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SPECIAL ISSUE

Geostatistical Seismic Inversion: One Nugget from the Tróia Conference

Amílcar Soares[1]

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Abstract One of the characteristics that struck me most about André when I first met him in 1980 was the constant desire to expose, discuss and, above all, make accessible the emerging ideas of geostatistics. It was easy for André's first group of Stanford students to learn and have a great deal of confidence in what they learned. It was clearly a breakthrough in relation to the existing geostatistics teaching and research culture. It was in this context that the Fourth International Geostatistics Conference was organised in 1992 in Tróia, and André was the main driving force in the organisation of that event. It was an impressive showcase of the brilliant ideas of the newly created Stanford Center for Reservoir Forecasting. The resolution of an inverse problem with the integration of seismic in the high-resolution stochastic subsurface models was one of the most remarkable nuggets presented at that conference. Louis Bortoli, Francois Alabert, André Haas and André Journel are the authors of the seminal paper on geostatistical inversion. As an interface methodology between two areas of knowledge, it was not easy to get this new type of model accepted by the geophysics community, which at the time was dominated by deterministic models of inversion. This paper presents a summary of this path and the main geostatistical methods for seismic inversion that have been developed since then, and which have today become a set of practical tools for characterising mineral resources in the petroleum and mining industry.

Keywords Journel · Geostatistics · Seismic inversion

✉ Amílcar Soares
asoares@tecnico.ulisboa.pt

1 CERENA/DECivil, Pavilhão de Minas, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 Lisbon, Portugal

1 Geostatistical Culture After André Journel

In the 1970s, geostatistics, geomathematics, operational research and other applied mathematics disciplines had a tremendous impact on the mining industry and, in particular, on the mining engineering and geologist profiles.

For a young graduate in mining engineering like I was in the late 1970s, learning geostatistics was quite challenging—not because of the many new methods that appeared, but because learning them was an initiation ritual. Geostatistical research was practically confined to a small group, whose dissemination of emerging methods was not at the centre of their concerns. At that time, Fontainebleau (Centre de Geostatistique et de Morphologie Mathématique de Fontainebleau) was a must-visit for geostatistical courses and for the bookstore, which I visited many times. It was where we looked for news, usually in the form of unfriendly texts in the *Notes de Geostatistique*. Presentations of new methods at conferences usually employed a closed language that seemed designed to make life difficult for the audience. The exceptions were application case studies, mostly mining applications that were presented and published in the conference proceedings. As for the software, the environment was similar: those who wanted to calculate a variogram first had to develop and write the code. For me, a young initiate, these were the main impressions of the dominant culture of the geostatistics research environment of that time.

In my opinion, one of the most innovative breakthroughs of André Journel's scientific life was precisely to promote and stimulate a movement against that dominant culture, by constantly fighting to expound, discuss and make accessible his emerging ideas about geostatistics. *Mining Geostatistics* (Journel and Huijbreghts 1978) was the first book that contained both clear geostatistical concepts and the code to compute variograms and kriging. In fact, it was the first open geostatistical source code to which I had access.

There was the birth of geostatistics, to which the names of Georges Matheron and the Fontainebleau school are linked; and there is geostatistics, after André Journel and Stanford University opened the discipline to a broad range of applications and to a diverse range of scientific communities.

2 Tróia Conference

It was in this context that the Fourth International Geostatistics Conference (Tróia 92) took place in 1992, with André one of the main driving forces behind the event's organisation. It was an impressive showcase of the brilliant ideas of the newly created Stanford Center for Reservoir Forecasting (SCRF).

The solving of an inverse problem with the integration of seismic in high-spatial-resolution stochastic models was among the more remarkable of the nuggets first presented at that conference. Louis Bortoli, Francois Alabert, André Haas and André Journel are the authors of the seminal paper on geostatistical inversion (Bortoli et al. 1992).

Based on a forward physical model that converts acoustic impedance (linked directly to the petrophysical properties one intends to model) into amplitude, the idea was to solve an inverse problem: from a set of indirect measurements of subsurface properties, the real seismic cube, one intends to predict the spatial distribution of the acoustic impedance models that generate a synthetic seismic volume similar to the real seismic. The purpose of that paper's algorithm (Bortoli et al. 1992) was to solve this inverse problem by following a sequential simulation approach: (i) in a random path, a set of realisations of the same seismic trace were simulated; (ii) after calculating the deviations between the synthetic seismic trace of the different realisations and the corresponding real seismic trace, the trace with the smallest deviations was selected and retained as 'experimental' data. In this way, one succeeds in generating a set of impedance models that are conditioned by well and seismic data.

The algorithm contained convergence problems with respect to the intended solution, but the brilliance of that workflow was that it created a stochastic framework for the seismic inversion problem while opening the door to other geostatistical seismic inversion alternatives.

Here we aim to present the latest advances in seismic inversion, most of which are supported by André Journel's scientific legacy.

