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Momentum and heat transfer in the magnetohydrodynamic stagnation-point flow of a viscoelastic fluid toward a stretching surface

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Abstract An analysis is made of the steady two-dimensional stagnation-point flow of an incompressible viscoelastic fluid over a flat deformable surface when the surface is stretched in its own plane with a velocity proportional to the distance from the stagnation-point. It is shown that for a viscoelastic conducting fluid of short memory (obeying Walters' B′ model), a boundary layer is formed when the stretching velocity of the surface is less than the inviscid free-stream velocity and velocity at a point increases with increase in the Hartmann number. On the other hand an inverted boundary layer is formed when the surface stretching velocity exceeds the velocity of the free stream and the velocity decreases with increase in the Hartmann number. A novel result of the analysis is that the flow near the stretching surface is that corresponding to an inviscid stagnation-point flow when the surface stretching velocity is equal to the velocity of the free stream. Temperature distribution in the boundary layer is found when the surface is held at constant temperature and surface heat flux is determined. It is found that in the absence of viscous and Ohmic dissipation and strain energy in the flow, temperature at a point decreases with increase in the Hartmann number.

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1 Introduction

Flow of an incompressible viscous fluid over a stretching surface has important applications in polymer industry. For instance a number of technical processes concerning polymers involves the cooling of continuous strips (or filaments) extruded from a die by drawing them through a stagnant fluid with controlled cooling system and in the process of drawing, these strips are sometimes stretched. The quality of the final product depends to a large extent on the rate of heat transfer at the stretching surface. Crane [1] gave a similarity solution in closed analytical form for steady two-dimensional incompressible boundary layer flow caused by the stretching of a sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point. Carragher and Crane [2] investigated heat transfer in the above flow in the case when the temperature difference between the surface and the ambient fluid is proportional to a power of distance from the fixed point. Temperature distribution in the flow over a stretching surface subject to uniform heat flux was studied by Dutta et al. [3]. Recently, Chiam [4] analyzed steady two-dimensional stagnation-point flow of an incompressible viscous fluid towards a stretching surface.

All the above investigations are, however, confined to flows of Newtonian fluids. In recent years, non-Newtonian fluids have become more important industrially. For instance in certain polymer processing applications, one deals with flow of a non-Newtonian fluid over a stretching surface. Similarity solutions for the velocity distribution in the flow of a power-law fluid past a stretching surface were given by Andersson and Dandapat [5]. The same flow was examined by Siddappa and Khapate [6] for a special class of non-Newtonian fluids known as second-order fluids, which are viscoelastic in nature. Bhattacharyya et al. [7] investigated the temperature distribution in the steady boundary layer flow of a second-order fluid (viscoelastic) past a stretching surface. Chen et al. [8] studied the flow and heat transfer in the boundary layer of a viscoelastic fluid of Walters' liquid B′ model (which characterizes a viscoelastic fluid with short memory) over a stretching surface subject to either constant temperature or uniform heat flux. Interestingly, the boundary layer equations for the steady two-dimensional flow studied in Refs. [7] and [8] are identical. This coincidence is, however, not fortuitous. It stems from the fact that the constitutive equation for the second-order fluid [9] is valid for low shear rates such that the characteristic time scale associated with the flow is very large compared with the memory of the fluid. Mahapatra and Gupta [10] investigated steady, two-dimensional stagnation-point flow of a viscoelastic fluid (obeying Walters' liquid B′ model) toward a stretching surface. Temperature distribution in the boundary layer was found in the case when the stretching surface is kept at constant temperature and heat flux at the surface was also calculated. Flow of an electrically conducting non-Newtonian fluid past a stretching surface was studied by Andersson [11], Abel et al. [12] when a uniform magnetic field acts transverse to the surface.

In this paper, we investigate steady, two-dimensional magnetohydrodynamic stagnation-point flow of a viscoelastic fluid (obeying Walters' liquid B′ model) toward a stretching surface. Temperature distribution in the boundary layer is found in the case when the stretching surface is kept at constant temperature.

2 Flow analysis

Consider the steady two-dimensional flow near a stagnation point at a surface coinciding with the plane $y = 0$, the flow being in the region $y > 0$. Two equal and opposing forces are applied along the x-axis so that the surface is stretched keeping the origin fixed as shown in Fig. 1. The velocity distribution in the frictionless flow in the neighborhood of the stagnation point at $x = y = 0$ is given by

$$u = ax, \quad v = -ay, \tag{1}$$

where the constant $a(> 0)$ is proportional to the free stream velocity far away from the stretching surface.

