TEACHING AIDS

Surface-Based 3D Modeling of Geological Structures

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Abstract Building a 3D geological model from field and subsurface data is a typical task in geological studies involving natural resource evaluation and hazard assessment. However, there is quite often a gap between research papers presenting case studies or specific innovations in 3D modeling and the objectives of a typical class in 3D structural modeling, as more and more is implemented at universities. In this paper, we present general procedures and guidelines to effectively build a structural model made of faults and horizons from typical sparse data. Then we describe a typical 3D structural modeling workflow based on triangulated surfaces. Our goal is not to replace software user guides, but to provide key concepts, principles, and procedures to be applied during geomodeling tasks, with a specific focus on quality control.

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1 Introduction

Understanding the spatial organization of subsurface structures is essential for quantitative modeling of geological processes. It is also vital to a wide spectrum of human activities, ranging from hydrocarbon exploration and production to environmental engineering. Because it is not possible to directly access the subsurface except through digging holes and tunnels, most of this understanding has to come from various indirect acquisition processes. 3D subsurface modeling is generally not an end, but a means of improving data interpretation through visualization and confrontation of data with each other and with the model being created, as well as a way to generate support for numerical simulations of complex phenomena (i.e., earthquakes, fluid transport) in which structures play an important role. As the interpretation goes, the 3D framework forces us to make interpretive decisions that would be left on the side in map or cross-section interpretations. Skilled geologists know how to translate 3D into 2D and vice versa, but, no matter how experienced one can be, this mental translation is bound to be qualitative, hence inaccurate and sometimes incorrect. 3D model building calls for a complex feedback between the interpretation of the data and the model. Such a feedback can only be partial when seeing only the interpretation on a section plane.

In most application fields, 3D modeling is also a means of obtaining quantitative subsurface models from which information can be gathered. Such a 3D Geological Information System can be used, for instance, in mineral potential mapping (Bonham-Carter 1994) and geo-hazard assessment (Culshaw 2005). 3D structural models can be meshed to solve (geo)physical problems and assess or predict production of natural resources, solve geomechanical problems, better understand mechanisms that trigger earthquakes, etc. In this case, one main concern is to estimate petrophysical properties of rocks such as porosity or seismic wave velocity, in order to simulate physical processes. Traditionally, these estimations are performed on regular grids typically using geostatistical methods (Goovaerts 1997; Chilès and Delfiner 1999). Yet, geologists know that the distribution of petrophysical properties is mostly determined by rock types. Therefore, a clear understanding of how rocks are spatially laid out in 3D is a prior to any geostatistical study or simulation of a physical process. A 3D structural model is a numerical representation of this structural information. As any model, this 3D structural model is at best a simplified view of reality depending on the choice of a representation as determined by the scale of study, the prior hypotheses about the features of geological objects being described and the application intended for the model, the quantity and quality of available information, and the limitations of the computing device (computing power, memory, precision). At worst, the model may be grossly wrong, displaying unrealistic fault geometries or variations of layer thickness. Unfortunately, in our experience, beginners with 3D modeling too often lose their critical sense about their work, mostly due to a combined effect of dazzling graphics and suboptimal human–machine communication.

The goal of this teacher's aide is to provide some practical clues and guidelines about the integration of surface and subsurface data into a consistent 3D structural model made of a set of geological interfaces. These 3D surfaces are used to model the main discontinuities of the domain of study, such as horizons, faults, unconformities, intrusion boundaries, etc. Such a structural model may be updated when new data becomes available or perturbed to account for structural uncertainties, and can be used as a framework to generate 3D meshes conforming to geological structures. We propose to intuitively present the main methodological and numerical approaches which can be used to generate 3D surfaces, and to give practical rules and clues about elementary quality control on the resulting 3D models. This paper is mostly based on our experience of teaching 3D structural modeling with the Gocad® geomodeling software; however, most general rules provided here should be applicable to other software platforms. Additional insight on surface-based 3D structural modeling methods is also available from de Kemp and Sprague (2003), Dhont et al. (2005), Fernández et al. (2004), Gjøystdal et al. (1985), Groshong (2006), Kaufman and Martin (2008), Lemon and Jones (2003), Mallet (1997, 2002), Sprague and de Kemp (2005), Turner (1992), Wycisk et al. (2009), Wu et al. (2005), Zanchi et al. (2009). After a rapid overview of the typical data available in a 3D modeling project (Sect. 2, Fig. 1(A)–(B)), elementary general rules of structural modeling are presented (Sect. 3). Then, Sect. 4 presents additional guidelines for appropriate representation of structural interfaces with numerical surfaces. The structural modeling process is described in Sect. 5 (Fig. 1(C)–(D)), with a focus on the main technical choices and quality controls to be made to obtain a consistent model. Before concluding this teacher's aide, Sect. 6 briefly presents some recent and ongoing research topics on 3D structural modeling.

2 Data Management

2.1 A Quick Overview of Earth Data

The typical input data for a 3D structural modeling project can be quite diverse and may include field observations (for instance, stratigraphic contacts and orientations, fault planes), interpretive maps and cross-sections, remote sensing pictures, and, for high budget projects, LIDAR outcrop data (Bellian et al. 2005), reflection seismic, and borehole data. Each data type has its specific features, which will act upon how it is integrated in the modeling process and affect the quality of the model. The resolution qualifies the smallest observable feature from a given type of data. For instance, the seismic resolution usually varies between 10 to 40 m, while direct observations on the field or on well cores can be made at a millimeter resolution. The accuracy relates to how much a datum approximates reality. Causes for deviations can typically originate from measurement errors, smoothing due to limited resolution, approximate positioning or georeferencing, database errors, incorrect interpretation or processing parameters, etc. Knowing how much data is reliable and interpreted is essential for weighting its contribution to the final model versus that of the other data types and one's interpretation. For instance, whereas geological cross-sections are often considered as hard data, 3D structural modeling may

