each successive time-point starting from time zero. The spacing between successive time-points (time-step) is controlled by Pspice to ensure the accuracy of the solution.

The set of ordinary differential equations is obtained by spatial discretization of the mathematical model, which is defined by a set of equations that includes: (i) heat conduction equation, (ii) boundary conditions, (iii) initial condition and (iv) special conditions. With this end in view, the whole 2-D region is divided into a number of volume elements, which are not necessarily similar. A network model for an elementary cell is designed from this set of equations, associating different kinds of electrical ports to each one of the terms that make up the differential equations: resistors, capacitors, and non-linear electrical devices. The model for the whole medium is obtained by connecting N elemental networks in series. Boundary conditions are implemented by additional electrical devices connected to the boundaries. This numerical method has been used by Zueco [18] to solve the problem of a semi-infinite vertical flat plate submitted to a constant heat flux in the presence of a magnetic field and by Zueco and Alhama [19] to develop an iterative algorithm to estimate the temperature-dependent thermo-physical properties of fluids.

As regards the problem studied here, the cells are rectangular, with, a surface section dimension of ΔY_f, ΔY_s and ΔX. The numbers of cells in the radial and axial directions are $N_{Y,f}$, $N_{Y,s}$ and N_X. The following expressions are applied: $\Delta Y_f = L_0/N_{Y,f}$, $\Delta Y_s = Y/N_{Y,s}$, and $\Delta X = L/N_X$, where $N_{Y,f} = 30$, $N_{Y,s} = 10$ and $N_X = 150$.

Discretization of (1) for the channel and (2) for the fluid yield the following ordinary differential equations in dimensionless form

Solid:

$$\begin{aligned}
&[\theta_{i,j-\Delta Y/2} - \theta_{i,j}]/(\Delta Y_s/2k_{sf}) \\
&\quad - [\theta_{i,j} - \theta_{i,j+\Delta Y/2}]/(\Delta Y_s/2k_{sf}) \\
&\quad + [\theta_{i-\Delta X/2,j} - \theta_{i,j}]/\Delta X^2/(2k_{sf}/\Delta Y_s) \\
&\quad - [\theta_{i,j} - \theta_{i+\Delta X/2,j}]/\Delta X^2/(2k_{sf}/\Delta Y_s) \\
&= \Delta Y_s k_{sf}/A d\theta_{i,j}/d\tau. \qquad (12)
\end{aligned}$$

Fluid:

$$\begin{aligned}
&[\theta_{i,j-\Delta Y/2} - \theta_{i,j}]/(\Delta Y_f/2) \\
&\quad - [\theta_{i,j} - \theta_{i,j+\Delta Y/2}]/(\Delta Y_f/2) + 9Bry^2\Delta Y_f \\
&= 1.5[1 - Y^2]Pe\Delta Y_f[\theta_{i+1,j} - \theta_{i,j}]/\Delta X \\
&\quad + Pe\Delta Y_f d\theta_{i,j}/d\tau \qquad (13)
\end{aligned}$$

where $\theta_{i,j}, \theta_{i,j+\Delta Y/2}, \theta_{i,j-\Delta Y/2}, \theta_{i+\Delta X/2,j}, \theta_{i-\Delta X/2,j}$, are the temperatures in the centre and ends of the cell, respectively. Each cell is electrically connected to contiguous cells to make up the whole model of the medium. Figure 2 shows the cells for the wall and the fluid in $Y = 1$, where the boundary conditions $\theta_s = \theta_f$ and $k_{sf}\partial\theta_s/\partial Y = \partial\theta_f/\partial Y$ are applied. The first condition is implemented directly with the union of the solid and the fluid, while the condition second is implemented by means of a measurement-current ($I_{mea,i,j}$) and a voltage-control current-generator ($G_{w,i,j}$).

Finally, to implement the other boundary conditions, voltage sources are used for the constant and variable temperature values. As regards the initial condition, voltages $\theta_s = \theta_f = 0$ are applied to the capacitors $C_{s,i,j}$ and $C_{f,i,j}$.

