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Mineral Deposit Densities for Estimating Mineral Resources

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Abstract Estimates of numbers of mineral deposits are fundamental to assessing undiscovered mineral resources. Just as frequencies of grades and tonnages of well-explored deposits can be used to represent the grades and tonnages of undiscovered deposits, the density of deposits (deposits/area) in well-explored control areas can serve to represent the number of deposits. Empirical evidence presented here indicates that the processes affecting the number and quantity of resources in geological settings are very general across many types of mineral deposits. For podiform chromite, porphyry copper, and volcanogenic massive sulfide deposit types, the size of tract that geologically could contain the deposits is an excellent predictor of the total number of deposits. The number of mineral deposits is also proportional to the type's size. The total amount of mineralized rock is also proportional to size of the permissive area and the median deposit type's size. Regressions using these variables provide a means to estimate the density of deposits and the total amount of mineralization. These powerful estimators are based on analysis of ten different types of mineral deposits (Climax Mo, Cuban Mn, Cyprus massive sulfide, Franciscan Mn, kuroko massive sulfide, low-sulfide quartz-Au vein, placer Au, podiform Cr, porphyry Cu, and W vein) from 108 permissive control tracts around the world therefore generalizing across deposit types. Despite the diverse and complex geological settings of deposit types studied here, the relationships observed indicate universal controls on the accumulation and preservation of mineral resources that operate across all scales. The strength of the relationships ($R^2 = 0.91$ for density and 0.95 for mineralized rock) argues for their broad use. Deposit densities can now be used to provide a guideline for expert judgment or used directly for estimating the number of most kinds of mineral deposits.

Keywords Mineral resource assessment · Number of deposits

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Introduction

Undiscovered mineral resources are the primary source of the long-term supply of minerals. Their estimation can be captured by three components: the number of mineral deposits, the deposit grades and tonnages, and the deposit locations. For many kinds of deposits, geologic controls on general locations and grades and tonnages can be represented by proper use of mineral deposit models (Cox et al. 1986). Estimates of the number of undiscovered deposits are presented in probabilistic terms to capture the uncertainty inherent in such estimates (Singer 1993). Typically these estimates have been made by expert judgment. However, estimating numbers of undiscovered mineral deposits could be based on deposit density models (deposits/area), or expert judgment, or some combination. Mineral deposit densities provide models useful to guide estimates of the number of undiscovered deposits in the same way that tonnage and grade frequencies are models of sizes and qualities of undiscovered deposits (Singer 1993; Singer et al. 2001). Estimates of deposit density are based on frequencies of deposits per unit of permissive area in well-explored control areas around the world. Deposit density estimates are made for a particular deposit type within a delineated region, the geology which is permissive for the deposit type, and that has a grade and tonnage model for the same deposit type.

Results from previous studies on deposit densities (Bliss and Menzie 1993; Singer 1994; Singer et al. 2001, 2005; Mosier et al. 2007) reveal several consistent patterns. One pattern is the relationship of permissive control areas to deposit densities, which is examined here for the three deposit types with the most information about variability within types. A second pattern is that deposit sizes are related to deposit densities. The power of combining sizes of permissive tracts and deposits to predict deposit densities or total resources in a tract regardless of deposit type is next demonstrated. Physical processes that might account for the strong relations between deposit density or total tract resources and permissive tract sizes and deposit sizes are then briefly examined. First, it is necessary to discuss the data used for deposit densities.

Nature of the Data

Unbiased examination of mineral deposit densities requires that the empirical evidence be collected in a consistent manner. Only grades and tonnages from well-explored deposits that meet the rules appropriate to the deposit type are used. All of the deposits used here were properly typed. For example, the operational spatial rule for porphyry copper deposits used to determine which orebodies should be combined was that all mineralized rock or alteration separated by less than two kilometers of unmineralized or unaltered rock was combined into one deposit. Thus, if alteration zones of two deposits are within two kilometers of each other, the deposits are treated as a single deposit. Permissive control areas exclude geologic settings where a deposit type could not exist. Because most permissive areas that are covered are poorly explored, the numbers of deposits that are reported from covered areas are typically biased downward and therefore are not representative of the densities (deposits/area) or spatial distributions of deposits. For most deposits and deposit densities used in this study, parts of the permissive area covered were excluded from analysis. These rules are applied to all deposits used to construct control tracts.

Data used herein (Table 1) is from Singer et al. (2001), Lisitsin et al. (2007), and Mosier et al. (2007) with the following exceptions: (1) the chromite deposit data of Singer (1994) was supplemented by adding deposits as small as one ton; (2) diamond pipes were not used because there are questions about the boundary of the permissive tract (Bliss 2007); and (3) the porphyry copper control areas are from (Singer et al. 2005) except those from South America were superseded by more recent studies (Cunningham et al. 2007). The South American control tracts contain the total tract areas, the number of known deposits and estimates of the number of undiscovered deposits. From all sources, there are ten different types of mineral deposits (Climax Mo, Cuban Mn, Cyprus massive sulfide, Franciscan Mn, kuroko massive sulfide, low-sulfide quartz-Au vein, placer Au, podiform Cr, porphyry Cu, and W vein) from 108 permissive control tracts around the world.

