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Energy Efficiency in a Base Station of 5G Cellular Networks using *M/G/1* Queue with Multiple Sleeps and *N*-Policy

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Abstract

Reducing energy consumption is the vital goal of green communication. Base station (BS) is a radio receiver/transmitter that serves as the hub of the local wireless network. It is a gateway between a wired network and the wireless network. BS consumes high energy to receive and transfer the signals. Power consumption in base station can be minimized by using effective sleep and wake-up/setup operations with a tolerable delay. In this research work, the service process of the BS is considered as an *M/G/1* queue with close down, sleep and setup. The strategy *N*-Policy is introduced to awake the BS from multiple sleeps (MS) after a predefined number *N* of user requests (URs) accumulated in the system. The supplementary variable technique is used to obtain the probability-generating functions and the steady-state probabilities for different states of the BS. The mean delay of the UR and mean power consumption of the BS are also derived. Also, the comparative analysis of the proposed model with the existing model has been presented. Computational results show that multiple sleeps with *N*-policy consumes less power than multiple sleeps without *N*-policy.

Keywords Mobile Network · Wireless Network · Energy Consumption · Multiple Sleeps · *N*- Policy

Mathematics Subject Classification (2020) 60K25 · 90B22 · 68M20

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1 Introduction

A cellular network or mobile network is a radio network distributed over the geographical region called cells. The cells which are distributed over the region may be of any perfect shape like square, circle, hexagon. Among the above mentioned shapes, Baltzis (2011) proved the hexagon shape cells are conventional, because it covers the entire area without overlapping. The communication over each of the cells are carried out by at least one fixed transceiver in that cell called Base Station (BS).

Due to tremendous demand, cellular networks have dramatically evolved in today's world. In the late 1970s, with successive generations (2 G to 4 G), cellular networks and technology had considerably evolved. In the first generation (1 G) cellular networks, the channel capacity was 30KHz and the data speed was 2.4 kbps. 1 G cellular networks only permitted voice calls to be made. Then in 1991, the second generation (2 G) cellular networks were introduced with bandwidths of 30KHz to 200KHz and permitted users to send messages (SMS and MMS), although at low speeds, up to 64 kbps. To increase the bandwidth as well as data speed, in 2000 and 2008, the third generation (3 G) and the fourth generation (4 G) cellular networks were introduced. The bandwidth of 3 G cellular networks and 4 G cellular networks was 5 MHz to 20 MHz but, 4 G users got speeds of up to 100 Mbps, while 3 G only promised a peak speed of 14 Mbps. This happens because the data rate in 4 G is higher than that of 3 G. Although 4 G cellular networks provided higher capacity, high speed of data rate but the latency was more Al-Falahy and Alani (2017) shown the difference between 5G and old mobile network generations, listing changes to several features. To reduce the latency and to get higher multi-Gbps peak data speeds, more reliability, massive network capacity and availability, in early 2019, the 5 G cellular networks were introduced.

Parvez et al. (2018) listed out some basic features that a 5 G cellular network is composed of namely core network, Software Defined Network (SDN), Small Cell BS, macro cell BS, User Equipment (UE), enhanced broadband and ultra reliable low latency communication etc. In 5 G cellular networks, Radio Access Network (RAN) connects the end-user devices and the other applications. This connection is achieved by transmitting information via radio waves from the end-user devices to a RAN's transceivers (which may be small cell or macro cell BS) and from RAN's transceivers to the core network which further it connects to the global internet. Since the user requests (URs) end-to-end delay (latency) in RAN and core network along with backhaul between RAN and core network is more, the 5 G cellular networks is involved to reduce the latency SDN. To achieve higher data speed rate with lower latency, enhanced mobile broadband and ultra-reliable low-latency communications are two new expected features in 5 G cellular networks. Hao (2021) discussed, enhanced mobile broadband provides higher speed data rate and ultra-reliable low latency communications provide lower latency in 5 G cellular networks. Ultra-reliable low-latency communications include some applications which need lower latency namely smart grid, smart factory, intelligent transport system and robotic surgery.

- The required end-to-end delay (latency) in smart grid is maximum 1 ms in a synchronous co-phasing of power suppliers (i.e., generators). For ON or OFF switching suppliers (i.e., solar panel, windmill, etc.) in smart grid, an end-to-end delay is maximum 100 ms.
- The required latency in smart factory is less than 1 ms.
- The required latency in intelligent transport system is 10 ms to 100 ms with packet loss rate of 10^{-3} to 10^{-5}.
- Robotic surgery also needs ultra reliable low latency communication.

The above mentioned features were already mentioned in 5 G networks as the key services Parvez et al. (2018).

Chih-Lin et al. (2020) proved that, in 5 G cellular networks, the power consumption with a bandwidth of 100 MHz is five times higher than the power consumption in a 4 G networks with a bandwidth of 20 MHz, presuming the power spectrum density to be the same. Hence, high power consumption is one of the major issue in 5 G cellular networks. To overcome this issue, energy efficiency has attracted many attentions in 5 G cellular networks which contains the following two types of cells: macro cell and small cell. In each cell, there is a BS, and all mobile devices in the same cell communicate with the BS in the same area. Small cell BSs reduces the load of macro cell BSs. To achieve more energy efficiency from small cell or macro cell BSs of 5 G cellular networks, is to adapt N limited scheme. Let Q be the number of URs in the BS queue. In N limited scheme, the BSs starts serving the URs when there are N URs accumulated in the 5 G cellular networks, i.e., ($Q \geq N$), which is shown in Fig. 1. Yang et al. (2017) showed that, in N limited scheme, when the BS woke up or shut off, it experienced a state transition delay. They also showed that the impact of state transition delay on the power consumption and traffic delay performance. They also discussed the tradeoff between the traffic delay requirements and power-saving. To get more energy efficiency from 5 G cellular networks, a new sleeping strategy is proposed, i.e., Multiple Sleeping scheme using N limited scheme by considering multiple sleeps (MSs) (where $Q < N$ or $Q > N$ till the duration is over), close down (with $Q < 1$) and setup time (where $Q \geq N$ till the duration is over) which are shown in Figs. 2 and 3.

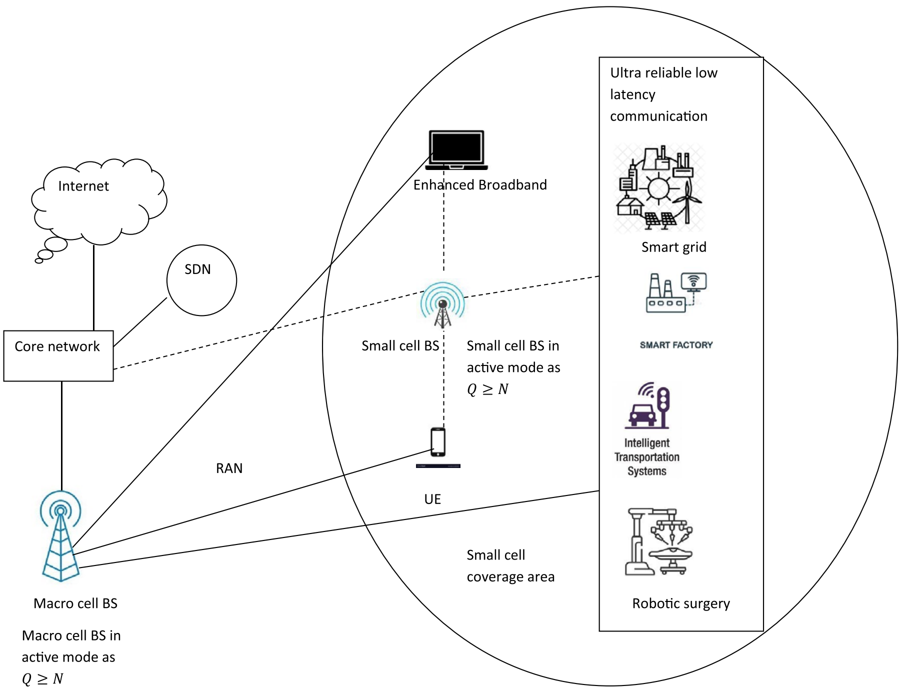


Fig. 1 5 G cellular networks architecture involving SDN with Base Station active mode and N limited scheme

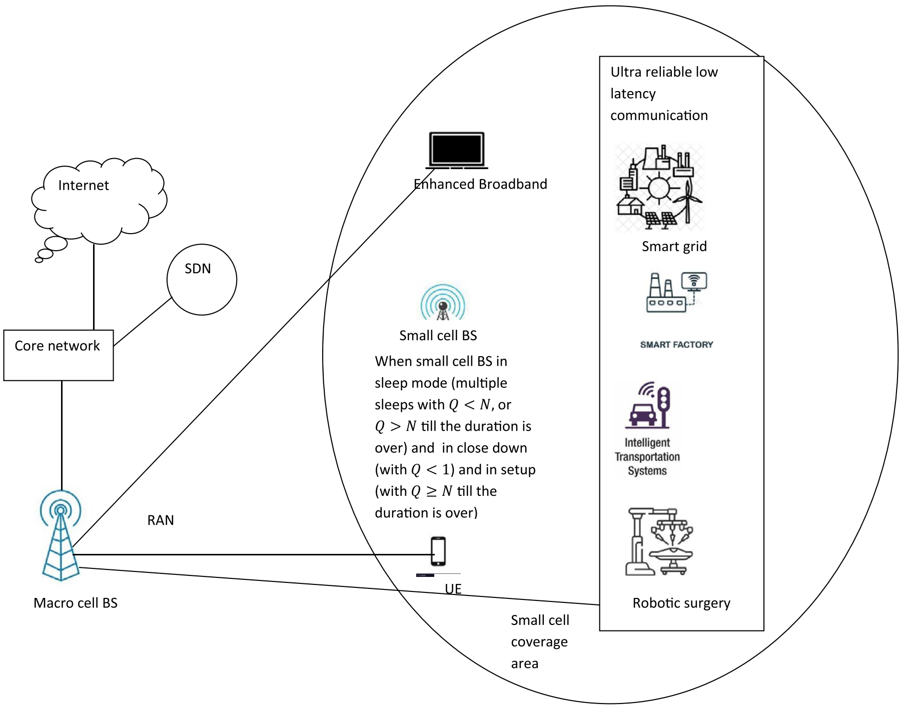


Fig. 2 5 G cellular network architecture involving SDN with small cell Base Station sleeping strategy and N limited scheme

In the literature, many sleeping and awaking strategies were proposed. However, the implementation of these strategies had some constraints like close down, set up. It is obvious that the analysis of different BSs sleeping strategies based on the energy saving and delay are essential. Some familiar sleeping strategies are single sleep (SS), MSs, N-limited scheme, light sleep, deep sleep etc. Generally the sleeping strategies depends either on time or the number of UR accumulated in the queue for service.