4 Numerical results

The prime objective of the present investigation is to study the combined effects of the viscous dissipation (Br, Brinkman number), wall thickness ratio ($\Gamma = L_1/L_0$) and thermo-physical properties ratio $k_{sf} = k_s/k_f$ and $A = \alpha_s/\alpha_f$, when the surface

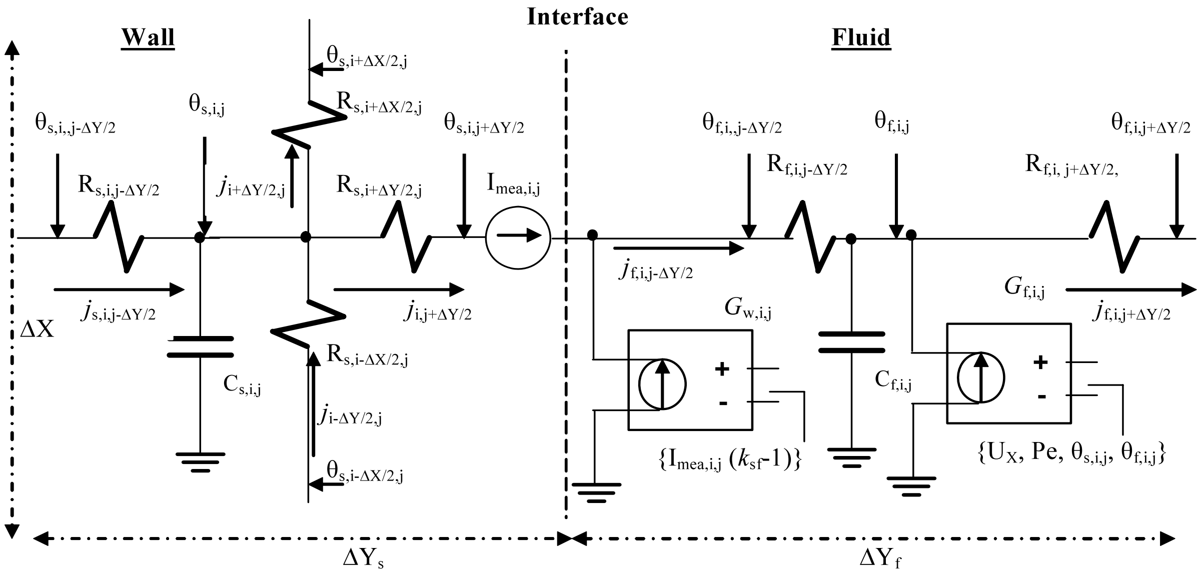

Fig. 2 Network model for channel and fluid and boundary condition in the interface

temperature of the horizontal channel varies with the axial distance. The Brinkman number represents a measure of the effect of the viscous dissipation. In all the computations presented in this paper we have taken Pr (Prandtl number) $= B$ (dimensionless angular frequency) $= 100$. The following physically realistic range of values of the parameters was considered: $Br = 0$–1.0, $\Gamma = 0.2$–0.5, $k_{sf} = 0.5$–3.0 and $A = 0.01$–1.0. The numerical values employed in each case are included in each figure. To obtain the stationary response, the condenser of the network is merely omitted. The effects of the parameters B and Pe were studied by Barletta et al. [5], and so are not studied in this paper.

The effects of Br (viscous dissipation) and Γ (wall thickness) on the axial steady dimensionless temperature profiles in the solid-fluid interface are depicted in Fig. 3 for $k_{sf} = 0.5$. It can be seen that the temperature increases when the Brinkman number increases and Γ decreases. Beside, the oscillation amplitude of the dimensionless interface temperature is greater when fewer values of Γ are used. In Fig. 4, the dimensionless temperature profiles in steady-state at the solid-fluid interface is reported for $\Gamma = 0.2$ and different values of Br and k_{sf}. This case confirms that the presence of viscous dissipation increases the temperature. The oscillation amplitude of the interface dimensionless temperature increases when k_{sf} increases.

