

Review of AdS/CFT Integrability, Chapter IV.2: Deformations, Orbifolds and Open Boundaries

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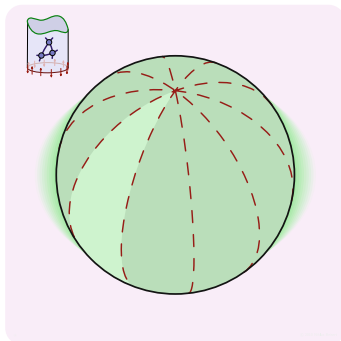
Received: 15 December 2010 / Revised: 18 April 2011 / Accepted: 14 May 2011

Published online: 23 July 2011 – © Springer 2011

Abstract. We review the role of integrability in the planar spectral problem of four-dimensional superconformal gauge theories besides $\mathcal{N}=4$ SYM. The cases considered include the Leigh–Strassler marginal deformations of $\mathcal{N}=4$ SYM, quiver theories which arise as orbifolds of $\text{AdS}_5 \times \text{S}^5$ on the dual gravity side, as well as various theories involving open spin chains.

Mathematics Subject Classification (2010). 81R50, 81T30, 81T60, 82B23.

Keywords. integrability, AdS/CFT, Bethe ansatz, marginal deformations, orbifolds, open spin chains.



1. Introduction

The fascinating integrable structures of the $\mathcal{N}=4$ SYM theory, reviewed in other contributions to this collection, highlight the unique position that this theory occupies among quantum field theories in four dimensions. Planar integrability is just the latest addition to a long list of remarkable properties, such as exact (perturbative and non-perturbative) conformal invariance, Montonen–Olive S-duality,

as well as the celebrated AdS/CFT correspondence, stating its equivalence to IIB string theory on the $\text{AdS}_5 \times \text{S}^5$ background.

The price to pay for these unique features is that the theory is highly unrealistic, and arguably very far removed from QCD, the theory of the strong interactions. It is thus natural to wonder whether the recent great advances in the understanding of $\mathcal{N}=4$ SYM are of any use when studying less supersymmetric theories. In the specific context of AdS/CFT integrability, one can ask whether there exist other four-dimensional field theories with similar integrability structures, where the techniques developed in the $\mathcal{N}=4$ SYM context can also be applied.

In this short review, we will attempt to provide a guide to the current state of affairs regarding AdS/CFT integrability in less supersymmetric situations. We will restrict ourselves to the very special class of four-dimensional supersymmetric field theories with similar finiteness properties to $\mathcal{N}=4$ SYM, which are therefore also superconformal.¹ We will see that, despite many similarities to the $\mathcal{N}=4$ SYM case, there also appear significant differences in the way integrability is manifested. Therefore, although there still is quite a long way to go from these theories to QCD, their study is worthwhile and can be expected to provide a useful stepping stone towards unraveling the implications of integrability in more realistic field theories.

2. The Marginal Deformations of $\mathcal{N}=4$ SYM

For any conformal field theory, it is interesting to study its space of *exactly marginal deformations*, all the ways to deform the theory preserving quantum conformal invariance. It has been known since the early eighties that $\mathcal{N}=4$ SYM admits $\mathcal{N}=1$ supersymmetric marginal deformations, with a non-perturbative proof given by Leigh and Strassler in 1995 [3] (where references to the earlier literature can also be found).

In $\mathcal{N}=1$ superspace language, the Leigh–Strassler deformations can be obtained purely by deforming the superpotential of the $\mathcal{N}=4$ SYM theory. The relevant part of the $\mathcal{N}=4$ SYM lagrangian is (with g being the gauge coupling)

$$\mathcal{L}_{\text{sup}} = \int d^2\theta \mathcal{W}_{\mathcal{N}=4}, \quad \text{where} \quad \mathcal{W}_{\mathcal{N}=4} = g \text{Tr}(X[Y, Z]). \quad (2.1)$$

Here X, Y and Z are the three adjoint chiral superfields of the $\mathcal{N}=4$ theory. It is not difficult to see that $\mathcal{W}_{\mathcal{N}=4}$ possesses an $\text{SU}(3) \times \text{U}(1)_R$ global invariance, the maximal part of the $\text{SU}(4)$ R-symmetry of the theory which can be made explicit in $\mathcal{N}=1$ superspace. Now consider the following more general $\mathcal{N}=1$ superpotential:

$$\mathcal{W}_{LS} = \kappa \text{Tr} \left(X[Y, Z]_q + \frac{h}{3} (X^3 + Y^3 + Z^3) \right) \quad (2.2)$$

¹We will thus not touch the topic of integrability in QCD, which is covered in [1] in this collection or discuss integrability in the three-dimensional ABJM theory, referring instead to [2].

where κ, q and h are a priori complex parameters and the q -commutator is defined as $[X, Y]_q = XY - qYX$. The $\mathcal{N}=4$ SYM theory can be recovered by the choice $(\kappa, q, h) = (g, 1, 0)$. Generically, the only continuous symmetry of \mathcal{W}_{LS} is the $U(1)_R$, which is always present in an $\mathcal{N}=1$ superconformal theory. When $h=0$, it is standard to express $q = \exp(2\pi i\beta)$ and call this case the β -deformation.² Here, apart from $U(1)_R$, \mathcal{W}_{LS} has an extra $U(1) \times U(1)$ symmetry acting by phase rotations on the scalars. The β -deformation with β real (i.e. q a phase) is variously known as the real- β or the γ -deformation.

There are several more marginal terms one could add to the superpotential; however (2.2) is special in that it preserves an important set of discrete symmetries:

$$\begin{aligned} \text{(a)} \quad & X \rightarrow Y, Y \rightarrow Z, Z \rightarrow X, \\ \text{(b)} \quad & X \rightarrow X, Y \rightarrow \omega Y, Z \rightarrow \omega^2 Z \end{aligned} \tag{2.3}$$

with ω a third root of unity. The first of these symmetries is particularly crucial, because it ensures that all scalar anomalous dimensions are equal. This observation allowed Leigh and Strassler to argue that finiteness imposes a *single* complex constraint on the four couplings (g, κ, q, h) , implying the existence of a three-dimensional parameter space of finite gauge theories. On this space, the superpotential (2.2) is thus *exactly* marginal. The finiteness constraint can be calculated at low loop orders, but its exact form is unknown, and its determination, even in the planar limit, would be a major step in our understanding of superconformal gauge theory. Here, we give it at one loop (see e.g. [4] for a derivation):

$$2g^2 = \kappa \bar{\kappa} \left[\frac{2}{N^2} (1+q)(1+\bar{q}) + \left(1 - \frac{4}{N^2}\right) (1+q\bar{q} + h\bar{h}) \right]. \tag{2.4}$$

Note that the constraint simplifies considerably in the planar ($N \rightarrow \infty$) limit, and that for the real β -deformation it reduces to $g^2 = \kappa \bar{\kappa}$, precisely the same as that for $\mathcal{N}=4$ SYM. It has been shown [5] that in this real- β case, the one-loop constraint is not modified at any higher order in the perturbative expansion. This is a first indication that, in the planar limit, the theory will share many of the properties of $\mathcal{N}=4$ SYM, including, as we will see, integrability.