3 Integration of Seismic Data in Stochastic Models

3.1 Forward Models for Integrating Seismic Data

The integration of auxiliary variables as full images in geostatistical models has gained great potential for application following the stochastic simulation methods with collocated co-kriging and Markov approximations described in Xu et al. (1992). In this context, seismic reflection data was soon viewed as a privileged window for the characterisation of subsurface properties, since it has high spatial representativeness that covers the full 3D spatial extent of the area of interest. Several studies were based on a forward simulation approach: the joint simulation of petrophysical properties with seismic reflection data (or any other seismic attribute like, for example, acoustic impedance) as a secondary variable (Dubrule 2003; Doyen 2007). However, the results of most of these studies did not meet expectations, largely because seismic reflection has much greater support compared to well log data, and much higher uncertainty, resulting in a poor relationship between co-variables which are crucial for the joint simulation outline.

3.2 Seismic Inversion

Seismic inversion is an approach that integrates seismic data into a stochastic model of subsurface properties by the solution of an inverse problem. In an inverse physical problem, such as the seismic inversion problem, we know only the response of the Earth to a limited set of indirect measurements (i.e., the seismic data), and we try to infer from this the model parameters—rock properties such as porosity, acoustic impedance—of the system being examined giving rise to the observed data (Tarantola 2005).

Geophysical inverse problems seek to imply the physical properties of the subsurface geology, the model parameters ($\mathbf{m} \in \mathbf{R^n}$), from a set of indirect geophysical measurements/observations ($\mathbf{d_{obs}} \in \mathbf{R^s}$), which are normally contaminated by measurement errors ($\mathbf{e}$) from different sources. The observed data ($\mathbf{d_{obs}}$) and the subsurface properties of interest ($\mathbf{m}$) are related by a forward model ($\mathbf{F}$). If the forward models can be mathematically described and the model parameters are known, the observed data may be synthesised by Eq. (1) (Tarantola 2005).

$$\mathbf{d_{obs}} = \mathbf{F}\left(\mathbf{m}\right) + \mathbf{e}. \tag{1}$$

Concerning the case of seismic inversion problems, $\mathbf{d_{obs}}$ represents the recorded seismic reflection data and available well log data, and $\mathbf{m}$ is the model parameter space for the properties to invert. These properties depend on the object of the inversion: acoustic and/or elastic impedance or density, P-wave and S-wave velocity models. The forward model, $\mathbf{F}$, can be described in the following form.

$$\mathbf{A} = \mathbf{r} * \mathbf{w}, \tag{2}$$

where $\mathbf{A}$ is the recorded seismic amplitude obtained by the convolution of $\mathbf{r}$, the subsurface reflection coefficients, which are dependent on the elastic properties (P-wave and S-wave velocities and density) of the subsurface geology, with an estimated wavelet $\mathbf{w}$.

Although this paper is focused on stochastic inversion methods anchored on the seminal idea of Bortoli's paper (Bortoli et al. 1992), it is worth mentioning different alternative approaches, namely the stochastic seismic inversion algorithms based on linearized Bayesian inverse methodologies. These are based on a particular solution of the inverse problem using the Bayesian framework and assuming the model parameters and observations as multi-Gaussian-distributed as well as the data error, which allows the forward model to be linearized (Buland and Omre 2003; Eidsvik et al. 2004; Hansen et al. 2006; Grana and Della Rossa 2010; Grana et al. 2017; Grana et al. 2017; De Figueiredo et al. 2018, 2019).

4 Geostatistical Seismic Inversion

4.1 Trace-by-Trace Seismic Inversion

Deterministic algorithms were the first attempt to solve Eq. (1), such as 'band-limited' or the integration of seismic trace (Lindseth 1979), coloured inversion (Lancaster and Whitcombe 2000), sparse-spike and model-based (Russel 1988). These frameworks usually result in a smooth representation of the Earth's subsurface properties, with much less spatial variability compared to the real and complex petrophysical and geological subsurface properties (Russell and Hampson 1991).

The first geostatistical seismic inversion methodology was introduced in the seminal paper by Bortoli et al. (1992), in which the solution of Eq. (1) is based on the sequential simulation approach. They proposed a sequential trace-by-trace approach:

at each step along the random path, a set of *Ns* realisations of one acoustic impedance trace is simulated using sequential Gaussian simulation (Deutsch and Journel 1996), taking the well log data and previously visited/simulated traces into account. Then, for each individual simulated impedance trace, the corresponding reflection coefficient is derived and convolved by a wavelet according to Eq. (2), which results in a set of *Ns* synthetic seismic traces. Each of the *Ns* synthetic traces is compared in terms of a mismatch function with the recorded/real seismic trace. The acoustic impedance realisation that produces the best match between the real and the synthetic seismic traces is retained in the reservoir grid as conditioning data for the simulation of the next acoustic impedance trace at the new location following the pre-defined random path.

This SCRF group's simple idea was the first successful attempt to generate one solution of acoustic impedance within a stochastic model framework, allowing for the assessment of uncertainty associated with the most probable images of subsurface properties (e.g., acoustic impedance, porosity).

The algorithm contained convergence problems in relation to the intended solution, minimum deviations to the real seismic in regions of poor signal-to-noise ratio (Azevedo and Soares 2017). More recent versions of trace-by-trace models try to overcome this drawback by avoiding noisy areas in the early stages of the inversion procedure (Grijalba-Cuenca and Torres-Verdin 2000; Connolly and Hughes 2016).