reveal some inconsistencies in interpretive cross-sections, and motivate reinterpretations. Another example is in the late fitting of seismic-derived structural models to pierce points observed along boreholes to correct for errors in seismic picking or velocity. The measurement precision is linked to the sensitivity of the acquisition method and corresponds to the range of error around the true value. For example, GPS tools give generally the errors about the acquired coordinates. The numerical storage (Bonham-Carter 1994) may be achieved in a matrix (or raster) format, used typically for images. Raster data typically result from some systematic acquisition or imaging procedure, and, inherently, have limited resolution. Alternatively, vector format used for lines, points, and polygons is sharper, and is hence the preferred format for punctual observations such as well cores, GPS-generated field measurements, and most interpreted data and models (cross-sections, maps, etc.). For convenience, data storage is often achieved using so-called 2.5D data structures, in which a single elevation value is given for a given map location. This type of representation, widely used in 2D GIS software, is appropriate for remotely sensed topographic surfaces, but may raise problems for representing general 3D geological structures such as recumbent folds, inverse faults, etc. Modern geomodeling software usually deals with true 3D representations for vector objects (de Kemp and Sprague 2003; Dhont et al. 2005; Mallet 1997, 2002).

2.2 Management and 3D Visualization of Earth Data

Georeferencing is a first step before starting geomodeling, whereby all available information is combined and organized in a common coordinate system (e.g., Culshaw 2005; Kaufman and Martin 2008; Zanchi et al. 2009), see Fig. 1(A). The choice of a good coordinate system is a crucial step in the modeling process, it has to cover the entire studied zone and be precise enough not to lose or distort information. To georeference an image, at least three control points must be defined. Their coordinates both in the original local and final coordinate systems are input by the user to compute the georeferencing transformation. The control points must be chosen as close as possible from georeferenced points and picked on precise geographic coordinate systems to minimize errors. When the number of control points is larger than the required minimum, residuals between the local and global control point coordinates provide a measure of georeferencing accuracy. A wrong georeference typically comes from errors in the selection of control points or from distortions produced by the scanning of paper documents. Early detection of such errors is paramount for building a consistent structural model. For this, 3D visualization functionalities available in geomodeling packages should be used extensively. The main tool at hand for this visual quality control is a 3D virtual camera, whose direction, viewing volume, and proximity to the 3D scene can be modified in real time to visually inspect the data (Möller and Haines 1999). Several objects can be displayed simultaneously, providing a simple and effective way of visually checking for possible inconsistencies. It is often useful to project a raster map (geological map, aerial picture) onto the corresponding digital elevation model using texture mapping. This results in a so-called Digital Terrain Model, or DTM (Fig. 1(A)). The DTM highlights the relationships between geological structures and topography, and provides georeferencing quality control by overlaying topographic contour lines onto the raster map. The
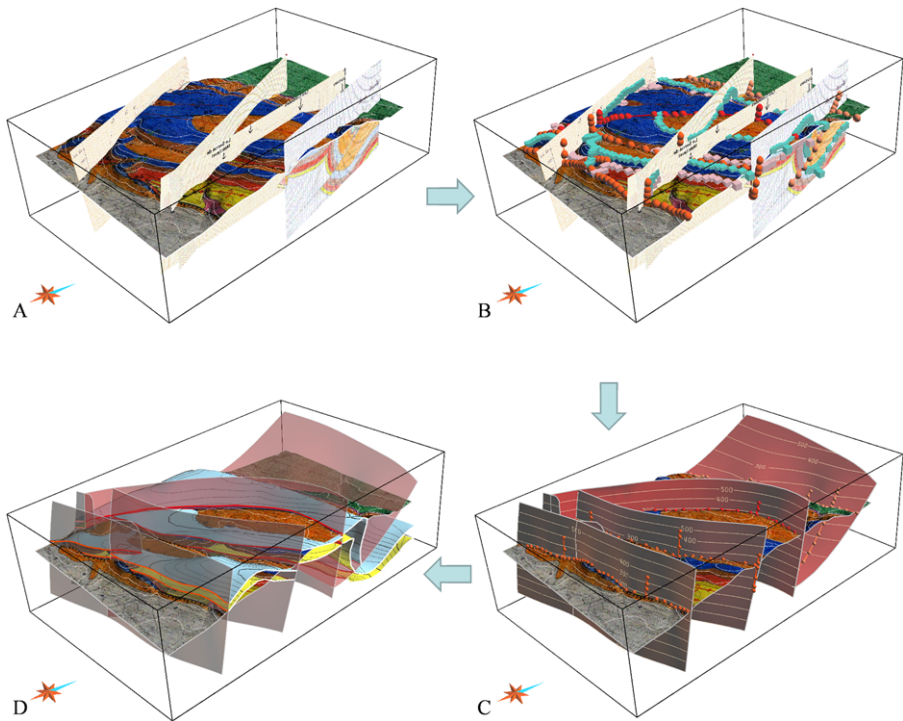
Fig. 1 Typical structural modeling steps applied to an undergraduate student's map and cross-sections of the Ribaute area, Southern France. The interpretation is extrapolated above the topographic surface. (**A**) Data georeferencing (Sect. 2.2). (**B**) Picking of relevant structural objects (Sect. 2.2); *Curves with spherical nodes* denote faults; *curves with cubic nodes* denote stratigraphic contacts. (**C**) Creation of fault network (Sects. 3, 4, 5.1 and 5.2; Figs. 8, 9); *contour lines* of fault surfaces are displayed. (**D**) Horizon modeling (Sects. 3, 4, 5.1 and 5.3; Figs. 8, 10); DTM has been hidden for clarity, and faults are displayed with transparency. 3D model can be viewed at http://www.gocad.org/~caumon/Research/Papers/MathGeo09_3DScreenshot.pdf

second step of data management consists of data preparation and cleaning. Raster images are not directly exploitable, and features of interest must be picked as vector objects (Fig. 1(B)). When these vector data have been imported from a 2D GIS system, projection onto the topographic surface and segmentation may also be needed to transform map polygons into lines which have a unique geological meaning. This projection (or a careless picking of lines on a DTM) can introduce significant artifacts in the line geometry, for the DEM resolution is often coarse as compared to the size of the features of interest. Detecting (and fixing) such problems early on is extremely important to keep a high signal/noise ratio, especially when data are sparse. Lastly, co-located points and outliers in the vector data should be checked, because they may introduce modeling artifacts during further steps. As a final refinement, it may be preferable to homogenize line sampling, to avoid alternation of short and long segments. Indeed, some surface construction techniques described in Sect. 5 are sensitive to line sampling.

