Meccanica (2006) 41: 299–310
DOI 10.1007/s11012-005-5899-z

Chaos in Acoustic Subspace Raised by the Sommerfeld–Kononenko Effect

T.S. KRASNOPOLSKAYA*
Institute of Hydromechanics National Academy of Sciences of Ukraine, 03680 Kiev, Ukraine

(Accepted: 9 September 2005)

Abstract. This paper deals with vibrations of an infinite plate in contact with an acoustic medium where the plate is subjected to a point excitation by an electric motor of limited power-supply. The whole system is divided into two: "exciter - foundation" and "foundation-plate-medium". In the system "motor-foundation" three classes of steady state regimes are determined: stationary, periodic and chaotic. The vibrations of the plate and the pressure in the acoustic fluid are described for each of these regimes of excitation. For the first class they are periodic functions of time, for the second they are modulated periodic functions, in general with an infinite number of carrying frequencies, the difference between which is constant. For the last class they correspond to chaotic functions. In another mathematical model where the exciter stands directly on an infinite plate (without foundation) it was shown that chaos might occur in the system due to the feedback influence of waves in the infinite hydro-elastic subsystem in the regime of motor shaft rotation. In this case the process of rotation can be approximately described as a solution of the fourth order nonlinear differential equation and may have the same three classes of steady state regimes as the first model. That is the electric motor may generate periodic acoustic waves, modulated waves with an infinite number of frequencies or chaotic acoustic waves in a fluid.

Key words: Acoustic Medium, Vibrations of an Infinite Plate, The Sommerfeld–Kononenko Effect.

1. Introduction

The coupling effect between an excitation machine and vibrational loads, found by Sommerfeld [14, 15], is a universal phenomenon and a manifestation of the law of conservation of energy. It always exists to a certain degree. The effect of the load is especially significant when the output power of the machine is comparable with the power consumed by load. This is the case of the so-called "limited power-supply" machine, where the load is under "limited" (nonideal) excitation. At first equations of motion with explanation the phenomena observed in Sommerfeld's experiments were obtained by Blekhman [2]. However, a rather complete study of the Sommerfeld effect has been given in the works of Kononenko [5], so that we call these phenomena as Sommerfeld–Kononenko effect [11].

In the stationary case the various interaction effects are caused by the consumption of energy in the internal damping of the vibrational system. For the particular

*Author for Correspondence: kras@ihm.kiev.ua

case where chaos occurs in the interaction between the exciting machine and the vibrating systems this effect was further considered in papers [7–9].

A different principle of interaction between the loads and the machine also applies. All machines are sources of noise, and the radiation of acoustic energy is an undesirable factor which should be controlled. Energy dissipation by radiation of sound and elastic waves into surrounding objects is significant in the dynamics of machines. The characteristic features of the limited excitation of hydro-elastic systems, where a significant fraction of the consumed energy is transported by waves, were studied in the works [4, 6, 10, 11]. In these studies the vibrations of an infinite plate in contact with an acoustic medium (a fluid) were considered where the plate was subjected to a point excitation, and one along a line using an electric motor of limited power. With the condition of a limited power-supply the dynamics of the machine will be described by an incomplete system of equations, which can be completed only after adding equations representing the behaviour of the load. In accordance with the language of modern science (which the discovery of chaos in regular dynamic systems has undoubtedly enriched), in this case parameters of machine functioning not only quantatively feel the effect of loading, but will be modified according to completely different laws. Before the existence of determinated chaos was discovered the reduction principle had been used in the analysis of complicated system by dividing it into parts and analysing each subsystem separately. The existence of chaotic regimes led to the conclusion that a complete, complex system may have a complicated behaviour pattern only because of the interaction between several components. For example, there is chaos in two-degree-of-freedom system describing a fluid-elastic vibrations of a constrained pipe conveying fluid [12].

In the present study the appearance of chaos in the process of interaction between an electric motor of limited power-supply and vibrations of an elastic infinite plate in contact with an acoustic medium is considered. Two different mathematical models of the considered physical system will be used. In the first the plate is subjected to a point excitation by a motor through an elastic foundation, in the second model the plate is excited directly without the motor foundation.

2. Mathematical Models of the "Exciter-Foundation-Infinite Hydro-elastic Subsystem"

The practical conditions in which a machine functions are usually such that it is mounted on a foundation and generates noise to the surroundings, so that it interacts with several different subsystems. These one modelled in the following way. As an exciter an electric motor (electromotor) with limited power-supply is [5, 14] chosen, which stands on a foundation. The foundation is modelled by a one-degree-of-freedom elastic system, for example, by a nonlinear spring (see Figure 1). It stands on a hydro-elastic infinite system, which includes an infinite elastic plate and an acoustic half-space. We can consider an elastic plate of thickness h, density ρ_0, and a mean surface coinciding with the plane $x = 0$. We assume that the half-space $x < 0$ is occupied by a fluid of density ρ where the velocity of sound is c (Figure 1). We further assume that the foundation, the spring with rigidity $c_0 - \gamma u^2$ (where u is the deformation of the spring), is placed at the origin O of a cylindrical coordinate system r, φ, x. The motor (static mass M) with an unbalanced mass m at a distance a from the shaft axis stands on the foundation. When the shaft rotates, the vertical component of