However, the idea of the workflow opened the door to other geostatistical seismic inversion alternatives.

4.2 Geostatistical Seismic Inversion

Soares et al. (2007) introduced the global stochastic inversion methodology based on an iterative approach that, in each iteration, generates, by stochastic simulations and co-simulation, a new ensemble of full 3D models. Convergence is assured iteratively through an optimiser towards the desired solution.

The general outline of this new family of geostatistical inversion algorithms is synthesised in Fig. 1. The acoustic impedance model generation and perturbation is performed, recurring to direct sequential simulation and co-simulation (Soares et al. 2007; Caetano 2009). At each iteration after the misfit, between synthetic and real seismic, an optimisation is calculated based on cross-over genetic algorithms that ensure the next generation of simulated models are closer to the desired objective function.

4.2.1 The Use of Direct Sequential Co-simulation for Global Transformation of Subsurface Earth Models

The geostatistical inversion uses the sequential direct co-simulation as the method of perturbing 3D impedance models in the iterative process. The global and local correlation coefficients between the transformed traces and the real seismic traces of different simulated models are used as the affinity criterion between real and inverted seismic reflection data to create the next generation of models. The iterative procedure contin-

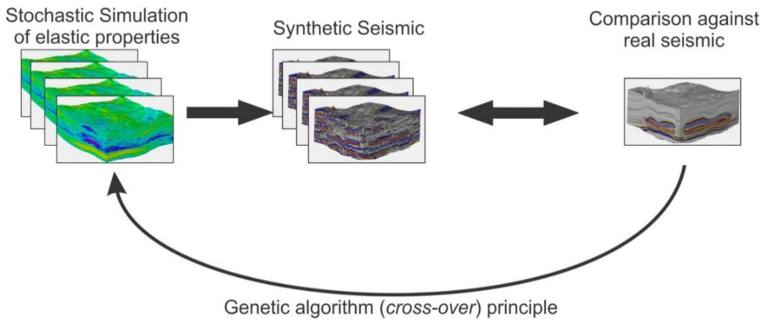


Fig. 1 General outline for iterative geostatistical seismic inversion methodologies with a global approach

ues until a stopping criterion is reached: frequently the global correlation coefficient between real and inverted seismic reflection data.

The direct sequential simulation (DSS) method was born from one of André's ideas. In 1994, Journel showed that in a sequential simulation, the covariance model is always reproduced if, at each step, the value generated centres on a local mean and variance that is identified by the simple kriging estimator and estimation variance, regardless of the cumulative distribution function used at that point (Journel 1994). The first step was taken towards a simulation method that did not require a Gaussian transformation of the data. The reproduction of the probability distribution function derived from the experimental data, the second-most important aim of any existing spatial simulation method, remained unresolved.

Soares (2001) proposes a direct sequential simulation (DSS) method containing the solution of both issues: the reproduction of variogram models and the global cdfs. The use of direct sequential simulation and co-simulation as the model parameter space perturbation during the iterative geostatistical seismic inversion procedure is a key concept in all geostatistical seismic inversion methodologies presented here.

4.2.2 Global Geostatistical Inversion

The global stochastic inversion (GSI; Soares et al. 2007; Caetano 2009) allows the inversion of post-stack seismic reflection data for acoustic impedance (Ip) models.

The general outline of the global stochastic inversion can be described in the following sequence of steps, illustrated in Fig. 2:

(i) For the entire seismic grid, use DSS to simulate a set of *Ns* acoustic impedance models that are conditioned to the available acoustic impedance well log data and which assume a spatial continuity pattern as revealed by a variogram model.
(ii) Derive a set *Ns* of synthetic seismic volumes by calculating the corresponding normal incidence reflection coefficients (RC) from the impedance models simulated in the previous step (Eq. 3); convolve these RC with an estimated wavelet for that particular seismic dataset.

$$\mathrm{RC} = \frac{\mathrm{Ip}_2 - \mathrm{Ip}_1}{\mathrm{Ip}_2 + \mathrm{Ip}_1}. \tag{3}$$

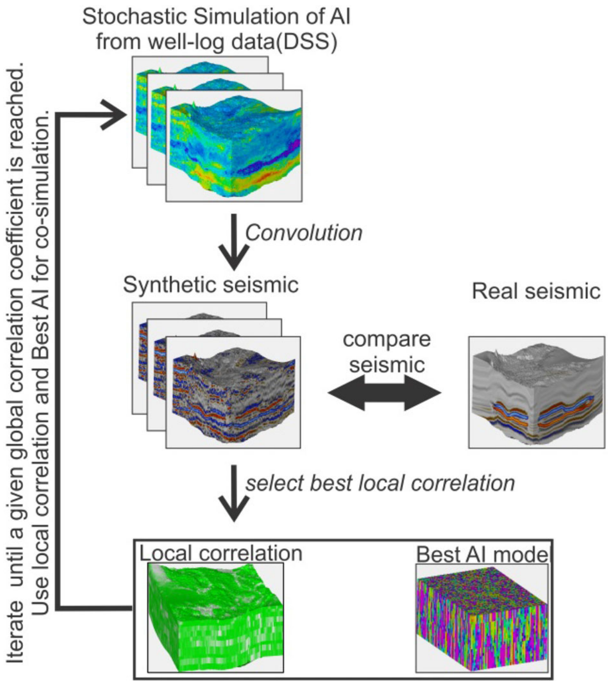


Fig. 2 Schematic representation of the global stochastic acoustic inversion methodology