2.1. THE GRAVITY DUAL OF THE β -DEFORMATION

If the $\mathcal{N}=4$ SYM theory admits exactly marginal deformations, the same must be true for its dual gravity background. Since the deformations preserve the conformal group, the AdS_5 factor of the geometry will not be affected, but we expect the S^5 part to be deformed, reflecting the reduction of the R-symmetry group to a subgroup of $SU(4)_R \simeq SO(6)$. For the β -deformation, the metric of this deformed S^5 was found by Lunin and Maldacena [6]. Focusing first on the *real- β* deformation,

²There exist several other conventions in the literature, related by relabellings of β and κ .

these authors showed that it can be obtained from S^5 by a sequence of T-duality, angle shift and T-duality, called a TsT transformation. To make this a bit more explicit, let us start with the five-sphere embedded in \mathbb{R}^6 as $\bar{X}X + \bar{Y}Y + \bar{Z}Z = 1$, and parametrise

$$X = \cos \gamma e^{i\varphi_1}, \quad Y = \sin \gamma \cos \psi e^{i\varphi_2}, \quad Z = \sin \gamma \sin \psi e^{i\varphi_3} \quad (2.5)$$

to obtain the five-sphere metric in terms of angle coordinates

$$ds^2 = d\gamma^2 + \cos^2 \gamma d\varphi_1^2 + \sin^2 \gamma (d\psi^2 + \cos^2 \psi d\varphi_2^2 + \sin^2 \psi d\varphi_3^2). \quad (2.6)$$

There are three explicit $U(1)$ isometries related to the angles φ_i , with the diagonal shift $\varphi_i \rightarrow \varphi_i + a$ corresponding to the $U(1)_R$, which is required by $\mathcal{N} = 1$ superconformal invariance. The TsT procedure starts by T-dualising along the other two isometry directions, then shifting the dual angles as $\tilde{\varphi}_2 \rightarrow \tilde{\varphi}_2 + \beta \tilde{\varphi}_3$, and finally T-dualising back. This breaks the $SO(6)$ implicit in (2.6) and results in a geometry preserving just a $U(1)^3$ isometry group, the right amount of symmetry for the dual to the β -deformation. We refer to [6,7] for more details and for the explicit IIB solution.³ Starting from the real- β background, a sequence of S-dualities leads to the dual of the complex- β deformation [6]. However, the geometry dual to the most general deformation (with $h \neq 0$) is still unknown.

2.2. THE REAL- β DEFORMATION AND INTEGRABILITY

In this section, we focus on the real- β deformation, which has received the most attention in literature. The integrability properties of this theory were first investigated in [9], where it was shown that the one-loop planar dilatation generator in the two-scalar $SU(2)_\beta$ sector corresponds to the hamiltonian of the integrable XXX $SU(2)_\beta$ spin chain. This was extended to the $SU(3)_\beta$ sector in [10]. In the latter work it was also noted that a suitable site-dependent transformation can map the hamiltonian of the deformed theory to that of the undeformed one (i.e. $\mathcal{N} = 4$ SYM), but with *twisted* boundary conditions. Building on [6], where a simple star product was introduced to keep track of the additional phases appearing in the real- β theory compared to the undeformed case, the work [11] showed that given an undeformed R -matrix satisfying the Yang–Baxter equation, the twisted one will do so as well.⁴

The conclusion is that the real- β deformation is just as integrable as $\mathcal{N} = 4$ SYM. It should thus be possible to find a Bethe ansatz encoding the spectrum of the theory. This can indeed be done by introducing appropriate phases (“twisting”) in the $\mathcal{N} = 4$ SYM Bethe ansatz. In the $SU(2)_\beta$ sector, this was performed at one

³It should also be noted that for $\beta = 1/k$ (i.e. q being a k -th root of unity), the dual background is actually an $AdS_5 \times S^5 / \mathbb{Z}_k \times \mathbb{Z}_k$ orbifold [8].

⁴The effect of the twist on other algebraic structures of the theory, such as the Yangian (reviewed in [12]), was considered in [13].

loop in [10], at higher loops in [14], while the higher-loop twist for all sectors was obtained in [11]. For simplicity, here we display just the one-loop, $SU(2)_\beta$ sector case:

$$e^{-2\pi i\beta L} \left(\frac{u_k + i/2}{u_k - i/2} \right)^L = \prod_{j=1, j \neq k}^M \frac{u_k - u_j + i}{u_k - u_j - i}, \quad \prod_{k=1}^M \frac{u_k + i/2}{u_k - i/2} = e^{2\pi i\beta M} \quad (2.7)$$

where the second equation is the cyclicity condition. Very recently, [15] provided a deeper understanding of the all-loop twisted Bethe equations by deriving them from a suitable *Drinfeld-twisted* S-matrix combined with a twist of the boundary conditions.

Integrability and the LM Background. Integrability of the IIB Green–Schwarz sigma model in the real- β deformed case was demonstrated in [7] by explicit construction of a Lax pair for the LM background. A Lax pair was also constructed for the pure-spinor sigma model in [16]. Therefore, just as in the undeformed case (reviewed in [17,18]), one can attempt to compare gauge theory results with strong coupling ones by considering semiclassical strings moving on the LM background. This was done in [19,20], with the construction of several semiclassical string solutions, which were matched to specific configurations of roots of the twisted Bethe ansatz. Their energies precisely agree with the gauge theory anomalous dimensions.

Giant magnons [21] on the LM background were constructed in [22,23], with the latter considering multispin configurations, while [24] considered more general rigid string solutions on the S_γ^3 subspace, with the giant magnons and spiky strings as special cases. The first finite-size correction to the giant magnon energy was computed in [25] and takes the following form:⁵

$$E - J = 2g \sin \frac{p}{2} \left(1 - \frac{4}{e^2} \sin^2 \frac{p}{2} \cos \left[\frac{2\pi(n - \beta J)}{2^{3/2} \cos^3 \frac{p}{4}} \right] e^{-\frac{J}{g \sin p/2}} + \dots \right) \quad (2.8)$$

where n is the unique integer for which $|n - \beta J| \leq \frac{1}{2}$. This expression exhibits the expected exponential falloff, but the momentum dependence is highly unusual and indeed reproducing it from the Lüscher correction techniques discussed in [27] is still an open problem.

