Mathematical Geology, Vol. 39, No. 1, January 2007 (© 2007)
DOI: 10.1007/s11004-006-9065-5

Fractal Modelling of the Microstructure Property of Quartz Mylonite During Deformation Process[1]

**Zhijing Wang,[2,3] Qiuming Cheng,[2,3] Li Cao,[3,4]
Qingling Xia,[2] and Zhijun Chen[2]**

Mylonite is the result of the dynamic metamorphism and minerals in mylonite are deformed gradually with an increase in the degree of metamorphism. Quantifying the degree of deformation including the irregularities of shapes and the frequency distribution of the minerals becomes one of the most challenging efforts in mylonite analysis. Fractal modelling has been demonstrated in this paper to be an effective mean to achieve the above goal. Perimeter-Area fractal model was used to quantify the irregularities in the geometries and Cumulative Number-Area model is used to characterize the irregularities of distribution of quartzs in mylonites, respectively. Examples of quartz from five types of mylonites with different degree of deformation within the foreland of the Moine Thrust Zone in NW Scotland are chosen to study the evolution processes of deformation. As the main mineral component of quartzite mylonite, patterns are extracted from digital photomicrographics of the multiscale-grey image grid data to show quartz grains with different degree of deformation, The areas and perimeters of the quartz grains were calculated by GIS-based image processing technologies. From type one to type five, with an increase in degree of deformation, the corresponding Perimeter-Area exponent D_{AP} *increases from 1.20, 1.28, 1.38, 1.46, to 1.60, respectively, the fractal dimension* D_{P} *of the perimeter from 1.07, 1.08, 1.17, 1.23, to 1.44, as well as the exponent of Cumulative Number- Area from 0.50, 0.51, 0.58, 0.82, to 0.85, respectively. The result has shown that as increase of the intensity of deformation, the shape of quartz grains tends to be more irregular, grain size tends to be smaller, and the number of grains increases. The results obtained using GSI model has indicated that as an increase in the intensity of deformation, the patterns of quartz grains tends to be more stratified and randomness increases.*

KEY WORDS: fractal model, microstructure, quartz mylonite, deformation.

---

[1] Received 4 February 2005; accepted 27 June 2006; Published online: 21 February 2007.

[2] State Key Lab of Geological Processes and Mineral Resources, China University of Geosciences, China 430074; e-mail: zhijingw@yorku.ca, qiuming@yorku.ca

[3] Department of Earth and Space Science and Engineering, York University, 4700 Keele Street, Ontario, Canada M3J1P3.

[4] China Geological Survey, Beijing, China 100011.

INTRODUCTION

Since the concept of fractal was proposed by Mandelbrot (1977), various fractal techniques have been developed to quantify the irregularities in geometries as well as patterns. These include the Number-Area model characterizing the irregular frequency distribution of sizes of objects such as the number of mineral deposits and the sizes of mineral deposits, the Perimeter-Area model characterizing irregular morphological characteristics of similarly shaped geological features, such as contours created from geochemical element concentration data in a mineral district (Cheng, 1994), multifractal model for quantifying singularity of spatial patterns (Cheng, 1994), to name a few. Among these, the Perimeter-Area model was first developed by Mandelbrot (1977, 1983) to associate the perimeters and areas of a set of similarly shaped sets. It was the so-called "slit island" model and it has been applied for quantifying irregular surfaces, including estimating the fractal dimension of a surface. Lovejoy (1982) used this Area-Perimeter relation to investigate the geometry of rain and cloud area and to characterize the degree of complexity of the shape of cloud and rain. Mandelbrot, Passoja, and Paullay (1984) used this model to study the fracture surface of a piece of metal. In this work the model was used to measure the surface toughness of metals. The limitation of this model was found in that it sometimes gives discrepant results with the simulations (Goodchild, 1988). Lovejoy and Schertzer (1991) have extended this model from the multifractal point of view. Cheng (1995) proposed a general form associating perimeters and areas of similarly shaped fractal sets with fractal areas (A) and perimeters (P). According to this model, the original model becomes a special case where the areas of the sets are regular set instead of fractals. This modified model has been applied to separate geochemical anomaly from background (Cheng, 1995), to characterize trace element distribution on the mineral surface (Zhang, Ma, and Cheng, 2001), and to model surface stream patterns (Cheng, 2001). Other examples of using perimeter-area model include morphometry of quartz aggregates in granite (Gulbin and Evangulova, 2003). In this paper, the general Perimeter–Area model will be used to characterize the deformation of quartz grains from mylonites of different types with different degrees of metamorphism.