Permissive Tracts

To consistently assess the undiscovered mineral resources of regions, areas are delineated where geology permits the existence of specific deposit types. These permissive tracts can be considered natural sampling control areas for the specific mineral deposit types rather than arbitrary cells. Permissive tracts are based on geological criteria derived from deposit models that are themselves based on studies of known deposits outside and perhaps within the study area. Permissive boundaries are defined such that the probability of deposits of the type delineated occurring outside the boundary are negligible; that is, generally less than 1 in 100,000.

Areas are excluded from permissive tracts only on the basis of geology, knowledge about unsuccessful, but comprehensive exploration, or the presence of barren overburden exceeding some predetermined thickness. A geologic map is the primary source of local information for delineating tracts or control areas and identifying which tracts are permissive for different deposit types. Descriptive deposit models identify the tectonic setting and geologic environment of each deposit type. Identification of a quality map for this purpose is a function of whether the map differentiates rock and age units that have a bearing on recognizing the geologic setting of a deposit type. Although it is not required that the control area be completely explored, it is important that the proportion of the number of exposed deposits and areas explored be estimated so that an adjustment can be made.

An example of using rock and deposit ages is the delineation of porphyry copper deposits that are formed in island and continental volcanic-arc subduction-boundary zones. The delineated control areas used for density estimates of porphyry copper deposits were outlined using maps with sufficient detail to identify borders of magmatic arcs while taking into consideration the deposit ages and distributions of major structures (Singer et al. 2005). Permissive tracts for podiform chromite deposits are delineated from maps showing where the ultramafic parts of ophiolite complexes are exposed in accreted terranes (Albers 1986).

Table 1 Control areas listed by deposit type, control area location, area permissive in km^2, number of deposits, median tonnage of type in tract in millions of metric tons, total tonnage in tract in millions of metric tons, and data source for deposit densities used here

Deposit type	Control area location	Area km^2	Number	Median tons (m)	Total tons (m)	Source
Climax porphyry Mo	USCO	12,000	4	240.1	960	Singer et al. 2001
Climax porphyry Mo	USNM	3,200	1	423.9	424	Singer et al. 2001
Cuban Mn	Cuba	1,200	120	0.00293	7.97	Mosier and Page 1988
Cuban Mn	Fiji	2,720	18	0.00369	0.170	Mosier and Page 1988
Cyprus m. sulfide	Betts Cove, CNNF	433	4	2.147	20.3	Mosier et al. 2007
Cyprus m. sulfide	Big Mike, USNV	420	1	0.100	0.100	Mosier et al. 2007
Cyprus m. sulfide	Cyprus	1,016	14	0.925	43.6	Mosier et al. 2007
Cyprus m. sulfide	Lokken, NRWY	949	2	1.581	25.1	Mosier et al. 2007
Cyprus m. sulfide	Smartville, USCA	518	3	0.128	1.51	Mosier et al. 2007
Cyprus m. sulfide	Sunro, CNBC	480	1	2.780	2.78	Mosier et al. 2007
Franciscan Mn	N. USCA	23,700	450	0.00015	0.904	Mosier and Page 1988
Franciscan Mn	Hokaido, Japan	3,890	117	0.00293	2.04	Mosier and Page 1988
Kuroko m. sulfide	Ammonoosuc, USME-USCT	1,136	3	0.136	0.650	Mosier et al. 2007
Kuroko m. sulfide	Ashland, USAL	81	1	1.300	1.30	Mosier et al. 2007
Kuroko m. sulfide	Binghampton, USAZ	26	1	0.363	0.363	Mosier et al. 2007
Kuroko m. sulfide	Buchans, CNNB	1,051	4	4.514	20.1	Mosier et al. 2007
Kuroko m. sulfide	Castine Fm., USME	236	3	0.182	1.00	Mosier et al. 2007
Kuroko m. sulfide	Chestatee, USGA	27	1	1.100	1.10	Mosier et al. 2007
Kuroko m. sulfide	Copper Hill, USCA	424	4	0.315	1.82	Mosier et al. 2007
Kuroko m. sulfide	Dominican Republic, DMRP	338	3	1.004	4.74	Mosier et al. 2007
Kuroko m. sulfide	East Shasta, USCA	73	2	0.306	0.771	Mosier et al. 2007
Kuroko m. sulfide	Flin Flon-Snow Lake, CNMN-CNSK	2,656	15	0.809	79.2	Mosier et al. 2007
Kuroko m. sulfide	Gopher Ridge, USCA	1,343	8	0.140	3.67	Mosier et al. 2007
Kuroko m. sulfide	Hawley-Bernard, USMS-USVT	297	1	0.902	0.90	Mosier et al. 2007
Kuroko m. sulfide	Hillabee, USAL-USGA	218	2	0.497	2.84	Mosier et al. 2007
Kuroko m. sulfide	Hokoruku, JAPN	900	8	10.444	129	Singer et al. 2001
Kuroko m. sulfide	Hornblende Gneiss, USGA	587	2	0.278	0.706	Mosier et al. 2007
Kuroko m. sulfide	Iberian Pyrite Belt, SPAN-PORT	1,300	48	2.046	1022	Mosier et al. 2007
Kuroko m. sulfide	Jerome, USAZ	24	1	29.000	29.0	Mosier et al. 2007
Kuroko m. sulfide	Kunitomi, JAPN	416	5	2.100	2.10	Mosier et al. 2007
Kuroko m. sulfide	Kutcho Creek, CNBC	243	1	22.600	22.6	Mosier et al. 2007
Kuroko m. sulfide	Myra Falls, CNBC	1,117	2	1.176	5.45	Mosier et al. 2007