3 Basic Structural Modeling Rules

A 3D structural model is made of geological interfaces such as horizons and faults honoring available observation data. These surfaces should fit the data within an acceptable range, depending on data precision and resolution. For instance, a 2 meter misfit between 3D seismic picks and a stratigraphic horizon is acceptable, for it can filter out noise present in the data. Well data, however, should generally be honored much more accurately. Strategies to correct for data misfit in this context will be discussed in Sect. 4.3. Each 3D surface represents a geological discontinuity due to the changes of depositional conditions, erosion, or tectonic events like faulting or intrusion. A consistent structural model comprises not only surfaces fitting observation data, but also correct relationships between the geological interfaces. For this purpose, some basic modeling rules have to be observed in the modeling output. In most cases, these constraints are enforced by distinguishing the macro-topology, or frame, which is used to model the borders of an object, and the micro-topology, or lattice, which deals with the mesh of the object. In this section, we focus on rules related to the macro-topology. In principle, these rules are similar to those used when drawing a 2D cross-section. In practice, however, the third dimension makes it difficult to detect areas where these rules have been infringed. Therefore, we now explicitly stress some topological requirements which always hold in structural modeling. Some of these rules may automatically be enforced by software implementation, but all are discussed here for generality.

3.1 Surface Topological Self-consistency

A 3D surface that is legal from a mathematical perspective does not necessarily represent a valid natural object. Indeed, a geological surface is a boundary between two volumes of rocks with different characteristics (seismic impedance, hanging wall, metamorphic isograd, lithology, etc.). Therefore, the surface orientation rule states that a geological surface is always orientable, i.e., has two well-defined sides. A corollary is that a surface shall not self-intersect, for it would suggest that the volumes separated by this surface overlap each other.

3.2 Relation Between Structural Interfaces

Most of the 3D structural modeling endeavor amounts to figuring out how faults and horizons interact with each other. A main topological requirement in volume modeling is that surfaces should only intersect along common borders (Mäntylä 1988). In 3D structural modeling, it is possible and convenient to use a relaxed variant of this surface non-intersection rule, stating that any two surfaces should not cross each other, except if one has been cut by the other (Mallet 2002, p. 272). This means, for instance, that a fault surface needs not be cut along horizon tear lines, which makes model updating much easier when new data becomes available. Naturally, specific conditions depending on the type of geological interfaces can also be stated (Caumon et al. 2004). The rock unit non-intersection rule states that for any two rock boundaries H_i and H_j, H_i may lie on one side only of H_j, and conversely. If not, this means that layers overlap each other, hence are ill-defined (see
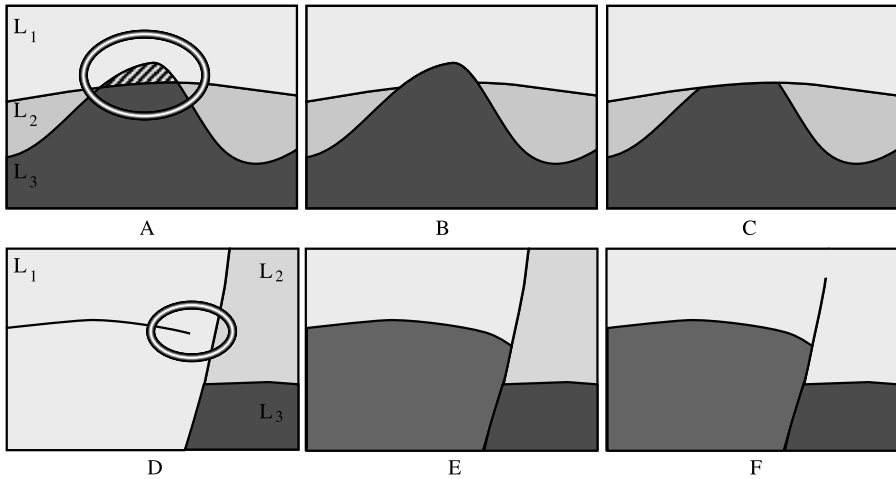
Fig. 2 Basic surface intersection rules: Overlapping layers (**A**, *hatched area*) and leaking layers (**D**) are invalid; whereas (**B**), (**C**), (**E**), (**F**) are consistent models

hatched part in Fig. 2(A)). This observation forms the basis of erosion or down-lap/intrusion rules available in many geomodeling packages (Calcagno et al. 2008; Mallet 2002). Figure 2(B)–(C) provides a simple 2D example of choice between erosion or downlap/intrusion to be made to correct the model shown in Fig. 2(A). Additional consistency conditions rely on the notion of logical borders on a surface, which describes the macro-topology. A surface border is defined by a set of con-nected border edges. For geomodeling needs, the border of a surface can be split into several pieces called logical borders, depending on their origin or role. For instance, a fault tear line on a horizon consists of two logical borders for the hanging wall and footwall. From this definition, the free border rule states that only fault surfaces may have logical borders not connected onto other structural model interfaces (Caumon et al. 2004). Indeed, stratigraphic surfaces necessarily terminate onto faults, uncon-formities or model boundaries (Fig. 2(D)–(E)); faults only may terminate inside rock units when the fault displacement becomes zero (Fig. 2(F)).