(iii) Each seismic trace from the *Ns* synthetic seismic volumes is then compared in terms of correlation coefficient against the real seismic trace from the same location. Two auxiliary volumes are created: one with the best acoustic impedance traces; the other with the corresponding local correlation coefficients. These are used as secondary variables and as local regionalised models for the generation of the new set of acoustic impedance models for the next iteration.

The new set of *Ns* acoustic impedance models is created using direct sequential co-simulation.

The iterative procedure stops when the global correlation coefficient between the full synthetic and real stacked seismic volumes is above a certain threshold (see details of the method in Azevedo and Soares 2017).

The GSI methodology allows the retrieval of high-resolution Ip models honouring the distribution function as estimated from the available well log data and the spatial continuity model as retrieved from a variogram model. It has been tested successfully on seismic datasets from very different geological contexts with diverse qualities.

The GSI outline was generalised for the characterisation of elastic properties, direct inversion of petrophysical properties, integration of rock physics relationship, and seismic and electromagnetic joint modelling, by taking advantage of the characteristics of direct sequential simulation of the joint simulation of multiple distributions (Horta and Soares 2010) and simulations with local probability distribution functions (Soares et al. 2017).

4.2.3 Geostatistical Elastic Inversion

The acoustic inversion algorithm was extended for the simultaneous inversion of partial angle stacks, for acoustic and elastic impedance models resulting in richer subsurface models. Acoustic and elastic impedance, Ip and Is, are jointly simulated (step i of the previous outline and co-simulated by using the direct sequential simulation with joint distributions of probability (Horta and Soares 2010). This simulation method succeeds in reproducing the bivariate distribution function (Ip, Is) as estimated from the experimental log data. Detailed application examples of this method can be found in the studies by Nunes et al. (2012) and Azevedo and Soares (2017).

4.2.4 Pre-stack Seismic AVA Inversion

The quality of seismic reflection data has increased enormously in recent decades, together with a reduction in the costs of acquisition. Pre-stack seismic data with high signal-to-noise ratio and considerably high fold is a reality nowadays, and one that enables better subsurface characterisation. Pre-stack seismic data is achieved by interpreting the changes in amplitude versus the offset (AVO), or with the angle of incidence (AVA). Geostatistical seismic AVA inversion (Azevedo et al. 2018a, b) relies on an identical general framework, but with the perturbation of the model parameters for density and P-wave and S-wave velocities performed sequentially using stochastic sequential co-simulation with joint distributions (Horta and Soares 2010). Details of the method and application results in real case studies can be found in Azevedo et al. (2018a, b, 2019).

4.2.5 Joint Inversion of Seismic and Electromagnetic Data

This geostatistical seismic inversion framework was extended for the joint inversion of seismic and electromagnetic data in which rock properties, such as porosity and water saturation, are simulated and co-simulated through direct sequential simulation and co-simulation (Horta and Soares 2010). Ensuring the complex relationships between rock properties per facies is key to the success and plausibility of the inverted models. These are converted into elastic properties using pre-calibrated rock physics modelling that links the rock and elastic domains. Properties from both domains are then forward-modelled into synthetic seismic and electromagnetic data. Each geophysical data uses a specific forward model. The mismatch between observed and synthetic geophysical data is used to update simultaneously all the rock properties of interest in the subsequent iteration.

The advantages of this method lie in the fact each data type is sensitive to a different but complementary petro-elastic property (i.e., electromagnetic data in relation to the type of pore fluid and seismic in relation to elastic properties).