Table 1 (*Continued*)

Deposit type	Control area location	Area km^2	Number	Median tons (m)	Total tons (m)	Source
Kuroko m. sulfide	North Haven, USME	36	1	0.050	0.050	Mosier et al. 2007
Kuroko m. sulfide	Orient, CUBA	3,390	2	2.391	11.9	Mosier et al. 2007
Kuroko m. sulfide	Pecos, USNM	149	1	2.090	2.09	Mosier et al. 2007
Kuroko m. sulfide	Quoddy Fm., USME	233	2	0.545	1.15	Mosier et al. 2007
Kuroko m. sulfide	Rudny–Altai, Russia–China	20,539	8	1.242	206	Mosier et al. 2007
Kuroko m. sulfide	Snake River, USOR	1,071	2	4.469	39.5	Mosier et al. 2007
Kuroko m. sulfide	Standing Pond, USMS-USVT	123	1	0.050	0.050	Mosier et al. 2007
Kuroko m. sulfide	Tasmania, AUTS	825	4	14.804	157	Mosier et al. 2007
Kuroko m. sulfide	Urals, Russia	81,615	19	12.366	634	Mosier et al. 2007
Kuroko m. sulfide	West Shasta, USCA	117	8	0.266	11.5	Mosier et al. 2007
Kuroko m. sulfide	Winterville Fm., USME	621	1	33.000	33.0	Mosier et al. 2007
Kuroko m. sulfide	Yavapai, USAZ	89	1	1.429	1.43	Mosier et al. 2007
Low sulfide qtz-gold	Bendigo, AUVT	7,000	23	0.427	60.9	Lisitsin et al. 2007
Placer gold	Kenai, USAK	3,260	20	1.07152	21.4	Singer et al. 2001
Placer gold	Weisman, USAK	2,760	15	1.07152	16.1	Singer et al. 2001
Podiform chromite	Alameda, USCA	31	2	0.000260	0.00155	This study, Singer 1994
Podiform chromite	Amador, USCA	19	8	0.000120	0.00119	Singer 1994
Podiform chromite	Butte, USCA	100	37	0.000085	0.0290	Singer 1994
Podiform chromite	Calaveras, USCA	66	17	0.000254	0.00765	Singer 1994
Podiform chromite	Del Norte, USCA	711	47	0.000101	0.0570	Singer 1994
Podiform chromite	El Dorado, USCA	94	30	0.000136	0.142	Singer 1994
Podiform chromite	Fresno, USCA	133	31	0.000121	0.0492	Singer 1994
Podiform chromite	Glenn, USCA	80	6	0.000540	0.0360	Singer 1994
Podiform chromite	Humboldt, USCA	271	16	0.000048	0.00399	Singer 1994
Podiform chromite	Lake, USCA	277	14	0.000034	0.00232	This study, Singer 1994
Podiform chromite	Mendocino, USCA	140	9	0.000021	0.00104	This study
Podiform chromite	Monterey, USCA	60	3	0.000017	0.000308	This study, Singer 1994
Podiform chromite	Napa, USCA	238	11	0.000137	0.00761	Singer 1994
Podiform chromite	Nevada, USCA	39	38	0.000044	0.00740	Singer 1994
Podiform chromite	Placer, USCA	51	86	0.000065	0.0215	Singer 1994
Podiform chromite	Plumas, USCA	208	26	0.000029	0.00209	Singer 1994
Podiform chromite	San Louis Obispo, USCA	187	16	0.000110	0.0572	This study, Singer 1994
Podiform chromite	Santa Barbara, USCA	50	6	0.000014	0.000195	This study, Singer 1994
Podiform chromite	Santa Clara, USCA	112	6	0.000135	0.00103	This study

Table 1 (*Continued*)