3.3 Geometric Constraints

In addition to data compliance, realism of the structural model geometry, though more difficult to characterize objectively, should always be assessed. This may be done visually in the complete or clipped 3D scene, and by extracting cross-sections. An important step in quality control is to use fault juxtaposition diagrams to check that fault displacement does not vary abruptly. Strike variations may be inspected by displaying horizon cutoff lines on the fault hanging wall and footwall. Likewise, vertical variations should be looked at by checking layer thickness variations on both sides of a fault and its compatibility with fault kinematics (Walsh et al. 2003).

Additionally, local surface orientation may be checked visually to detect mod-eling artifacts such as saddle geometries between cross-sections. Quantitative ap-proaches may also be used, such as surface curvature analysis (Groshong 2006;

Mallet 2002; Pollard and Fletcher 2005; Samson and Mallet 1997). For instance, one may deem that a horizon is realistic only if it can be unfolded without deformation, i.e., if its Gaussian curvature is null everywhere (Thibert et al. 2005). Another possibility for fault surfaces is to check whether their geometry allows for displacement using a thread criterion (Thibault et al. 1996). When it comes to assessing the likelihood not of a surface but of the whole structural model, simple apparent or normal thickness of sedimentary formations may be used. Another more rigorous but more difficult approach is to restore the structural model into depositional state (Maerten and Maerten 2006; Moretti 2008; Muron 2005; Rouby et al. 2000); strain analysis can then be used to judge on the model likelihood. More generally, structural models should be compatible with all types of observation data which result from physical processes in the earth, for instance as seismograms or reservoir production data. The assimilation of such data to reduce 3D structural uncertainty is an active research topic (Sect. 6).

4 Practical Modeling Guidelines

In addition to the general rules formulated above, practical modeling choices must be made when building a computer-based geometric model. Therefore, we will now present the notions of model resolution and mesh quality, which are both essential for a good 3D structural modeling study.

4.1 Finding the Appropriate Model Resolution

The discrete structural model is a piecewise approximation of an ideal continuous object. The discrete model is all the more accurate than it is closer to that ideal continuous object. The accuracy of a discrete surface is determined by the precision of its points (usually, simple or double floating point precision), and the density of its points, which provides more degrees of freedom to approximate the continuous surface by polygons. When building a 3D structural model, the question of mesh density is often to be raised, independently of how this surface interacts with other objects. For instance, the resolution of a surface can be modified while maintaining the definition of its logical borders. Visualization and processing of an excessively dense model are inefficient; conversely, coarse objects may be too rigid to account for complex 3D shapes. A common misunderstanding is that model resolution should be adapted to data density. In the presence of redundant data, as often encountered in Geosciences, this practice can possibly lead to inefficient representations and suboptimal performance. Conversely, when data are sparse, oversimplified models may lead to severe simplifications and to an understatement of uncertainties. Therefore, a structural model should ideally have the minimal resolution to reflect the desired geometric complexity of the structures. Obviously, one's understanding of geometric complexity is related to data features. Therefore, model resolution should be at least such that the misfit between the model and the data is within the range of data uncertainty. Model resolution may also be higher to ensure mesh quality and to account for interpretive input and analog reasoning. In many cases, it is useful and appropriate
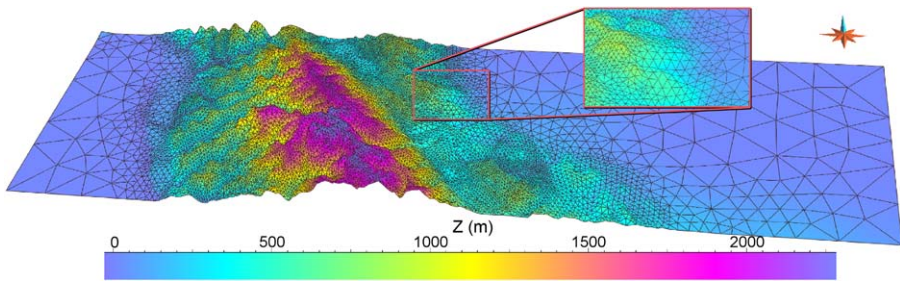
Fig. 3 Triangulated surfaces allow for varying resolution depending on the needed level of detail. This topographic surface (width 60 km) was created by adaptive triangulation of a digital elevation model
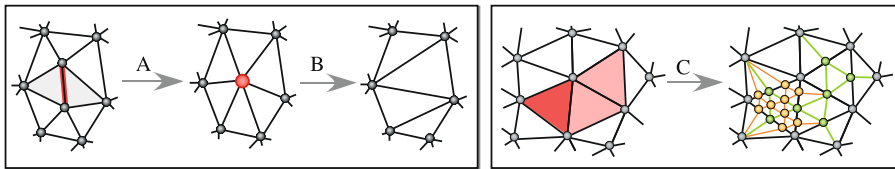

Fig. 4 Examples of local operations modifying the resolution of a triangulated surface. The *edge collapse* (**A**) and *node collapse* (**B**) operations coarsen the triangular mesh (elements to collapse highlighted in red). Triangle subdivision (**C**) refines the mesh: all *red triangles* are subdivided once (resulting in green nodes and edges), and the *dark red triangle* is subdivided twice (resulting in orange nodes and edges)

to allow for spatially varying resolution on geological surfaces (e.g., few points in smoothly varying areas, and high densities in high curvature areas, Fig. 3). This need for adaptive resolution is a motivation for using triangulated surfaces (also known as triangulated irregular networks, TINs) as compared to rigid computer representations such as 2D grids (Fernández et al. 2004; Lemon and Jones 2003; Mallet 1997, 2002). In practice, the resolution of a geological surface can be locally adapted to meet the appropriate density. Decimation of a triangulated surface removes nodes carrying redundant information. Decimation is based on node collapse (Fig. 4(A)) or edge collapse (Fig. 4(B)) operations. Conversely, densification (Fig. 4(C)) increases surface resolution. Densification can be performed arbitrarily or semi-automatically by considering the misfit between the surface and the data or using subjective assessment. Hierarchical surfaces such as quad trees provide another means of dealing with locally variable resolution, but call for special processing to handle faults.