The effects of Br and Γ on axial steady local heat flux profiles in the solid-fluid interface are studied in Fig. 5 for $k_{sf} = 0.5$. It can be seen that the local heat flux values obtained become negative as Br increases (the opposite of the case of temperature). Besides, the oscillation amplitude of the dimensionless interface temperature increases the fewer the values of Γ used (as in the case of the temperature in Fig. 3). In Fig. 4, the local heat flux profiles in steady-state at the solid-fluid interface is reported for $\Gamma = 0.2$ and different values of Br and k_{sf}. This case confirms that as viscous dissipation increases negative values of the local heat flux are obtained. The oscillation amplitude of the interface local heat flux increases when k_{sf} increases.

One conclusion that can be reached from the above is that the oscillation amplitude of both interface temperature and heat flux does not depend on Brinkman number. Figure 7 shows the steady axial evolution of Nu_x for different values of Br for $\Gamma = 0.2$ and $k_{sf} = 2.0$. We can see that the local Nusselt number has vertical asymptotes (singularities) due to the disappearance of the difference $(\theta_w - \theta_b)$ at an axial distance from the entrance cross-section, which depends on the Brinkman number. For the cases plotted, the singularity that first appears is for $Br = 1.0$, and so, as Br increases, the singularity appears closer to the entry of the channel. For an axial position approximately

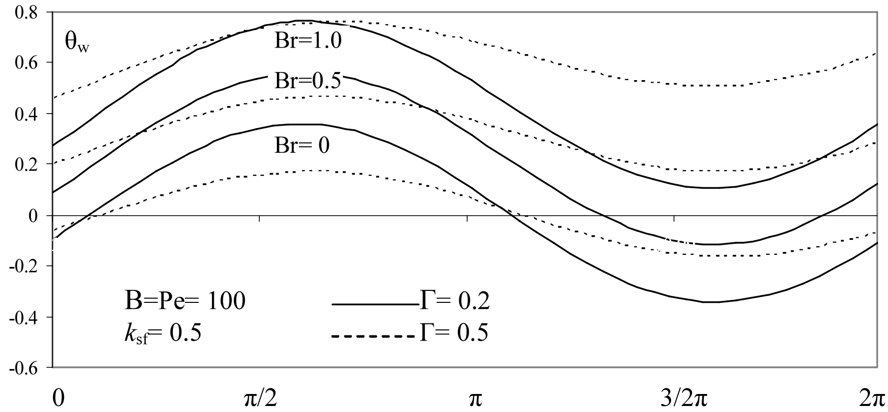

Fig. 3 Interfacial temperature θ_w at the steady-state for $B = P_e = 100$, $k_{sf} = 0.5$ and different values of Br (0, 0.5 and 1.0) and Γ (0.2 and 0.5)

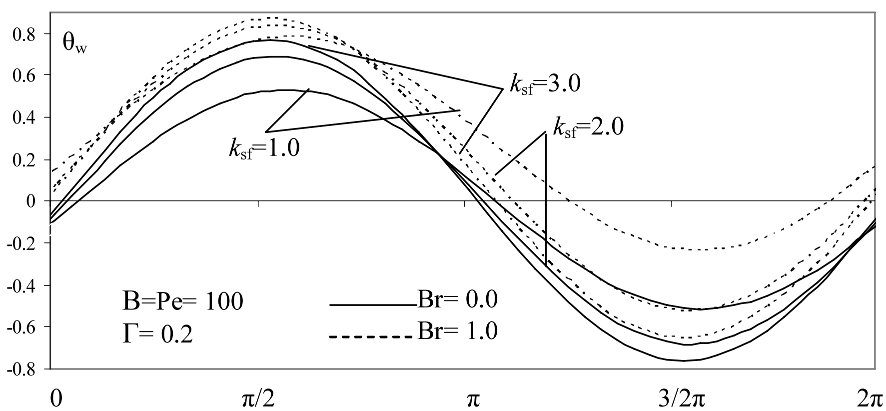

Fig. 4 Interfacial temperature θ_w at the steady-state for $B = P_e = 100$, $\Gamma = 0.2$ and different values of k_{sf} (1.0, 2.0 and 3.0) and Br (0 and 1.0)