Deposit type	Control area location	Area km^2	Number	Median tons (m)	Total tons (m)	Source
Podiform chromite	Shasta, USCA	261	19	0.000130	0.0217	Singer 1994
Podiform chromite	Sierra, USCA	35	17	0.000061	0.00591	Singer 1994
Podiform chromite	Siskiyou, USCA	1,221	87	0.000046	0.0917	Singer 1994
Podiform chromite	Sonoma, USCA	147	6	0.000331	0.00531	This study, Singer 1994
Podiform chromite	Stanislaus, USCA	38	6	0.000216	0.00333	Singer 1994
Podiform chromite	Tehama, USCA	151	33	0.000072	0.0116	Singer 1994
Podiform chromite	Trinity, USCA	1,034	46	0.000032	0.00646	Singer 1994
Podiform chromite	Tulare, USCA	69	7	0.000242	0.00445	Singer 1994
Podiform chromite	Tuolumne, USCA	107	23	0.000190	0.0511	Singer 1994
Porphyry copper	West Philippine 1	47,100	15	125.8	3,300	Singer et al. 2005
Porphyry copper	USAZ–Mexico 7	216,700	46	290.5	51,038	Singer et al. 2005
Porphyry copper	Carpathian-Balkan, SE Eur.15	77,400	21	403.2	4,330	Singer et al. 2005
Porphyry copper	Central Philippine 2	76,580	15	99.8	2,638	Singer et al. 2005
Porphyry copper	Cret. Peru (7)	107,296	6	75.3	2,738	Cunningham et al. 2007
Porphyry copper	East China 17	146,320	4	330.1	2,077	Singer et al. 2005
Porphyry copper	East Phillipine 3	90,480	9	90.7	3,202	Singer et al. 2005
Porphyry copper	E-Central Kazakhstan 16	236,390	15	159.7	2,086	Singer et al. 2005
Porphyry copper	Eocene Colombia-Panama (1)	51,632	12	700.0	7,725	Cunningham et al. 2007
Porphyry copper	Eocene-l. Olig. CILE (10)	21,200	16	1,852.9	60,864	Cunningham et al. 2007
Porphyry copper	Jurassic Ecuador-Colombia (3)	67,761	17	230.2	9,376	Cunningham et al. 2007
Porphyry copper	l Eocene-u K AGTN (15)	83,201	2	194.0	563	Cunningham et al. 2007
Porphyry copper	l Eocene Peru (9a)	30,154	11	205.8	5,062	Cunningham et al. 2007
Porphyry copper	m Miocene CILE (14a)	17,597	8	186.5	4,373	Cunningham et al. 2007
Porphyry copper	m Miocene Chile–AGTN (13a)	70,587	12	1,120.0	8,620	Cunningham et al. 2007
Porphyry copper	Miocene Central Peru (6)	53,235	27	435.0	18,278	Cunningham et al. 2007
Porphyry copper	Miocene Ecuador-Colombia (5)	58,787	16	153.6	8,525	Cunningham et al. 2007
Porphyry copper	Miocene–Pliocene CILE–AGTN (13b)	41,800	8	1,940.8	8,741	Cunningham et al. 2007

Table 1 (*Continued*)

Deposit type	Control area location	Area km^2	Number	Median tons (m)	Total tons (m)	Source
Porphyry copper	Molong, AUNS 19	11,500	4	265.2	666	Singer et al. 2005
Porphyry copper	N. Sulawesi, INDO 4	34,075	3	56.2	344	Singer et al. 2005
Porphyry copper	USNV	32,800	5	173.3	1,319	Singer et al. 2001
Porphyry copper	Oligiocene, CILE (11)	2,429	2	553.0	1,363	Cunningham et al. 2007
Porphyry copper	Paleocene-E. S. Peru–N. CILE (8)	69,088	24	482.5	18,045	Cunningham et al. 2007
Porphyry copper	Permian, AGTN (16)	29,085	6	308.6	2,931	Cunningham et al. 2007
Porphyry copper	Puerto Rico 8	1,552	2	165.7	344	Singer et al. 2005
Porphyry copper	Quesnellia, CNBC-USWA 5	221,840	25	129.8	8,291	Singer et al. 2005
Porphyry copper	Stikinia, CNBC-USWA	255,700	11	128.0	4,185	Singer et al. 2005
Porphyry copper	u Eocene, CILE (12)	6,913	6	540.0	3,440	Cunningham et al. 2007
Porphyry copper	u Miocene–Pliocene, AGTN (14d)	5,770	5	300.0	2,700	Cunningham et al. 2007
Porphyry copper	u Miocene–Pliocene, CILE (14c)	24,048	8	650.8	6,116	Cunningham et al. 2007
Porphyry copper	u Miocene–Pliocene, CILE (14b)	6,901	4	7,681.2	25,800	Cunningham et al. 2007
Porphyry copper	Yulong China 18	93,400	14	160.9	1,533	Singer et al. 2005
Tungsten vein	SE China	140	4	0.55976	2.24	Singer et al. 2001