4.2 Mesh Quality

While local mesh editing is extremely useful to locally adapt surface resolution, it can introduce elongated triangles (Fig. 4(C)). However, many numerical codes running on polygonal surface meshes are sensitive to mesh quality. In the case of TINS, triangles should have the largest possible minimal angle. This geometric consideration has an incidence on the surface topology. In an ideal surface made only of equilateral triangles, each internal node has exactly six neighbors, separated by angles of 60°. Of course, such a surface is of little practical interest, for it can only represent a plane.
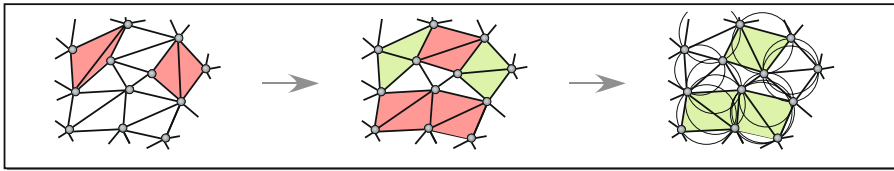
Fig. 5 Three alternative triangulations of the same set of points. *Arrows* and *colors* indicate how the edge flipping operation transforms one triangulation into another (*red*, before edge flipping; *green*, after edge flipping). The rightmost triangulation honors the Delaunay criterion

When representing a specific 3D shape, the departure from that ideal mesh should remain as small as possible. For a given geometry of surface points, the triangulation maximizing mesh quality honors the Delaunay condition, which states that the circumscribed circle of every triangle should not contain any point of the surface (Delaunay 1934). From any given triangulated surface, edge flipping can be used to match this criterion (Fig. 5). Other topological operations such as node relocation or node collapse (Fig. 4(B)) may also be used to improve mesh quality. Automatic mesh improvement tools are often proposed by geomodeling software to avoid tedious manual mesh editing. Such automated tools are very convenient to combine adaptive surfaces resolution and acceptable mesh quality (Fig. 3). Nevertheless, these processes can be time-consuming, so it is good to keep an eye on mesh quality throughout the 3D structural modeling process.

4.3 Data Misfit

As mentioned in Sect. 4.1, a possible cause for data misfit lies in a poor surface resolution. Global or local mesh refinement may then be a strategy to increase the surface accuracy. A common situation is to honor approximately soft data (e.g., seismic picks) and exactly hard data (e.g., well pierce points). Kriging the hard data with locally varying mean supplied by soft data is a possible way to tackle the problem. Alternatively, least-squares interpolation such as Discrete Smooth Interpolation (Mallet 1992) can affect different weights to each type of information. The hard data may also be inserted into the mesh and fixed in later steps (in this case, spikes on the surface can be avoided by interpolating the error on the surface, then displacing the surface nodes so that the error becomes null).

5 Structural Modeling Process

Structural modeling is generally achieved in two steps (Fig. 1(C)–(D)). First, fault surfaces are built to partition the domain of study into fault blocks. Then, stratigraphic horizons are created, following the rules described in Sect. 3. In general, this process takes geological data into account. Therefore, we will first describe some surface construction strategies to account for typical data types.
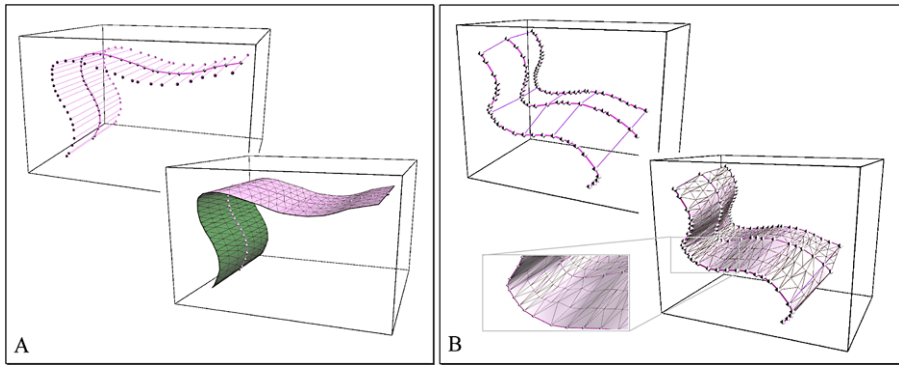
Fig. 6 Direct surface construction from curves. A curve and an expansion vector can be used to generate a cylindrical surface (**A**). Simple surfaces may also be generated by associating a series of cross sections lines (**B**). In both cases, the mesh quality depends on the regularity of the line sampling; in addition to line regularization strategies, later improvement of the surface mesh may be needed

5.1 Surface Construction

5.1.1 Direct Triangulation

Surface construction strategies vary depending on the type of geological surface created and structural complexity. In some cases, surfaces may be assumed cylindrical and can be created from a polygonal line and an expansion vector (Fig. 6(A)). This type of hypothesis is often convenient when creating fault surfaces from map traces. The expansion vector is often the dip vector $\mathbf{v} = [v_x \ v_y \ v_z]^T$. Following the right-hand rule convention, this vector is obtained from the average surface strike angle θ (between 0 and 2π, where 0 denotes the northing direction $[0\ 1\ 0]^T$) and dip angle φ (between 0 and π)

$$v = \begin{bmatrix} \cos\theta \cdot \cos\varphi \\ -\sin\theta \cdot \cos\varphi \\ -\sin\varphi \end{bmatrix}. \tag{1}$$