For the constitutive equation of viscoelastic fluid we adopt Walters' liquid B′ [13] model given by (in cartesian tensor notation)

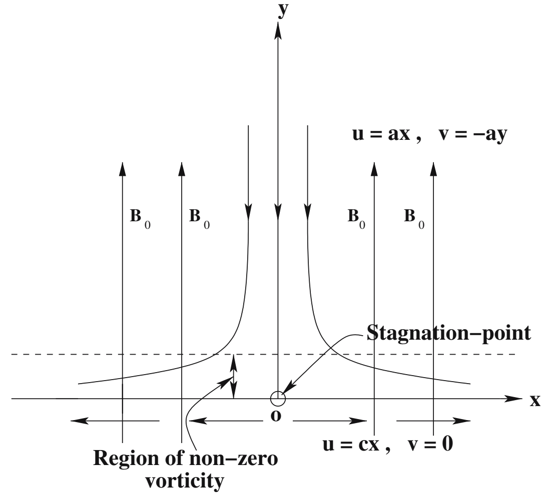

Fig. 1 A sketch of the physical problem

$$\tau_{ik} = -p\delta_{ik} + 2\mu e_{ik} - 2k_0 \times \left[\frac{\partial e_{ik}}{\partial t} + v_j \frac{\partial e_{ik}}{\partial x_j} - e_{ij}\frac{\partial v_k}{\partial x_j} - e_{jk}\frac{\partial v_i}{\partial x_j}\right], \quad (2)$$

where τ_{ik}, p and e_{ik} denote, respectively, the stress tensor, pressure and the rate-of-strain tensor. Further μ and k_0 stand for the limiting viscosity at small rate of shear and the elastic coefficient, respectively. It should be noted that the above model for the fluid is valid for a viscoelastic fluid with short memory.

Using (1) and (2), the steady two-dimensional boundary layer equations for the electrically conducting viscoelastic fluid flow are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (3)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = a^2 x + \nu\frac{\partial^2 u}{\partial y^2} - \frac{k_0}{\rho}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right] - \frac{\sigma B_0^2}{\rho}(u - ax), \quad (4)$$

$$\frac{\partial p}{\partial y} = O(\delta), \quad (5)$$

where (u, v) are the velocity components with $\nu = \mu/\rho$, ρ being the fluid density. In deriving these equations it is tacitly assumed that in addition to the usual boundary layer approximations, the contribution due to the normal stress is of the same order of magnitude as that due to shear stresses so that both ν and k_0/ρ are of $O(\delta^2)$, δ being the boundary layer thickness. Note that $k_0 > 0$. Further in (4), it is assumed that the induced magnetic field is negligible. This is a valid assumption for flow at small magnetic Reynolds number as in the case of a viscoelastic electrically conducting fluid like Olive oil.

The boundary conditions are

$$u = cx, \quad v = 0 \quad \text{at} \quad y = 0, \quad (6)$$

$$u \to ax \quad \text{as} \quad y \to \infty, \quad (7)$$

where c is a positive constant.

A little inspection shows that the boundary layer equations (3) and (4) admit of a similarity solution

$$u(x, y) = cxf'(\eta), \quad v(x, y) = -(c\nu)^{\frac{1}{2}} f(\eta), \quad (8)$$

where

$$\eta = (c/\nu)^{\frac{1}{2}} y. \quad (9)$$

With u and v given by (8), we find that (3) is identically satisfied and (4) leads to

$$f'^2 - ff'' = a^2/c^2 + f''' - k\left(2f'f''' - f''^2 - ff^{iv}\right) - M^2\left(f' - \frac{a}{c}\right), \quad (10)$$

where

$$k = \frac{k_0 c}{\rho\nu} \quad \text{and} \quad M = \left(\frac{\sigma B_0^2}{\rho c}\right)^{1/2}. \quad (11)$$

Here M is Hartmann number which is a measure of the strength of the magnetic field. The boundary conditions (6) and (7) become

$$f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = a/c. \quad (12)$$

For a viscous fluid ($k = 0$), (10) is of third order and the boundary conditions (12) suffice to determine the solution of (10). But for a viscoelastic fluid ($k \neq 0$), these boundary conditions are not enough to obtain the solution of (10) which is of fourth order. However implicit in the derivation of the constitutive equation (2) is the neglect of terms of order k_0^2. Thus k as defined in (11), which is a measure (dimensionless) of relaxation time, is very small for a viscoelastic fluid with small memory. This, of course, means that the characteristic time scale of the fluid flow is large compared with the relaxation time of the fluid.