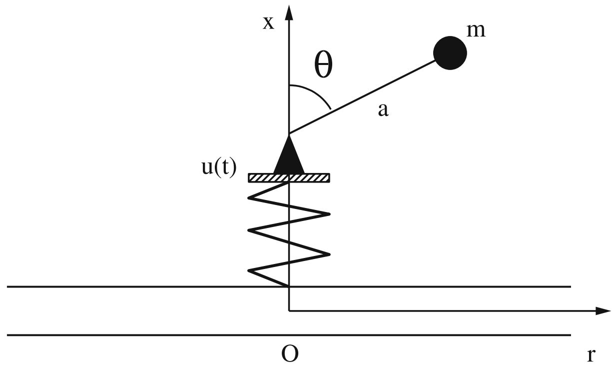

Figure 1. Scheme of the system.

the inertial force of the mass m is $ma\mathrm{d}^2/\mathrm{d}t^2(1-\cos\Theta)$, where Θ is the rotational displacement angle of the shaft, measured with respect to upward vertical. The angular velocity of the shaft $\dot{\Theta}$ is not a fixed value but an additional unknown in the considered problem where the excitation is by an electromotor with limited power-supply. The shaft rotation is affected by the feedback influence of the foundation vibrations and the waves in the hydroelastic system. Therefore it is necessary to include an additional equation for the process of shaft rotation in the mathematical model of the foundation and the plate vibrations with "limited" excitation (or under the influence of the Sommerfeld–Kononenko effect).

We further consider the elastic vibration $u(t)$ of the foundation and bending vibration $w(r,t)$ of the plate. The equations describing the behaviour of the whole system can be written as

$$\begin{aligned} &M\left(\ddot{u}+\frac{\partial^2 w(0,t)}{\partial t^2}\right)+\kappa_0\dot{u}+c_0u-\gamma u^3=ma\frac{\mathrm{d}^2}{\mathrm{d}t^2}(1-\cos\Theta);\\ &I\ddot{\Theta}=L(\dot{\Theta})-H(\dot{\Theta})+ma\sin\Theta\left[g+\ddot{u}+\frac{\partial^2 w(0,t)}{\partial t^2}\right];\\ &D\Delta^2 w(r,t)+\rho_0 h\frac{\partial^2 w(r,t)}{\partial t^2}=(c_0u-\gamma u^3)\frac{\delta(r)}{2\pi r}+p(r,0,t); \end{aligned} \tag{1}$$

where κ_0 is the damping coefficient; I is the moment of inertia of the rotor shaft; $L(\dot{\Theta})$ is the driving torque; $H(\dot{\Theta})$ is the moment of resistive forces of electromotor [5, 14]; g is the acceleration due to gravity; D is the bending rigidity of the plate; $\Delta=\partial^2/\partial r^2+1/r\,\partial/\partial r$; $\delta(r)$ is the Dirac function; $p(r,x,t)$ is the acoustic pressure of the fluid, which satisfies a wave equation of the form

$$\Delta p(r,x,t)+\frac{\partial^2 p}{\partial x^2}=\frac{\partial^2 p}{c^2\partial t^2}. \tag{2}$$

To simplify, we can consider the case of resonant vibrations of the foundation and assume that $u(t)>w(r,t)$. So the system described in (1) can be divided into two. The first is given by

$$\begin{aligned} &M\ddot{u}+\kappa_0\dot{u}+c_0u-\gamma u^3=ma\frac{\mathrm{d}^2}{\mathrm{d}t^2}(1-\cos\Theta),\\ &I\ddot{\Theta}=L(\dot{\Theta})-H(\dot{\Theta})+ma\sin\Theta[g+\ddot{u}]; \end{aligned} \tag{3}$$

and the second, linked to the first, by

$$D\Delta^2 w+\rho_0 h\frac{\partial^2 w}{\partial t^2}=(c_0u-\gamma u^3)\frac{\delta(r)}{2\pi r}+p(r,0,t);\qquad \Delta p+\frac{\partial^2 p}{\partial x^2}=\frac{\partial^2 p}{c^2\partial t^2}. \tag{4}$$

We can consider the steady state regimes of the systems given in (3) and (4). System (3) is nonlinear and describes a complicated process of energy redistribution from the electromotor into the vibrating foundation. It is necessary to point out that the interaction between the vibration $u(t)$ and the shaft rotation with respect to the angle $\Theta(t)$ is itself nonlinear since a connection with the periodic influence of the electromotor is always a nonlinear function of the the rotational angle Θ.

In order to obtain the solution of the system (3), we introduce a small positive parameter

$$\epsilon=\frac{m}{M}. \tag{5}$$

The rotational velocity of the shaft $\dot{\Theta}$ is regarded as being close to the natural frequency of the foundation $\omega_0=(\frac{c_0}{M})^{\frac{1}{2}}$, and we may then write

$$\dot{\Theta}=\omega_0+\frac{1}{2}\epsilon^{\frac{2}{3}}\omega_0\nu(\tau); \tag{6}$$

where $\tau=\frac{1}{2}\epsilon^{\frac{2}{3}}\Theta(t)$-slow time. We propose a solution for u in the form

$$u(t)=\epsilon^{\frac{1}{3}}a[\alpha(\tau)\cos\Theta+\beta(\tau)\sin\Theta]. \tag{7}$$

For the new coordinates $\alpha(\tau)$ and $\beta(\tau)$ after the procedure of averaging in fast time $\Theta(t)$ we obtain the equations

$$\begin{aligned}\frac{d\alpha}{d\tau}&=-\eta\alpha-\nu\beta-\gamma_1(\alpha^2\beta+\beta^3);\\ \frac{d\beta}{d\tau}&=-\eta\beta+\nu\alpha+\gamma_1(\alpha^3+\alpha\beta^2)+1;\end{aligned} \tag{8}$$

where $\eta=\frac{\kappa_0}{M\epsilon^{\frac{2}{3}}\omega_0}$; $\gamma_1=\frac{3\gamma}{4M\omega_0^2}$.