In SS scheme, the BS goes to sleep when there are no UR is waiting in the queue. After a particular time, it will awake unaware of number of UR waiting in the queue. A known fact is that the longer sleeping time saves more energy, but increases the delay time. Thus, it is necessary to show the trade off between the power saving and delay. In MS scheme, the BS sleeps when no UR is waiting in the queue. After the first sleep, the BS wakes up in case of at least one UR arrives and then the BS will start to process the UR. Otherwise, BS resumes the next sleep. This process will continue until the BS finds at least one UR in the queue. This sleeping strategy is better than SS in energy efficiency perspective. Whereas in N-limited scheme, the BS wakes up when N number of URs is accumulated. A component in BS or the Control BS (CBS) will count the number of URs accumulated in the queue. The BS will start to setup or wake-up when the accumulated number of UR in the queue is greater than or equal to the predefined number called N. Larger N value will result lesser power consumption but the delay will be more.

In the cases of SS and MS, BS will start to setup when atleast one UR is found in the system. It may decrease the delay, but increases the power consumption. Instead of awaking

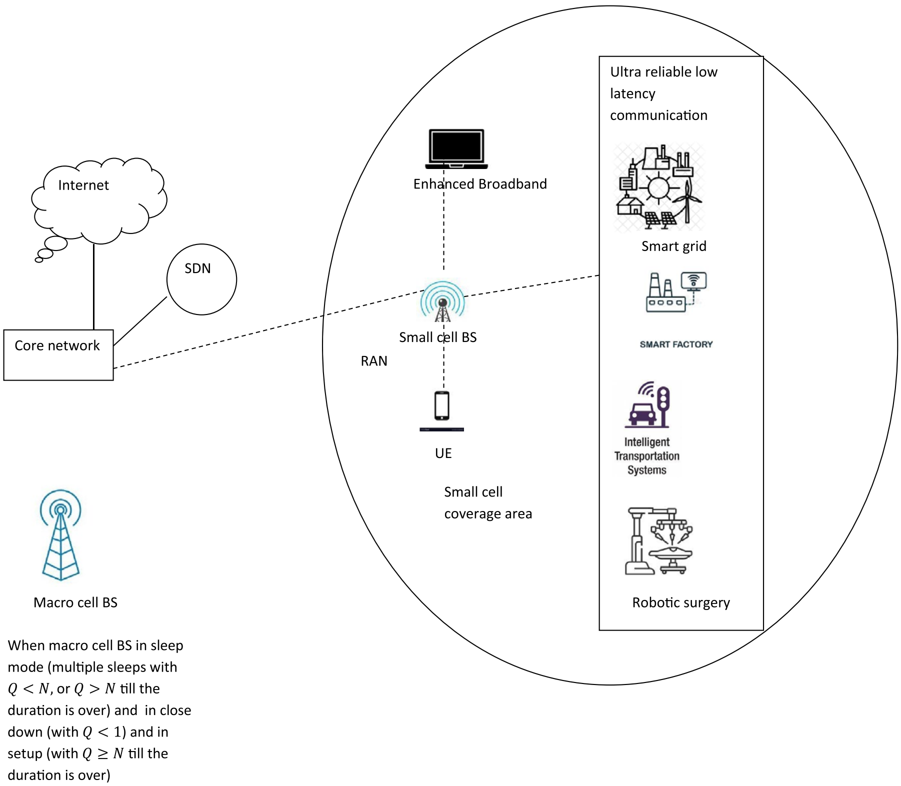


Fig. 3 5 G cellular network architecture involving SDN with macro cell Base Station sleeping strategy and N limited scheme

the BS for one UR in SS and MS scheme, the scheme MS with N-policy will awake the BS only when N number of URs have accumulated in the queue. Thus, it gives more energy saving. Also introducing close down time before starting the SS and MS can reduce the delay of the URs. This motivates us to analyse the impact of applying MS scheme using N-limited scheme on a macro cell or small cell BS in 5 G cellular networks with close down and setup time.

The proposed sleeping strategy, i.e., MS scheme using N-limited on a BS with close down and setup time is better than the SS scheme using N-limited scheme. When $N = 1$ the scheme becomes the MS scheme only and when $N \geq 1$, the scheme becomes the MS scheme with N-limited scheme. In MSs with N-limited scheme, the BS wakes up after N $(Q \geq N, N \geq 1)$ URs have gathered in the BS, whereas in MSs without N-limited scheme, the BS wakes up whenever a $(Q = N, N = 1)$ UR comes.

By achieving energy efficiency using N-limited scheme, one can move forward to the green communication in 5G cellular networks. Niu (2011) described, with the increasing rate of carbon footprint, the unfolding cellular technology has increased the energy consumption in mobile networks. This is one of the cause of an adverse effect on the environment and human health. Keeping this point in mind, Gandotra et al. (2017) studied a survey on techniques to make the 5G cellular networks GREEN. In the BS of 5G cellular networks, by considering the high deployment density, the overall power consumption may be 12 times higher than the 4 G networks. Chih-Lin et al. (2020) described that, this power consumption estimate was intolerable due to involved economic and environmental costs. Hence, by gaining energy efficiency, green communication is possible in 5G cellular networks.

The work flow in this article is as follows: Section 2 presents the literature review. Section 3 elaborates the mathematical model description with queue size distribution. Section 4 presents some theorems and essential performance measures. Section 5 describes the numerical illustrations, the impact of the parameters and a comparative analysis. Finally, Section 6 draws the conclusions and future work.

2 Literature Review

López-Pérez et al. (2022) listed the following three technologies are employed for reducing energy usage at the 5 G RAN level: massive multiple-input multiple-output (mMIMO), the lean carrier design, and 5 G sleep modes. A survey on the power consumption or energy efficiency in BS under different sleeping schemes and wake up techniques has been analyzed by Wu et al. (2015). Wu et al. (2013) studied the power matching and energy efficiency for a BS incorporated with the *N*-based and V-based sleeping schemes. Lie et al. (2016) analysed the energy efficiency in BS with light sleep and deep sleep under random sleeping and strategic sleeping policies. Yang et al. (2016) discussed the light sleep, deep sleep in BS and their energy consumption. Four different sleep modes based on traffic via deactivating the components or configured to a power saving mode was studied by Debaillie et al. (2015). The analytical frame work of power consumption model with four sleep depths was performed by Onireti et al. (2017).

By default BS took some time to wake up, but in all the research articles mentioned above they considered sleep without setup time. Also, Kamitsos et al. (2010) derived optimal trade-off between energy saving and average response and proved that it had a hysteretic structure. Further, set up time was also ignored here. Niu et al. (2015) analysed the sleep mode operations by modelling the BS as an $M/G/1$ queue with setup and close down times. A comparative study was taken over the three different wake up schemes namely single sleep, multiple sleeps and *N*-limited schemes by Guo et al. (2016). In which they considered the service process in base station as an $M/G/1$ with close down and setup times. Yang et al. (2017) analysed the impact of state transition delay on the power consumption and traffic delay performance under the scheme *N*-limited sleeping. Also they discussed the trade-off between energy saving and traffic delay requirements. Wu et al. (2020) analysed the power consumption and grade of service in BS with sleeping under isolated, cooperative and hybrid schemes.

Research progress on essential green tradeoffs such as delay against power in 5G communication technologies was analysed by Zhang et al. in (2016). Li et al. (2020) provided an overview of the power-saving measures enabled by 5G NR. Gupta (2012) proposes delay-tolerant traffic to improve system energy-efficiency. Zhao et al. (2014) studied the energy-delay tradeoffs of a virtual base station with sleeping. With four different sleep patterns, Salem et al. (2019) calculated the energy delay tradeoff for a 5 G network.

Many authors had used a combinations of close down time, setup time, and *N*-policy to conduct research in a queue. Haridass and Arumuganathan (2012) obtained steady-state performance measures for multiple vacations with setup time. Parthasarathy and Sudhesh (2008) established a transient solution for the Markov queueing model with *N*-policy. Krishna Reddy et al. (1998) analysed the steady state behaviour of a bulk arrival, bulk service queueing system with N-policy, multiple vacations and setup times. Among this, the supplementary variable technique was used in Haridass and Arumuganathan (2012) and Krishna Reddy et al. (1998) to obtain the queue size distribution. In this paper, the authors applied the supplementary variable technique to analyse the UR queue in a 5 G BS with close down, setup time and N-policy in MS scheme.

3 Model Description

To analyse the power consumption in a BS with multiple sleeps and N-policy, delay was also an important factor to study. Consequently, a stochastic model has been developed for the BS with multiple sleeps and N-policy. The main goals of the queueing model was to obtain the steady state probability of different BS states and the mean delay of URs.

The arrival of URs to the BS follows the Poisson process with the rate λ. Each user request is individually processed in the BS. The service time of the UR is iid random variable S and follows a general distribution. URs are processed based on FIFO queue discipline in the BS. The server will close down when there is no UR in the queue to reduce the power consumption. The close down time C follows a general distribution. During close down, if any UR arrives, the server immediately starts its service, else the server will be switched to sleep mode. The sleeping time is an iid random variable V that follows a general distribution. The wake-up scheme is defined as multiple sleeps with N-Policy. In that, the sleeping BS wakes up only when N or more URs have been accumulated at the end of any sleeping period. At the end of any sleep, if there are at least N number of URs waiting in the queue the server starts to set up (or called warm-up). The server starts to serve the URs after the completion of setup. The server setup time R is also generally distributed. The transition diagram of the proposed model is shown in Fig. 4.

An $M/G/1$ queue with sleep, close down and setup time is used to model the user request queue in the base station.