Since $k \ll 1$, we seek a solution in the form as

$$f(\eta) = f_0(\eta) + kf_1(\eta) + O(k^2). \quad (13)$$

Substituting (13) in (10) and equating the coefficients of k^0 and k, we get

$$f_0'^2 - f_0 f_0'' = a^2/c^2 + f_0''' - M^2\left(f_0' - \frac{a}{c}\right), \quad (14)$$

$$f_1''' + f_0 f_1'' - 2f_0' f_1' + f_0'' f_1 = 2f_0' f_0''' - f_0''^2 - f_0 f_0^{iv} + M^2 f_1'. \quad (15)$$

Using (12), the boundary conditions for f_0 and f_1 are

$$f_0(0) = 0, \quad f_0'(0) = 1, \quad f_0'(\infty) = a/c, \quad (16)$$

$$f_1(0) = 0, \quad f_1'(0) = 0, \quad f_1'(\infty) = 0. \quad (17)$$

It is interesting to note that when $a = c$, (10) subject to the boundary conditions (12) admits of an exact analytical solution given by

$$f(\eta) = \eta. \tag{18}$$

This leads to from (8), $u = ax, v = -ay$. From this we can infer that when $a = c$, the velocity distribution near the stretching surface is the same as that of a frictionless flow away from the surface so that no boundary layer is formed near the surface. Equations (14) and (15) along with the boundary conditions (16) and (17) are solved numerically by finite-difference method using Thomas algorithm [14].

Figure 2 shows the variation of the horizontal velocity component with distance from the surface for several values of the Hartmann number M with $a/c = 3.0$ and $k = 0.001$. It can be seen that for a fixed value of a/c (with $a/c > 1$), the velocity increases with increase in Hartmann number. It can also be seen that when $a/c > 1$, the flow has a boundary layer structure and the thickness of this boundary layer decreases with increase in M. It is interesting to note from Fig. 3 that when $a/c < 1$, the flow has an inverted boundary layer structure. This is due to the fact that when $a < c$, the stretching velocity cx of the surface exceeds the velocity ax of the external stream. Further it is observed from this figure that for a fixed value of a/c (with $a/c < 1$), the horizontal velocity at a point decreases with increase in M in contrast with the corresponding velocity distribution for $a/c > 1$ shown in Fig. 2.

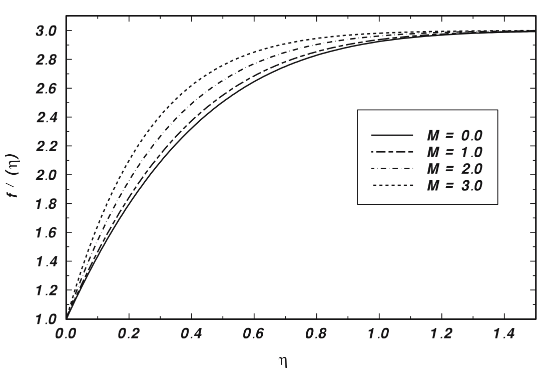

Fig. 2 Variation of $f'(\eta)$ with η for several values of M with $k = 0.001$ and $a/c = 3.0$

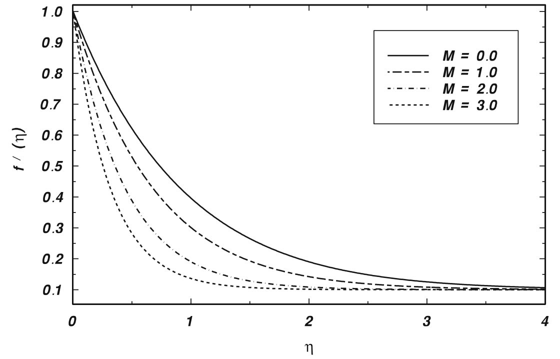

Fig. 3 Variation of $f'(\eta)$ with η for several values of M with $k = 0.001$ and $a/c = 0.1$

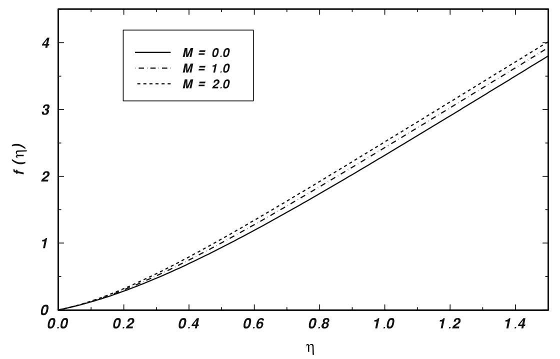

Fig. 4 Variation of $f(\eta)$ with η for several values of M with $k = 0.001$ and $a/c = 3.0$