PERIMETER-AREA FRACTAL MODEL

The perimeter-area model is a mathematical model associating the relationship between the perimeter (P) and area (A) of similarly shaped fractals. Taking quartz grains in a mylonite as an example, the expression is (Cheng, 1995)

$$P \propto A^{\frac{1}{2}D_{\mathrm{AP}}} \tag{1}$$

where P is the perimeter, A is the area of quartz grain in a thin section, and $\propto$ stands for "proportional to." D_{AP} is the exponent of the power-law relationship. It can be further related to $D_{\mathrm{AP}} = 2D_{\mathrm{P}}/D_{\mathrm{A}}$ where D_{P} and D_{A} are the fractal dimensions of the perimeter P and the fractal dimension of the area A, respectively. If one deals with only sets with "normal area" with $D_{\mathrm{A}} = 2$ then $D_{\mathrm{AP}} = D_{\mathrm{P}}$, Therefore, the model (1) becomes the original form developed by Mandelbrot (1983). Quartz grains from mylonite are often irregular due to deformation that may show fractal area with dimension $D_{\mathrm{A}} < 2$. In this case $D_{\mathrm{AP}} > D_{\mathrm{P}}$. Therefore, in order to quantify the irregularities of mineral crystals "normal area" should not be assumed. For this reason the general model (1) must be used. To determine Area-Perimeter exponent D_{AP}, the data set of P and A plotted as log–log scale will show linear relationship between the Log$A$ and Log$P$, which can be fitted with straight line by Least Square method. The slope of linear regression can be taken as the estimation of $1/2D_{\mathrm{AP}}$

$$\log P = C + \frac{1}{2} D_{\mathrm{AP}} \log A \tag{2}$$

In general, D_{AP} is a value ranging between 1 and 2. If $D_{\mathrm{AP}} = 1$, then $P \propto A^{0.5}$, it implies regularly shaped sets, i.e. the squares or circles. The higher the value of D_{AP}, the greater flattening shape the grains become; If $D_{\mathrm{AP}} = 2$, then $P \propto A$, it implies extremely stratified sets so that the perimeter changes as the same rate as area, in other words, the perimeter acts as area. For example, a normal quartz grain with no deformation tends to be globular grain perpendicular to its c-axis as quartz belongs to hexagonal system. For the regular quartz grains with different sizes there exists a power-law relationship between their perimeters and areas with the exponent $D_{\mathrm{AP}} = 1$. However, quartz grains in metamorphosed rock such as mylonites are often gone through deformation and recrystalization and they often become irregular in shapes and sizes which may correspond to $D_{\mathrm{AP}} > 1$. In general D_{AP} can primarily characterize the irregularity of perimeters. In the deformation of the quartz in mylonites the deformation may be dominated by stratification/flattening which may show anisotropic scaling properties. There are other ways to characterize this type of deformation, for example, one may use semivariogram to characterize the anisotropic property or Generalized Scaling Invariance (GSI) techniques (Lovejoy and Schertzer, 1991). In this paper we use the perimeter-area model to characterize the deformation. The advantage of the perimeter-area model is that it not only applicable to stratification/flattening but also more irregular deformation as well. In addition the tortuous of the boundary can be characterize by the D_{P} according to $D_{\mathrm{AP}} = 2D_{\mathrm{P}}/D_{\mathrm{A}}$ if D_{A} is estimated by other methods such as Box-counting method. This study will demonstrate that the component of quartz grains from different types of mylonites can be characterized by Box-Count dimension D_{A}, so do the stratification/flattening quartz grains by the exponent D_{AP} of the Perimeter-Area model and stratification parameter c using GSI technique, and the irregularity of perimeter by fractal dimension D_{P}.