Permissive tracts that are well explored and are used to develop deposit density models are called control areas. Boundaries of mapped rock units form the primary basis for drawing limits of permissive control areas. Preliminary tract boundaries are extended using interpolated geology and geophysical surveys, such as aeromagnetics, to identify where younger rocks or sediments conceal permissive rocks. The scale of the maps can have a strong effect on the extent of cover portrayed—detailed maps commonly show more cover than regional maps. In most cases, covered areas are excluded from control areas because typically they are poorly explored and, therefore, would provide biased estimates of density. Estimates of deposit densities were made to a depth of 1 km beneath the surface. For some deposit types like podiform chromite, the extent to which undiscovered deposits might exist at depth in control areas is not known. Deposit density estimates are formed from known deposits that are consistent with the grade and tonnage models and are located in most cases in exposed permissive control areas.

Deposit Density and Permissive Area

Deposit density models are designed to be used within a three-part assessment system (Singer 1993), which affects how the models should be constructed. In this system, grade and tonnage models have the form of frequency distributions of tonnages and average grades of well-explored deposits of each type—they serve as models for grades and tonnages of undiscovered deposits of the same type occurring in geologically similar settings. Biases can be introduced into three-part assessments either by a flawed grade and tonnage model or by lack of consistency of the number-of-deposits estimates with the grade and tonnage model. Also, it is important in developing mineral-deposit density models that any rules used in the descriptive or the grade and tonnage model also are applied to the deposit densities.

As an example of the use of rules in models, for volcanogenic massive sulfide deposit models, the density control areas contained exposed deposits with tonnages and grades consistent with the grade-tonnage models for kuroko- or Cyprus-type deposits, each deposit was separated from other deposits by at least 500 meters of barren rock, and exposure of deposits included the weathered gossan zones or part of the massive sulfide bodies themselves (Mosier et al. 2007). In addition, surficial areas of permissive host rocks were well explored and all deposits that are exposed at the surface are believed to have been found. Although it is not required that the control area be completely explored, it is important that the proportion of the number of exposed deposits and areas explored be estimated so that an adjustment can be made.

Densities of podiform chromite, porphyry copper, and volcanic-hosted massive sulfide deposits were estimated per 100,000 km^2 of exposed permissive rock. Use of the 100,000 km^2 scaling is so that estimates may be more easily discussed and remembered because the estimates are then represented by whole numbers of deposits, which should be easier to remember than small decimal numbers. Each deposit density histogram is skewed to high densities of deposits (Fig. 1). The few high-density control areas for each type are relatively small in area. Deposit density is inversely related to the area of permissive rock in well-explored tracts for three of the studied deposit types (Fig. 2). Density of deposits decreases with increasing size of delineated tract for each of the types.

Remarkably, despite the diverse origins of these deposit types, each follows approximately the same power law function of permissive area (Fig. 2). Henley and Berger (2000) suggest that the total contained metal within a deposit type follows a power law implying similarity in the processes of formation. In the power law relations presented here, area permissive, A, is the base for a power law scaling of the form

$$\text{Density}\ \left(\text{d/km}^2\right) = S_t A^{\beta}. \tag{1}$$

The exponent, β, is the general scalar which is approximately 0.6 for the three deposit types plotted in Fig. 1. The intercept without the 100,000 km^2 scaling used in the figure, S_t, varies with deposit type and is approximately equal to the inverse of the average aerial extent of mineralization and alteration for each of the three deposit types plotted.

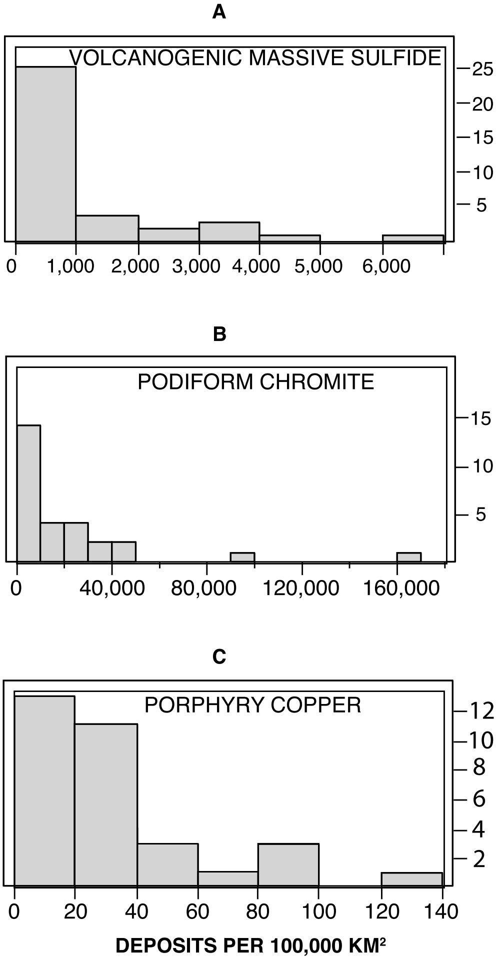

Fig. 1 Histograms of the number of deposits per 100,000 km^2 of permissive rock. *Top* (**A**), volcanogenic (Cyprus + kuroko) massive sulfide deposits; midle (**B**), podiform chromite deposits; *bottom* (**C**), porphyry copper deposits. Data listed in Table 1