Wrapping corrections. To calculate wrapping corrections to the spectrum (due to interactions whose span is greater than the length of the spin chain), one needs to go beyond the asymptotic Bethe ansatz. It turns out that the techniques developed for $\mathcal{N}=4$ SYM (reviewed in [27–29] in this collection) can be applied with relative ease to the β -deformed theory. In particular, it was argued in [30] that the β -deformation is described by the *same* Y-system as $\mathcal{N}=4$ SYM. The β parameter arises by appropriately modifying the asymptotic (large L) solution, exploiting

⁵Recently, this result was extended to the case of dyonic, or two-spin, giant magnons [26].

the freedom to twist it by certain complex numbers. The authors of [30] showed that this procedure correctly reproduces the higher-loop asymptotic Bethe ansatz of [11] (for all sectors, and more general twists) and derived generalised Lüscher formulae for generic operators in the β -deformed theory.

Turning to results for specific operators, an interesting feature of the β -deformed theory compared to $\mathcal{N}=4$ SYM (first noted in [31]) is that one-impurity operators

$$O_{1,L} = \text{Tr} \phi Z^{L-1}, \quad \phi \in \{X, Y\} \quad (2.9)$$

are not protected by supersymmetry and thus acquire anomalous dimensions. Because of this, the real- β theory provides an excellent setting for the perturbative study of wrapping effects for short operators (reviewed in [32] and also in [33]): Apart from the calculations being simpler (compared to two-impurity cases like the Konishi operator), it also allows for a clean separation of the effects of wrapping from those due to the dressing factor, since the latter does not contribute at all for these states. Wrapping effects for such states, at critical wrapping (where the loop level equals the length of the operator) have been computed up to 11 loops in [34,35] (who also provided a recursive formula for calculating them at higher loop orders), and have recently been reproduced in [30] via the twisted solution to the Y-system and in [36] using generalised Lüscher formulae.⁶

A very special case arises when $\beta = \frac{1}{2}$ and one considers even length operators. Then the (higher-loop version of the) Bethe ansatz (2.7) becomes the same as that for $\mathcal{N}=4$ SYM, apart from a sign in the cyclicity constraint. In this case, a closed (instead of iterative) form for the critical wrapping correction at any L was found in [37]. Also working at $\beta = \frac{1}{2}$, and using the Lüscher techniques reviewed in [27], the work [38] calculated the wrapping corrections to the single-impurity operator with $L=4$ up to five loops, i.e. the first two nontrivial orders:⁷

$$\begin{aligned} \Delta_w^{4\text{-loop}} &= 128(4\zeta(3) - 5\zeta(5)), \\ \Delta_w^{5\text{-loop}} &= -128(12\zeta(3)^2 + 32\zeta(3) + 40\zeta(5) - 105\zeta(7)). \end{aligned} \quad (2.10)$$

The four-loop result agrees with the perturbative calculations in [34], while at the time of writing there does not exist a perturbative result for the five-loop (subleading wrapping) correction. In [39], the wrapping corrections at $\beta = \frac{1}{2}$ were used to argue for the equivalence [suggested by (2.7) for the asymptotic spectrum] of the full (non-asymptotic) spectra of the β -deformed theory at β and $\beta + 1/L$, with the recent leading finite-size results of [36] in complete agreement with this.

Moving on to the two $L=4$ two-impurity operators ($\text{Tr}(XYXY)$ and $\text{Tr}(XXYY)$), their anomalous dimensions were found to four-loop order through explicit

⁶Note that no TBA equations (see [28]) have yet been constructed for the β -deformed theory.

⁷Here Δ_w denotes the wrapping contribution to the anomalous dimension, i.e. the difference of the exact result from the asymptotic one.

calculation in [34,35].⁸ They were also computed and matched (for arbitrary β) using Lüscher methods in [40] as well as through the Y-system in [30]. Essentially the same calculation (starting from a slightly different perspective) was performed in [36].

Finally, there exists at the moment a prediction [36], coming from Lüscher methods, for the leading finite-size correction to the energy for one- and two-impurity $sl(2)$ -sector operators, which is yet to be checked by explicit perturbative calculations.⁹

Amplitudes. As reviewed in [43], one manifestation of integrability in the $\mathcal{N}=4$ SYM context is the appearance of iterative structures (which go by the name of the *BDS conjecture*) expressing multiloop amplitudes in terms of one-loop ones. One might therefore expect that amplitudes in the real- β theory satisfy such relations as well. It has indeed been shown [44] that all (MHV and non-MHV) planar amplitudes in the real- β theory are proportional to the corresponding $\mathcal{N}=4$ SYM ones, differing only in phases affecting the tree-level part of the amplitude. Thus the BDS conjecture for $\mathcal{N}=4$ SYM extends straightforwardly to the real- β deformation. At strong coupling (where the tree-level part is not visible), gluon amplitudes in the real- β theory have been shown to equal those for $\mathcal{N}=4$ SYM [45].

2.3. INTEGRABILITY BEYOND THE REAL- β DEFORMATION?

In the above, we focused on a very special subset of the marginal deformations, those where $h=0$, while q is just a phase. One can ask whether there exist other integrable values of the parameters (q, h) . Keeping $h=0$ but passing to complex β , the hamiltonian in the two-holomorphic scalar sector is that of the $SU(2)_q$ XXZ model and is thus integrable [9]. However, this generically ceases to be the case beyond this simple sector [10]: contrary to initial expectations, the one-loop hamiltonian in the *full* scalar sector is not that of the integrable $SO(6)_q$ XXZ spin chain, but of a type not matching any known integrable hamiltonians. It was also shown in [10] that, unlike the real- β case, it is not possible to transfer the deformation to the boundary conditions by site-dependent redefinitions.¹⁰ The conclusion was that the one-loop hamiltonian for the *generic* LS deformation is not integrable.¹¹

⁸Note that in $\mathcal{N}=4$ SYM one linear combination of these operators is BPS, while the other is a descendant of the Konishi operator. However, in the deformed theory, there is no BPS combination.

⁹See also [41,42] for more recent results on wrapping for twist operators in the β -deformed theory.

¹⁰Note, furthermore, that the star-product techniques of [6] do not apply beyond real β , their naive extension giving rise to a non-associative product.

¹¹The question of whether higher-loop integrability persists in the (all-loop closed) $SU(2)_q$ sector remains unanswered, with some progress towards constructing the required higher charges reported in [46].