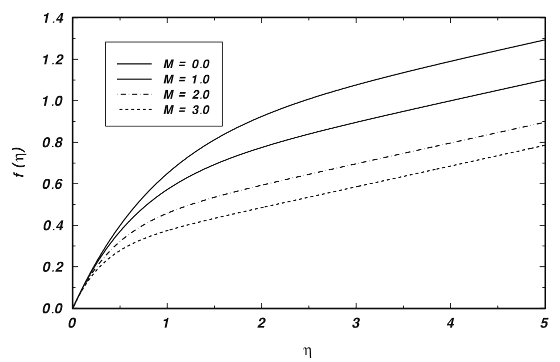

Fig. 5 Variation of $f(\eta)$ with η for several values of M with $k = 0.001$ and $a/c = 0.1$

Figure 4 shows the variation of $f(\eta)$, the vertical component of velocity, with η for several values of M with $a/c = 3.0$ and $k = 0.001$. It can be seen that the vertical velocity at a point increases with increase in M for $a/c > 1$. But the vertical velocity at a point decreases with increase in M for $a/c < 1$ as can be seen in Fig. 5.

Table 1 Values of $(1-3k)f''(0)$

M	$a/c \to 0.1$	0.2	0.5	1.0	1.1	1.2	1.5
0.0	−0.9414	−0.8919	−0.6492	0	0.1604	0.3300	0.8907
1.0	−1.2802	−1.1786	−0.8081	0	0.1872	0.3823	1.0132
2.0	−1.9711	−1.7789	−1.1608	0	0.2508	0.5077	1.3138
3.0	−2.7539	−2.4676	−1.5790	0	0.3302	0.6651	1.6980

Table 2 Values of $-f''(0)$

k	$-f''(0)$	
	Bhattacharyya et al.[7]	Present results
0.001	1.0005	0.9964
0.005	1.0025	0.9984
0.01	1.0050	1.0009
0.03	1.0153	1.0108
0.05	1.0260	1.0208
0.07	1.0370	1.0307
0.09	1.0483	1.0406
0.17	1.0976	1.0803
0.25	1.1547	1.1200
0.33	1.2217	1.1598

The shear stress at the stretching surface $y=0$ is given by (2) and (8) as

$$\left(\tau_{xy}\right)_{y=0} = \rho c (c\nu)^{\frac{1}{2}} x \left[1-3k\right] f''(0). \qquad (19)$$

The dimensionless form of the wall shear stress, i.e., skin-friction coefficient $C_f \left(= \left(\tau_{xy}\right)_{y=0} / \rho c (c\nu)^{\frac{1}{2}} x\right)$ is then obtained from (19) as

$$C_f = (1-3k) f''(0). \qquad (20)$$

Table 1 gives the values of the dimensionless wall shear stress $(1-3k)f''(0)$ for $k=0.01$ and several values of a/c and M.

It can be seen that for a fixed value of the elastic parameter k and a/c, the magnitude of the wall shear stress increases with increase in M. In fact it can be seen from Figs. 2 and 3 that increase in M results in increase in the magnitude of velocity gradient at the surface in both the cases $a/c > 1$ and $a/c < 1$.

In order to have numerical checks with previous results from the literature, we compare the results of the present study with the corresponding results of Bhattacharyya et al. [7] and Andersson [11]. Note that although the constitutive equation in Walters' B$'$ model for fluid of small memory in our investigation is slightly different from the constitutive equation for second-order fluid also based on small memory fluid, the boundary layer equations for both the fluids are identical. This can be seen from (5) in Bhattacharyya et al. and (4) in the present study in the case $M=0$. Similarly when $M \neq 0$, we get the same boundary layer equations in both the cases.

Table 2 gives the values of $-f''(0)$ for several values of the elastic parameter k when $a/c = 0.0$ and $M = 0.0$.

Table 3 shows the comparision in the values of $-f''(0)$ for different values of k and M in the present study with the corresponding values obtained by Andersson [11].

The small discrepancy in the values of $-f''(0)$ for different values of k and M as shown in Tables 2 and 3 is explained as follows. While the values of $-f''(0)$ as computed by Bhattacharyya et al. [7] in the case $a/c = 0$ and $M = 0$ and Andersson [11] in the case $a/c = 0$ and $M \neq 0$ are based on the exact analytical solution for $f(\eta)$ found by them, the corresponding values of $-f''(0)$ in the present study are based on the approximate solution for $f(\eta)$ as given by (13), which is correct to $O(k)$. This explains why the discrepancy increases with increase in k.