The notations used in the queueing model of the proposed work are listed in Table 1. Let $N(t)$ be the size of the BS queue at any time t. The state of the BS at any time t is indicated by $M(t)$ as follows:

$$M(t)=\begin{cases}0 & \text{if the BS is in close down}\\ 1 & \text{if the BS is in busy}\\ 2 & \text{if the BS is in setup}\\ j+2 & \text{if the BS is in } j^{th} \text{ sleep}, j=1,2,\dots\end{cases}.$$

Then $X(t)=\{M(t),N(t),t\geq 0\}$ is a stochastic process with state space $\Omega=\{(0,0),(1,0),(1,1),\dots,(2,N),(2,N+1),\dots,(3,0),(3,1),\dots,(4,0),(4,1),\dots\}$

The system state probabilities are defined as follows:

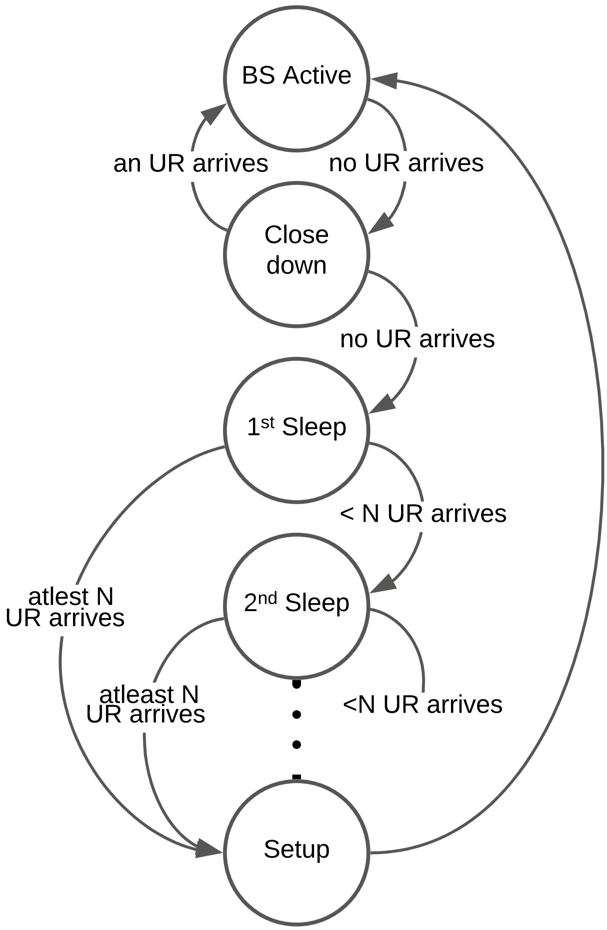
$C_0(x,t)$ - Probability that the BS in close down process with remaining close down time is lying in between x and $x+dx$, and no URs.

$$i.e., C_0(x,t)=Pr\{N(t)=0,\ x\leq C^0\leq x+dt\ ,M(t)=0)\}.$$

$P_{1,n}(x,t)$ - The probability that there are exactly $n(n=0,1,\dots)$ URs in buffer at time t and one in service with remaining service time is lying in between x and $x+dx$.

$$i.e., P_{1,n}(x,t)=Pr\{N(t)=n,\ x\leq S^0\leq x+dt\ ,M(t)=1)\}.$$

Fig. 4 Transition diagram for the MS scheme using *N*-policy on a 5G BS with close down and setup time



$R_n(x,t)$ - The probability that there are exactly $n(n = N, N+1 \ldots)$ URs in buffer at time t with remaining setup time of BS is lying in between x and $x + dx$.

$$i.e., R_n(x,t) = Pr\{N(t) = n,\ x \le R^0 \le x + dt\ , M(t) = 2)\}.$$

Table 1 Notations used in the proposed model

Description	Notation			
	Service time	Close down time	Sleep time	Setup time
Random variable	S	C	V	R
Probability density function	$s(x)$	$c(x)$	$v(x)$	$r(x)$
Cumulative distribution function	$S(x)$	$C(x)$	$V(x)$	$R(x)$
Laplace Stieltjes transforms	$\widetilde{S}(\theta)$	$\widetilde{C}(\theta)$	$\widetilde{V}(\theta)$	$\widetilde{R}(\theta)$
Remaining time at time t	$S^0(t)$	$C^0(t)$	$V^0(t)$	$R^0(t)$

$Q_{jn}(x, t)$ - The probability of exactly $n(n = 0, 1, \ldots)$ URs in buffer on $j^{th}(j = 1, 2, \ldots)$ sleep at time t with some remaining sleeping time which lies between x and $x + dx$.

$$i.e., Q_{jn}(x, t) = Pr\{N(t) = n,\ x \le V^0 \le x + dt\ , M(t) = j + 2\}.$$

3.1 The Queue Size Distribution

The queue size equations were obtained using supplementary variable technique. As per the queueing diagram given in Fig. 5 all possible system states are identified and corresponding equations are written for an infinitesimal Δt of the proposed model as follows:

$$\begin{aligned} P_{1,0}(x - \Delta t, t + \Delta t) &= P_{1,0}(x, t)[1 - \lambda\Delta t] + P_{1,1}(0, t)s(x)\Delta t \\ &\quad + \int_0^\infty C_0(y,\ t)dys(x)[\lambda\Delta t] \end{aligned} \tag{1}$$

$$\begin{aligned} P_{1,n}(x - \Delta t, t + \Delta t) &= P_{1,n}(x, t)[1 - \lambda\Delta t] + P_{1,n+1}(0, t)s(x)\Delta t \\ &\quad + P_{1,n-1}(x, t)[\lambda\Delta t], \qquad 1 \le n \le N - 2 \end{aligned} \tag{2}$$

$$\begin{aligned} P_{1,n}(x - \Delta t, t + \Delta t) &= P_{1,n}(x, t)[1 - \lambda\Delta t] + P_{1,n+1}(0, t)s(x)\Delta t \\ &\quad + P_{1,n-1}(x, t)[\lambda\Delta t] + R_{n+1}(0, t)s(x)\Delta t, \\ &\quad n \ge N - 1 \end{aligned} \tag{3}$$

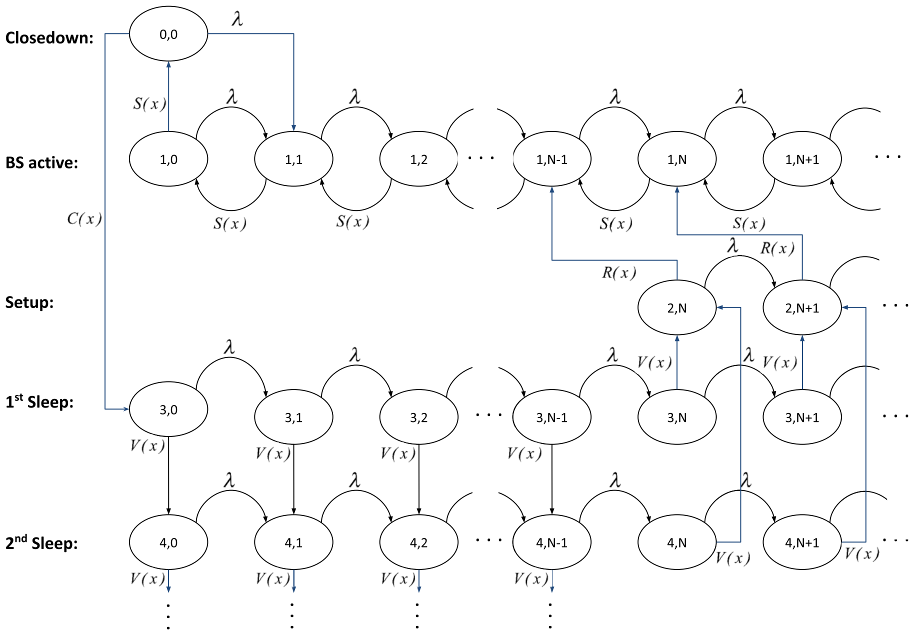


Fig. 5 State transition diagram for the $M/G/1$ queue with N-policy, close down, sleep and setup in a BS

$$C_0(x-\Delta t,t+\Delta t)=C_0(x,t)[1-\lambda\Delta t]+P_{1,0}(0,t)c(x)\Delta t \tag{4}$$

$$Q_{10}(x-\Delta t,t+\Delta t)=Q_{10}(x,t)[1-\lambda\Delta t]+C_0(0,t)v(x)\Delta t \tag{5}$$

$$Q_{1n}(x-\Delta t,t+\Delta t)=Q_{1n}(x,t)[1-\lambda\Delta t]+Q_{1(n-1)}(x,t)[\lambda\Delta t],\quad n\geq 1 \tag{6}$$

$$Q_{j0}(x-\Delta t,t+\Delta t)=Q_{j0}(x,t)[1-\lambda\Delta t]+Q_{(j-1)0}(0,t)v(x)\Delta t,\quad j=2,3,\ldots \tag{7}$$

$$\begin{aligned}Q_{jn}(x-\Delta t,t+\Delta t)&=Q_{jn}(x,t)[1-\lambda\Delta t]+Q_{j(n-1)}(x,t)[\lambda\Delta t]\\&\quad+Q_{(j-1)n}(0,t)v(x)\Delta t,\quad 1\leq n\leq N-1,j=2,3,\ldots\end{aligned} \tag{8}$$

$$\begin{aligned}Q_{jn}(x-\Delta t,t+\Delta t)&=Q_{jn}(x,t)[1-\lambda\Delta t]+Q_{j(n-1)}(x,t)[\lambda\Delta t],\\&\quad n\geq N,j=2,3,\ldots\end{aligned} \tag{9}$$

$$R_N(x-\Delta t,t+\Delta t)=R_N(x,t)[1-\lambda\Delta t]+\sum_{j=1}^{\infty}Q_{jN}(0,t)r(x)\Delta t \tag{10}$$

$$\begin{aligned}R_n(x-\Delta t,t+\Delta t)&=R_n(x,t)[1-\lambda\Delta t]+R_{n-1}(x,t)[\lambda\Delta t]\\&\quad+\sum_{j=1}^{\infty}Q_{jn}(0,t)r(x)\Delta t,\qquad n\geq N+1.\end{aligned} \tag{11}$$