As shown by Singer and Menzie (in press), map scales tend to be correlated with sizes of permissive tracts for two deposit types. Part of the relations between scale and area of permissive tracts could be due to effects of map scale on information portrayed. More general map scales mean that areas of geologic units that could contain a particular deposit type appear to be larger than the same areas on detailed maps because of the inclusion of units that are not permissive, and are not differentiated on the more general geologic map. It is also possible that the relationship between permissive area and deposit density is due to the nature of the geologic settings of different deposit types. More specifically, the slopes of the regressions by deposit type shown in Fig. 2 are similar; it is their intercepts that are different. Those intercepts are higher for the types of deposits that are smaller in size. Based on the three deposit types shown in Fig. 2, the kinds of deposits that are smaller (such as podiform chromite) are located in spatially less extensive geologic settings.

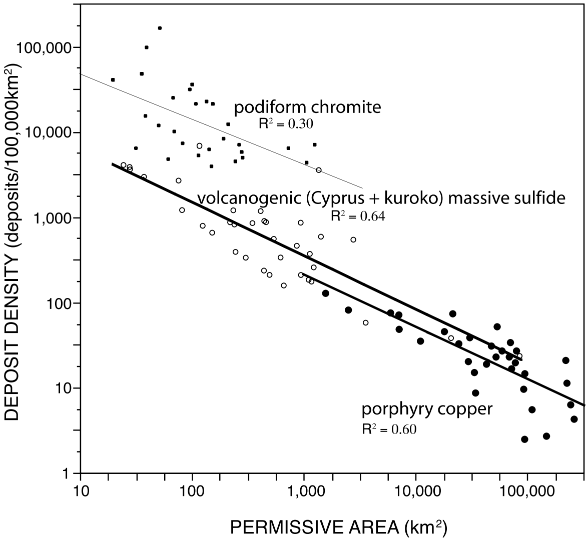

Fig. 2 Density of deposits per 100,000 km^2 versus the permissive area in km^2. *Top* podiform chromite deposits; *middle*, volcanogenic (Cyprus + kuroko) massive sulfide (VMS) deposits; *bottom*, porphyry copper deposits with their respective regression lines. Densite of deposits per km^2 (without the 100,000 km^2 scaling used in the plot) are: 1.64 Area$^{-0.53}$, 0.26 Area$^{-0.62}$, and 0.14 Area$^{-0.61}$ for podiform chromite, VMS, and porphyry copper types, respectively. Data listed in Table 1

Density and Deposit Sizes

The relationship between the size of the permissive area and the deposit density shown for the three deposit types also applies across deposit types. The association of density and deposit size also exists across types. The average aerial extent of mineralization and alteration is not well known for most deposit types so a surrogate that is known, median deposit tonnage, is used to examine its relationship to deposit density (Fig. 3). Tonnages used in this example are typically the median tonnages from the control areas. In cases where only one deposit is known in a tract, the median for the deposit type was used. Across the ten different deposit types plotted, the general scaling from the power law function is −0.39. Thus, deposit density decreases with increasing deposit size as would be expected. Within a deposit type, smaller deposits are more common than large deposits. This is another way to state that tonnages within a deposit type have a highly skewed frequency distribution. The frequency distribution of deposit sizes across deposit types may not be represented by a lognormal distribution, but it certainly is highly skewed with the largest deposits being rare. It is not surprising that the density of deposits is inversely related to deposit size.

General Estimators

The area of permissive tracts and deposit size are the basis for two power law functions predicting mineral deposit density. The two predictors can be used together to predict the density of deposits across deposit types using all of the data in Table 1

$$\log(\text{Density}) = 4.2474 - 0.5146 \log(A) - 0.2254 \log(\text{Deposit Size}), \qquad (2)$$

where Density is the number of deposits per 100,000 km^2, A is the permissive area in km^2, and log(Deposit Size) is the mean of the logged tonnage of interest (in mil-