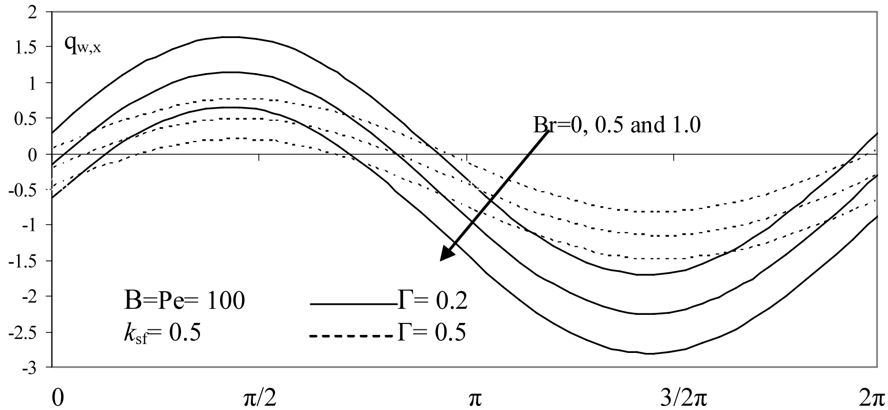

Fig. 5 Interfacial heat flux $q_{w,x}$ at the steady-state for $B = P_e = 100$, $k_{sf} = 0.5$ and different values of Br (0, 0.5 and 1.0) and Γ (0.2 and 0.5)

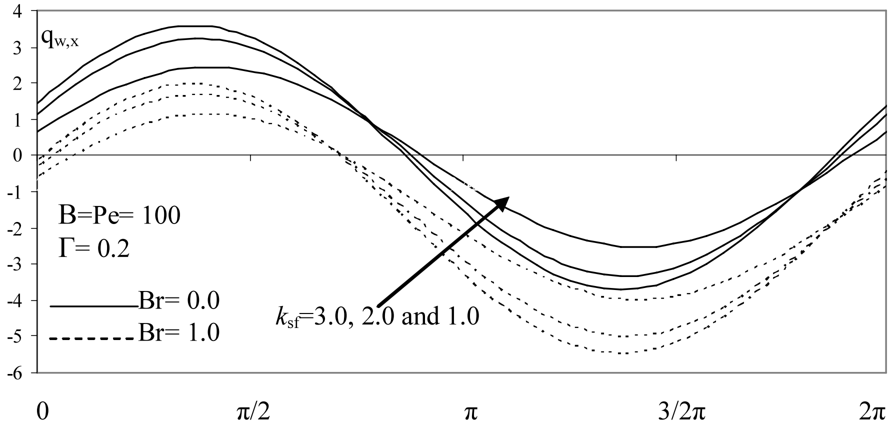

Fig. 6 Interfacial heat flux $q_{w,x}$ at the steady-state for $B = P_e = 100$, $\Gamma = 0.2$ and different values of k_{sf} (1.0, 2.0 and 3.0) and Br (0 and 1.0)