Nevertheless, as demonstrated in [47], there *do* exist certain special choices of parameters for which the one-loop hamiltonian is integrable:

$$(q, h) = \left\{ (0, 1/\bar{h}), \left((1 + \rho) e^{\frac{2\pi i m}{3}}, \rho e^{\frac{2\pi i n}{3}} \right), \left(-e^{\frac{2\pi i m}{3}}, e^{\frac{2\pi i n}{3}} \right) \right\}. \quad (2.11)$$

Some of these choices were also discovered via the study of amplitudes in [48]: They correspond to special cases where the one-loop planar finiteness condition (2.4) does not receive corrections at higher loops, similarly to the real- β deformation.

In [49], a unifying framework for all these integrable cases was proposed: Their corresponding one-loop hamiltonians can be related to the real- β case by *Hopf twists*. These are a way of modifying the underlying R -matrix, leaving the integrability properties unaffected. Since (as shown in [11]) the real- β hamiltonian is itself related to the undeformed hamiltonian by such a twist, all these integrable cases can be seen to be nothing but Hopf-twisted $\mathcal{N}=4$ SYM.

Another special (one-loop) integrable sector beyond real β was found in [50]. It is an $SU(3)$ sector composed of two holomorphic and one antiholomorphic scalar, for instance $\{X, Y, \bar{Z}\}$. The hamiltonian in this sector actually turns out to be $XXZ SU(3)_q$, the standard (integrable) q -deformation of $SU(3)$. However, this sector is not closed beyond one loop, complicating the discussion of higher-loop integrability.

Apart from these special cases, the deformed hamiltonian is not integrable. An intuitive explanation for this [14] is that the construction of the dual gravity background for the complex β deformation involves a sequence of S-duality transformations on the LM background [6]. The strong–weak nature of S-duality means that the intermediate stages involve interacting strings, which (as reviewed in [51]) are unlikely to preserve integrability.

A more direct argument for this lack of integrability was recently given in [49], who noticed that there exists a Hopf algebraic deformation of the global $SU(3)$ R-symmetry group of the $\mathcal{N}=4$ theory under which the full LS superpotential (2.2) is invariant. However, this symmetry, defined through a suitable R -matrix depending on the deformation parameters q and h is not a “standard” quantum-group deformation of $SU(3)$. In particular, apart from the special cases discussed above, the (q, h) - R -matrix does *not* respect the Yang–Baxter equation, and consequently the corresponding Hopf algebra is not quasitriangular. Thus the construction (reviewed in [52]) of the transfer matrices and eventually of the integrable S-matrix of the theory would not be expected to go through.

2.3.1. More General TsT Transformations

A different way of generalising the Lunin–Maldacena solution is by performing TsT transformations along all three available $U(1)$ ’s within the S^5 [7]. Since one of these corresponds to the R-symmetry, this procedure will completely break the superconformal symmetry. However, it can be shown that these γ_i -deformations

preserve integrability: The Lax pair construction goes through in this case as well [7] and in [53] it was argued that the Green–Schwarz action on TsT-deformed backgrounds is the same as the undeformed one, but with twisted boundary conditions. In [54], string energies were shown to match anomalous dimensions coming from the corresponding three-phase deformed spin chain. In addition, [55] showed that the action for three-spin strings in the “fast string” limit admits a Lax pair and thus that string motion is integrable. The integrability properties of the γ_i theories are thus very similar to the real- β case.¹²

One can also perform integrability-preserving TsT transformations along one AdS_5 and one S^5 direction, leading to dipole-type deformations in the gauge theory [57], as well as purely along the AdS_5 directions, leading to a noncommutative deformation on the gauge theory side [58] (see [59] for a review of the latter case).

2.3.2. Non-field Theory Deformations

As was first noted in [10], there exist integrable deformations of the algebraic structures at the $\mathcal{N} = 4$ SYM point, which do not have a good field theory interpretation, in the sense of arising as the one-loop hamiltonian of a deformed field theory. A large class of such deformations was presented in [11]. More recently, q -deformations of the $\mathfrak{psu}(2|2) \times \mathbb{R}^3$ algebra were considered in [60]. The role of such deformations in the AdS/CFT correspondence is not well understood, but their further study can be expected to provide a deeper understanding of the $\mathcal{N} = 4$ integrable structures by embedding them in a larger framework.¹³

3. Integrability and Orbifolds of $\mathcal{N} = 4$ SYM

Besides adding marginal operators, another way of obtaining CFTs with less supersymmetry from $\mathcal{N} = 4$ SYM is by orbifolding [61,62]. On the gauge theory side, this involves picking a discrete subgroup Γ of the R -symmetry group and performing the following projection on the fields (here for $\Gamma = \mathbb{Z}_M$):

$$\phi \rightarrow \omega^{s_\phi} \gamma \phi \gamma^{-1}, \quad \text{where } \gamma = \text{diag}(1, \omega, \omega^2, \dots, \omega^{M-1}), \quad \omega = e^{\frac{2\pi i}{M}}. \quad (3.1)$$

The integer s_ϕ is related to the $SU(4)_R$ charge of the field ϕ . The resulting theories have a quiver structure: starting with an $U(MN)$ theory, one obtains a product gauge group $U(N)_1 \times \dots \times U(N)_M$ with matter fields in bifundamental representations. The amount of supersymmetry preserved can be $\mathcal{N} = 2, 1$ or 0, depending on the subgroup of $SU(4)_R$ on which Γ acts: $SU(2)$, $SU(3)$ or the whole $SU(4)_R$, respectively. For instance, a choice of s_ϕ resulting in an $\mathcal{N} = 2$ theory is $(s_X, s_Y, s_Z) = (1, -1, 0)$.

¹²As was the case for the β deformation, it is possible to generalise the γ_i -deformations to complex values of γ_i while preserving integrability, but only for very special values, similar to (2.11) [56].

¹³For a simple illustrative example of how considering a deformed theory can clarify aspects of the undeformed one, the reader is referred to section 1.2 of [52] in this collection.