NUMBER-AREA MODEL

It will be demonstrated that the following simple Number-Size model showing the relationship between cumulative number of objects and the size of the objects can be used to study the distribution of quartz grain sizes observed in a thin section

$$N(>A) \propto A^{-D} \tag{3}$$

where A is the area of grains, N is the cumulative number of grains with the area greater than A, and $\propto$ stands for "proportional to". To determine exponent D, for the data set of $N\ (>A)$ and A, the slope coefficient of linear regression can be used

$$\log N(>A) = C - D \log A \tag{4}$$

The value of the exponent determines the changing rate of the number of grains and their sizes. Large D implies few grains with large size or more grains with small size. This model will be used to characterize the relationships between number of quartz grains and their sizes undergone different degrees of deformation.

DEFORMATION OF QUARTS IN MYLONITES FROM THE MOINE ZONE OF NORTHWESTERN SCOTLAND

Mylonite is formed due to dynamic metamorphism. During the formation procession, minerals will be subjected to changes in both shape and in size, due to the flattening deformaiton of globular detrital quartz grain as well as the recrystallization of quartz in constraint spaces. Characterizing the property of such type of change of quartz grain with different degree of deformation by fractal modeling is of interest to the petrologists and structural geologists.

The study area is the Moine zone located in Northwestern Scotland which consists of a series of thrusts, of which the Moine thrust is considered to be the oldest (Elliott, 1976; Elliott and Johnson, 1980), and mylonites lie immediately beneath the Moine outcrop. At the Stack of Glebcoul in the northern Assynt in Cambrian Pipe Rock quartzite within the foreland of the Moine thrust zone, the mylonite can be classified into five types briefly, according to the degree of deformation, prefered alignment of elongation and the dynamically recrystallization of

Table 1. The Properties of Mylonites at the Stack of Glebcoul in the Northern Assynt in Cambrian Pipe Rock Quartzite within the Foreland of the Moine Thrust Zone

Mylonite	Location Beneath the thrust (m)	Properties of individual relict quartz grain	Ratio of major to minor axis	Recrystalization of quartz	Deformation degree
Type 1	70	Slightly flattened, quartz displays undulose extinction, deformation bands and sub-basal deformation lamellae	2:1	Confined to a few deformation band boundaries and detrital grain margins	Weakly deformed
Type 2	40	Foliation defined by preferred alignment of flattened detrital quartz grains	4:1	Up to 10% of the total rock volumn	Moderately deformed
Type 3	9.6	Ribbon-like quartz grains, 2 mm, variably flattened	50:1–100:1	Account for up to 40–70%, with size of 10–15 μm	Intensively deformed quartzite
Type 4	0.15	Relict grains is only locally preserved dimensions of up to 5 mm. SB is appear s to be parallel to the SA.	80:1	Over 60–100% with size about 10 μm	Intensively deformed quartzite
Type 5	0.15	A prefered alignment (SB) obligue to the mylonite foliation (SA) with the section angle about (±30°)	80:1	Over 60–100% with size about 10 μm	Intensively deformed quartzite

quartz. On the basis of the research results by previous geologists (Law, Casey, and Knipe, 1986; Law and Potts, 1987), the properties of mylonites of five types can be summarized in Table 1.

The photomicrographs (Fig. 1A–E) show the microstructure of mylonites of five types mentioned above, respectively. From Type 1 to Type 5, as the Moine thrust is approached from below, both finite strain magnitude and degree of recrystallisation with the quartz mylonites increase progressively (Weathers and others, 1979). Driven by strain or shear heating, strain rates increase towards the Moine Thrust, and the deformation occurred over a long period of time near the thrust surface (Weathers and others, 1979). As illustrated in Fig. 1 both the degree of deformation and that of recrystalization on the quartz grain increases from the bottom to the top of the Moine Thrust.