Since the system does not change over time in steady state, the assumptions $P_{1,n}(x)=\lim_{t\to\infty}P_{1,n}(x,t)$, $C_0(x)=\lim_{t\to\infty}C_0(x,t)$, $Q_{jn}(x)=\lim_{t\to\infty}Q_{jn}(x,t), R_n(x)=\lim_{t\to\infty}R_n(x,t)$ are taken to obtain the steady state probabilities. Using these assumptions in Eqs. from (1) to (11), the steady state equations are obtained as follows:

$$-\frac{d}{dx}P_{1,0}(x)=-\lambda P_{1,0}(x)+P_{1,1}(0)s(x)+\lambda s(x)\int_0^{\infty}C_0(y)dy \tag{12}$$

$$\begin{aligned}-\frac{d}{dx}P_{1,n}(x)&=-\lambda P_{1,n}(x)+P_{1,n+1}(0)s(x)+\lambda P_{1,n-1}(x),\\&\quad 1\leq n\leq N-2\end{aligned} \tag{13}$$

$$\begin{aligned}-\frac{d}{dx}P_{1,n}(x)&=-\lambda P_{1,n}(x)+P_{1,n+1}(0)s(x)+\lambda P_{1,n-1}(x)+R_{n+1}(0)s(x),\\&\quad n\geq N-1\end{aligned} \tag{14}$$

$$-\frac{d}{dx}C_0(x)=-\lambda C_0(x)+P_{1,0}(0)c(x) \tag{15}$$

$$-\frac{d}{dx}Q_{10}(x)=-\lambda Q_{10}(x)+C_0(0)v(x) \tag{16}$$

$$-\frac{d}{dx}Q_{1n}(x) = -\lambda Q_{1n}(x) + \lambda Q_{1(n-1)}(x), n \geq 1 \tag{17}$$

$$-\frac{d}{dx}Q_{j0}(x) = -\lambda Q_{j0}(x) + Q_{(j-1)0}(0)v(x), \quad j = 2, 3, \ldots \tag{18}$$

$$-\frac{d}{dx}Q_{jn}(x) = -\lambda Q_{jn}(x) + \lambda Q_{j(n-1)}(x) + Q_{(j-1)n}(0)v(x), \\ 1 \leq n \leq N-1, j = 2, 3, \ldots \tag{19}$$

$$-\frac{d}{dx}Q_{jn}(x) = -\lambda Q_{jn}(x) + \lambda Q_{j(n-1)}(x), n \geq N, j = 2, 3, \ldots \tag{20}$$

$$-\frac{d}{dx}R_N(x) = -\lambda R_N(x) + \sum_{j=1}^{\infty} Q_{jN}(0)r(x) \tag{21}$$

$$-\frac{d}{dx}R_n(x) = -\lambda R_n(x) + \lambda R_{n-1}(x) + \sum_{j=1}^{\infty} Q_{jn}(0)r(x), n \geq N+1. \tag{22}$$

Taking Laplace Stieltjes transform on both sides of Eqs. (12) to (22),

$$-[\theta\widetilde{P}_{1,0}(\theta) - P_{1,0}(0)] = -\lambda\widetilde{P}_{1,0}(\theta) + P_{1,1}(0)\widetilde{S}(\theta) + \lambda\widetilde{S}(\theta)\int_0^{\infty} C_0(y)dy \tag{23}$$

$$-[\theta\widetilde{P}_{1,n}(\theta) - P_{1,n}(0)] = -\lambda\widetilde{P}_{1,n}(\theta) + P_{1,n+1}(0)\widetilde{S}(\theta) + \lambda\widetilde{P}_{1,n-1}(\theta), \\ 1 \leq n \leq N-2 \tag{24}$$

$$-[\theta\widetilde{P}_{1,n}(\theta) - P_{1,n}(0)] = -\lambda\widetilde{P}_{1,n}(\theta) + P_{1,n+1}(0)\widetilde{S}(\theta) + \lambda\widetilde{P}_{1,n-1}(\theta) \\ + R_{n+1}(0)\widetilde{S}(\theta), n \geq N-1 \tag{25}$$

$$-[\theta\widetilde{C}_0(\theta) - C_0(0)] = -\lambda\widetilde{C}_0(\theta) + P_{1,0}(0)\widetilde{C}(\theta) \tag{26}$$

$$-[\theta\widetilde{Q}_{10}(\theta) - Q_{10}(0)] = -\lambda\widetilde{Q}_{10}(\theta) + C_0(0)\widetilde{V}(\theta) \tag{27}$$

$$-[\theta\widetilde{Q}_{1n}(\theta) - Q_{1n}(0)] = -\lambda\widetilde{Q}_{1n}(\theta) + \lambda\widetilde{Q}_{1(n-1)}(\theta), \quad n \geq 1 \tag{28}$$

$$-[\theta\widetilde{Q}_{j0}(\theta) - Q_{j0}(0)] = -\lambda\widetilde{Q}_{j0}(\theta) + Q_{(j-1)0}(0)\widetilde{V}(\theta), \quad j = 2, 3, \ldots \tag{29}$$

$$-[\theta\widetilde{Q}_{jn}(\theta) - Q_{jn}(0)] = -\lambda\widetilde{Q}_{jn}(\theta) + \lambda\widetilde{Q}_{j(n-1)}(\theta) + Q_{(j-1)n}(0)\widetilde{V}(\theta), \\ 1 \leq n \leq N-1, j = 2, 3, \ldots \tag{30}$$

$$-[\theta\widetilde{Q}_{jn}(\theta)-Q_{jn}(0)]=-\lambda\widetilde{Q}_{jn}(\theta)+\lambda\widetilde{Q}_{j(n-1)}(\theta),n\geq N,j=2,3,\ldots \tag{31}$$

$$-[\theta\widetilde{R}_N(\theta)-R_N(0)]=-\lambda\widetilde{R}_N(\theta)+\widetilde{R}(\theta)\sum_{j=1}^{\infty}Q_{jN}(0) \tag{32}$$

$$-[\theta\widetilde{R}_n(\theta)-R_n(0)]=-\lambda\widetilde{R}_n(\theta)+\lambda\widetilde{R}_{n-1}(\theta)+\widetilde{R}(\theta)\sum_{j=1}^{\infty}Q_{jn}(0),n\geq N+1. \tag{33}$$

The following probability generating functions are defined:

$$\begin{aligned}
&\widetilde{P}_1(z,\theta)=\sum_{n=0}^{\infty}\widetilde{P}_{1,n}(\theta)z^n; && P_1(z,0)=\sum_{n=0}^{\infty}P_{1,n}(0)z^n;\\
&\widetilde{C}(z,\theta)=\widetilde{C}_0(\theta); && C(z,0)=C_0(0)\\
&\widetilde{R}(z,\theta)=\sum_{n=N}^{\infty}\widetilde{R}_n(\theta)z^n; && R(z,0)=\sum_{n=N}^{\infty}R_n(0)z^n.\\
&\widetilde{Q}_j(z,\theta)=\sum_{n=0}^{\infty}\widetilde{Q}_{jn}(\theta)z^n; && Q_j(z,0)=\sum_{n=0}^{\infty}Q_{jn}(0)z^n,j=1,2,\ldots\\
&P(z)=\widetilde{P}_1(z,0)+\widetilde{C}(z,0)+\widetilde{Q}_1(z,0)+\sum_{j=2}^{\infty}Q_j(z,0)+\widetilde{R}(z,0).
\end{aligned} \tag{34}$$

Multiplying Eq. (23) by z^0, (24) by $z^n(n=1,2,\ldots,N-2)$, (25) by $z^n(n=N-1,N,N+1,\ldots)$, and taking summation from 0 to ∞, the following equation is obtained

$$\begin{aligned}
&-\theta\sum_{n=0}^{\infty}\widetilde{P}_{1,n}(\theta)z^n+\sum_{n=0}^{\infty}P_{1,n}(0)z^n=-\lambda\sum_{n=0}^{\infty}\widetilde{P}_{1,n}(\theta)z^n+\lambda\sum_{n=1}^{\infty}\widetilde{P}_{1,n-1}(\theta)z^n\\
&+\widetilde{S}(\theta)\Big(\sum_{n=0}^{\infty}P_{1,n+1}(0)z^n+\sum_{n=N-1}^{\infty}R_{n+1}(0)z^n+\lambda\int_0^{\infty}C_0(y)dy\Big)\\
&\quad z[\theta-\lambda+\lambda z]\widetilde{P}_1(z,\theta)=[z-\widetilde{S}(\theta)]P_1(z,0)-\widetilde{S}(\theta)\Big(-P_{1,0}(0)+R(z,0)+\lambda z\widetilde{C}(z,0)\Big).
\end{aligned} \tag{35}$$

Using Eqs. (34), (26) can be written as

$$\begin{aligned}
&-\theta\widetilde{C}(z,\theta)+C(z,0)=-\lambda\widetilde{C}(z,\theta)+P_{1,0}(0)\widetilde{C}(\theta)\\
&\Rightarrow[\theta-\lambda]\widetilde{C}(z,\theta)=C(z,0)-P_{1,0}(0)\widetilde{C}(\theta).
\end{aligned} \tag{36}$$

Multiplying Eq. (27) by z^0, (28) by $z^n(n\geq 1)$ and taking summation from 0 to ∞, the following equation arrives

$$\begin{aligned}
-\theta\sum_{n=0}^{\infty}\widetilde{Q}_{1n}(\theta)z^n+\sum_{n=0}^{\infty}Q_{1n}(0)z^n&=-\lambda\sum_{n=0}^{\infty}\widetilde{Q}_{1n}(\theta)z^n+\lambda\sum_{n=1}^{\infty}\widetilde{Q}_{1(n-1)}(\theta)z^n\\
&+C_0(0)\widetilde{V}(\theta)\\
&\Rightarrow[\theta-\lambda+\lambda z]\widetilde{Q}_1(z,\theta)=Q_1(z,0)-C(z,0)\widetilde{V}(\theta).
\end{aligned} \tag{37}$$

Multiplying Eq. (29) by z^0, (30) by $z^n(n=1,2,\ldots,N-1)$, (31) by $z^n(n=N,N+1,\ldots)$ taking summation from 0 to ∞, we get

$$
\begin{aligned}
-\theta \sum_{n=0}^{\infty} \widetilde{Q}_{jn}(\theta)z^n + \sum_{n=0}^{\infty} Q_{jn}(0)z^n = & -\lambda \sum_{n=0}^{\infty} \widetilde{Q}_{jn}(\theta)z^n + \lambda \sum_{n=1}^{\infty} \widetilde{Q}_{j(n-1)}(\theta)z^n \\
& + \widetilde{V}(\theta) \sum_{n=0}^{N-1} \widetilde{Q}_{(j-1)n}(0)z^n \\
& \Rightarrow [\theta - \lambda + \lambda z]\widetilde{Q}_j(z,\theta) = Q_j(z,0) - \widetilde{V}(\theta) \sum_{n=0}^{N-1} Q_{(j-1)n}(0)z^n.
\end{aligned} \tag{38}
$$

Multiplying Eq. (32) by z^N, (33) by $z^n (n = N+1, N+2, \dots)$ and taking summation from N to ∞, the following equation is obtained

$$
\begin{aligned}
-\theta \sum_{n=N}^{\infty} \widetilde{R}_n(\theta)z^n + \sum_{n=N}^{\infty} R_n(0)z^n = & -\lambda \sum_{n=N}^{\infty} \widetilde{R}_n(\theta)z^n + \lambda \sum_{n=N+1}^{\infty} \widetilde{R}_{n-1}(\theta)z^n \\
& + \widetilde{R}(\theta) \sum_{n=N}^{\infty} \sum_{j=1}^{\infty} Q_{jn}(0)z^n \\
& \Rightarrow [\theta - \lambda + \lambda z]\widetilde{R}(z,\theta) = R(z,0) \\
& - \widetilde{R}(\theta) \sum_{j=1}^{\infty} \left[Q_j(z,0) - \sum_{n=0}^{N-1} Q_{jn}(0)z^n \right].
\end{aligned} \tag{39}
$$