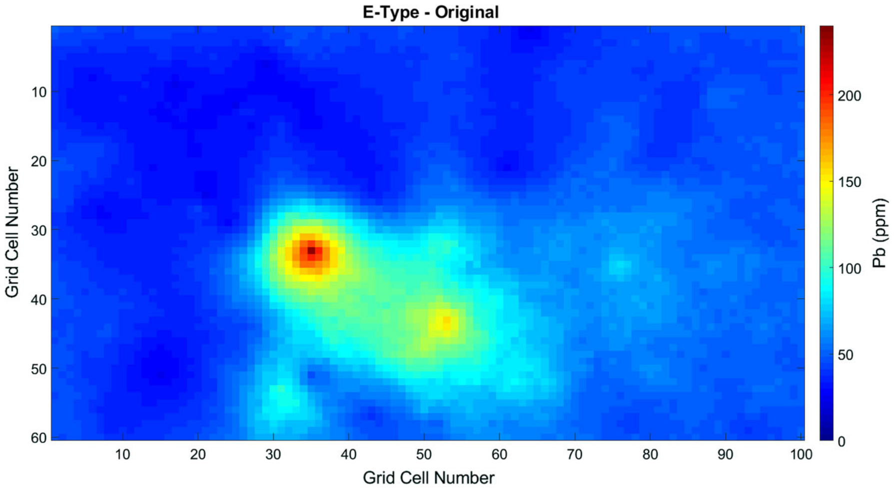


Fig. 3 E-type of the ensemble of 100 realizations of a stochastic simulation

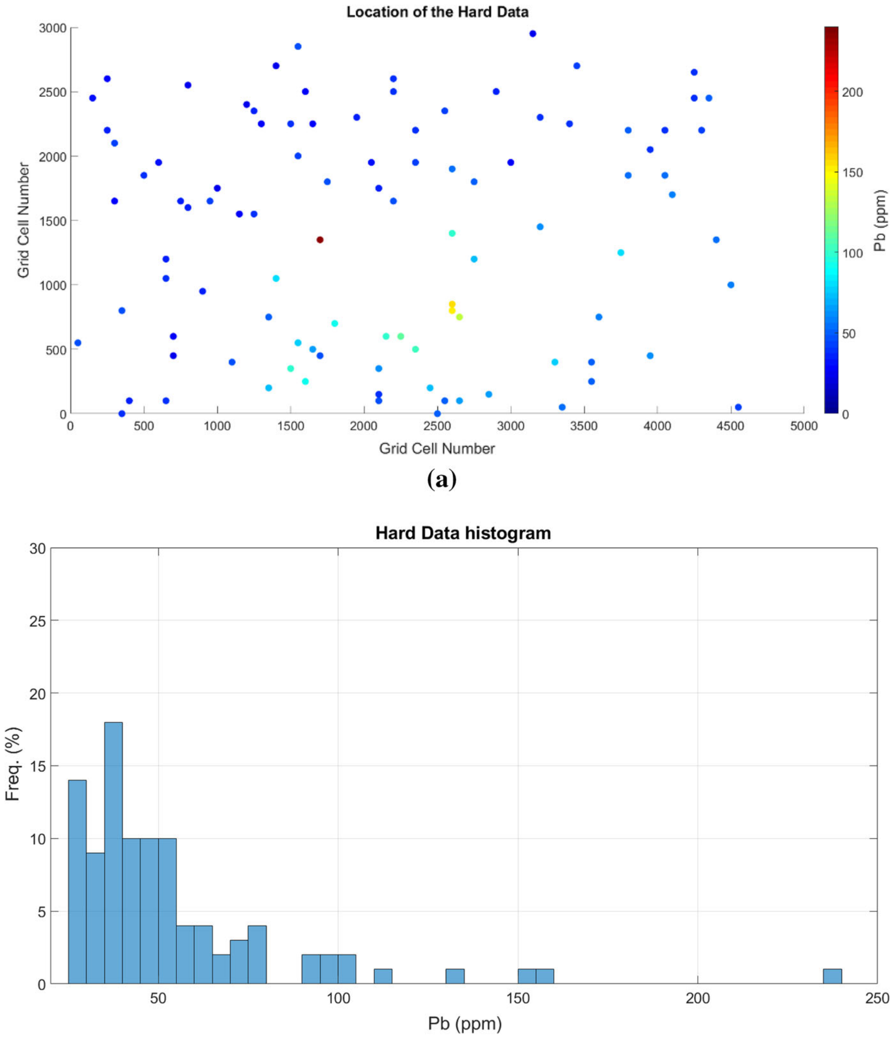
5 Seismic Inversion with Self-Updating Local Models

5.1 Conditional Simulations to Update Local Models

Direct sequential simulation methods with local probability distribution function models (Soares et al. 2017) enabled conditional simulations to be viewed as methods for updating these statistics and integrating them into the simulation process.

A sequential simulation begins with global variogram and pdf models, which can change locally during the conditioning process. The main idea in this approach [presented by Soares et al. (2017)] is to use these local models rather than the global models to generate a new set of realisations. For example, Fig. 3 represents the average model of the ensemble of 100 realisations of a soil contaminant concentration (Pb), generated via a stochastic sequential simulation and based on an estimated probability distribution function with the experimental soil data set (Fig. 4).

The conditioning effect of the experimental data reproduces local zones of high values and variability and other zones with low values that are spatially homogeneous. Figure 5 shows the variance map of the same set of realizations and the histograms for two different local areas. Since the conditioning information (i.e., the experimental data samples) generates simulated realisations with different local models, the main propose of this algorithm is to generate a new set of realisations by using these local models. This self-updating framework is illustrated in two different geostatistical seismic inversion approaches: one with self-updating local distributions; the other updating local anisotropy variogram models.