Table 3 Values of $-f''(0)$

k	M	$-f''(0)$	
		Andersson [11]	Present results
0.001	0.0	1.0005	0.9964
	1.0	1.4149	1.4058
	2.0	2.2372	2.2132
	3.0	3.1639	3.1151
0.01	0.0	1.0050	1.0009
	1.0	1.4213	1.4121
	2.0	2.2473	2.2231
	3.0	3.1782	3.1289
0.05	0.0	1.0260	1.0208
	1.0	1.4509	1.4401
	2.0	2.2942	2.2670
	3.0	3.2444	3.1905
0.07	0.0	1.0369	1.0307
	1.0	1.4665	1.4541
	2.0	2.3187	2.2889
	3.0	3.2791	3.2214
0.09	0.0	1.0483	1.0406
	1.0	1.4825	1.4681
	2.0	2.3440	2.3109
	3.0	3.3150	3.2522
0.17	0.0	1.0976	1.0803
	1.0	1.5523	1.5240
	2.0	2.4544	2.3987
	3.0	3.4710	3.3754
0.25	0.0	1.1547	1.1200
	1.0	1.6330	1.5800
	2.0	2.5820	2.4866
	3.0	3.6515	3.4987
0.33	0.0	1.2217	1.1598
	1.0	1.7277	1.6359
	2.0	2.7318	2.5744
	3.0	3.8633	3.6219

3 Heat transfer

Let us now consider the heat transfer equation in the flow of a viscoelastic electrically conducting fluid. To this end it is necessary to establish the energy balance for a fluid element in motion and to consider it in conjunction with the equation of motion. It is to be noted that during the motion of a viscoelastic fluid, a certain amount of energy is stored up in the fluid as strain energy and some energy is lost due to viscous and Ohmic dissipation. Thus for such a fluid, the energy balance is determined by the internal energy, the conduction of heat, the convection of heat with the flow, the generation of heat through viscous and Ohmic dissipation and the strain (or deformation) energy stored in the fluid due to its elastic properties. Using boundary layer approximations, the transfer of heat in the steady two-dimensional flow of a viscoelastic fluid can be expressed in the form of the energy equation given by

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\lambda\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{k_0}{\rho c_p}\frac{\partial u}{\partial y}\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+\frac{\sigma B_0^2}{\rho c_p}(u-ax)^2, \tag{21}$$

where T, λ and c_p denote the temperature, the thermal diffusivity and the specific heat of the fluid, respectively. Note that the second and the third terms on the right hand side of (20) represent the terms due to viscous dissipation and strain energy, respectively, while the last term represents Ohmic dissipation. The boundary conditions are

$$T = T_w \quad \text{at} \quad y = 0, \quad T \to T_\infty \quad \text{as} \quad y \to \infty, \tag{22}$$

where T_w and T_∞ are constants with $T_w > T_\infty$. Introducing the dimensionless temperature θ as

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{23}$$

and using (8), we get from (21)

$$xf' \frac{\partial \theta}{\partial x} - f \frac{\partial \theta}{\partial \eta} = \frac{\lambda}{\nu} \frac{\partial^2 \theta}{\partial \eta^2} + \frac{c^2 x^2}{c_p (T_w - T_\infty)} f''^2 - \frac{k_0 c^3 x^2}{\mu c_p (T_w - T_\infty)} (f' f''^2 - f f'' f''') + \frac{\sigma B_0^2 c x^2}{\rho c_p (T_w - T_\infty)} \left(f' - \frac{a}{c}\right)^2. \tag{24}$$

Setting

$$\theta(x, \eta) = \theta_0(\eta) + \frac{c x^2}{\nu} \theta_1(\eta) \tag{25}$$

in (24) and equating the coefficients of x^0 and x^2, we obtain the following equations for $\theta_0(\eta)$ and $\theta_1(\eta)$

$$\theta_0'' + Prf\theta_0' = 0, \tag{26}$$

$$\theta_1'' + Prf\theta_1' - 2Prf'\theta_1 = PrEk(f'f''^2 - ff''f''') - PrEf''^2 - M^2 PrE \left(f' - \frac{a}{c}\right)^2, \tag{27}$$

where Pr and E are the Prandtl number and Eckert number defined by

$$Pr = \frac{\nu}{\lambda}, \quad E = \frac{\nu c}{c_p (T_w - T_\infty)}. \tag{28}$$

The boundary conditions for $\theta_0(\eta)$ and $\theta_1(\eta)$ are derived from (22), (23), and (25) as

$$\theta_0(0) = 1, \quad \theta_0(\infty) = 0, \tag{29}$$

$$\theta_1(0) = 0, \quad \theta_1(\infty) = 0. \tag{30}$$

It is clear from above that the temperature distribution depends on five dimensionless parameters: (a) the viscoelastic parameter k, (b) the Prandtl number Pr, (c) the Eckert number E, (which characterizes viscous dissipation in the flow) , (d) the parameter a/c and (e) Hartmann number M. Equations (26) and (27) subject to the boundary conditions (29) and (30) have been solved numerically by finite difference method using Thomas algorithm [14].