In the following section the steady regimes of interaction will be analysed. For this reason the static characteristic of an electromotor [5] as the torque $L(\dot{\Theta})$ will be used, and accordingly , we assume $(L-H)I^{-1}=2\epsilon^{\frac{2}{3}}\omega_0^{-1}M_1(\dot{\Theta})$. The substitution $\dot{\Theta}=\Omega(\tau)$ is introduced, and then on the basis of (3) and the procedure of averaging in the fast time Θ we can write an equation for Ω

$$\frac{\mathrm{d}\Omega}{\mathrm{d}\tau}=M_1(\Omega)-\mu\beta; \tag{9}$$

where $\mu=m^{\frac{1}{3}}M^{\frac{2}{3}}a^2\omega_0I^{-1}$. Approximating the static characteristic of the electromotor by a linear function $M_1(\Omega)=N_0-N_1\Omega$ (N_0, N_1-constants) then, the following equation is valid for the frequency offset ν

$$\frac{\mathrm{d}\nu}{\mathrm{d}\tau}=N_2-N_1\nu-\mu_1\beta. \tag{10}$$

Here $N_2 = 2M^{\frac{2}{3}}m^{-\frac{2}{3}}(N_0\omega_0^{-1} - N_1)$; $\mu_1 = 2M^{\frac{4}{3}}m^{-\frac{1}{3}}a^2I^{-1}$.

The complex interaction process between the foundation vibrations and the shaft rotation is thus described by three nonlinear coupled equations (8) and (10). If the foundation itself is a linear system ($\gamma_1 = 0$) the process of the interaction will also be a nonlinear one as well as the system of equations (8) and (10).

We should emphasize that with an ideal (unlimited) excitation of the foundation vibrations, the system of two averaged equations (8) will become a two parameter one having two constant parameters η and ν, and no chaotic regimes. However, with the problem considered here, where ν is an additional unknown variable, the system of equations (8) and (10) may have chaotic regimes due to the nonlinear interaction.

3. Steady State Regimes of the Interaction

Here the steady solutions of system of equations (8) and (10), which may represent equilibrium states, periodic and also chaotic solutions are analyzed ([7, 8]). In the three-dimensional phase-space (α, β, ν) these solutions correspond to such asymptotic trajectories as the point, the limit cycle and chaotic attractor, respectively. Asymptotic trajectories of the system (8) and (10) may be onstructed using numerical methods. In this paper the fourth order Runge–Kutta method is used. The system of equations (8) and (10) contains five parameters $(\eta, \gamma_1, N_2, N_1, \mu_1)$ which, together with the initial conditions, determine its behaviour in a steady regime. We assume

$$\eta = 0.1; \quad \gamma_1 = 0.125; \quad N_2 = 0.04; \quad \mu_1 = 0.5; \quad \alpha(0) = \beta(0) = \nu(0) = 0. \tag{11}$$

Parameter N_1 was varied with the purpose of determining all the possible classes of asymptotic trajectories. In Figure 2 the dependence on parameter N_1 is shown of the maximal nonzero Lyapunov exponent λ determined by the algorithm of Benettin [1]. When the largest of the Lyapunov exponent is greater than zero, then chaos is observed in the system.

Evidently, there are two such regions of chaotic motion at $0.489 \leqslant N_1 \leqslant 0.506$ and $0.631 \leqslant N_1 \leqslant 0.671$.

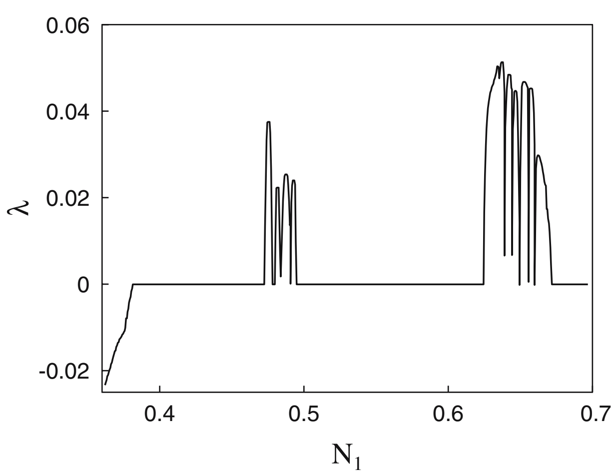

Figure 2. Dependence on N_1 of the maximal Lyapunov exponent λ.

When $0 < N_1 < 0.382$ the system has a stable equilibrium position, which corresponds to the first type of the steady state regime, namely the stationary regime.

Periodic solutions are realized in the intervals $0.382 \leqslant N_1 < 0.489, 0.506 < N_1 < 0.631$ and $0.671 < N_1 < 3.0$.

When $N_1 = 0.70$ the coordinates of the equations system (8) and (10) are periodic functions that correspond to a two-turn cycle in phase-space (Figure 3(a)). Results for the dimensionless power of the motor $P_1 = N_2 - N_1\nu$, the power consumed by the foundation damping force $P_2 = \eta\mu_1(\alpha^2 + \beta^2)$, the total power $P = P_1 + P_2$ are shown in Figure 3(b). For the case considered the powers show typical periodic behaviour.