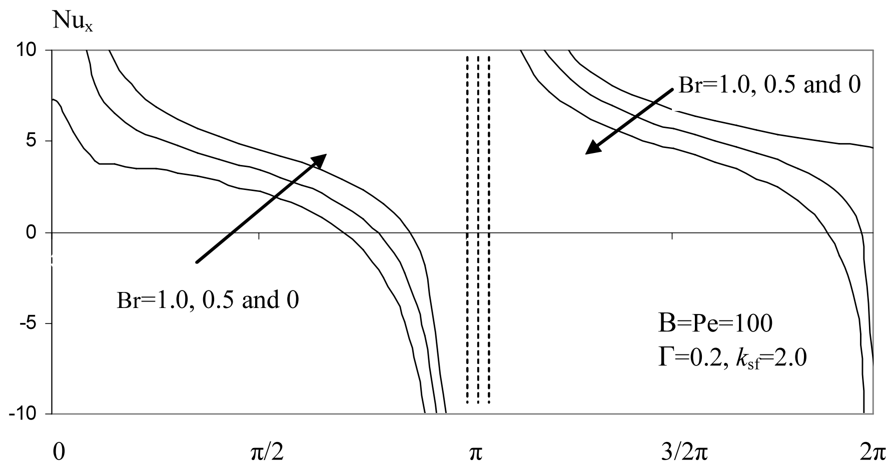

Fig. 7 Local Nusselt number Nu_x at the steady-state for $B = P_e = 100$, $\Gamma = 0.2$, $k_{sf} = 2.0$ and different values of Br (0, 0.5 and 1.0)

$0 < X < 7/8\pi$, the greater the local Nusselt number, the fewer the values of Br that are used, while the opposite occurs for approximately $9/8\pi < X < 2\pi$.

The transient dimensionless temperature profiles (various values of τ and steady-state) are shown in Fig. 8, for $Br = 0$, $\Gamma = 0.2$ and 0.5, $k_{sf} = 0.5$ and 3.0, and $A = 0.01$ and 1.0. The magnitude achieved in steady-state is always independent of the value of the diffusivity ratio. However, when A increases, the value of the wall capacitor $C_{s,i,j} = \Delta Y_s k_{sf}/A$ decreases, the energy storage capacity of the wall decreases and the time required to reach the steady-state also decreases. The time required to reach the steady-state increases for smaller values of k_{sf} (because of the great thermal capacity of the channel) and for greater values of Γ. For the case $\Gamma = 0.5$ and $k_{sf} = 3.0$, the times required to reach the steady-state are 34 for $A = 0.01$ and 9 for $A = 1$. This case shows the greatest difference between times, and, for $\Gamma = 0.2$ and $k_{sf} = 3.0$ the times are 8 for $A = 0.01$ and 5 for $A = 1$.

Figure 9 shows the axial evolution of the local heat flux $q_{w,x}$ in steady-state and in transient situation, for $Br = 0$, $\Gamma = 0.2$ and 0.5, $k_{sf} = 0.5$ and 3.0, and $A = 0.01$ and 1.0. The local heat flux decreases as the time necessary to reach the steady-state increases for the case of $A = 1.0$. However, for the case of $A = 0.01$, the local heat flux increases as the time to reach the steady-state increases. Furthermore, the

(a)

(b)

(c)