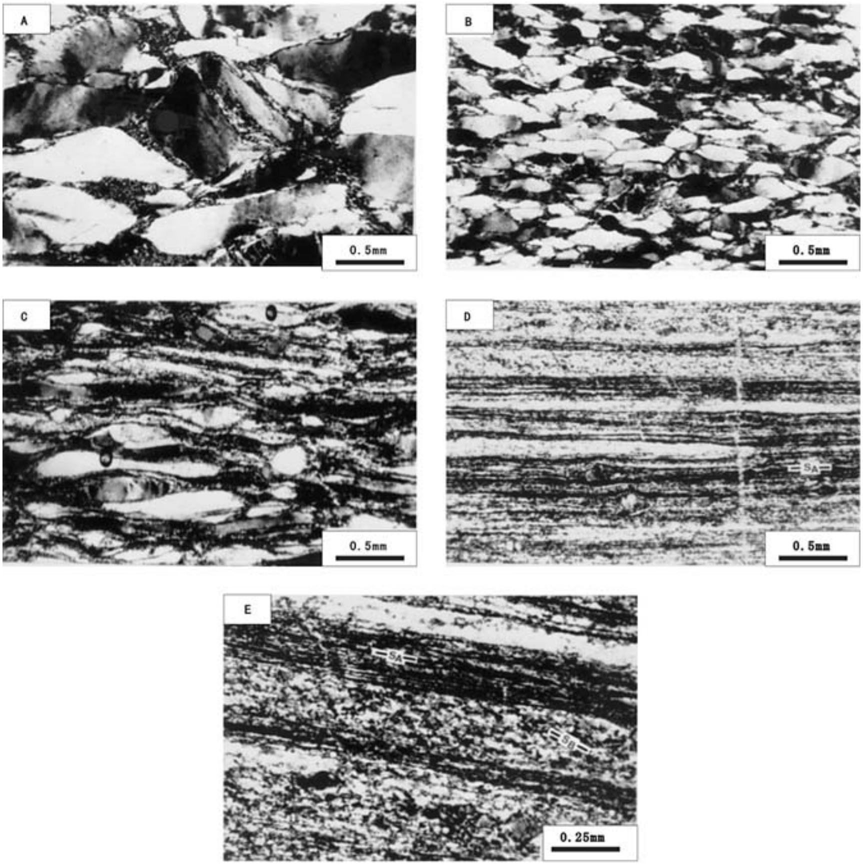

Figure 1. Photomicrographs of mylonites, crossed polarized light (from Law, Casey, and Knipe, 1986). Image A, B, C, D, and E are images of the mylonites of Type 1, 2, 3, 4 and 5, respectively. The micrographs are viewed towards NNE and were cut normal to foliation and parallel to lineation.

DATA PROCESSING

The photomicrographics of mylonite are digitized as a 256-greyscale black-white image and then converted into contour map. The quartz features are extracted from the image and then converted to vector data to calcluate the values of area and perimeter by using ArcGIS technique (ESRI INC., 2004). The image of mylonite of Type 2 is used as an example to illustrate the processes of image processing (Fig. 2A–D). Image A is the original photo taken with polarized light. The image is converted to grid data and the contour map is created on the basis of the grey scale. Several empirical experiments have indicated that the outlines with 180-scale