The chaotic trajectory for $N_1 = 0.64$ is shown in Figure 4(a). Power curves for this case are shown in Figure 4(b). The total power also oscillates around zero (as in the periodic regimes), but no constant period exists in slow time, for which the average power will be zero.

Summarizing, in the system "electromotor-foundation" three classes of steady state regimes are determined. The first class (I) consists of the stationary regimes, when vibrations of the foundation occur with constant amplitude and frequency and the electromotor shaft rotates with a constant speed. The second class (II) contains

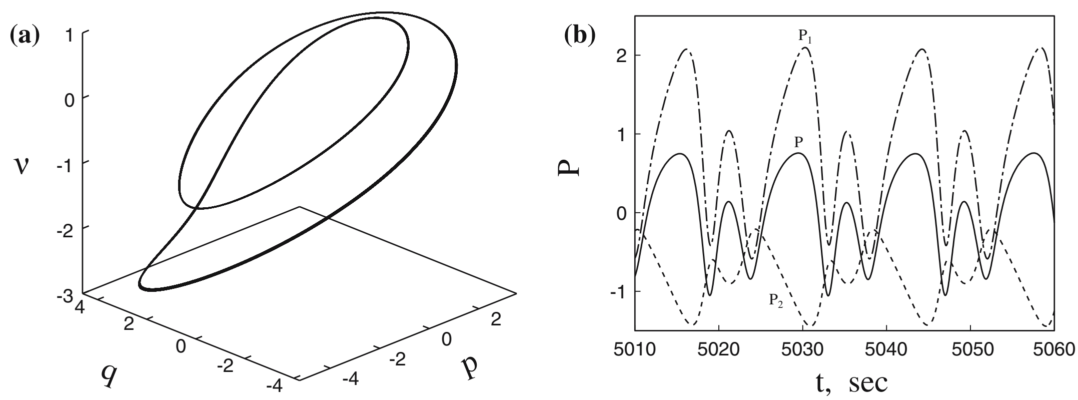

Figure 3. Graphs of (a) the trajectory and (b) the powers P_1, P_2 and the total power P at $N_1 = 0.70$ for the periodic regime.

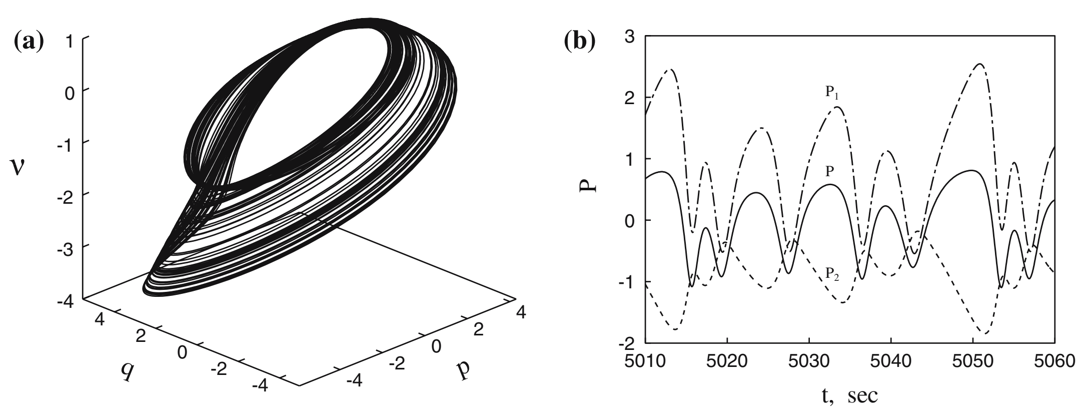

Figure 4. Phase portrait of (a) the chaotic attractor and (b) graphs of the powers P_1, P_2 and the total rower P for $N_1 = 0.64$.

regimes with periodically changing amplitude and frequency of foundation vibrations and the shaft speed periodically changes with time. Finally, the third class (III) corresponds to chaotic regimes when amplitude and frequency of vibrations and the electromotor speed change in time chaotically. The last regime is asymptotically established in the system. The system cannot leave this regime without assistance or be approximated by regimes of the first two classes.

4. Regular and Chaotic Waves in the Infinite Hydro-Elastic System

Now we can consider all possible types of the solution to the system (4). The boundary condition is that the normal component of the velocity must be continuous in passing from the plate (where it is $\partial w(r,t)/\partial t$) to the fluid (where it is v_x). Therefore we have $\partial w/\partial t = v_x$ (when $x=0$). Using the relation $\partial p(r,x,t)/\partial x = -\rho\partial v_x/\partial t$, which is satisfied in the acoustic field, the boundary condition becomes

$$-\rho\frac{\partial^2 w(r,t)}{\partial t^2} = \frac{\partial p(r,x,t)}{\partial x}; \quad x=0. \tag{12}$$