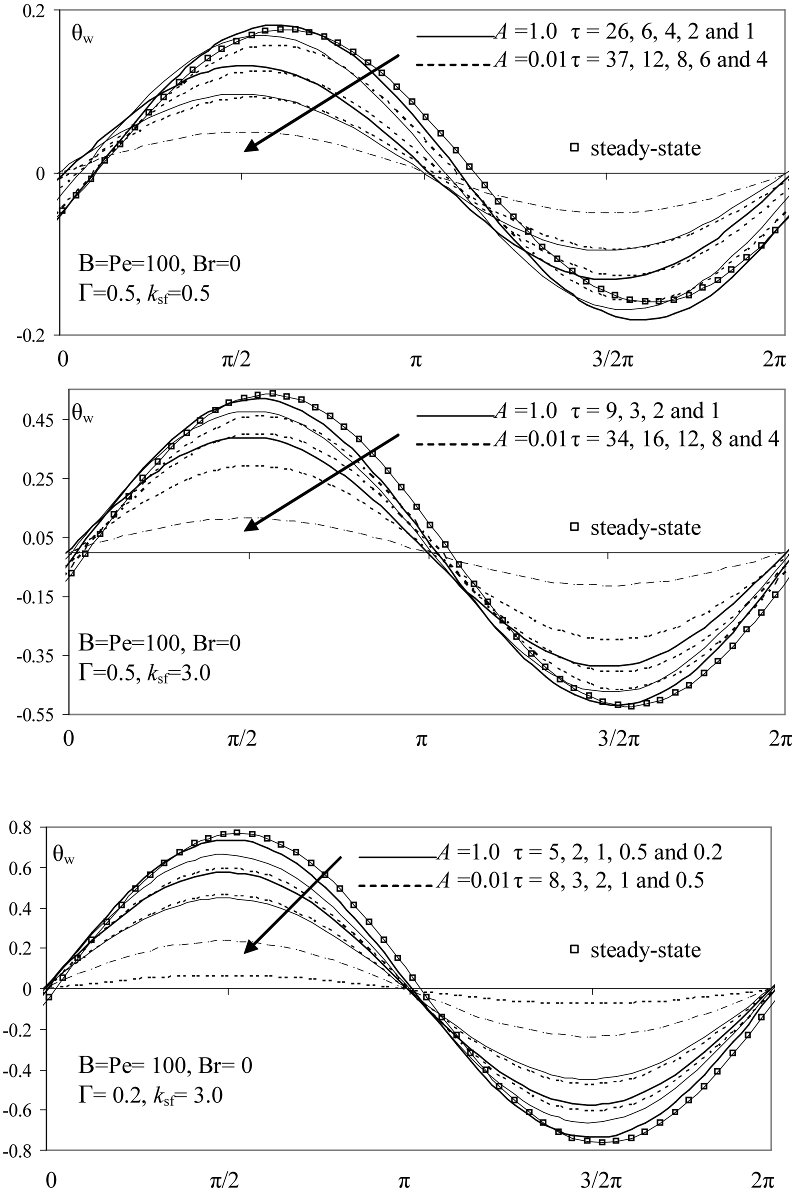

Fig. 8 Transient interfacial temperature θ_w for $B = P_e = 100$, $Br = 0$ and different values of A (0.01 and 1.0): **a** $k_{sf} = 0.5$ and $\Gamma = 0.5$; **b** $k_{sf} = 3.0$ and $\Gamma = 0.5$; **c** $k_{sf} = 3.0$ and $\Gamma = 0.2$

magnitude achieved in steady-state is always independent of the value of the diffusivity ratio. For the case $\Gamma = 0.2$ and $k_{sf} = 3.0$, the times required to reach the steady-state are 12 for $A = 0.01$ and 11 for $A = 1$. This case shows the least difference between times, as in the previous case.

When A increases, the time required to reach the steady-state decreases. This time increases for larger values of k_{sf} (the opposite of the case of temperature) and of Γ. An exact knowledge of the transient evolution is important for designing devices for engineering applications, for example in heat exchange, heat recovery steam generation, ..., because in a non-stationary situation strong local heat flux may be extremely dangerous for the operation of such devices.