One can easily keep track of gauge invariant operators by writing them in terms of the unorbifolded fields but with suitable phases inserted in the trace:

$$\text{Tr}(\gamma_m XYXZ \cdots), \quad \text{where } \gamma_m = \text{diag}(1, \omega^m, \dots, \omega^{(M-1)m}), \quad m = 1, \dots, M-1. \tag{3.2}$$

Operators for different choices of m do not mix with each other and correspond to different *twisted sectors* on the string side ($m=0$ being the untwisted sector). It is easy to see that the parent and orbifolded theory will only differ by additional phases in the Bethe equations, as well as a modification of the cyclicity condition. The one-loop Bethe equations in various $SU(2)$ sectors were considered in [63], while their structure for the full scalar sector was derived in [64], who also argued that the higher-loop $\mathcal{N}=4$ SYM equations can easily be adapted to the orbifold case.¹⁴ For the (X, Y) m -twisted $SU(2)$ sector, the one-loop equations take the form

$$e^{-\frac{4\pi im}{M}} \left(\frac{u_k + \frac{i}{2}}{u_k - \frac{i}{2}} \right)^J = \prod_{j \neq k}^K \frac{u_k - u_j + i}{u_k - u_j - i}, \quad \prod_{k=1}^K \left(\frac{u_k + \frac{i}{2}}{u_k - \frac{i}{2}} \right) = e^{\frac{2\pi im}{M}}. \tag{3.3}$$

Note the strong similarity to the Bethe ansatz (2.7) for the β -deformation. The Bethe ansatz for more general (e.g. non-abelian) orbifolds has been presented in [65].

On the string side, one considers an $AdS_5 \times S^5/\mathbb{Z}_M$ background,¹⁵ constructed via the following identifications (here for an $\mathcal{N}=2$ orbifold):

$$(X, Y, Z) \sim (e^{\frac{2\pi i}{M}} X, e^{-\frac{2\pi i}{M}} Y, Z). \tag{3.4}$$

An analysis of two-spin semiclassical strings on this and more general backgrounds was performed in [63] and their energies were successfully compared to the corresponding solutions of the orbifolded Bethe ansatz above.

An advantage of the orbifold theory compared to the parent one is that a *single* giant magnon is a physical state. This was used in [67] to settle an issue of gauge non-invariance (dependence of the magnon energy on the light-cone gauge fixing parameter, once finite-size effects are considered), which had previously arisen in the $AdS_5 \times S^5$ case [68]. It was later argued that single magnons in $\mathcal{N}=4$ SYM can always be thought of as living on the orbifolded theory [69]. Recently, TBA equations and wrapping effects (up to next-to-leading order) were considered for a particular orbifold theory in [36].

Another interesting application of orbifold theories is that, having a new parameter M , one can consider novel scaling limits. One such limit produces the “winding state” [70], where one starts with a string winding around an S^3/\mathbb{Z}_M in an $\mathcal{N}=2$ orbifold and takes $M \rightarrow \infty$ while also taking large J , keeping M^2/J finite.

¹⁴These authors also exhibited the Bethe equations for a *combination* of orbifolding and twisting.

¹⁵Integrability for $AdS_5 \times S^5/\mathbb{Z}_p \times \mathbb{Z}_q$ orbifolds has been considered in [66].

In [71], finite-size corrections to this state, as well as to orbifolded circular strings, were calculated up to order $1/J^2$ and shown to match with the Bethe ansatz results. In a related $M \rightarrow \infty$, BMN-type limit [72], the first finite-size corrections to two-impurity operators in the $\mathcal{N}=2$ theory were computed in [73], both directly using the dilatation operator (to two loops) and using the higher-loop version of the twisted Bethe ansatz 3.3. They were shown to agree with each other and, given the appropriate choice of dressing factor, with the dual pp-wave string result, calculated using DLCQ methods (see [74] for related earlier work).

Starting from the $\mathcal{N}=2U(N) \times U(N)$ quiver theory, one can move away from the orbifold point by varying the two gauge couplings independently, while preserving superconformal invariance. In [75] this was shown to break integrability, but in the extremal case where one of the two couplings vanishes (and we obtain an $SU(N)$ gauge theory with $N_f=2N$ flavors), it appeared that integrability might be recovered. This result, if confirmed, would provide a first example of an integrable theory in the Veneziano limit ($N, N_f \rightarrow \infty$ with N/N_f constant) instead of the usual 't Hooft limit. Recently, [76] considered magnon propagation on such interpolating non-integrable chains.

On the amplitude side, it is known that orbifold theories are planar equivalent to the parent theory to all orders in perturbation theory [77]. Thus, the BDS iterative conjecture is expected to immediately transfer to the orbifold theories.

3.1. OTHER BACKGROUNDS

Apart from the orbifold theories discussed above, there exist several AdS/CFT setups with reduced supersymmetry in the literature, and one can ask whether integrability appears in those cases as well. Perhaps the best-known example of this kind [78] is constructed by taking the near horizon limit of a stack of D-branes situated at the tip of the *conifold*, a noncompact six-dimensional Calabi–Yau manifold, which can be written as a cone over the five-dimensional *Sasaki–Einstein* manifold known as $T^{1,1}$. The near horizon geometry of this system is $\text{AdS}_5 \times T^{1,1}$ and corresponds to an $\mathcal{N}=1$ superconformal $U(N) \times U(N)$ gauge theory with bifundamentals, which is an infrared limit of a \mathbb{Z}_2 orbifold theory of the type discussed above.

There has been intense activity in constructing semiclassical string solutions on $T^{1,1}$, as well as generalisations known as $T^{p,q}$, $Y^{p,q}$ and $L^{p,q,r}$ [79–86]. However, these conformal fixed points only appear at strong coupling and thus do not correspond to perturbatively finite field theories. It is therefore far from obvious that one should expect to find integrability. Indeed, no Lax pair construction is known for these backgrounds. Furthermore, as observed in [85] for $T^{1,1}$ and its β -deformed analogue, the dispersion relation for magnons and spiky strings is transcendental, in stark contrast to the $\text{AdS}_5 \times S^5$ case. This is a clear indication that integrability, if it appears at all, would have to do so in a much more complicated way than in $\mathcal{N}=4$ SYM. On the other hand, it was shown in [87] that,

for the cases mentioned above, the bosonic sector in the near-flat space limit [88] is the same as for S^5 . Thus the full sigma models do at least possess an integrable subsector.

4. Open Spin Chain Boundary Conditions

One can also investigate integrability in a less supersymmetric setting by considering systems involving spin chains with *open* boundary conditions. This clearly signals the presence of open strings, and therefore D-branes, on the dual string side. After reviewing some universal aspects of open spin chains, we will proceed to discuss several different situations where they make an appearance in the AdS/CFT context.

As reviewed in [52] in this collection, in the algebraic approach to integrability for closed spin chains one begins by considering the RTT relations for the monodromy matrix, defined in terms of an R -matrix satisfying the Yang–Baxter equation:

$$R_{12}(u, v)R_{13}(u, w)R_{23}(v, w) = R_{23}(v, w)R_{13}(u, w)R_{12}(u, v). \tag{4.1}$$

For open chains, these equations still hold, but have to be supplemented (at each boundary) with the *reflection*, or *boundary Yang–Baxter* equation [89]:

$$R_{12}(u, v)K_1(u)R_{21}(v, -u)K_2(v) = K_2(v)R_{12}(u, -v)K_1(u)R_{21}(-u, -v). \tag{4.2}$$

Here, the $K_{1,2}(u)$ are known as the boundary reflection matrices. See e.g. [90] for a discussion of various boundary conditions, and the corresponding reflection matrices, for $\mathfrak{sl}(n)$ and $\mathfrak{sl}(m|n)$ spin chains, as well as further references to the open-chain literature. In the special case where the boundary conditions preserve the same $\mathfrak{gl}(n)$ symmetry as the bulk chain (which is often not the case in the setups to be considered below), the general form of a perturbatively long-range integrable $\mathfrak{gl}(n)$ spin chain with open boundary conditions was given in [91].