Using the Laplace transform with respect to time and the Hankel transform with respect to the radial coordinate, general expressions for the bending deflection and pressure can be represented in the form [3, 4]:

$$\begin{aligned} w(r,t) &= \frac{1}{2\pi\mathrm{i}}\int_{\sigma-\mathrm{i}\infty}^{\sigma+\mathrm{i}\infty}\int_0^\infty \zeta_1(\lambda,s)J_0(\lambda r)\lambda \mathrm{e}^{st}\,\mathrm{d}\lambda\mathrm{d}s; \\ p(r,x,t) &= \frac{1}{2\pi\mathrm{i}}\int_{\sigma-\mathrm{i}\infty}^{\sigma+\mathrm{i}\infty}\int_0^\infty \zeta_2(\lambda,s)J_0(\lambda r)\mathrm{e}^{x\sqrt{\lambda^2+s^2c^{-2}}}\lambda \mathrm{e}^{st}\,\mathrm{d}\lambda\mathrm{d}s \end{aligned} \tag{13}$$

Substituting these expressions into the boundary condition (12) and into (4) leads to the relation $\zeta_1(\lambda,s)=\Phi(s)/\xi(\lambda,s)$ where $\xi(\lambda,s)=D\lambda^4+\rho_0 hs^2+\rho s^2/\sqrt{\lambda^2+s^2c^{-2}}$; $\Phi(s)$ is the Laplace transform of the function $(c_0u-\gamma u^3)$ and hence

$$\begin{aligned} \Phi(s) &= \int_0^\infty [c_0u(t)-\gamma u^3(t)]\mathrm{e}^{-st}\mathrm{d}t \\ &= \int_0^\infty [\epsilon^{1/3}ac_0(\alpha\cos\Theta+\beta\sin\Theta)-\epsilon a^3\gamma(\alpha\cos\Theta+\beta\sin\Theta)^3]\mathrm{e}^{-st}\mathrm{d}t \end{aligned} \tag{14}$$

The relationships (13) can then be written as

$$w(r,t) = \frac{1}{2\pi\mathrm{i}}\int_{\sigma-\mathrm{i}\infty}^{\sigma+\mathrm{i}\infty}\int_0^\infty \frac{\Phi(s)}{\xi(\lambda,s)}J_0(\lambda r)\lambda\mathrm{e}^{st}\,\mathrm{d}\lambda\,\mathrm{d}s;$$

$$p(r,x,t) = -\frac{1}{2\pi\mathrm{i}}\int_{\sigma-\mathrm{i}\infty}^{\sigma+\mathrm{i}\infty}\int_0^\infty \frac{\Phi(s)}{\xi(\lambda,s)}J_0(\lambda r)\lambda\mathrm{e}^{st}\frac{\mathrm{e}^{x\sqrt{\lambda^2+s^2c^{-2}}}}{\sqrt{\lambda^2+s^2c^{-2}}}\,\mathrm{d}\lambda\,\mathrm{d}s. \tag{15}$$

For the first class (I) of the steady state regimes, where $u(t)$ is the periodic function with constant amplitude (α_0,β_0) and frequency Ω_0, the vibrations of $w(r,t)$ have the form

$$
\begin{aligned}
w(r,t) = {} & \frac{c_1\alpha_0 - \frac{3}{4}\gamma_2\alpha_0^3}{2\pi}\Re\left[\mathrm{e}^{-\mathrm{i}\Omega_0 t}\int_0^\infty \frac{J_0(\lambda r)\lambda}{\xi(\lambda,\mathrm{i}\Omega_0)}\mathrm{d}\lambda\right] \\
& + \frac{c_1\beta_0 - \frac{3}{4}\gamma_2\beta_0^3}{2\pi}\Im\left[\mathrm{e}^{-\mathrm{i}\Omega_0 t}\int_0^\infty \frac{J_0(\lambda r)\lambda}{\xi(\lambda,\mathrm{i}\Omega_0)}\mathrm{d}\lambda\right] \\
& - \frac{\gamma_2\alpha_0^3}{8\pi}\Re\left[\mathrm{e}^{-3\mathrm{i}\Omega_0 t}\int_0^\infty \frac{J_0(\lambda r)\lambda}{\xi(\lambda,3\mathrm{i}\Omega_0)}\mathrm{d}\lambda\right] \\
& + \frac{\gamma_2\beta_0^3}{8\pi}\Im\left[\mathrm{e}^{-3\mathrm{i}\Omega_0 t}\int_0^\infty \frac{J_0(\lambda r)\lambda}{\xi(\lambda,3\mathrm{i}\Omega_0)}\mathrm{d}\lambda\right]; \qquad (16)
\end{aligned}
$$

where $c_1 = \epsilon^{1/3}ac_0$; $\gamma_2 = \epsilon a^3\gamma$; and $\Re$ is the real part and $\Im$ is the imaginary part of a function.

For the second class (II) of the steady state regimes $u(t)$ appears to be a modulated periodic function whose amplitude and frequency are also periodic functions. Vibrations of plate and pressure in an acoustic fluid will be described by regular but complicated functions of time, having infinite number of Fourier harmonics. We consider the following. Let a solution of the system of equations (8) and (10) have the most simple periodic form

$$
\begin{aligned}
&\alpha = \alpha_1\cos\omega\tau = \alpha_1\cos(\epsilon^{2/3}\omega_0\omega t/2); \\
&\beta = \beta_1\sin\omega\tau = \beta_1\sin(\epsilon^{2/3}\omega_0\omega t/2); \qquad (17) \\
&\nu = \nu_1\cos\omega\tau = \nu_1\cos(\epsilon^{2/3}\omega_0\omega t/2),
\end{aligned}
$$