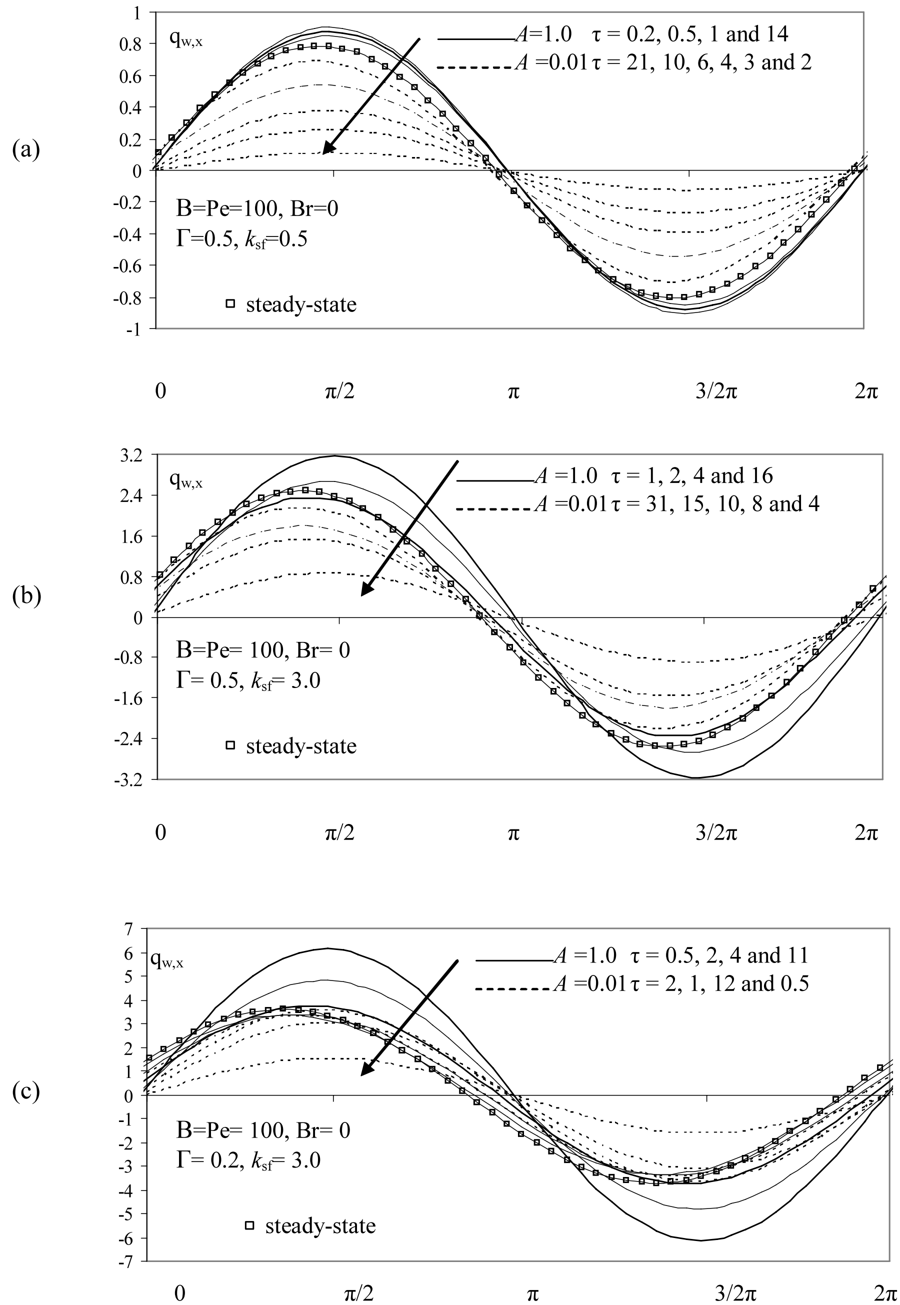

Fig. 9 Transient interfacial heat flux $q_{w,x}$ for $B = P_e = 100$, $Br = 0$ and different values of A (0.01 and 1.0): **a** $k_{sf} = 0.5$ and $\Gamma = 0.5$; **b** $k_{sf} = 3.0$ and $\Gamma = 0.5$; **c** $k_{sf} = 3.0$ and $\Gamma = 0.2$

5 Concluding remarks

The unsteady conjugated heat transfer problem for laminar flows in a horizontal channel (including bi-dimensional conduction in the wall and viscous dissipation) is studied. This study uses the network simulation method, which is especially useful when hard non-linearities are present in the equations of the process. This numerical tool requires none of mathematical manipulations inherent in the resolution of finite-difference equations. Instead, the software selected to solve the circuits does this work. In this work, the computation time was a few seconds. Furthermore it is very easy and rapid to make changes in the boundary and initial conditions of the problem. The main results of the investigation may be briefly summarized as follows:

– It is important to consider wall conduction because it plays a significant role in unsteady conjugated heat transfer in a horizontal channel.
– Exact knowledge of transient evolution is necessary for the design of devices in engineering applications.
– The time required to reach the steady-state of the temperature is longer as the values of k_{sf} and A decrease or the values of Γ increase.
– The time required to reach the steady-state of the local heat flux is longer as the values of A decrease or the values of Γ and k_{sf} increase.
– The effect of wall conduction on heat transfer increases as k_{sf} decreases and Γ increases.
– The presence of viscous dissipation leads to an increase in the temperature profile, but values more negative of the local heat flux.
– The oscillation amplitude of both interface temperature and heat flux does not depend on the Brinkman number, and the amplitude increases when k_{sf} increases in both cases.

References

1. Pagliriani G (1988) Effect of axial conduction in the wall and the fluid on conjugate heat transfer in thick-walled circular tubes. Int Commun Heat Mass Transfer 55:5881–591
2. Vick B, Özisik MN (1981) An exact analysis of low Péclet number heat transfer in laminar flow with axial conduction. Lett Heat Transfer 8:1–10
3. Wijeysundera NJ (1986) Laminar forced convection in circular and flat ducts with wall axial conduction and external convection. Int J Heat Mass Transfer 29:797–807
4. Guedes ROC, Cotta RM, Brum NCL (1990) Heat transfer in laminar flow with wall axial conduction and external convection. J Thermophys 5:4
5. Barletta A, Rossi di Schio E, Comini G, Agaro PD (2007) Conjugate forced convection heat transfer in a plane channel: Longitudinally periodic regime. Int J Thermal Sci (in press)
6. Cotta RM, Mikhailov MD, Özisik MN (1986) Transient conjugated forced convection in ducts with periodically varying inlet temperature. Int J Heat Mass Transfer 30(10):2073–2082
7. Weigong L, Kakac S (1991) Unsteady thermal entrance heat transfer in laminar with a periodic variation inlet temperature. Int J Heat Mass Transfer 34(10):2581–2592
8. Olek S, Elias E, Wacholder E, Kaizerman S (1991) Unsteady conjugated heat transfer in laminar pipe flow. Int J Heat Mass Transfer 34(6):1443–1450
9. Yan WM (1993) Transient conjugated heat transfer in channel flows with convection from the ambient. Int J Heat Mass Transfer 36(5):1295–1301
10. Yan WM, Lee KT (1995) Unsteady conjugated mixed convection in a vertical channel. ASME J Heat Transfer 117:234–238
11. Lee KT, Yan WM (1996) Transient conjugated mixed convection inside Ducts with convection from the ambient. Int J Heat Mass Transfer 39:1203–1211
12. Zueco J, Alhama F, González-Fernández CF (2004) Analysis of laminar forced convection with network simulation in thermal entrance region of ducts. Int J Thermal Sci 43:443–451
13. Zueco JJ (2006) Network model to study the transient heat transfer problem in a vertical channel with viscous dissipation. Int Commun Heat Mass Transfer 33:1079–1087
14. Barletta A, Magyari E (2007) Forced convection with viscous dissipation in the thermal entrance region of a circular duct with prescribed wall heat flux. Int J Heat Mass Transfer 50:26–35
15. Liskovets OA (1965) The method of lines (review). Differ Equ 1:1308–1323
16. Microsim Corporation (1994) PSPICE, versión 6.0: 20 Fairbanks, Irvine, CA 92718
17. L.W. Nagel, SPICE, a computer program to simulate semiconductor circuits. Memo UCB/ERL M520, University of California, Chaps. 4, 5, 6
18. Zueco JJ (2006) Numerical study of unsteady free convection magneto-hydrodynamic flow of a dissipative fluid along a vertical plate submitted to a constant heat flux. Int J Eng Sci 44:1380–1393
19. Zueco J, Alhama F (2007) Simultaneous inverse determination of the temperature-dependent thermophysical properties of fluids using the network simulation method. Int J Heat Mass Transfer 50:3234–3243