Substituting $\theta = \lambda - \lambda z$ in Eqs. (35), (37), (38), (39) and $\theta = \lambda$ in (36) the following equations are obtained

$$
P_1(z,0) = \frac{z\widetilde{S}(\lambda - \lambda z)}{[z - S(\lambda - \lambda z)]} \Big(-P_{1,0}(0) + R(z,0) + \lambda z \widetilde{C}(z,0) \Big), \tag{40}
$$

$$
C(z,0) = \widetilde{C}(\lambda) P_{1,0}(0), \tag{41}
$$

$$
Q_1(z,0) = C(z,0)\widetilde{V}(\lambda - \lambda z), \tag{42}
$$

$$
Q_j(z,0) = \widetilde{V}(\lambda - \lambda z) \sum_{n=0}^{N-1} Q_{(j-1)n}(0)z^n, \qquad j \geq 2 \tag{43}
$$

$$
R(z,0) = \widetilde{R}(\lambda - \lambda z) \sum_{j=1}^{\infty} \left[Q_j(z,0) - \sum_{n=0}^{N-1} Q_{jn}(0)z^n \right]. \tag{44}
$$

The following equation is obtained by substituting Eq. (40) in (35)

$$
\widetilde{P}_1(z,\theta) = \frac{z[\widetilde{S}(\lambda - \lambda z) - \widetilde{S}(\theta)] \Big[\Big(-P_{1,0}(0) + R(z,0) + \lambda z \widetilde{C}(z,0) \Big) \Big]}{[\theta - \lambda + \lambda z][z - \widetilde{S}(\lambda - \lambda z)]}. \tag{45}
$$

Substituting Eq. (41) in (36), the following equation is obtained

$$\widetilde{C}(z,\theta)=\frac{\widetilde{C}(\lambda)-\widetilde{C}(\theta)}{[\theta-\lambda]}P_{1,0}(0). \tag{46}$$

Using Eqs. (41) and (42) in (37), the following equation is obtained

$$\widetilde{Q}_1(z,\theta)=\frac{\widetilde{V}(\lambda-\lambda z)-\widetilde{V}(\theta)}{[\theta-\lambda+\lambda z]}\widetilde{C}(\lambda)P_{1,0}(0). \tag{47}$$

Substituting Eq. (43) in (38), we get

$$\widetilde{Q}_j(z,\theta)=\frac{\widetilde{V}(\lambda-\lambda z)-\widetilde{V}(\theta)}{[\theta-\lambda+\lambda z]}\sum_{n=0}^{N-1}Q_{(j-1)n}(0)z^n. \tag{48}$$

Substituting Eq. (44) in (39), we obtain

$$\widetilde{R}(z,\theta)=\frac{\widetilde{R}(\lambda-\lambda z)-\widetilde{R}(\theta)}{[\theta-\lambda+\lambda z]}\sum_{j=1}^{\infty}\left[Q_j(z,0)-\sum_{n=0}^{N-1}Q_{jn}(0)z^n\right]. \tag{49}$$

The probability generating function of the queue size $P(z)$ using Eq. (34) is obtained as

$$\begin{aligned}P(z)=&\frac{\widetilde{S}(\lambda-\lambda z)}{-\lambda[z-\widetilde{S}(\lambda-\lambda z)]}\Bigg[\Big(1-\widetilde{C}(\lambda)\widetilde{R}(\lambda-\lambda z)\widetilde{V}(\lambda-\lambda z)-z(1-\widetilde{C}(\lambda))\Big)P_{1,0}(0)\\&+\widetilde{R}(\lambda-\lambda z)[1-\widetilde{V}(\lambda-\lambda z)]\sum_{j=1}^{\infty}\sum_{n=0}^{N-1}Q_{jn}(0)z^n\Bigg].\end{aligned} \tag{50}$$

Remark 1 The probability generating function $P(z)$ must satisfy $P(1)=1$. Applying L'Hospital's rule, evaluating $\lim_{z\to 1}P(z)$ and equating the expression to 1, $\big(1-\lambda E(S)\big)>0$ is obtained to meet the requirement. As a result, the suggested model's stability requirement is $\rho<1$, where $\rho=\lambda E[S]$.

4 Performance Measures

This article has proposed a MS with N-policy for a BS. It is vital to investigate the proposed scheme's performance before adopting it. As a result, the required probabilities, steady state probabilities, mean delay, and mean power consumption has been derived in this section, and the influence of parameter on these performance measures has been examined in Section 5.

4.1 Computation of Probability of Number of URs in the Queue

The probability generating function $P(z)$ consists of the constants $Q_{jn}(0)$ and $P_{1,0}(0)$ respectively, which denote the probability of n URs in the queue at the end of j^{th} sleep and the probability of no UR in the queue at the completion epoch of a service.

The following theorems are used to express $Q_{jn}(0)$ in terms of $P_{1,0}(0)$:

Theorem 1 *Let α_i be the probability of i URs arrive during a sleep. Corresponding probability generating function is given by*

$$\sum_{i=0}^{\infty} \alpha_i z^i = \widetilde{V}(\lambda - \lambda z).$$

Proof It is given that α_i is the probability of i URs arrive during a sleep.

$$\alpha_i = \int_0^{\infty} \frac{e^{-\lambda t}(\lambda t)^i}{i!}\, dV(t).$$

Multiplying both the sides of the above equation by z^i and taking the summation from $i = 0$ to ∞, the following equation is obtained

$$\begin{aligned}
\sum_{i=0}^{\infty} \alpha_i z^i &= \sum_{i=0}^{\infty} \int_0^{\infty} \frac{e^{-\lambda t}(\lambda t)^i}{i!} z^i\, dV(t) \\
&= \int_0^{\infty} \sum_{i=0}^{\infty} \frac{e^{-\lambda t}(\lambda t)^i}{i!} z^i\, dV(t) \\
&= \int_0^{\infty} \sum_{i=0}^{\infty} e^{-\lambda t} e^{\lambda t z}\, dV(t) \\
&= \widetilde{V}(\lambda - \lambda z).
\end{aligned}$$

□

Theorem 2 *Let α_i be the probability of i URs arrive during a sleep and let $q_n = \sum_{j=1}^{\infty} Q_{jn}(0)$ be the probability of n URs in the queue at the end of any j^{th} sleep. Then,*

$$q_n = \beta_n \widetilde{C}(\lambda) P_{1,0}(0),\ n = 0, 1, \dots, N-1$$

where

$$\beta_n = \frac{\alpha_n + \sum_{i=1}^{n} \beta_i \alpha_{n-i}}{1 - \alpha_0}.$$

$$\begin{aligned}
\sum_{j=1}^{\infty} Q_j(z,0) &= Q_1(z,0) + \sum_{j=2}^{\infty} Q_j(z,0) \\
&= \widetilde{V}(\lambda - \lambda z)\left[\widetilde{C}(\lambda) P_{1,0}(0) + \sum_{j=2}^{\infty} \sum_{n=0}^{N-1} Q_{(j-1)n}(0) z^n\right]
\end{aligned}$$

$$\sum_{n=0}^{\infty} \sum_{j=1}^{\infty} Q_{jn}(0) z^n = \sum_{n=0}^{\infty} \alpha_n z^n \left[\widetilde{C}(\lambda) P_{1,0}(0) + \sum_{j=1}^{\infty} \sum_{n=0}^{N-1} Q_{jn}(0) z^n\right].$$

Proof

Let $q_n = \sum_{j=1}^{\infty} Q_{jn}(0)$. Hence, the above equation becomes

$$\sum_{n=0}^{\infty} q_n z^n = \sum_{n=0}^{\infty} \alpha_n z^n \left[\widetilde{C}(\lambda) P_{1,0}(0) + \sum_{n=0}^{N-1} q_n z^n\right].$$

Equating the coefficient of z^n on both the sides, we get

$$q_n = \alpha_n \widetilde{C}(\lambda) P_{1,0}(0) + \sum_{i=0}^{n} \alpha_i q_{n-i}, \qquad n = 0, 1, \ldots, N-1.$$

$$\text{When} \quad n = 0, \quad q_0 = \frac{\alpha_0 \widetilde{C}(\lambda) P_{1,0}(0)}{1-\alpha_0} = \beta_0 \widetilde{C}(\lambda) P_{1,0}(0),$$

$$\text{where} \quad \beta_0 = \frac{\alpha_0}{1-\alpha_0}.$$

$$\text{When} \quad n = 1, \quad q_1 = \frac{[\alpha_1 + \alpha_1 \beta_0] \widetilde{C}(\lambda) P_{1,0}(0)}{1-\alpha_0} = \beta_1 \widetilde{C}(\lambda) P_{1,0}(0),$$

$$\text{where} \quad \beta_1 = \frac{\alpha_1 + \alpha_1 \beta_0}{1-\alpha_0}.$$

$$\text{When} \quad n = 2, \quad q_2 = \frac{[\alpha_2 + \alpha_2 \beta_0 + \alpha_1 \beta_1] \widetilde{C}(\lambda) P_{1,0}(0)}{1-\alpha_0} = \beta_2 \widetilde{C}(\lambda) P_{1,0}(0),$$

$$\text{where} \quad \beta_2 = \frac{\alpha_2 + \alpha_2 \beta_0 + \alpha_1 \beta_1}{1-\alpha_0}.$$

$$\text{In general} \quad q_n = \beta_n \widetilde{C}(\lambda) P_{1,0}(0), \text{ where} \quad \beta_n = \frac{\alpha_n + \sum_{i=1}^{n} \alpha_i \beta_{n-i}}{1-\alpha_0}.$$

□

Corollary 2.1 *The probability of no URs in the queue at the end of a service completion epoch is given by*

$$P_{1,0}(0) = \frac{\lambda(1 - \lambda E[S])}{1 - \widetilde{C}(\lambda) + \lambda E[R]\widetilde{C}(\lambda) + \lambda E[V]\widetilde{C}(\lambda) + \lambda E[V]\widetilde{C}(\lambda) \sum_{n=0}^{N-1} \beta_n}. \tag{51}$$

Proof The result obtained by letting $z \to 1$ in Eq. (50) and equating the expression to 1. □

4.2 Some Special Cases

The PGF of the proposed model in Eq. (50) is compared with the PGF of some of the existing models and are discussed below as special cases of the proposed model.