where α_1, β_1, ν_1 are constants. Then

$$
\begin{aligned}
\frac{u(t)}{a} &= \epsilon^{1/3}\alpha_1\cos(\epsilon^{2/3}\omega_0\omega t/2)\cos[\omega_0 t + \frac{\nu_1}{\omega}\sin(\epsilon^{2/3}\omega_0\omega t/2) - \frac{\nu_1}{\omega}] \\
&\quad + \epsilon^{1/3}\beta_1\sin(\epsilon^{2/3}\omega_0\omega t/2)\sin[\omega_0 t + \frac{\nu_1}{\omega}\sin(\epsilon^{2/3}\omega_0\omega t/2) - \frac{\nu_1}{\omega}] \\
&= F_1(t)\cos\left(\frac{\nu_1}{\omega}\sin(\epsilon^{2/3}\omega_0\omega t/2)\right) + F_2(t)\sin\left(\frac{\nu_1}{\omega}\sin(\epsilon^{2/3}\omega_0\omega t/2)\right) \\
&= F_1(t)\left[J_0\left(\frac{\nu_1}{\omega}\right) + 2\sum_{k=1}^{\infty} J_{2k}\left(\frac{\nu_1}{\omega}\right)\cos(k\epsilon^{2/3}\omega_0\omega t)\right] \\
&\quad + 2\,F_2(t)\sum_{k=1}^{\infty} J_{2k-1}\left(\frac{\nu_1}{\omega}\right)\sin\left(k - \frac{1}{2}\right)\epsilon^{2/3}\omega_0\omega t,
\end{aligned}
$$

where F_1, F_2 are periodic functions with frequency Ω_1; and

$$
\Omega_k = \omega_0 \pm \frac{k}{2}\epsilon^{2/3}\omega_0\omega; \qquad k = 1, 2, 3, \ldots, \infty.
$$

The power spectrum and the time dependent function $u(t)/a$ for $\epsilon = 0.1$, $f_0 = \omega_0/(2\pi) = 9.05/(2\pi)$ Hz and $N_1 = 0.70$ (the modulated periodic regime) are shown in Figure 5. The maximal peak of the power spectrum corresponds to f_0. Peaks are situated equidistantly with a frequency difference $f_d = \epsilon^{2/3}\omega f_0/2$. In this case, the solution (15) for w and p will represent an infinite sum of items

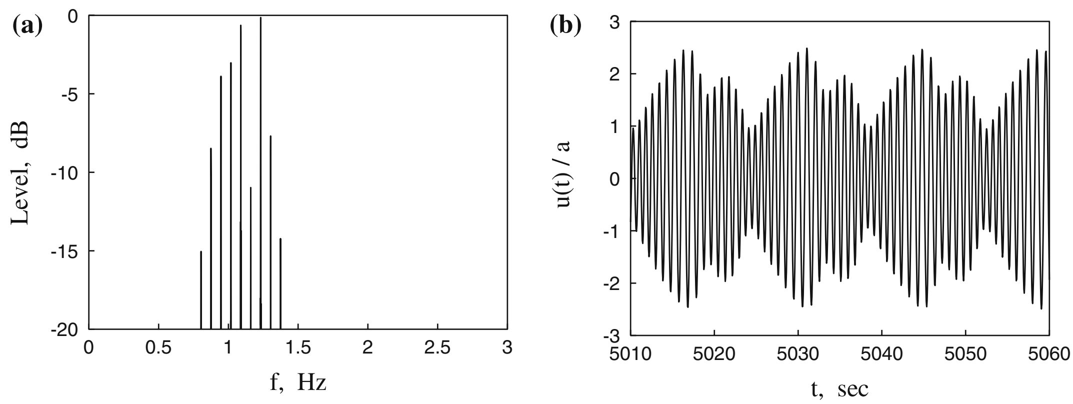

Figure 5. (a) Power spectrum and (b) the temporal realizations u/a for $N_1 = 0.70$ and $\epsilon = 0.1$.

$$p \ni e^{i\Omega_k t} \int_0^\infty - \frac{J_0(\lambda r)\lambda\rho\Omega_k^2}{\xi(\lambda, i\Omega_k)\sqrt{\lambda^2 - \frac{\Omega_k^2}{c^2}}} e^{x\sqrt{\lambda^2 - \frac{\Omega_k^2}{c^2}}} d\lambda; \tag{18}$$

$$w \ni e^{i\Omega_k t} \int_0^\infty \frac{J_0(\lambda r)\lambda}{\xi(\lambda, i\Omega_k)} d\lambda; \quad k = 1, 2, 3, \ldots, \infty \tag{19}$$

For every wave frequency Ω_k it is necessary to solve the problem of the wave formation that this frequency had excited in space. In order to achieve this the inverse transformations (19) and (18) must be made, taking into account the theory of singularities [4, 6]. In Figure 5 nine peaks are shown at the levels greater than -20 dB comparatively to the maximum. So for w and p in this interval we will have the sum of aproximately nine solutions of the stationary type.