The generic structure of any putative open string Bethe ansatz is

$$e^{2ip_k L} = B_1(p_k)B_2(p_k) \prod_{j=1, j \neq k}^M S_{jk}(p_j, p_k)S_{kj}(-p_j, p_k) \tag{4.3}$$

where the S_{jk} are the bulk S-matrices, and $B_{1,2}$ are the boundary reflection matrices. To understand the above structure (see also [91] for a nice exposition), note that a given excitation moving with momentum p_i will scatter with a number of other excitations, reflect from the boundary, scatter with the other excitations again (but with opposite momentum) and reflect from the second boundary before finally returning to its original position. Assuming that the bulk theory is integrable, the question of integrability hinges on the precise form of the boundary matrices $B_{1,2}$.

In the closed-chain case, the Bethe ansatz is normally accompanied by a cyclicity condition (which on the string side arises from the closed-string level-matching

condition). However, this is absent for the open-chain case. An immediate consequence of this is that single-impurity states are physical, even for non-zero momentum.

As in the closed spin-chain case, new effects arise when considering long-range *short* open spin chains, in particular *spanning* terms, which are the analogues of the closed-chain wrapping interactions for finite-length open spin chains. Little is known at present about their structure from the field theory side, though a study of such terms in [91] suggests that they are not strongly constrained by integrability, which would therefore appear to lose some of its predictive power for short chains.

4.1. OPEN SPIN CHAINS WITHIN $\mathcal{N}=4$ SYM

Although this review is mainly concerned with integrable theories beyond $\mathcal{N}=4$ SYM, there exist several interesting cases where integrable open spin chains arise *within* the $\mathcal{N}=4$ theory itself. We will thus first discuss this class of theories, which arise through the consideration of non-trivial backgrounds within $\mathcal{N}=4$ SYM.

4.1.1. Open Strings on Giant Gravitons

The first case of this type is that of open strings ending on maximal giant gravitons [92] in $\text{AdS}_5 \times S^5$. These are $D3$ -branes wrapping three-cycles inside the five-sphere. The gauge theory picture is that of an open spin chain word attached to a baryon-like (determinant) operator in $\mathcal{N}=4$ SYM, formed out of one of the scalars in the theory, here denoted Φ_B :

$$\epsilon_{i_1 \dots i_N} \epsilon^{j_1 \dots j_N} (\Phi_B)_{j_1}^{i_1} \dots (\Phi_B)_{j_{N-1}}^{i_{N-1}} (\Phi_{k_1} \Phi_{k_2} \dots \Phi_{k_L})_{j_N}^{i_N}. \quad (4.4)$$

In the large- N limit, the determinant part becomes very heavy and has no dynamics of its own, so this system behaves as a spin chain with open boundary conditions.

The one-loop hamiltonian for this type of chain was considered in [93] and shown to be integrable. It was further investigated at two loops in [94], with the final two-loop result, in the $\text{SU}(2)$ sector, given in [95]:

$$H = (2g^2 - 8g^4) \sum_{i=1}^{\infty} (I - P_{i,i+1}) + 2g^4 \sum_{i=1}^{\infty} (I - P_{i,i+2}) \\ + (2g^2 - 4g^4) q_1^{\Phi_B} + 2g^4 q_2^{\Phi_B} \quad (4.5)$$

with $q_i^{\Phi_B} = 1$ if $\Phi_i = \Phi_B$ and 0 otherwise. The first two terms are the same as the bulk hamiltonian, the third is the naive boundary contribution coming from all the derivatives in the dilatation operator acting outside the determinant, while the last term comes from one of the derivatives acting *on* the determinant. This term is naively $1/N$ suppressed, but survives in the planar limit, the suppression being compensated by the fact that it can act on any of the $N-1$ terms

in the determinant. As shown in [21], the hamiltonian (4.5) is consistent with integrability. On the string side, [96] constructed non-local conserved sigma model charges for classical open strings ending on maximal giant gravitons in the full bosonic sector, thus providing strong supporting evidence for all-loop integrability of the maximal graviton system.

For non-maximal giant gravitons (which correspond to sub-determinant-type operators in the gauge theory), the open spin chain becomes dynamical, in the sense that the number of sites can vary, even at one-loop level. This case was investigated in [97,98], where it was argued that the formalism of *Cuntz chains* provides a better description than the standard spin chain and some (numerical) evidence for integrability was provided. However, on the string side, the appearance of extra conditions hinders the construction of non-local sigma model charges [96]. Thus, the prospects for integrability in this case do not look particularly good.¹⁶

Reflecting Magnons. Giant magnons ending on maximal giant gravitons were considered in [95]. One can, without loss of generality, choose to consider open chains made up of a large number of Z fields, which, on the string side, correspond to semiclassical strings with a large angular momentum along the 5–6 plane within S^5 . One can then consider two different orientations of the giant magnon relative to this plane.

The $Y=0$ magnon: In this case, we choose the $D3$ -brane to wrap the three-sphere defined by $Y=0$, which corresponds to the operator $\det Y$ in the gauge theory. Attaching an open spin chain to this determinant, we are led to an operator of the form:

$$\epsilon_{i_1 \dots i_N} \epsilon^{j_1 \dots j_N} Y_{j_1}^{i_1} \dots Y_{j_{N-1}}^{i_{N-1}} (Z \dots Z \chi Z \dots Z)_{j_N}^{i_N}. \quad (4.6)$$

Here χ stands for any impurity, though it will need to be a Y field if we wish to stay within the $SU(2)$ sector. As explained in [95], this configuration has no boundary degrees of freedom, and there is a unique vacuum state. The boundary preserves an $SU(1|2)^2$ out of the bulk $SU(2|2)$ symmetry. The boundary scattering phase was found in [100], while commuting open-chain transfer matrices, necessary for the construction of the Bethe ansatz, were derived in [101].¹⁷ In [104], it was shown that part of the bulk Yangian symmetry persists for boundary scattering and can be used to determine the bound-state reflection matrices. This boundary Yangian has been further discussed in [105]. The higher-loop Bethe ansatz for this class of operators was proposed in [106], see also [107] for an earlier discussion. A different derivation, which agrees with the one above, is in [108].