As actual cylindrical surfaces are seldom encountered in nature, it is also possible to associate several lines interpreted on parallel cross-sections to obtain piecewise conical surfaces (Fig. 6(B)). In both cases, the number of line-parallel triangle strips to be inserted should be such that triangles are roughly isotropic. Direct surface construction from non-intersecting lines is often unsatisfactory. Indeed, one cannot directly account for intersecting lines or unstructured point sets. Moreover, the conical or cylindrical surfaces are just too simple to approximate the actual geometry of structural surfaces (Pollard and Fletcher 2005). Triangulation of the data points can overcome these limits. As seen in Sect. 4.2, the Delaunay triangulation of the data points maximizes the mesh quality of triangles. It is obtained by local or global projections of the data onto a plane (for instance the average plane) so that the empty circle condition can be checked. By definition, the boundary of the Delaunay triangulation is the convex hull of the points in the projective plane, which may yield border effects on the final result (Fig. 7(A)). A typical strategy is to also use a polygonal curve
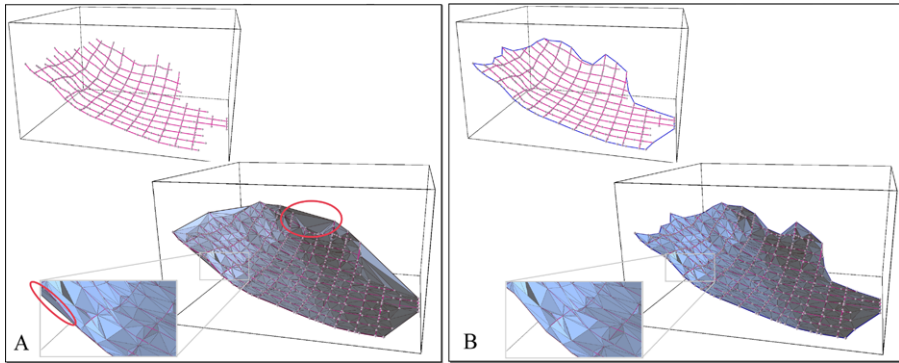
Fig. 7 Delaunay triangulation of 3D curve points (**A**) is bounded by the convex hull of the points, which may locally generate mesh elements orthogonal to the overall orientation of the surface (highlighted by *red ellipses*). This border effect may be addressed by using a surface outline (**B**). In both cases, mesh quality is poor due to irregular and noisy sampling

bounding the domain, and clipping away the Delaunay triangles outside of that curve (Fig. 7(B)). Direct triangulation, either from extrapolated curves or from points, exactly honors input points. This is both good and bad because any noise present in the data, e.g., due to picking errors, is incorporated in the surface geometry. Moreover, direct triangulation seldom produces a good quality mesh because data points are often irregularly sampled (Fig. 7). Mesh density is directly related to point density and not to geometric features such as surface curvature. For these reasons, automatic mesh improvement is often needed before further modeling steps.

5.1.2 Indirect Surface Construction

One way to tackle the limitations of the direct triangulation methods is to interpolate some initial surface under constraints to minimize the data misfit. Kriging may be used for that purpose, but often implies that only surface elevation is considered. Other interpolators able to integrate several types of constraints have also been described and used for structural modeling purposes (Haecker 1992; Kaven et al. 2009). Here, we will only cover Discrete Smooth Interpolation (DSI), which optimizes all three spatial coordinates of mesh vertices under a large set of constraints (Mallet 1992, 1997, 2002). Shortly, DSI solves for the optimal location of the surface nodes to minimize a weighted sum of the surface roughness and the constraint misfit. Roughness can be formulated as the discrete Laplacian computed over the surface, and ensures the convergence of the method, provided at least one fixed point per surface (Mallet 1992). Constraint is a generic term to describe how data and interpretations are accounted for. Strict constraints restrict the degrees of freedom of surface nodes during the interpolation. For instance, the Straight Line constraint allows a node to move only along a specified direction; the Cylinder constraint allows a node to move along a specified plane; a Control Node is frozen to a given location in space. In addition, soft constraints are honored in a least-squares sense by DSI. For example, a Control Point attracts the surface along a specific direction as a rubberband. Customizing this direction is handy to build complex geological surfaces such

as salt domes (Mallet 1997). Some surface border nodes may also be constrained to move along another surface, which is very useful to account for contacts between geological surfaces. Also, Thickness and Range Thickness constraints may be used to force the interpolated surface to lie at a given distance from another surface. This distance is computed on a vector field, so may be either the apparent vertical thickness or the true thickness when the vector field is normal to the surface. The indirect surface construction with DSI is illustrated in Fig. 8. In this example, the initial surface is obtained from the triangulation of a planar boundary curve (Fig. 8(A)). The curve sampling is regularized, and internal points are automatically inserted at the center of the triangle's circumscribed circles to ensure a homogeneous mesh density and a satisfying mesh quality. This initial surface is then interpolated with DSI using control points and boundary constraints (Fig. 8(B)), straight line constraints (green segments) are set on the axis-parallel surface borders, and cylinder constraints (green transparent planes) are used on the other borders; lines are used as control points, and attract the surface along a fixed direction (in red). The surface mesh is then refined, and the attraction direction is set to be locally orthogonal to the surface (Fig. 8(C)). To remove saddle effects due to roughness minimization, axial curves and local surfaces created with curve extrapolation techniques may be used as interpretive data to better constrain the interpolated geometry by fold axis orientation (Fig. 8(D)). The resulting surface (Fig. 8(E)) can be refined and interpolated again for a smooth aspect (Fig. 8(F)). In general, whatever the interpolation method retained, indirect surface
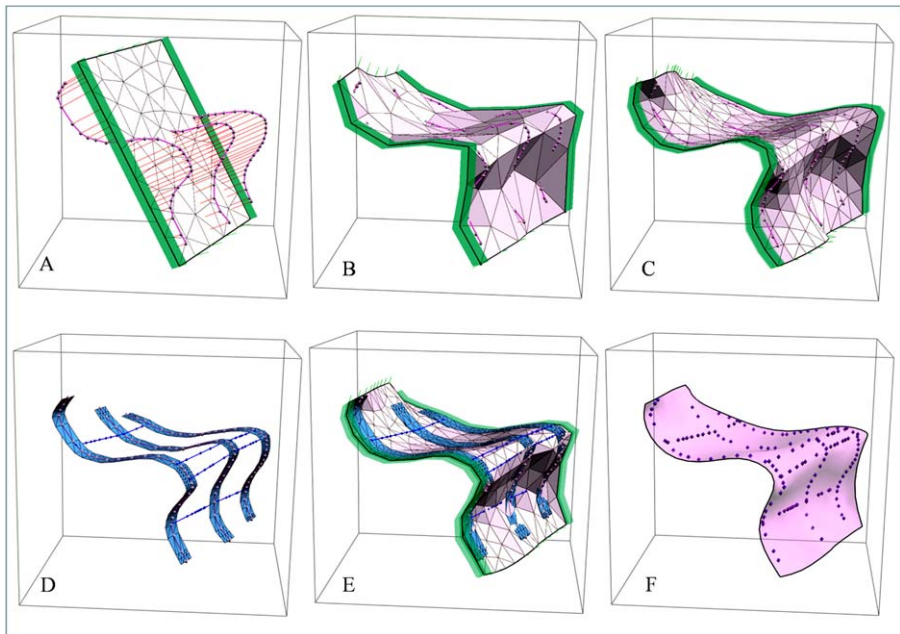