Finally, for the last class (III) of steady state regimes, when chaotic vibrations of the foundation with the chaotic changing frequency are realized, the function $u(t)/a$ (7) has a continuous spectrum. The power spectrum and the temporal realization of $u(t)/a$ for the chaotic regime at $\epsilon = 0.1$, $f_0 = 9.05/2\pi$ Hz and $N_1 = 0.64$ are shown in Figure 6 (for initial conditions (11)) and in Figure 7 (for $\alpha(0) = 0.01$ and other parameters as in (11)). All the temporal realizations were analysed after a prolonged time interval in order for the transitional regimes to be complete. The differences in power spectrums (clearly seen if compare Figure 6(a) to Figure 7(a)) and temporal realizations prove that small changes (about ϵ^2) in the initial conditions can significantly change the trajectories and the power spectrum because of the change to the chaotic value of ν. The power spectrums of the functions of w and pressure $p(r, x, t)$ in the fluid during the chaotic regimes of exciter will also be continuous and more complicated.

5. Chaos in the System "Exciter-Infinite Hydro-elastic Subsystem"

Let us consider the possibility of chaotic waves in the system "exciter-infinite hydro-elastic subsystem". In this case we assume that the electromotor stands directly on

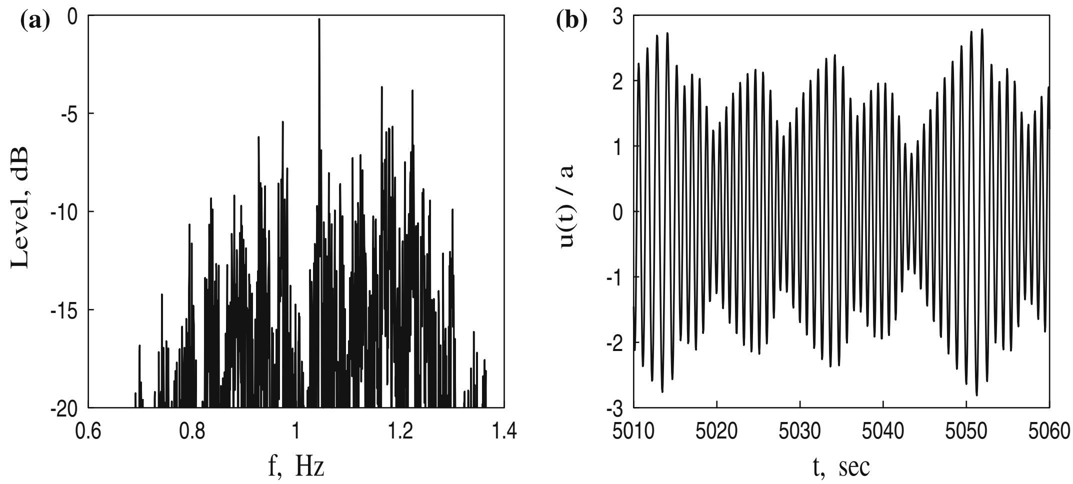

Figure 6. (a) Power spectrum and (b) the temporal realizations $u(t)/a$ for $N_1 = 0.64, \epsilon = 0.1$ and the initial conditions (11).

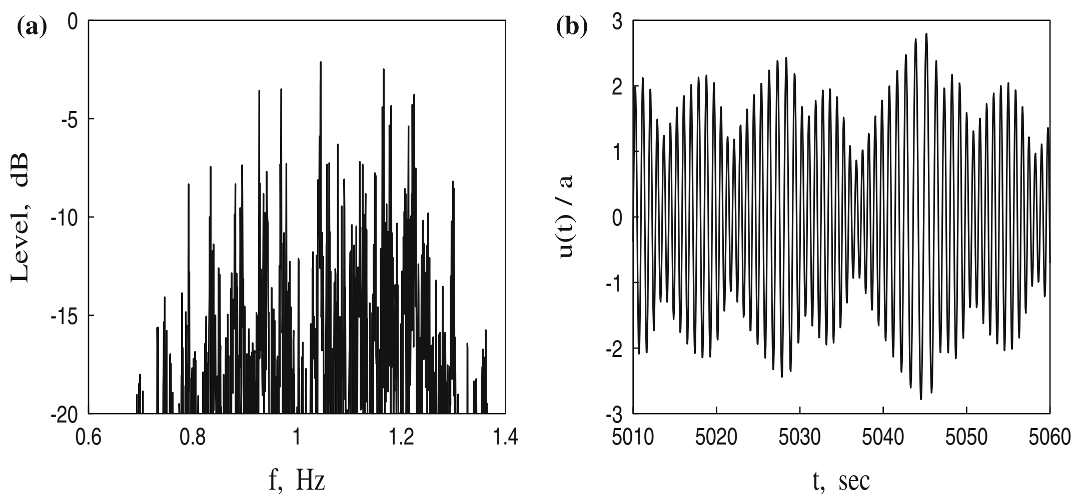

Figure 7. (a) Power spectrum and (b) the temporal realizations $u(t)/a$ for $N_1 = 0.64, \epsilon = 0.1$ and $\alpha(0) = 0.01, \beta(0) = 0$ and $\nu(0) = 0$.

the elastic infinite plate without a foundation. The equations describing the process of interaction have the form

$$I\ddot{\Theta} = L(\dot{\Theta}) - H(\dot{\Theta}) + ma \sin\Theta \left[g + \frac{\partial^2 w(0,t)}{\partial t^2} \right]; \tag{20}$$

$$D\Delta^2 w(r,t) + \rho_0 h \frac{\partial^2 w(r,t)}{\partial t^2} = (\dot{\Theta}^2 \cos\Theta + \ddot{\Theta} \sin\Theta) \frac{\delta(r)}{2\pi r} + p(r,0,t). \tag{21}$$