(a)

(b)

Fig. 4 **a** Experimental data location and **b** corresponding histogram

5.2 Seismic Inversion with Self-Updating Local Distributions

The methods presented above, chapter 4.2, share a common misfit control between seismic and seismograms through the correlation coefficients or equivalent misfit measures. In some applications these methods can reach a roadblock: high correlation coefficients may be critical when they happen at early stages of the iterative procedure, which may drive the convergence process towards local minima that are far from the global minimum solution. To overcome this, Azevedo et al. (2020) suggest using simulations with local distributions rather than the co-simulations to generate—for

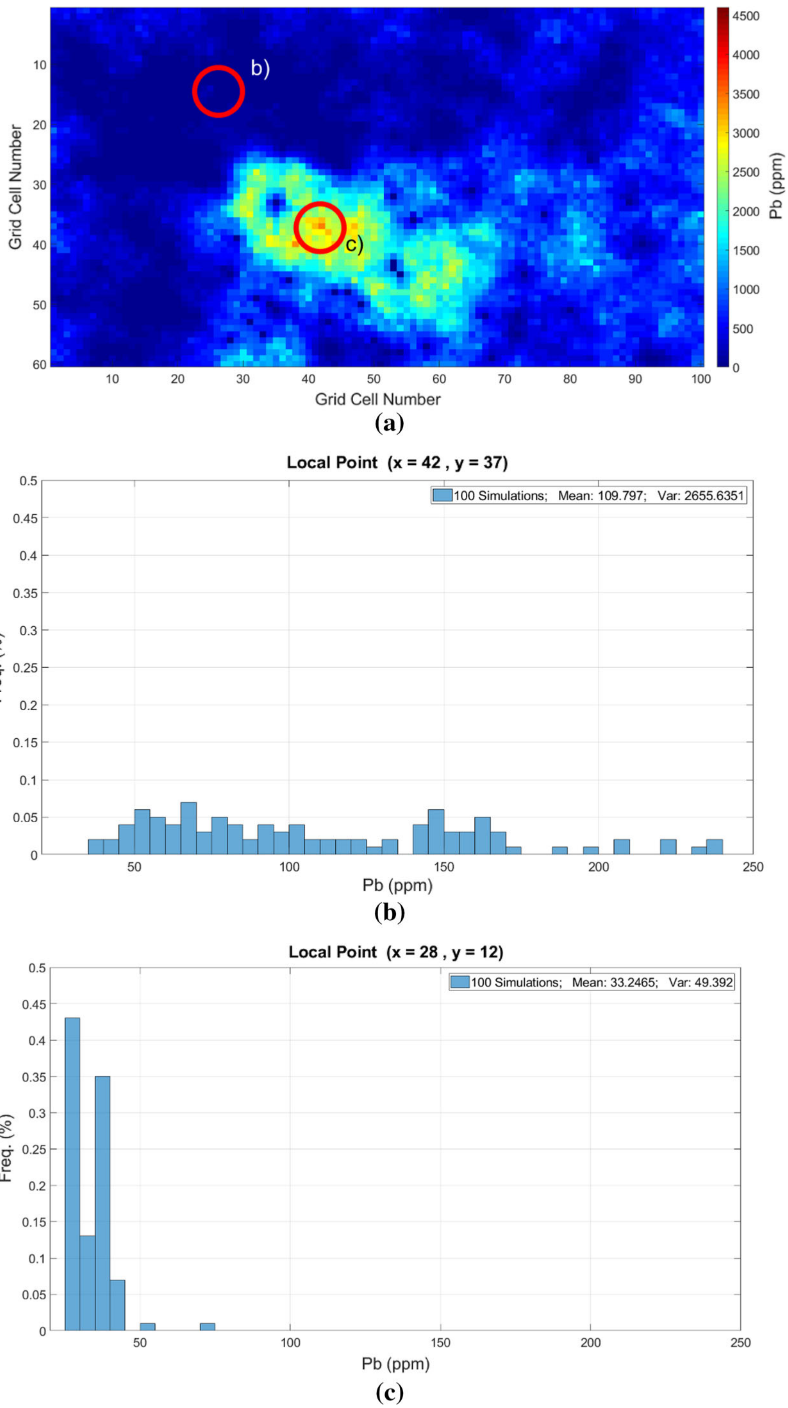


Fig. 5 **a** Variance model of ensemble of realizations. Red circles show the approximate location of the histograms of Pb shown in (**b**) and (**c**)