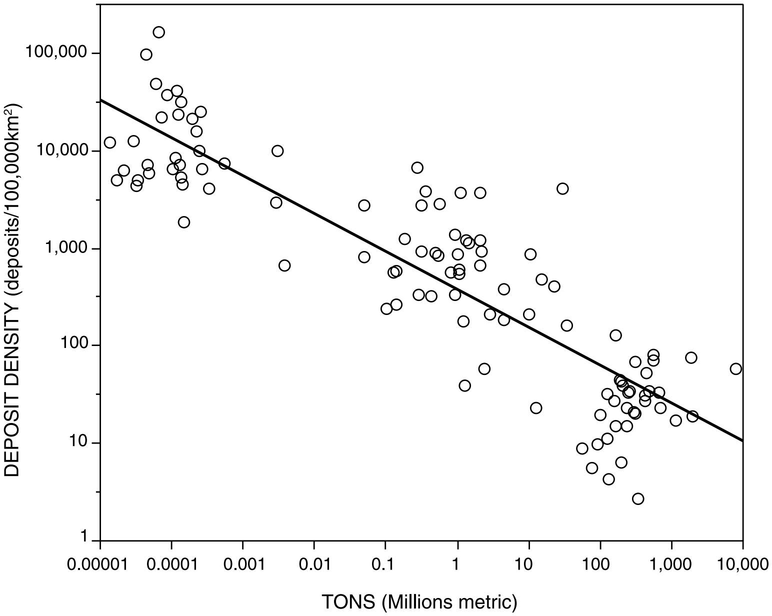

Fig. 3 Median deposit size in millions of tons for all deposit types versus deposit density in deposits per 100,000 km^2. Density od deposits per km^2 (without the 100,000 km^2 scaling used in the plot) is: 0.0036 $\text{Size}^{-0.39}$; $R^2 = 0.77$. Data listed in Table 1

lions metric tons). Over 90 percent of the variation in mineral deposit density is accounted for in this equation ($R^2 = 0.91$). The high R^2 value demonstrates the predictive power of (2). The regressions here are in log–log space. In arithmetic space, the scatter plots are clearly heteroscedastic thereby violating assumptions of tests of significance, so it is necessary to perform the regression in log space. Testing suggests that errors about the regressions can be approximated by (log) normal distributions. For lognormal distributions, the arithmetic median is estimated by 10^u (Aitchison and Brown 1963). From this regression, an estimate of the median number of deposits is

$$\text{Number deposits} = \text{permissive area}/100{,}000 \times 10^{\log(\text{Density})}. \tag{3}$$

Remarkably, area permissive and deposit size also allow prediction of the total amount of mineralized rock of the deposit type in a permissive tract

$$\log(\text{Total tonnage}) = -1.038 + 0.6784 \log(A) + 0.6193 \log(\text{Deposit Size}). \tag{4}$$

Total tonnage is an estimate of the total tonnage of all mineralized rock in the delineated tract of the deposit type in 'Deposit Size'. About 95 percent of the variability in total tonnage in a tract is explained by (4) ($R^2 = 0.95$). These results based on the ten different deposit types used in this analysis suggest that they are, in general, scaled versions of one another. It seems that these equations could be used to estimate number of deposits and the total mineralized rock for any deposit type.

Physical Processes Responsible for Observed Relations

Despite the diverse and complex geologic settings of deposit types studied here—varying from ultramafic rocks for podiform chromite, evolved granites for Climax Mo, and to detrital sediments for placer Au—the relationships observed here indicate universal controls on the accumulation and preservation of mineral resources. Understanding the underlying processes that generate these relationships may provide some insight into why the relations are so strong and perhaps guide applications other than resource assessments. Mineral deposit sizes are determined by both deposit accumulation and destruction through erosion and other means. Deposits range in size over ten orders of magnitude, whereas grades range over only three orders. Within each of the deposit types that are represented by more than two deposits in this study, the distribution of tonnages are approximately lognormal as are tonnages of most deposit types (Singer 1993).

Rigorous tests for lognormality of the tonnages of 67 deposit types by Singer (1993) show that only five of the 67 types tested are significantly different (either skewed or peaked) than lognormal at the one percent level. In addition to the above empirical results demonstrating that the lognormal distribution is an appropriate model for most observed mineral deposit tonnages, there exists a long and distinguished history of scientific publications providing a theoretical and empirical basis to these observations (Allais 1957; Aitchison and Brown 1963; Brinck 1967; Matheron 1959; Razumovsky 1940). Frequency distributions of the area of permissive control tracts of four deposit types with sufficient data (porphyry Cu, podiform Cr, Kuroko massive sulfide, and Cyprus massive sulfide) are also not significantly different than lognormal. Lognormal variables can be thought of as having been generated from the multiplicative products of many small independent factors. Many of the laws of chemistry and physics that control the formation and preservation of ore deposits are multiplicative, such as the flux of metal in hydrothermal systems (Henley and Berger 2000) and the velocity of simple chemical reactions that depend on the product of concentrations of chemical species involved. These processes are scale independent and are useful in understanding the relationship between deposit size and the number of deposits. The correlation of size of permissive tracts and the size of deposits is consistent with some constraints such as the difficulty of making and preserving large deposits in spatially small settings. Large deposits require the availability of large quantities of materials that would typically be available in large geologic settings.

Conclusions

Mineral deposits in this study range over more than ten orders of magnitude in size and ten different types of deposits provide a basis for generalization. An extensive body of empirical evidence shows power law functions of the density of deposits and the total amount of mineralization to size of geologic settings that could contain the different deposit types and to the median size of the deposit types. These results indicate that the general processes responsible for the formation and preservation of these diverse deposits are similar and operate across all scales.