General expressions for the bending deflection and pressure variation can be written as (15), where the function $\Phi(s)$ is now

$$\Phi(s) = \frac{ma}{2\pi} \int_0^\infty [\dot{\Theta}^2 \cos\Theta + \ddot{\Theta} \sin\Theta] \mathrm{e}^{-st} \mathrm{d}t \tag{22}$$

The electromotor shaft rotation can be described by the equation

$$I\ddot{\Theta} = L(\dot{\Theta}) - H(\dot{\Theta}) + ma\sin\Theta\left[g + \frac{ma}{(2\pi)^2\mathrm{i}}\frac{\partial^2}{\partial t^2}\int_{\sigma-\mathrm{i}\infty}^{\sigma+\mathrm{i}\infty}\right.$$
$$\left.\int_0^\infty \frac{\lambda e^{st}}{\xi(\lambda, s)}\left(\int_0^\infty (\dot{\Theta}^2\cos\Theta + \ddot{\Theta}\sin\Theta)e^{-st}dt\right)\mathrm{d}\lambda\mathrm{d}s\right]; \tag{23}$$

This is a nonlinear integro-differential equation in the variable Θ. If the approximation of the Hankel transformation at the point $r = 0$ is used, this equation becomes a nonlinear fourth order equation with respect to time. This means that it may have its own chaotic regimes because, in general, chaos can appear in solutions to nonlinear equations of the third or higher orders [13]. It will also have two classes of regular solutions: stationary and periodic (or quasi-periodic). As in the first model considered here, these three classes of steady state regimes, in the process of the electromotor shaft rotation, generate a periodic force, a modulated periodic force with infinite number of carrying frequencies and a chaotic one. Each of these forces can generate a different class of waves in the infinite hydro-elastic system.

6. Conclusion

The interaction between an infinite hydro-elastic system and an exciter in a model where the exciter stands on a foundation and in a model where the exciter stands directly on the hydro-elastic system have been analysed.

The possibility of acoustic chaos in a regular system has been demonstrated. This chaos can arise from the “exciter-foundation” subsystem and then appear in the acoustic medium. The model “exciter-foundation” itself can be applied to describe some resonant vibrations of the finite plate (when using a low-dimensional model of the plate vibration) in an acoustic medium.

In the case where the exciter stands directly on the infinite plate, chaos might appear in the area of electromotor shaft rotation due to the feedback influence of the infinite hydro-elastic subsystem, leading to chaotic waves being generated in the hydro-elastic subsystem.

The results discussed in this paper should be applicable in a wide variety of situations. For example, they can be used to investigate the various types of energy radiation by an electromotor of limited power-supply as it interacts with finite elastic plates and constuctions in contact with an acoustic medium. Further development could include an extention of these results to fluid loading with mean flow.

References

1. Benettin, G., Galgani, L. and Strelcyn, J.M., ‘Kolmogorov entropy and numerical experiments’, *Phys. Rev. A* **14** (1976) 2338–2345.
2. Blekhman, I.I., ‘The self-synchronisation of thevibration of certain vibratory machines’. *Inzh, Sb.* **XVI** (1953) 49–72.
3. Crighton, D.G. and Innes, D., ‘Low frequency acoustic radiation and vibration response of locally excited fluid-loaded structures’, *J. Sound Vib.* **91** (1983) 293–314.

4. Frolov, K.V. and Krasnopol'skaya, T.S., 'Sommerfeld effect in system without internal damping', *Sov. Appl. Mech.* **23** (1987) 1122–1126.
5. Kononenko, V.O., *Vibrating System with a Limited Power-Supply*, Iliffe Books Ltd, London, 1969.
6. Krasnopol'skaya, T.S., 'Vibrations of an infinite plate, in contact with a fluid, when excited along a straight line by a motor of finite power', *Sov. Appl. Mech.* **24** (1988) 918–924.
7. Krasnopolskaya, T.S. and Shvets, A.Yu., 'Chaos in dynamics of machines with a limited power-supply', in: Okrolnick, M. and Pust, L. (eds.), *Eight World Congress on the Theory of Machines and Mechanisms*, Vol. 1, Czechoslovak Academy of Sciences, Prague 1991, pp. 181–184.
8. Krasnopolskaya, T.S. and Shvets, A.Yu., 'Chaos in vibrating systems with limited power-supply', *Chaos* **3** (1993) 387–395.
9. Krasnopolskaya, T.S. and Shvets, A.Yu., 'Chaotic surface waves in limited power-supply cylindrical tank vibrations', *J. Fluids Struct.* **8** (1994) 1–18.
10. Krasnopolskaya, T.S., 'Acoustic chaos caused by Sommerfeld effect', *J. Fluids Struct.* **8** (1994) 803–815.
11. Krasnopolskaya, T.S., 'Acoustic chaos caused by Sommerfeld–Kononenko effect'. *Acoustic Bull.* **5**(3) (2002) 35–45.
12. Païdoussis, M.P. and Moon, F.C., 'Nonlinear and chaotic fluidelastic vibrations of a flexible pipe conveying fluid,' *J. Fluids Struct.* **2** (1988) 567–591.
13. Moon, F.C., *Chaotic Vibrations*, Wiley, New York, 1987.
14. Sommerfeld, A., 'Beitrage zum dynamischen ausbau der festigkeislehre', *Zeitschrift des Vereins Deutscher Ingenieure* **46** (1902) 391–394.
15. Timoshenko, S., *Vibration Problems in Engineering*, Van Nostrand Co, NY, 1928.