¹⁶Nevertheless, integrability was recently demonstrated for giant magnons scattering off $Y=0$ non-maximal gravitons [99], indicating that integrable subcases do exist.

¹⁷The works [102,103] generalised the q -deformed S-matrix of [60] to the $Y=0$ and $Z=0$ magnon context, and studied open-chain transfer matrices for these cases.

The $Z=0$ magnon: Here we consider a $D3$ -brane wrapping the three-sphere defined by $Z=0$, which is dual to the gauge theory operator $\det Z$. The open chain is still made up mainly of Z s, but it is easy to see that they cannot be attached directly to the determinant: Such a configuration would factorise into a determinant plus a trace. To obtain a nontrivial open spin chain, there need to be impurities (fields other than Z) stuck to the boundary:¹⁸

$$\epsilon_{i_1 \dots i_N} \epsilon^{j_1 \dots j_N} Z_{j_1}^{i_1} \dots Z_{j_{N-1}}^{i_{N-1}} (\chi Z \dots Z \chi'' Z \dots Z \chi')_{j_N}^{i_N}. \quad (4.7)$$

In this case, there *are* boundary degrees of freedom, which (like the bulk magnon) fall into representations of $SU(2|2)^2$ [95]. There are thus 16 states living on each boundary, which were identified on the string side in [109], by considering fermionic zero modes around the finite-size string solution for an open string ending on the $Z=0$ graviton.¹⁹ The boundary scattering phase was found in [110]. One notable feature of the $Z=0$ case is the presence of poles in the reflection amplitude not corresponding to bound states, the origin of which was explained in [111]. As for $Y=0$, a boundary R -matrix was proposed in [95]; however, it did not directly satisfy the BYBE. This was reconsidered in [112], who found a suitable basis where the boundary R matrix does satisfy the BYBE. The higher-loop nested Bethe ansatz in this case was constructed in [113].

Finite-Size Effects. Considerable recent activity in the $\mathcal{N}=4$ SYM context has centered around understanding finite-size effects, or wrapping interactions on the gauge theory side (see the reviews [27–29,32] in this collection). There is an analogous formalism for the open-chain case, which was used in [114] to compute Lüscher-type corrections to open strings ending on giant gravitons (for vacuum states) and compare with explicit gauge theory results. The anomalous dimension of the $Y=0$ vacuum chain was shown to vanish (a result expected by supersymmetry), while in the $Z=0$ case it was non-trivial. The Lüscher formulae of [114] were extended to the multiparticle case in [115], allowing the computation of finite-size corrections to one-excitation states in the $Y=0$ case and leading to an explicit prediction to be checked by future gauge theory perturbative calculations. The analogous computation for the (more challenging) $Z=0$ brane has not yet been performed. Furthermore, no TBA or Y-system equations are available at present for the boundary case.

Classical solutions for finite-size magnons on $Z=0$ gravitons (generalising those in [109]) can be found in [116].

Other Graviton–Magnon Combinations. The work [117] studied open strings ending on giant gravitons in the AdS part of the geometry and, on the gauge side, identified the planar dilatation operator as the hamiltonian of an open $sl(2)$ spin chain.

¹⁸In the $SU(2)$ sector, all the χ s will have to be of the same type, e.g. Y fields.

¹⁹The string solution itself was previously found in an unpublished work by C. Ahn, D. Bak and S.J. Rey.

However, novel features such as a variable occupation number and continuous bands in the spectrum prevented a clear understanding of integrability in this case. Other configurations of strings on giant gravitons have been considered in [118] (in the BMN limit), as well as in [119], where gauge theory operators dual to a giant graviton/magnon bound state are proposed.

4.1.2. Operators with Very Large R -Charge

Giant gravitons are dual to baryonic operators in $\mathcal{N} = 4$ SYM, the dimensions of which grow linearly with N . One can consider other types of operators whose dimension grows as N^2 , which in the simplest case are of the form $(\det Z)^M$ (with $M \sim N$) but more generally are described by Schur polynomial operators related to the Young diagram encoding their symmetries. On the dual gravity side, the number of $D3$ -branes is so large that it is no longer possible to ignore backreaction, and this modifies the AdS geometry into an LLM-type background. Strings “attached” to the above operators²⁰ have recently been considered from the gauge theory side in [120]. It is possible to integrate out the effect of the background and construct an effective dilatation operator, which is integrable in a certain limit. Interestingly, this limit includes non-planar diagrams between the trace operator and the background. Although, as reviewed in [51], truly non-planar contributions (acting on the trace operator by splitting and joining) are still expected to spoil integrability, this novel integrable limit of $\mathcal{N} = 4$ SYM is still interesting and deserves further exploration.

4.1.3. Open String Insertions on Wilson Loops

In the absence of nontrivial background operators for the spin chain to end on, open string boundary conditions would not be gauge invariant. A way to avoid this problem is to consider open-chain insertions on Wilson loops [121]. As shown in that work, which considered such operators in the $SU(2)$ sector at one loop, the boundary conditions turned out to be purely reflective (Neumann). Thus the Bethe ansatz can be related to a closed-chain one by the method of images. The dual description of the Wilson loop (which has angular momenta on S^5 to account for the scalar insertions) was shown to reduce to “half” the standard closed folded string solution, whose energy precisely matches the Bethe ansatz computation. This setup is thus at least one-loop integrable (no higher-loop checks have been performed at present).

4.2. THEORIES WITH FUNDAMENTAL FLAVOR

One can also obtain open spin chains by extending the field content of $\mathcal{N} = 4$ SYM by adding flavors, i.e. fields in the fundamental representation of the gauge

²⁰Note that these are actually closed strings, since after the $D3$ -branes have backreacted there are no explicit open strings on the background.

group. Introducing such fields into the spectrum means that, apart from trace operators, one can construct gauge-invariant operators of the generic form:

$$\bar{Q}\Phi_{i_1}\Phi_{i_2}\cdots\Phi_{i_L}Q \quad (4.8)$$

where Q is one of the fundamental fields. This operator, having no cyclicity properties, will behave as an open chain. We will now review three distinct settings where these types of operators have been studied in an integrability context.