Estimates of the number of mineral deposits are directly related to the size of the permissive tracts within deposit types. These permissive tracts can be considered natural sampling units for particular mineral deposit types in that they are based on geology and not an artificial boundary such as a cell. Deposit density (deposits/area) decreases as the size of permissive tract increases for each of the three deposit types studied. Deposit density decreases as the map scale increases for both volcanic-hosted massive sulfide and porphyry copper deposit types. For most deposit types the relationships developed here represent robust methods to estimate the numbers of deposits and the total resources in delineated tracts. For some deposit types these predictors might not work properly because of the difficultly of delineating the boundaries of the permissive rocks. For example, permissive tracts for Mississippi Valley Zn–Pb deposits might need to be delineated over broad areas where the only geologic information is the presence of carbonate rocks. A related situation might occur where there are widespread covering materials and the permissive geology under the cover represents a small part of the total area but cannot be separately delineated. In such cases, the regression equations presented here would tend to overestimate the resources. Careful integration of geophysics and extrapolated geology would reduce the number of these problems.

The strength of the relationships ($R^2 = 0.91$ for density and 0.95 for mineralized rock) argues for the broad use of these predictors of number of deposits and total resources. Of course where specific deposit density models exist, they are likely to lead to better estimates in that they would have lower variances, and should be used rather than the general models presented here. Deposit densities can now be used to provide a guideline for expert judgment or used directly for estimates of the numbers of most kinds of mineral deposits.

References

Aitchison J, Brown JAC (1963) The lognormal distribution. Cambridge University Press, Cambridge, 176 p

Albers JP (1986) Descriptive model of podiform chromite. In: Cox DP, Singer DA (eds) Mineral deposit models. US Geological Survey Bulletin 1693, p 34

Allais M (1957) Method of appraising economic prospects of mining exploration over large territories—Algerian Sahara case study. Manag Sci 3:285–345

Bliss JD, Menzie WD (1993) Spatial mineral-deposit models and the prediction of undiscovered mineral deposits. In: Kirkham RV, Sinclair WD, Thorpe RI, Duke JM (eds) Mineral deposit modeling. Geological Association Canada Special Paper 40, pp 693–706

Bliss JO (2007) Personal communication

Brinck JW (1967) Note on the distribution and predictability of mineral resources. Euratom 3461, Brussels, 25 p

Cox DP, Barton PR, Singer DA (1986) Introduction. In: Cox DP, Singer DA (eds) Mineral deposit models. US Geological Survey Bulletin 1693, pp 1–10

Cunningham CG et al (2007) Quantitative mineral resource assessment of undiscovered porphyry copper deposits in South America. US Geological Survey, Open-File Report 2007-XX, 204 p (in press)

Henley RW, Berger BR (2000) Self-ordering and complexity in epizonal mineral deposits. Annu Rev Earth Planet Sci 28:669–719

Lisitsin V, Olshina A, Moore DH, Willman CE (2007) Assessment of undiscovered mesozonal orogenic gold endowment under cover in the northern part of the Bendigo Zone. GeoScience Victoria Gold Undercover Report 2, Department of Primary Industries, 98 p, www.dpi.vic.gov.au/minpet/store

Matheron G (1959) Remarques sur la loi de Lasky. Chron Mines O-M Rech Min, 27e annee 282:463–465

Mosier DL, Page NJ (1988) Descriptive and grade-tonnage models of volcanogenic manganese deposits in ocean environments—a modification. US Geological Survey Bulletin 1811, 28 p

Mosier DL, Singer DA, Berger VI (2007) Volcanogenic massive sulfide deposit density. US Geological Survey Scientific Investigations Report 2007-5082, 21 p, http://pubs.usgs.gov/sir/2007/5082/

Razumovsky NK (1940) Distribution of metal values in ore deposits. CR (Dokl) Acad Sci URSS 9:814–816

Singer DA (1993) Basic concepts in three-part quantitative assessments of undiscovered mineral resources. Nonrenew Resour 2(2):69–81

Singer DA (1994) Conditional estimates of the number of podiform chromite deposits. Nonrenew Res 3(3):200–204

Singer DA, Menzie WD Map scale effects on estimating the number of undiscovered mineral deposits. In: Qiuming C and Graeme B-C (eds) 40 years progress in mathematical geology. Elsevier, Amsterdam, 19 p (in press)

Singer DA, Menzie WD, Sutphin D, Mosier DL, Bliss JD (2001) Mineral deposit density—an update. In: Schulz KJ (ed) Contributions to global mineral resource assessment research. US Geological Survey Professional Paper 1640-A, A1–A13. (Also available online at http://pubs.usgs.gov/prof/p1640a/)

Singer DA, Berger VI, Menzie WD, Berger BR (2005) Porphyry copper density. Econ Geol 100(3):491–514