Case (i) When $N = 1$, the PGF given in Eq. (50), represents the PGF of $M/G/1$ queueing system with multiple sleeps, close down, setup and without N-policy as given by

$$P(z) = \frac{(\lambda E(S) - 1)\widetilde{S}(\lambda - \lambda z)\Big(\widetilde{C}(\lambda)\Big(z - \widetilde{R}(\lambda - \lambda z)\Big(\beta_0(\widetilde{V}(\lambda - \lambda z) - 1) + \widetilde{V}(\lambda - \lambda z)\Big)\Big) - z + 1\Big)}{(z - \widetilde{S}(\lambda - \lambda z))\Big(1 + \widetilde{C}(\lambda)\big(\lambda E(R) + \big(\beta_0 + 1\big)\lambda E(V) - 1\big)\Big)}. \tag{52}$$

Also note that the above result was a special case of the result obtained by Kalita et al. (2020).

Case (ii) When no close down, Eq. (50) becomes

$$P(z) = \frac{(1-\lambda E(S))\widetilde{S}(\lambda-\lambda z)\left(1-\widetilde{R}(\lambda-\lambda z)\left((1-\widetilde{V}(\lambda-\lambda z))\left(\sum_{n=0}^{N-1}\beta_n z^n\right)+\widetilde{V}(\lambda-\lambda z)\right)\right)}{(z-\widetilde{S}(\lambda-\lambda z))\left(\lambda E(V)\left(1+\sum_{n=0}^{N-1}\beta_n\right)+\lambda E(R)\right)}. \tag{53}$$

The above represents the probability generating function of $M/G/1$ queueing system with multiple sleeps, N-policy and setup. This result coincides with $M/G/1$ queueing system with multiple vacations, N-policy and setup times obtained by Krishna Reddy et al. (1998).

Case (iii) When no close down, no setup, Eq. (50) becomes

$$P(z) = \frac{(1-\lambda E(S))\widetilde{S}(\lambda-\lambda z)(\widetilde{V}(\lambda-\lambda z)-1)\left(\sum_{n=0}^{N-1}\beta_n z^n+1\right)}{\lambda E(V)(z-\widetilde{S}(\lambda-\lambda z))\left(\sum_{n=0}^{N-1}\beta_n+1\right)}. \tag{54}$$

The above expression represents the PGF of $M/G/1$ queueing system with multiple vacation and N-policy. This result matches with the special case of result obtained by Gautam et al. (2020) for $M/G/1$ queueing system with multiple vacation and N- policy.

Case (iv) When no close down, no sleeps and no setup, Eq. (50) reduces to the following equation

$$P(z) = \frac{(z-1)(1-\lambda E(S))\widetilde{S}(\lambda-\lambda z)}{z-\widetilde{S}(\lambda-\lambda z)}. \tag{55}$$

Equation (55) matches with the PGF of $M/G/1$ queueing system as in Shortle et al. (2018) and Pollaczek-Khinchin (P-K) formula obtained by Castañed et al. (2012).

4.3 Steady State Probabilities

Let $q_{AC}, q_{CD}, q_{SL}, q_{SU}$ denote the steady state probability of the BS in active, close-down, sleep and set up.

The steady state probability that the BS is in active is given by

$$\begin{aligned} q_{AC} &= \lim_{z\to 1}\widetilde{P}_1(z,0) = \lim_{z\to 1}\frac{[1-\widetilde{S}(\lambda-\lambda z)]}{[-\lambda+\lambda z][z-\widetilde{S}(\lambda-\lambda z)]}\Bigg[P_{1,0}(0)\Big(1-z[1-\widetilde{C}(\lambda)] \\ &\quad -\widetilde{R}(\lambda-\lambda z)\widetilde{C}(\lambda)\widetilde{V}(\lambda-\lambda z)\Big) \\ &\quad +\widetilde{R}(\lambda-\lambda z)[1-\widetilde{V}(\lambda-\lambda z)]\sum_{j=1}^{\infty}\sum_{n=0}^{N-1}Q_{jn}(0)z^n\Bigg] \\ &= \frac{E[S]}{1-\lambda E[S]}\Bigg[P_{1,0}(0)\Big(1+\lambda\widetilde{C}(\lambda)E[R] \\ &\quad +\lambda\widetilde{C}(\lambda)E[V]-\widetilde{C}(\lambda)\Big)+\lambda E[V]\sum_{j=1}^{\infty}\sum_{n=0}^{N-1}Q_{jn}(0)\Bigg]. \end{aligned} \tag{56}$$

The steady state probability that the BS is in close down

$$
\begin{aligned}
q_{_{CD}} = \lim_{z \to 1} \widetilde{C}(z,0) = \lim_{z \to 1} \frac{\widetilde{C}(\lambda) - 1}{[-\lambda]} P_{1,0}(0) \\
= \frac{1 - \widetilde{C}(\lambda)}{\lambda} P_{1,0}(0).
\end{aligned}
\tag{57}
$$

The steady state probability that the BS is in sleep

$$
\begin{aligned}
q_{_{SL}} = \lim_{z \to 1} \sum_{j=1}^{\infty} \widetilde{Q}_j(z,0) = \widetilde{Q}_1(z,0) + \sum_{j=2}^{\infty} \widetilde{Q}_j(z,0) \\
= E[V]\Big[\widetilde{C}(\lambda)P_{1,0}(0) + \sum_{j=1}^{\infty}\sum_{n=0}^{N-1} Q_{jn}(0)\Big].
\end{aligned}
\tag{58}
$$

The steady state probability that the BS is in set up

$$
\begin{aligned}
q_{_{SU}} = \lim_{z \to 1} \widetilde{R}(z,0) = \lim_{z \to 1} \frac{\widetilde{R}(\lambda - \lambda z) - 1}{[-\lambda + \lambda z]} \Big[[\widetilde{V}(\lambda - \lambda z) - 1] \sum_{j=1}^{\infty}\sum_{n=0}^{N-1} Q_{jn}(0) z^n \\
+ \widetilde{C}(\lambda)\widetilde{V}(\lambda - \lambda z) P_{1,0}(0) \Big] \\
= E[R]\widetilde{C}(\lambda)P_{1,0}(0).
\end{aligned}
\tag{59}
$$

4.4 Computation of Mean Delay

Delay is the time that an UR spends in the queue till the service begins. If the BS is already serving an UR, the newly arrived UR has to wait in the queue to get service, this is called as queue delay or busy period of the server. In the proposed model, the server will not provide service when the BS is on sleep or setup. This delay is called as additional delay or idle period Powell and Avi-Itzhak (1967) of the server. The close down period is not a factor for delay because the BS immediately processes the UR that arrives during the close-down.

Levy and Kleinrock (1986) showed that the delay in the queue with setup was composed of the direct sum of two independent variables namely queue delay and additional delay due to the setup. Krishna Reddy et al. (1998) calculated additional delay as the sum of idle period due to multiple vacation process and mean length of setup time for a $M^X/G(a,b)/1$ queueing system with N-policy, multiple vacations and setup. The proposed model presents the following factors of delay: queue, MS and setup time.

It is obvious that while BS is active, the URs are affected only by queue delays. The UR will not be delayed if they arrive during close down and are served immediately. The URs arrive while the BS that is sleeping gets affected by sleep, queue, and setup delays. The URs that arrive during setup will have setup and queue delays.

Hence, The mean delay for an UR is calculated as follows:

$$
E(D) = \frac{\lambda E[S^2]}{2(1 - \lambda E[S])} + (\text{mean sleep period}) q_{_{SL}} + (\text{mean setup duration}) q_{_{SU}}. \tag{60}
$$

In the above equation, the term $\frac{\lambda E[S^2]}{2(1-\lambda E[S])}$ represents the mean queuing delay in an $M/G/1$ queueing system Shortle et al. (2018).

The mean sleep period is calculated as follows:

Let I be the random variable denoting the Idle period due to multiple sleeps. To find $E[I]$, let us define another variable U as follows:

$U = 0$, if the server finds at least N URs after the first short sleep.

$U = 1$, if the server finds less than N URs after the first short sleep.

$$\begin{aligned} E[I] &= E[I/U=0]P[U=0] + E[I/U=1]P[U=1] \\ &= E[V]P[U=0] + (E[V]+E[I])P[U=1] \\ &= \frac{E[V]}{P[U=0]} \end{aligned} \tag{61}$$

where

$$\begin{aligned} P[U=0] &= P[\text{server finds at least N URs after the first sleep}] \\ &= 1 - \sum_{n=0}^{N-1} Q_{1n}(0). \end{aligned} \tag{62}$$

Actually,

$$\begin{aligned} \sum_{n=0}^{\prime\infty} Q_{1n}(0)z^n = Q_1(z,0) = C(z,0)\widetilde{V}(\lambda-\lambda z) &= \widetilde{V}(\lambda-\lambda z)\widetilde{C}(\lambda)P_{1,0}(0) \\ &= \sum_{n=0}^{\prime\infty} \alpha_n z^n \widetilde{C}(\lambda)P_{1,0}(0) \end{aligned}$$

where α_n represents probability of arrival of n URs during sleep. Equating the coefficients of z^n on both the sides, we get

$$Q_{1n}(0) = \alpha_n \widetilde{C}(\lambda)P_{1,0}(0). \tag{63}$$

Using Eqs. (62) and (63) in (61) we obtain

$$E(I) = \frac{E[V]}{1-\widetilde{C}(\lambda)P_{1,0}(0)\sum_{n=0}^{\prime N-1}\alpha_n}. \tag{64}$$

Using Eq. (64) in (60), the mean delay is given by

$$E(D) = \frac{\lambda E[S^2]}{2(1-\lambda E[S])} + q_{SL}\left(\frac{E[V]}{1-\widetilde{C}(\lambda)P_{1,0}(0)\sum_0^{N-1}\alpha_n}\right) + q_{SU}E[R] \tag{65}$$

where q_{SL} and q_{SU} are the steady state probabilities of the BS in sleep and setup respectively.