Fig. 8 Indirect surface construction. The initial coarse surface (**A**) is interpolated under constraints to yield a first approximate surface (**B**). Mesh is refined and constraint direction is optimized (**C**). Additional interpretive data are added (**D**), before final mesh refinement and interpolation (**E**). *Flat triangle shading* is used deliberately to highlight the effect of surface resolution

construction provides a practical way to adapt surface resolution to one's needs, and to add interpretive data to better constrain interpolation results.

5.2 Fault Network Modeling

Faults are very important in structural modeling, for they partition space into regions where stratigraphic surfaces are continuous. Therefore, it is important to generate faults and to determine how faults terminate onto each other before considering other geological surfaces. The methods described in Sect. 5.1 may be used to create fault surfaces. Defining the connectivity between these fault surfaces is probably the most important and the most consequential step in structural modeling. This can usually be done by considering the geometry of both fault data and surrounding horizon data to assess the fault slip. Indeed, the fault slip should always be null at dangling fault boundaries (Sect. 3.2). Therefore, when a horizon on either side of a fault is significantly offset near the fault boundary, this usually suggests that the fault border should be extrapolated or projected onto another fault (Fig. 9). This information can then be used to fill the gap between the branching fault and the main fault. In the DSI framework, this is achieved using a Border on Surface constraint. This extrapolation does not necessarily mean that meshes along the contact are coincident. Therefore additional processing may be needed to obtain a sealed contact (Caumon et al. 2004; Euler et al. 1998).

5.3 Horizon Modeling

Horizon construction may be achieved fault block by fault block, from horizon data using either direct or indirect surface building methods. The logical borders must then be defined interactively to ensure that horizon borders are properly located onto fault surfaces (for instance, with border on surface constraints in the DSI framework). This block-wise approach is adapted for simple models with few faults. Each step of the process is manually controlled, and can be specifically adjusted to the goal at hand. As a counterpart for this control, the process may be very tedious. Therefore, one
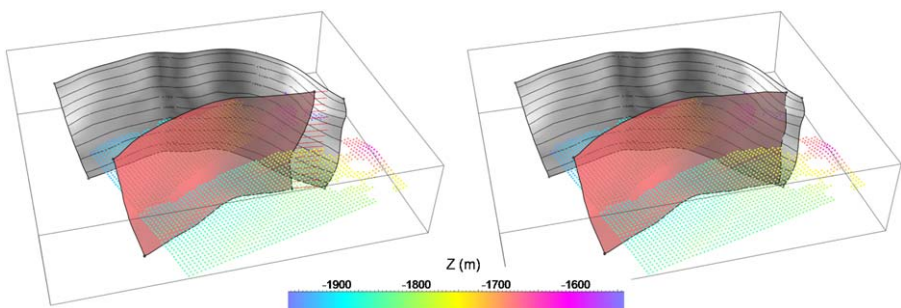

Fig. 9 Defining and enforcing a contact between faults. In this example, the slip is evaluated from the offset of neighboring horizon data (displayed with colored elevation Z). The contact between the initial red fault and the grey fault is highlighted by *red lines* (*left*). After interpolation, the red fault is extrapolated onto the main fault (*right*)