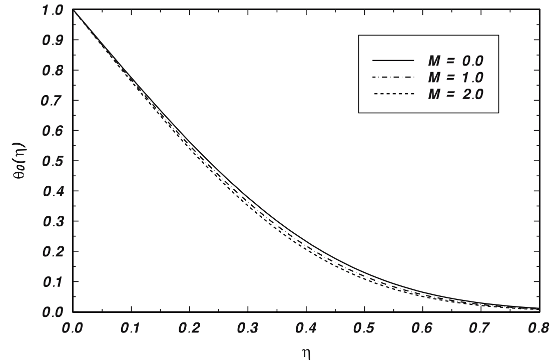

Fig. 6 Variation of $\theta_0(\eta)$ with η for several values of M with $k = 0.001$, $a/c = 3.0$, and $Pr = 5.0$

In the absence of any viscous and Ohmic dissipation and stored deformation energy in the flow ($E = 0$), the temperature distribution is given by $\theta_0(\eta)$ whose variation with η is shown in Fig. 6 for several values of M with $Pr = 5.0$, $a/c = 3.0$, and $k = 0.001$. It can be seen that for given values of $Pr, a/c$, and k, temperature at a point decreases with increase in the Hartmann number when $a/c > 1.0$. But when $a/c < 1.0$ it is evident from Fig. 7 that $\theta_0(\eta)$ increases with increase in Hartmann number for fixed values of k, Pr, and a/c.

Figure 8 shows the variation of $\theta_1(\eta)$ with η for several values of M with $k = 0.001$, $Pr = 5.0$, $E = 2.0$, and $a/c = 3.0$. It is seen that in the vicinity

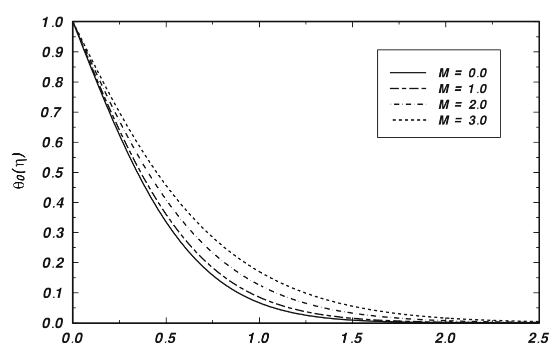

Fig. 7 Variation of $\theta_0(\eta)$ with η for several values of M with $k = 0.001$, $a/c = 0.1$, and $Pr = 5.0$

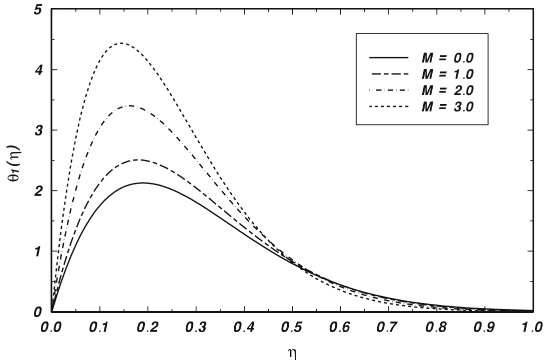

Fig. 8 Variation of $\theta_1(\eta)$ with η for several values of M with $k = 0.001$, $a/c = 3.0$, $E = 2.0$, and $Pr = 5.0$

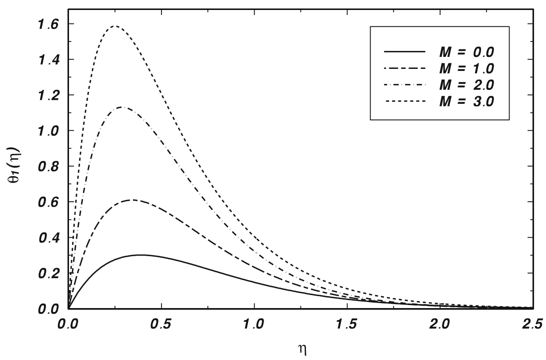

Fig. 9 Variation of $\theta_1(\eta)$ with η for several values of M with $k = 0.001$, $a/c = 0.1$, $E = 2.0$, and $Pr = 5.0$

of the stretching surface $\theta_1(\eta)$ increases with the increase in M but after a certain distance from the stretching sheet the opposite trend is observed in the variation. The corresponding variation of $\theta_1(\eta)$ with η for the same values of M, Pr, E, and k is displayed in Fig. 9 when $a/c = 0.1$. It can be clearly seen that at a given point, $\theta_1(\eta)$ increases with increase in M.