4.5 Energy Consumption Model

In the proposed model, the BS is modelled as an $M/G/1$ queue with close down, sleep and setup. The arrival of URs follows Poisson process with rate λ. The arrived URs are served with service time S in active state. During the active state the power consumption is assumed to be E_{AC}. The BS will be active until the system becomes empty. After that the BS wait for URs to arrive for some time called close down period before going to sleep mode. Power consumption during close down is E_{CD}. If no UR arrives during close down, the BS goes to sleep mode. The power consumption during sleep mode is E_{SL}. The BS sleep continues until it finds N URs at the end of any sleep. At the end of any sleep, the BS starts to setup if the number of accumulated URs reaches N. The power during setup is E_{SU}. Let P be the power consumed by BS per unit time. Hence, the mean power consumed per unit time is obtained as

$$\begin{aligned} E[P] &= \frac{1}{E[\text{cycle length}]}\big[E_{AC}E[\text{active period}] + E_{CD}E[\text{close down period}] \\ &\quad + E_{SL}E[\text{sleep period}] + E_{SU}E[\text{setup period}]\big] \\ &= q_{_{AC}}E_{AC} + q_{_{CD}}E_{CD} + q_{_{SL}}E_{SL} + q_{_{SU}}E_{SU}. \end{aligned} \tag{66}$$

where $q_{_{AC}}$, $q_{_{CD}}$ $q_{_{SL}}$ and $q_{_{SU}}$ signify the steady state probabilities of different states of the BS, which are available in Eqs. (56), (57), (58) and (59) respectively.

When busy BS consumes 100% power because all the components are active. During close down the BS stays idle and awaits for the arrival, it consumes power lesser than that of active state. If no UR arrives during close down, it will enter into the sleep mode. The energy consumption in sleep mode is very low but not zero because few components are active like the one component used to count the number of URs waiting in the queue. It also consumes some energy when N URs accumulated at the end of any sleep to start the system setup for service. The power consumption of the BS is shown in Fig. 6. Based on Woon et al. (2021) and Niu et al. (2015), the assumptions in Table 2 are taken for numerical illustration.

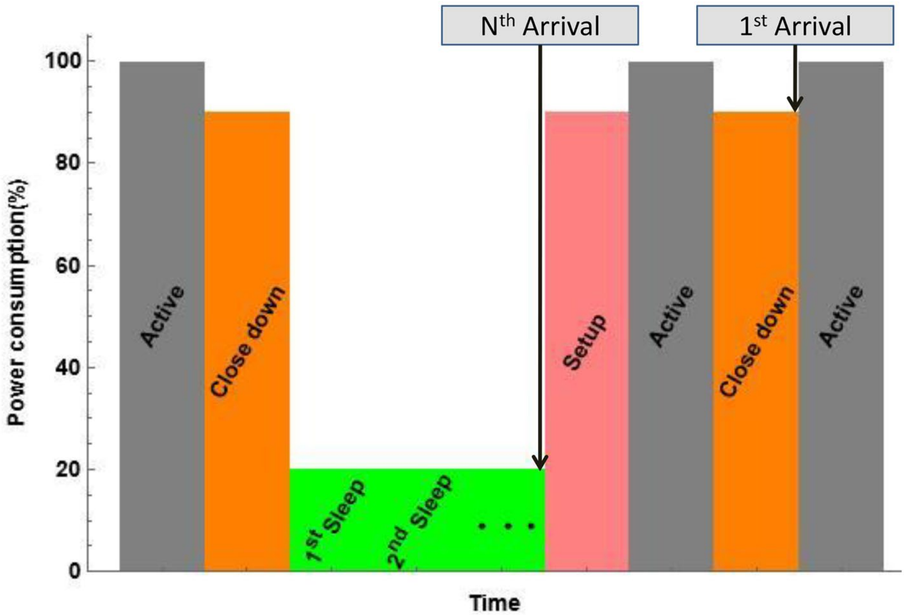


Fig. 6 Power consumption of the BS with multiple sleeps and N-policy

Table 2 Values for the power consumption in different states

Power consumption in different states	Values
E_{AC}	$11577J/s$
E_{CD} (90% of E_{AC})	$10419.3J/s$
E_{SL}(20% of E_{AC})	$2315.4J/s$
E_{SU}(90% of E_{AC})	$10419.3J/s$

5 Numerical Illustration

In order to analyse the model, it was essential to find the effect of each parameter on the measures of mean delay and mean power consumption. It was assumed that the service time S, close down time C, sleep time V and setup time R random variables followed deterministic distribution. The PMF of deterministic distribution was $P(X = x) = 1$ with mean K and variance 0. Here, the mean of the random variables S, C, V and R were taken as 0.5, 5, 6 and 5 respectively.

5.1 Impact of Arrival Rate λ on Mean Delay and Mean Power Consumption

Figure 7 shows the effect of arrival rate λ on mean delay and mean power consumption. When arrival rate increased, the frequency of BS entering into sleep decreased. Thus, the mean delay decreased upto certain λ value. After that due to traffic, the mean delay started to increase. On the other hand, the power consumption increased for increasing λ because the BS which was less often goes to sleep.

5.2 Impact of Mean Service Time on Mean Delay and Mean Power Consumption

As expected, the mean delay increased as the mean duration of service $E[S]$ increased (see Fig. 8(a)). At the same time, as the mean duration of service E[S] increased, so does the probability of the BS being in the active state. As a result, Fig. 8(b) shows an increase in mean power consumption.

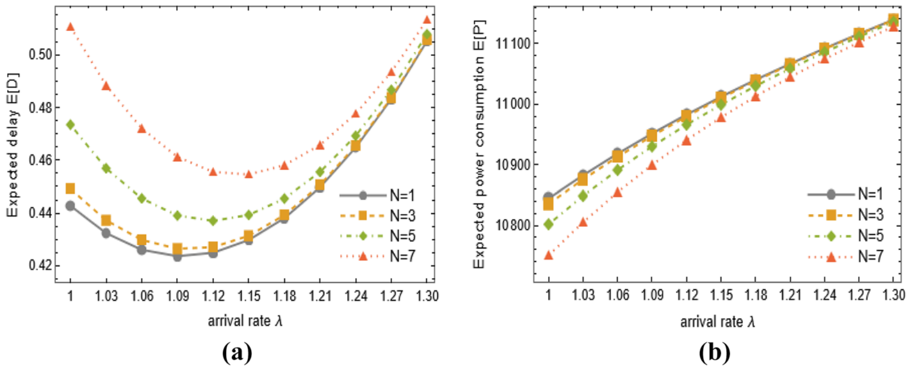


Fig. 7 Mean delay and mean power consumption vs. arrival rate λ for different N values

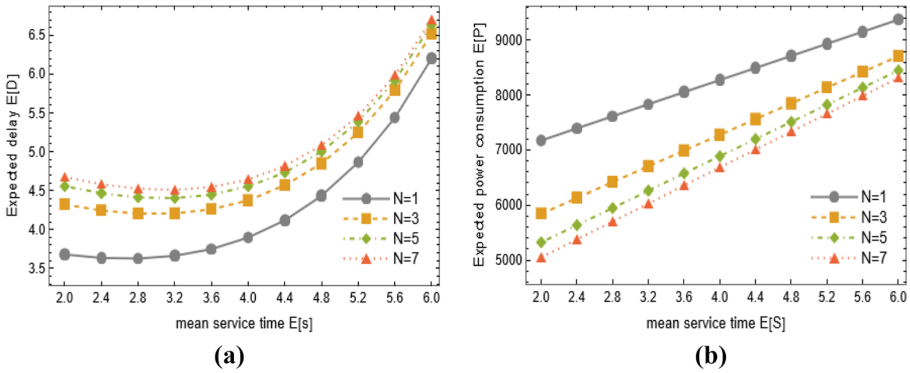


Fig. 8 Mean delay and mean power consumption vs. mean service time for different *N* values

5.3 Impact of the Mean Sleep Time on Mean Delay and Mean Power Consumption

In Fig. 9(a) when the mean sleep duration increased as theory says the mean delay increased. But, in Fig. 9(b) the mean power consumption decreased because the probability of activeness of the BS decreased when mean sleep duration increased.

5.4 Impact of the Mean Setup Time on Mean Delay and Mean Power Consumption

During setup, the BS did not provide service of the URs. Hence, as the mean setup time $E[R]$ increased, obviously the delay increased. Figure 10(a) also assures the same. During the setup, the BS started to activate its components, and thereby it consumed more power. Hence, from Fig. 10(b) it is noted that larger mean setup duration consume more power.

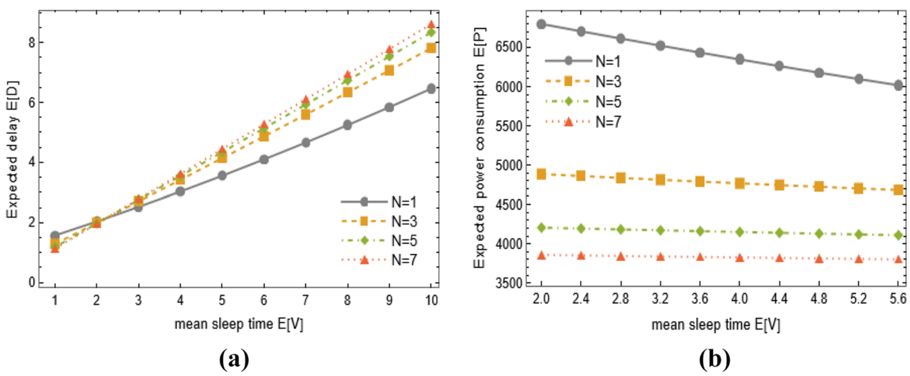


Fig. 9 Mean delay and mean power consumption vs. mean sleep time for different *N* values

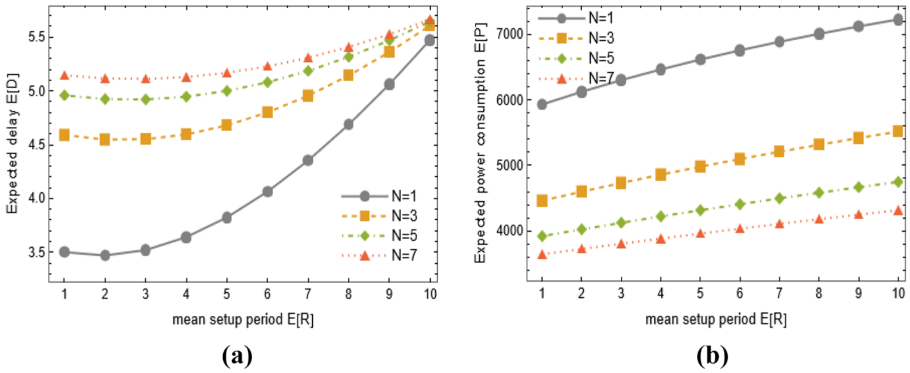


Fig. 10 Mean delay and mean power consumption vs. mean setup period for different N values

5.5 Impact of the Mean Close Down Duration on Mean Delay and Mean Power Consumption

Since close down time increased, the frequency of BS entering into sleep mode decreased. As a result, mean delay decreases as shown in Fig. 11(a) but mean power consumption increases as given in Fig. 11(b).