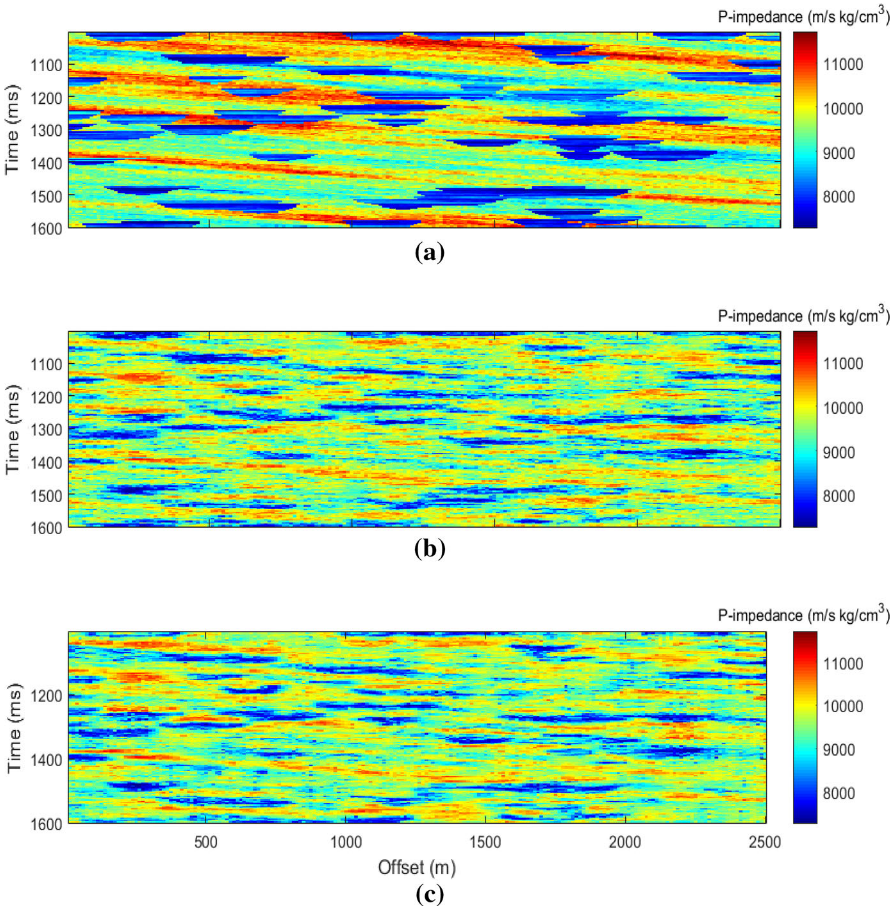


Fig. 6 **a** True Ip model; **b** mean model from global iterative geostatistical seismic inversion; **c** mean model from geostatistical seismic inversion with self-updating Ip distributions

example in geostatistical acoustic inversion—another realisation of Ip models in each iteration. The main idea behind this model is to update the local impedance pdfs during the iterative process. Hence, after the misfit evaluation, the best elastic samples (i.e., the elastic samples that result in lowest misfit) are used to build local elastic pdfs at each location $x_0 : F_{\mathrm{Ip}}(x_0)$. In the next iteration, a sequential simulation with local probabilities is used to generate the next set of models. Basically, at each location x_0, a simulated value is drawn from the local distribution $F_{\mathrm{Ip}}(x_0)$, rather than from the global pdf, as in ordinary stochastic sequential simulation. In this way, local distributions of the main variables Ip, $F_{\mathrm{Ip}}(x_0)$, are updated with the optimisation procedure and condition of the subsequent models of Ip. This self-updating model with seismic data integration can be summarised in the following sequence of steps:

(i) Generate a set of *Ns* realisations of $\mathrm{Ip}(x)$ with stochastic sequential simulation with global statistics;

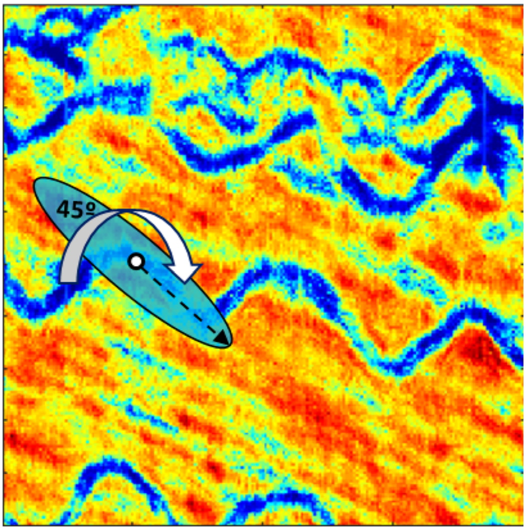


Fig. 7 Templates with different directions and angles of anisotropy are tested for each spatial location, and the best is chosen based on minimum variance of the point values inside the template

(ii) Follow steps (ii) to (iv) of the sequence of point three;
(iii) Create or update local pdfs with the best Ip values of this iteration;
(iv) Calculate a new set of Ns realisations of Ip (x) by using direct sequential simulation with local distributions;
(v) Repeat steps (ii) to (iv) until the global correlation coefficient between the synthetic and the real full-stack volumes reaches the desired threshold.

Figure 6 shows a vertical section retrieved from a synthetic application and compares the average model for Ip calculated from 32 realisations of Ip after six iterations. The true Ip model is compared with inverted sections retrieved from global iterative geostatistical acoustic inversion and the self-updating model and the true Ip model. The update of the local distributions allows one to better predict the shape, boundaries of the channels and the absolute values within the sand bodies. These effects are better observed when the true Ip model has contradictory dips.

5.3 Self-Updating of Local Models of Covariance to Enhance Non-stationary Geological Patterns

In non-stationary geological patterns, such as meander-form sand channels, after a set of realisations using a global model of covariance, through the inversion procedure the seismic data is able to update local spatial trends as they are quantified by local variogram anisotropies.