From (25), the dimensionless rate of heat transfer at the surface $\eta = 0$ given by $-\theta'(X,0)$ is evaluated as

$$-\theta'(X,0) = -\theta_0'(0) - X^2\theta_1'(0), \qquad (31)$$

where

$$X = (c/\nu)^{\frac{1}{2}}x. \qquad (32)$$

Table 4 gives the values of $\theta_0'(0)$ for several values of M and a/c with $Pr = 5.0$ and $k = 0.01$.

It can be seen that $-\theta_0'(0)$ is positive and this is consistent with the fact that in the absence of viscous and Ohmic dissipation as well as strain energy in the flow ($E = 0$), heat flows from the surface to the fluid as long as $T_w > T_\infty$. It is also seen that for a fixed value of $a/c(< 1)$, the surface heat flux $-\theta_0'(0)$ decreases with increase in M. On the other hand, when $a/c > 1$, the surface heat flux $-\theta_0'(0)$ increases with increase in M. Interestingly enough, there is no variation of the surface heat flux $-\theta_0'(0)$ with change in M for $a/c = 1$. This stems from the fact that when $a/c = 1$, the flow near the stretching surface is an inviscid flow (see (18)) unaffected by the Hartmann number of the fluid. It can further be observed from Table 4 that for a fixed value of M, the surface heat flux $-\theta_0'(0)$ increases with increase in a/c.

Table 6 shows the comparision in the values of the variation $\theta_1(\eta)$ with η for different values of k when $Pr = 10.0$, $E = 4.0$, $a/c = 0.0$, and $M = 0.0$ in the present study with the corresponding values obtained by Bhattacharyya et al.[7].

Values of $\theta_1'(0)$ are computed from the numerical solution of (27) for various values of Pr, E, k, a/c, and M and these values are all found to be positive. Then the values of the dimensionless heat flux at the surface $-\theta'(X,0)$ are computed from (31) and shown in Tables 7 and 8 corresponding to two distinct locations ($X = 0.1$ and $X = 2.5$) on the stretching surface.

Table 5 gives the comparision in the values of $-\theta_0'(0)$ for several values of k with $Pr = 10.0$ when $a/c = 0.0$ and $M = 0.0$ with the corresponding values computed by Bhattacharyya et al. [7]. The

Table 4 Values of $\theta_0'(0)$ with $Pr = 5.0$ and $k = 0.01$

M	$a/c \to 0.1$	0.2	0.5	1.0	1.1	1.2	1.5
0.0	−1.5813	−1.5965	−1.6587	−1.7841	−1.8101	−1.8362	−1.9143
1.0	−1.5124	−1.5431	−1.6353	−1.7841	−1.8129	−1.8413	−1.9248
2.0	−1.3819	−1.4425	−1.5900	−1.7841	−1.8186	−1.8521	−1.9474
3.0	−1.2566	−1.3474	−1.5467	−1.7841	−1.8244	−1.8631	−1.9712

Table 5 Values of $-\theta_0'(0)$

k	$-\theta_0'(0)$	
	Bhattacharyya et al.[7]	Present results
0.09	2.2975	2.3043
0.17	2.2867	2.2956
0.25	2.2742	2.2868
0.33	2.2597	2.2779

Table 6 Values of $\theta_1(\eta)$

k	η	$\theta_1(\eta)$	
		Bhattacharyya et al.[7]	Present results
0.001	0.2		0.7760
	0.4		0.8368
	1.0		0.3326
	1.2		0.2248
0.01	0.2		0.7743
	0.4		0.8342
	1.0		0.3303
	1.2		0.2229
0.05	0.2		0.7657
	0.4		0.8218
	1.0		0.3198
	1.2		0.2142
0.09	0.2	0.7567	0.7557
	0.4	0.8090	0.8080
	1.0	0.3097	0.3092
	1.2	0.2059	0.2055
0.17	0.2		0.7311
	0.4		0.7764
	1.0		0.2877
	1.2		0.1884
0.25	0.2	0.7195	0.7003
	0.4	0.7535	0.7392
	1.0	0.2641	0.2657
	1.2	0.1691	0.1714
0.33	0.2	0.6974	0.6628
	0.4	0.7212	0.6961
	1.0	0.2397	0.2431
	1.2	0.1500	0.1545

argument is found to be fairly close. As explained earlier, the discrepancy increases with increase in k.