5.6 Impact of *N* on Mean Delay and Mean Power Consumption

The BS wait for N number of URs to awaken before proceeding, the bigger N values, cause the BS to sleep many times. Figures 7, 8, 9, 10, 11 show that when N increases the mean delay increases whereas the mean power consumption decreases. When $N = 1$, the proposed model was the same as the MS model/MS without N-policy described in Guo et al. (2016).

In the above-mentioned study, the BS will set up if it finds at least one UR in the queue at the end of any sleep. As a result, in multiple sleeps without N-policy, energy usage is higher than in multiple sleeps with N-policy as shown in Table 3. It compares the mean delay and mean power consumption of MS with and without N-policy. Here, the % of deviation

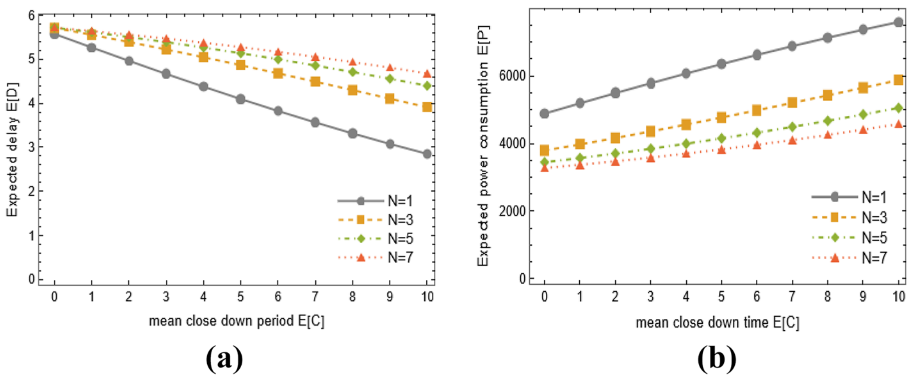


Fig. 11 Mean delay and mean power consumption vs. mean close down period $E[C]$ for different N values

Table 3 Comparison of mean delay and mean power consumption for MS scheme with and without N-policy

Category	N	Mean delay		Mean power consumption	
		$E[D]$	% of Deviation	$E[P]$	% of Deviation
MS scheme without N-policy	1	4.09534	NA	6346.57	NA
	2	4.59758	12.3	5342.55	15.8
	3	4.86525	18.8	4766.47	24.9
MS scheme with N-policy	4	5.02689	22.7	4401.55	30.6
	5	5.13611	25.4	4149.86	34.6
	6	5.21524	27.3	3965.75	37.5
	7	5.27528	28.8	3825.23	39.7

measures how much the value from MS with N-policy differs in percent from MS without N-policy. Percentage (%) of deviation is calculated using the following formula

$$\%\text{ of deviation} = \frac{\text{MS without } N\text{-policy measure - MS with } N\text{-policy measure}}{\text{MS without } N\text{-policy measure}}.$$

In MS with N-policy, the mean power consumption decreases as N value increases, but the delay increases. From the % of deviation, it is observed that large N values can effectively minimize the mean power consumption within a reasonable delay.

5.7 Comparative Analysis

In this sub-section, the performance results of the proposed model with the existing work are compared. Guo et al. (2016) analyzed BS queue as an $M/G/1$ with MS, close down and setup time. In Guo et al. (2016), if no UR arrives during the sleep, then the BS goes for the next sleep and it repeats until it finds at least one UR in the BS queue. The power can be conserved by waking up the BS after accumulating a certain number of URs rather than waking for one UR as in Guo et al. (2016). Note that, in Guo et al. (2016), no N-policy is considered in the MS scheme. The major goal of the proposed work is to implement the N-policy in the MS scheme to reduce energy consumption. Hence, the UR queue in the BS is modeled as an $M/G/1$ queue with setup, close down, and N-policy in the MS scheme. The proposed model with $N = 1$ is the same as the model presented in Guo et al. (2016). When $N = 1$, the mean delay and the mean power consumption given in Eqs. (65) and (66) respectively, become

$$E[D] = E[R]^2(-\Psi) + \frac{\lambda E[S^2]}{2(1 - \lambda E[S])} + \frac{E[V]^2\left(-\frac{\Psi\widetilde{V}(\lambda)}{1-\widetilde{V}(\lambda)} - \Psi\right)}{\Psi\widetilde{V}(\lambda) + 1}$$

$$E[P] = \frac{\Psi}{\lambda\widetilde{C}(\lambda)}\left(\frac{E_{\mathrm{AC}}\lambda E(S)(\widetilde{C}(\lambda)(\lambda E[R](\widetilde{V}(\lambda) - 1) - \lambda E(V) - \widetilde{V}(\lambda) + 1) + \widetilde{V}(\lambda) - 1)}{(\lambda E(S) - 1)(\widetilde{V}(\lambda) - 1)}\right.$$
$$\left. + E_{\mathrm{CD}}(-1 + \widetilde{C}(\lambda)) - \lambda E[R]E_{\mathrm{SU}}\widetilde{C}(\lambda) + \frac{\lambda E_{\mathrm{SL}}E(V)\widetilde{C}(\lambda)}{\widetilde{V}(\lambda) - 1}\right)$$

where,

$$\Psi = \frac{\lambda \widetilde{C}(\lambda)(\lambda E[S] - 1)}{-\widetilde{C}(\lambda) + \lambda E[R]\widetilde{C}(\lambda) + \lambda E[V]\widetilde{C}(\lambda) + \frac{\lambda E[V]\widetilde{C}(\lambda)\widetilde{V}(\lambda)}{1-\widetilde{V}(\lambda)} + 1}.$$

The above equations are analogous with the results obtained by Guo et al. (2016). The numerical results of the above equations as well as the numerical results of the proposed model with N-policy are given in Table 3. The results given in Table 3 justify the efficiency of the proposed model.

By defining the following two models, a numerical comparison with the prior work has been presented to demonstrate the efficiency of the proposed model:

Model 1: The BS queue was modeled in Guo et al. (2016) as an $M/G/1$ queue with close down, setup time and MS scheme without N-policy.

Model 2: The BS queue is modeled in this paper as an $M/G/1$ queue with close down, setup time, and MS scheme with N-policy (Proposed model).

The numerical comparison of expected power consumption and expected delay for Model 1 and Model 2, when the arrival rate (λ) varies is given in Fig. 12. From Fig. 12, the observations are as follows:

- As λ increases, the expected power consumption for Model 1 and Model 2 increases and expected delay decreases as it should be.
- As λ varies from 0.1 to 1.1, the expected power consumption for the Model 1 is higher than that of the Model 2 but the expected delay is less.

The numerical comparison of expected power consumption and expected delay for Model 1 and Model 2, when the mean close-down period ($E(C)$) varies is given in Fig. 13. From Fig. 13, the observations are as follows:

- As $E(C)$ increases, the expected delay for Model 1 and Model 2 decreases but expected power consumption increases as it should be.

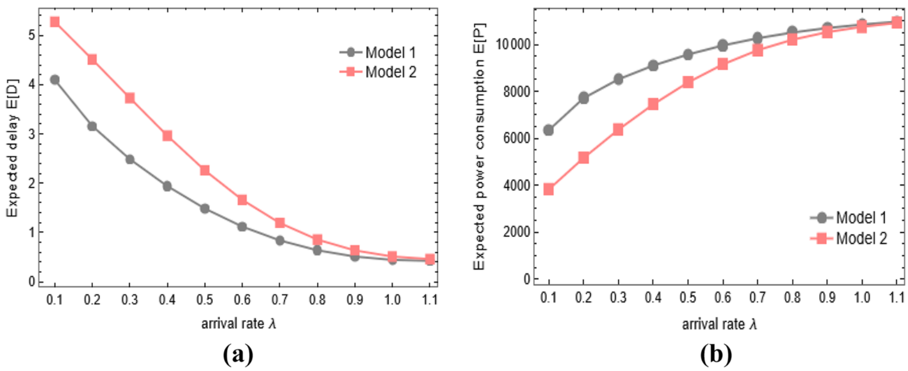


Fig. 12 Comparison between Model 1 and Model 2 with $N = 7$ when arrival rate varies

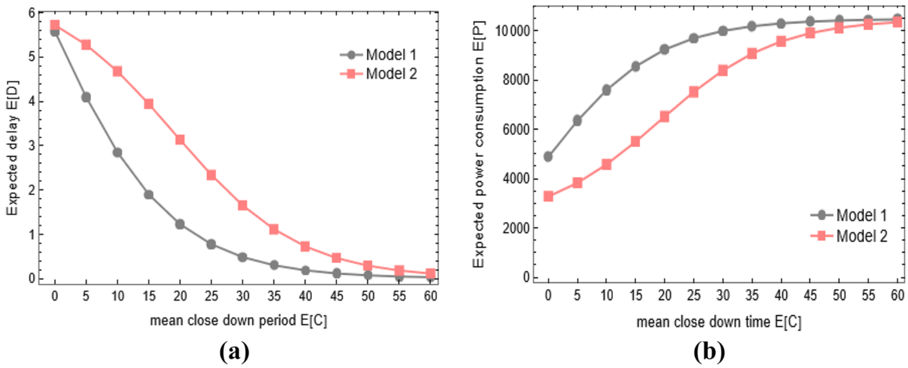


Fig. 13 Comparison between Model 1 and Model 2 with $N = 7$ when close down duration varies

- As $E(C)$ increases, the expected power consumption for Model 2 is less than that of Model 1 but the expected delay is more.

Based on the above comparative analysis, it is concluded that for the optimal value of N the proposed model will be more useful for real 5 G networks.

6 Conclusions and Future Work

5 G cellular networks is standard for higher data speed rate, high-reliability and low-latency communications. Communications in 5 G cellular networks can have a latency of less than 1 ms. To achieve the energy efficiency from 5 G cellular networks, an effective sleeping technique for small cell or macro cell BSs was required. Therefore, in this paper, BS (macro cell or small cell BS) queue is modeled as an $M/G/1$ queueing system with close down, setup time, and MS scheme with N-policy. Through comparative analysis with existing model, it is shown that the energy consumption of the BS can be reduced by introducing an MS scheme with N-policy. As N increased the power consumption decreased but delay increased. It is observed that, a short closed down duration and larger N value reduce the power consumption but increase the delay.

Further, to optimize the power consumption and delay, by considering an MS scheme with N-policy in 5 G cellular networks, the tradeoff between power consumption and delay can be analyzed. The optimization problem can be used to find the optimal value for N and the optimal value for the mean close-down period. Further, since the network traffic is not always consistent, the research work can be extended to the MS scheme with the randomized N-policy.

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Code Availability NA.

Declarations

Conflict of Interest NA.

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