In the misfit evaluation step of the seismic inversion workflow, local variograms are calculated with the best images (best misfit). Basically, an image analysis algorithm is applied to evaluate the ellipsoid that has a minimum variance of all points inside it, see the sketch of a xoy map of Fig. 7. This ellipsoid of covariance will replace the global model of covariance in the next iterations. The co-simulations with local

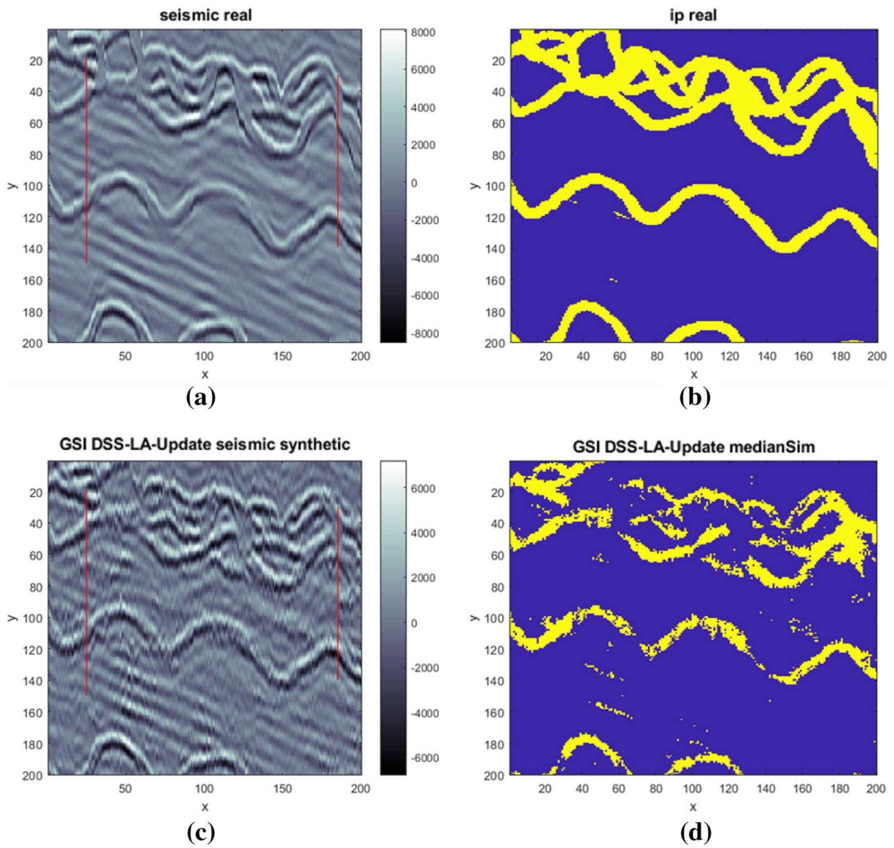


Fig. 8 **a** Real seismic data; **b** true Ip showing the low-impedance sand channels; **c** synthetic seismic of the final iteration; **d** Ip model obtained with self-learning algorithm

models of anisotropy follow the methods proposed by Soares (1990) and Luis and Almeida (1997).

Assuming $\theta(x_0)$ are the local covariance model parameters (directions and anisotropy ratio) at location x_0, the local mean and variance at x_0 are estimated by a simple kriging (SK). Here, in Eq. (1), the SK mean is represented in its dual form

$$z\,(x_0) * = \sum_{\alpha} \delta_\alpha C_\theta\,(x_0, x_\alpha)\,, \tag{4}$$

where the weights δ_α are the product of the data vector, and the covariance matrix between samples is constructed with the model $C_\theta(h)$ of x_0. Local models $C_\theta(h)$ are updated in each iteration, thereby revealing the non-stationary geological features.

This methodology was tested in a two-dimensional example of a synthetic case study. Figure 8 shows a two-dimensional horizontal section of the seismic cube of a turbidite reservoir; the true Ip model and the final Ip model obtained with the acoustic inversion with self-updating of local covariance models. The final synthetic seismic matches, satisfactorily, the real seismic. The spatial connectivity of sand channels is very well reproduced with the proposed algorithm, which is shown to be a very promising technique for characterizing these non-stationary environments.

6 Final Remarks

One final remark must be made about the geostatistical seismic inversion methods that can be considered a game changer in subsurface characterisation methods. They have a high potential for integrating geophysical data (e.g., seismic and electromagnetic) into stochastic models of rock properties. Geomechanical models can also be derived from velocity models, after inversion (Gray et al. 2012). Seismic inversion allows the integration of rock physics models with new insights in rock-typing (Azevedo et al. 2019), a final word to emphasise the potential applications of these methodologies to other fields like seismic oceanography (Azevedo et al. 2018a, b).

But the entire set of geostatistical seismic inversion methods presented in this paper are, by some means, the result of André's ideas. Moreover, this is just a small sample—an example—of his influence over an entire range of geostatistical methods for the evaluation of natural resources. In fact, when we think of geostatistics as it is today, it necessarily means remembering André Journel's unique role in transforming geostatistics into a widely recognised modern discipline supporting a set of efficient tools for solving practical problems within a wide range of natural resource applications.

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