It can be seen from Table 7 that for a small value of $X (X = 0.1)$, heat flows from the surface to the fluid because $-\theta'(X, 0)$ is positive. On the other hand from Table 8 it is observed that for $X = 2.5$, heat flows from the fluid to the stretching surface because $-\theta'(X, 0)$ is negative for all values of M and a/c except $a/c = 1$. This interesting result admits of a physical interpretation. For small enough values of X, both viscous and Ohmic dissipation and strain energy in the flow are small (see (24)) and hence no significant heat is generated in the flow. Thus for small values of X, heat flows from the surface to the fluid since $T_w > T_\infty$. But for large values of X, sufficient heat is generated inside the boundary layer due to the combined influence of viscous and Ohnic dissipation and stored deformation energy. Under such circumstances temperature very near the surface may exceeds the surface temperature T_w and heat then flows from the fluid to the surface even when $T_w > T_\infty$. The reason why in the exceptional case $(a = c)$, $-\theta'(X, 0)$ is positive $(-\theta'(X, 0) = 1.7841)$ even for a large value of X is as follows. For $a = c$, the flow near the stretching surface is the inviscid stagnation-point flow which is unaffected by the elasticity of the fluid and also there is neither any viscous nor Ohmic dissipation of energy in the flow. It is further observed from Table 7 that when there is heat transfer from the surface to the fluid, the surface heat flux (for given values of k, Pr, and E) increases with increase in a/c. However the variation of surface heat flux (for given values of a/c, Pr, k and E) with the Hartmann number M is non-monotonic. It should be noted, however, that whereas the viscous and Ohmic dissipation terms in (21) are always positive leading to dissipation of energy in the form of heat, the last but one term representing the strain energy is not everywhere positive in the flow. Since heat flows away from the surface to the fluid for small values of X and it flows from the fluid to the surface for moderate or large values of X, it is conceivable that at a certain location $X = X_0$ on the stretching surface, heat flux vanishes. Table 9 gives the values of X_0 for various values of a/c and M when $Pr = 5.0$, $k = 0.01$ and $E = 4.0$.

The reason why $X_0 = \infty$ for $a/c = 1$ follows from the fact that when $a = c$, the flow near the surface is the inviscid stagnation point flow so that heat always flows from the surface to the fluid so long as $T_w > T_\infty$.

Table 7 Values of $\theta'(X,0)$ for $X = 0.1$ with $Pr = 5.0$, $E = 4.0$, and $k = 0.01$

M	$a/c \to 0.1$	0.2	0.5	1.0	1.2	1.4	1.5
0.0	−1.5421	−1.5623	−1.6421	−1.7841	−1.8326	−1.8730	−1.8897
1.0	−1.4214	−1.4700	−1.6052	−1.7841	−1.8360	−1.8751	−1.8897
2.0	−1.1762	−1.2820	−1.5285	−1.7841	−1.8423	−1.8774	−1.8864
3.0	−0.9137	−1.0816	−1.4467	−1.7841	−1.8478	−1.8756	−1.8765

Table 8 Values of $\theta'(X,0)$ for $X = 2.5$ with $Pr = 5.0$, $E = 4.0$, and $k = 0.01$

M	$a/c \to 0.1$	0.2	0.5	1.0	1.2	1.4	1.5
0.0	22.9547	19.7856	8.7109	−1.7841	0.4154	7.6979	13.4983
1.0	55.3574	44.1845	17.1852	−1.7841	1.5208	11.9364	19.9879
2.0	127.1635	98.9095	36.8303	−1.7841	4.2305	22.4456	36.1589
3.0	213.0439	164.7321	60.9626	−1.7841	7.7075	36.0584	57.2038

Table 9 Values of X_0 for $Pr = 5.0$, $E = 4.0$, and $k = 0.01$

M	$a/c \to 0.1$	0.2	0.5	1.0	1.1	1.2	1.5
0.0	0.6347	0.6831	0.9999	∞	4.5604	2.2576	0.8811
1.0	0.4077	0.4593	0.7369	∞	3.6986	1.8501	0.7409
2.0	0.2592	0.2997	0.5086	∞	2.7342	1.3795	0.5652
3.0	0.1914	0.2252	0.3933	∞	2.1783	1.1030	0.4563

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