4.2.1. *The Orientifold Theory*

In this setup, one considers a D3–O7–D7 system, where one first performs an orientifold projection and then adds the required number of D7 branes (four, plus their mirrors) to cancel the orientifold charge. The result is $\mathcal{N}=2$ SYM with gauge group $\mathrm{Sp}(N)$, one hypermultiplet in the antisymmetric representation and four in the fundamental, which is known to be a finite theory.²¹ The $\mathcal{N}=2$ vector multiplet contains an adjoint chiral multiplet W , while the antisymmetric hypermultiplet has two chiral multiplets Z, Z' . The near-horizon geometry is that of an $\mathrm{AdS}_5 \times \mathrm{S}^5/\mathbb{Z}_2$ orientifold. Here, the \mathbb{Z}_2 acts as $Z \rightarrow -Z$ (or $\varphi_3 \rightarrow \varphi_3 + \pi$), leaving a fixed plane at $Z=0$. The worldsheet coordinate is also identified as $\sigma \rightarrow \pi - \sigma$.

Relatively few studies of integrability have been undertaken for this theory. The pp-wave spectrum was discussed in [123]. Several open-spinning-string solutions on the dual orientifold were considered in [124]. In [125], the one-loop hamiltonian for the $\mathrm{SU}(3)$ sector comprising W, Z, Z' was shown to be integrable and the corresponding one-loop Bethe ansatz constructed. In the $(Z, Z')\mathrm{SU}(2)$ sector, it is:

$$\left(\frac{u_k + \frac{i}{2}}{u_k - \frac{i}{2}}\right)^{2L} = \prod_{j \neq k}^K \frac{u_k - u_j + i}{u_k - u_j - i} \frac{u_k + u_j + i}{u_k + u_j - i} \quad (4.9)$$

Note that it is of the form (4.3). Applying the doubling trick, by means of which this Bethe ansatz can be related to a closed string one with the extra condition that the set of roots is symmetric under $u_j \rightarrow -u_j$, energies of two-spin open strings were successfully compared to gauge theory in [126]. At the time of writing, three-spin strings have not been compared, while the question of higher-loop integrability is still open.

4.2.2. *The D3–D7-Brane System*

Here, one considers $\mathrm{AdS}_5 \times \mathrm{S}^5$, with a D7-brane filling AdS_5 and wrapping an S^3 in S^5 . Unlike the case above, this theory is conformal only in the strict large- N

²¹A different type of orientifold which preserves $\mathcal{N}=4$ SYM but leads to gauge group $\mathrm{SO}(N)$ or $\mathrm{Sp}(N)$ was recently considered in [122], though in that case the focus was on non-planar corrections, the differences to $\mathrm{SU}(N)$ being relatively minor at the planar level.

limit, where the backreaction of the D7 brane can be ignored. On the gauge theory side, this corresponds to ignoring $1/N$ -suppressed processes with virtual fundamental flavors between bulk states (which would provide a non-zero contribution to the β -function).

The bulk hamiltonian is the same as for $\mathcal{N}=4$ SYM, so closed spin chains in this setup are automatically integrable. The one-loop open-chain hamiltonian is integrable as well, with trivial boundary terms [127]. The one-loop, SU(2)-sector Bethe ansatz is precisely the same as (4.9). The higher-loop reflection matrices for this case were studied in [128], where it was shown that integrability survives, largely thanks to the fact that the boundary respects the $\mathfrak{psl}(2|2) \times \overline{\mathfrak{psl}(2|2)}$ factorisation of the bulk theory. More recently, the work [129] extended these results by constructing the reflection matrices for boundary scattering of *bound states*.

On the gravity side, [96] showed integrability for the full bosonic sector by observing that the equations governing open string motion are practically the same as in the maximal giant graviton case discussed above. It is thus expected that this system exhibits higher-loop integrability.

4.2.3. Defect Theories

A different setup with fundamentals can be obtained by considering a D3–D5 system, with a single D5 sharing only three directions (say x^0, x^1 and x^2) with the stack of N D3 branes. The configuration thus has four Neumann–Dirichlet directions and preserves supersymmetry. Taking the D3-brane near-horizon limit, we obtain the usual $\text{AdS}_5 \times \text{S}^5$ geometry, but now the D5 brane wraps an $\text{AdS}_4 \times \text{S}^2$ in $\text{AdS}_5 \times \text{S}^5$. On the gauge theory side, we obtain $\mathcal{N}=4$ SYM coupled to a *defect* located at $x^3=0$. The matter content on the defect is a 3d SU(N) vector multiplet plus a 3d fundamental hypermultiplet (containing two chiral multiplets $q_{1,2}$).

As shown in [130], starting from a ground state of the form $\bar{q}_1 Z \cdots Z q_1$, there are two types of excitations one can consider: If the excitations are along the D5 brane, the boundary conditions are Dirichlet, which on the gauge theory translates to the boundary term being fixed. Otherwise, the string satisfies Neumann boundary conditions, which for the spin chain means that the boundary excitations are dynamical. The boundary state can flip from q_1 to q_2 , which effectively increases the length of the chain by 1. In both cases, the boundary matrix is trivial and the full bosonic sector is integrable at one loop. As before, there is no boundary phase in the SU(2) sector, though it does make an appearance in the SL(2) sector [131]. Spinning string solutions in this setup were considered in [132–134].

However, it was eventually understood that this one-loop integrability is an accident. The first indication came from the gravity side, when [96] showed that nonlocal charges could only be constructed in the SU(2) sector. Finally, by careful analysis of the symmetries, [128] constructed the all-loop reflection matrices (aspects of which were previously considered in [107]) with the result that they do *not* satisfy the BYBE.

5. Outlook

In this short review, we gave an overview of several different known ways of pushing integrability beyond the highly symmetric case of $\mathcal{N} = 4$ SYM. As we have seen, it is relatively easy to maintain integrability at the one-loop level in less supersymmetric (but still superconformal) situations, but all-loop integrability is a much more stringent requirement. Indeed, it appears that all non- $\mathcal{N} = 4$ SYM models where higher-loop integrability persists are really just $\mathcal{N} = 4$ SYM in disguise, in the sense that the bulk spin chain is undeformed, with differences arising only in the boundary conditions: twisted ones for the real- β deformations, orbifold ones for the quiver theories, and open ones for giant gravitons and theories with fundamentals.

This observation seems to reaffirm how special the $\mathcal{N} = 4$ SYM theory is, even within the already very restricted class of superconformal quantum field theories. On the other hand, the rich pattern of integrability breaking in the theories discussed above should help us better appreciate the implications (and limitations) of integrability for more realistic theories, in a more controllable setting than that of QCD. Even in those cases which *are* believed to be higher-loop integrable, there remain numerous open questions whose resolution can be expected to contribute to a deeper understanding of AdS/CFT integrability, and ultimately of the AdS/CFT correspondence itself.

Acknowledgements

I am grateful to N. Beisert, R. Janik, C. Kristjansen, T. Månsson, C. Sieg and A. Torrielli for useful discussions, comments and clarifications during the writing of this review. This work was supported by FNU through grant number 272-08-0329.

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