may also create each horizon at once as if faults did not exist, then cut the horizon by the faults and interpolate under constraints (Fig. 10). This approach is more appealing than the piecewise construction, because it automatically computes the topology of the horizon (i.e., the number of fault blocks and the definition of logical borders). Also, boundary conditions necessary to the model validity can be determined automatically. The tradeoff for this automation is twofold. First, it is very sensitive to the quality of the fault network representation. Small gaps between fault surfaces may lead to artificial ramps connecting two fault blocks. Second, the surface cut tends to over-refine the mesh of the cut surface along the intersection line. Mesh improvement is therefore needed before proceeding with further modeling steps (Fig. 10(C)). The reasoning made for modeling faulted horizons can be extended to stratigraphic unconformities. Once again, horizons truncated laterally because of onlap or erosion may be modeled conformably to the truncating surface. However, as layers most often pinch-out tangentially, it is often better in practice to model each stratigraphic surface as if no unconformity were present, and then trim the horizons later on, depending on truncation rules (Fig. 2(B)–(C)). During interpolation, data points located close to faults may attract the corresponding surface on the other side of the fault. This is typically observed with vertical interpolation in the presence of non-vertical
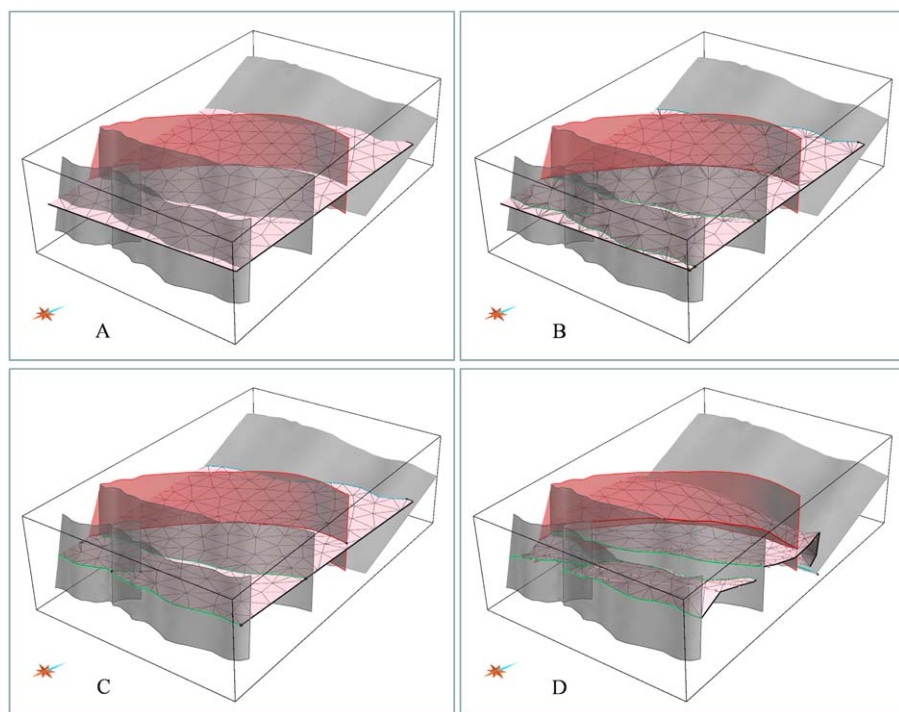

Fig. 10 Main steps of faulted horizon modeling. The initial horizon surface (**A**) is cut by the fault network (**B**). After mesh improvement around cut lines and removal of the unconstrained southern part (**C**), interpolation of the horizon is performed so as to maintain contacts between horizon borders and faults, and to honor map traces and cross-section data (**D**)

faults. The corresponding artifacts can often be checked on a fault juxtaposition diagram or Allan map, which highlights unrealistic variation of fault slip. Most often, this is corrected by manually re-drawing or editing the contact, or simply by ignoring data points in the neighborhood of the fault surface and re-interpolating. Another simple quality control on interpolated horizons is to check layer thickness. In the case of sparse data, unrealistic thickness variations may indeed originate from the lack of observation data. In this case, one may use interpretive data, manual surface updates or thickness constraints of the interpolation method (Mallet 2002, p. 269; Kaven et al. 2009).

6 Recent and Ongoing Research

3D structural modeling research mostly aims at better addressing the differences in quality and scale in data, and at incorporating more geological rules into modeling methods. For instance, Thibert et al. (2005) constrain horizon surfaces constructed from isoline contours to be developable. Another approach concerns the use of implicit surfaces corresponding to isovalues of a 3D scalar field. A major benefit of implicit surfaces is that they directly enforce the validity conditions described in Sect. 3.2, at the cost of larger memory usage. They also make model updating much easier than with surface-based methods. For instance, Chilès et al. (2004) and Calcagno et al. (2008) use dual kriging to create a 3D potential field whose equipotentials describe the geometry of horizons and faults. Mallet's (2004) GeoChron theory defines a mapping $u(x, y, z) = [u, v, t]^{\mathrm{T}}$ between the present subsurface geometry and geo-chronological space by representing paleo-geographic coordinates (u, v) at the time of deposition t. An implementation of this theory based on tetrahedral meshes conformable to faults is described by Frank et al. (2007), Moyen et al. (2004), Tertois and Mallet (2007). In addition to these two directions, we believe the next frontier of structural modeling is the creation of several structural models instead of one, all equally honoring available data at their specific scale (Caumon et al. 2007; Holden et al. 2003). Such a set of realizations could be filtered by validation codes such as balanced restoration and geomechanical modeling (Maerten and Maerten 2006; Moretti 2008; Muron 2005). Another avenue for further progresses also covers assimilation of complex data such as reservoir production history for discarding possible structural interpretations (Suzuki et al. 2008; Tarantola 2006).

7 Conclusions

We have reviewed a set of rules and guidelines to create consistent structural models made of free-form 3D surfaces. Typically structural modeling workflows start with georeferencing the data, then building the fault network, and finally generating 3D horizons which are consistent with faults and stratigraphic layering rules.

Throughout this process, three main elements must be born in mind. Firstly, the quality and reliability of available data should be considered to define the data integration strategy, and possibly guide choices when inconsistencies are observed or

become patent during modeling. Secondly, the care of the numerical representation of a model should be considered. Good mesh quality can often be obtained automatically thanks to progresses in geometry processing. However, modeler's input is critical for adapting model resolution to one's needs, in order to best exploit available computer hardware. Lastly, the basic volumetric consistency and the kinematic realism of the model should be observed. Although the direct generation of compatible geological structures often remains a problem, visual quality control is a must to detect inconsistencies. Additionally, quantitative restoration methods can be used to further check model realism and quantify deformations.

In many applied studies, 3D structural model building is not an end, but a means to address a natural resource estimation problem, for instance, the understanding of flow in an underground reservoir. In this case, a natural trend is to focus on the final modeling output, and to make approximations in the 3D structural model. This is very risky, and, when needed, should always be backed up by facts (well tests, reservoir production, sensitivity studies). Even so, a structural model directly controls gross rock volumes and connectivity of high and low values, provides clues to characterize strain, and defines the stationary regions, the distances, and possibly the spatial trends needed by geostatistics for petrophysical modeling. This makes it very difficult to predict the impact of a structural error on the final output. Accuracy about 3D structures is therefore a key factor in the successful design of predictive earth models.

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