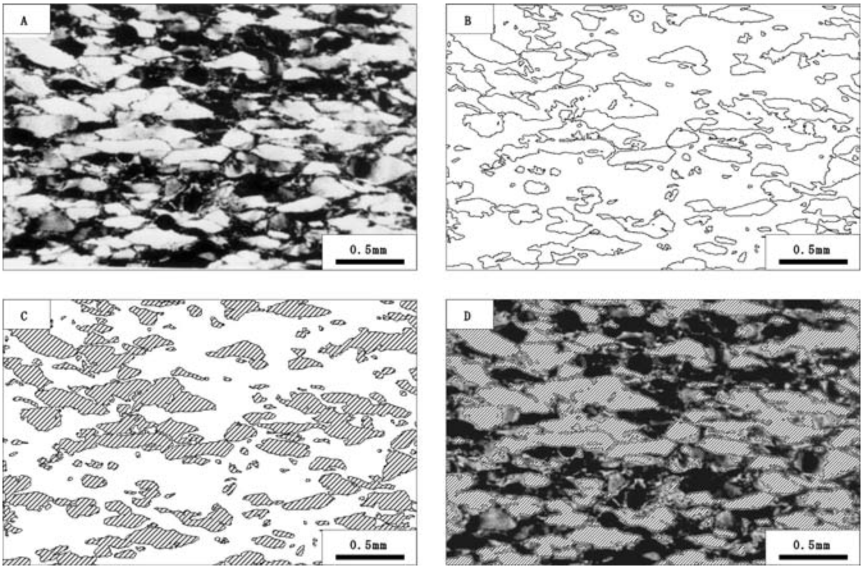

Figure 2. (A) The original image of mylonite Type 2; (B) Outlines of quartz grains with threshold 180-grey scale; (C) vector coverage converted from (B) by ArcGIS; and (D) superimposing the outlines on C over the image A for comparison.

(Fig. 2B) are good to represent the outlines separating quartz grains from subgrains which are too small so that they are not included in the calculation. The contour lines are converted into coverage using ArcGIS so that the areas and perimeters of these extracted quartz grains can be readily calculated with high precision. Some clean up processes have to be utilized in order to remove the noise related to the conversion. For example, vector lines are edited and small separate tangles were deleted. The result is shown in Fig. 2C. The comparison of image C and A (see Fig. 2D) shows that the outlines extracted on C can reasonably represent the outlines of quartz grains on A. Similarly, the images of quartz grain of the other four types of mylonites are analyzed by using the same GIS-based technique.

THE FRACTAL DIMENSION D_A OF THE AREA

By using the traditional Box-Counting method, we can obtain the fractal dimension D_A of the area of quartz grains of five types of mylonites. We still use the image (Fig. 2C) of mylonite of Type 2 (Fig. 1B) as an example. Four types of grids with an increase in box size δ from 2, 4, 8, to 16 pixels can be used to cover quartz grains' images (Fig. 2C) and these grids can be used for counting the corresponding number of grey boxes (N(δ)) occupied by quartz grains with the aid

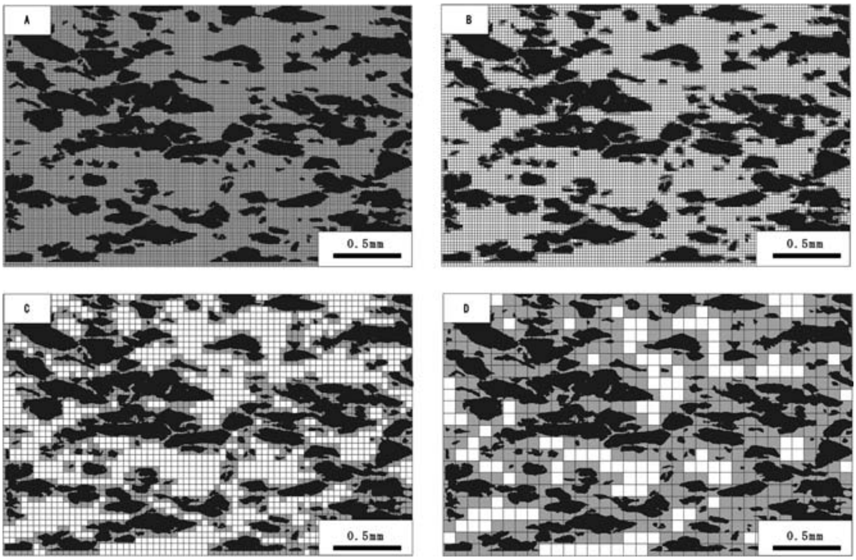

Figure 3. Diagrams showing the partition of image with variable grid sizes for calculating the fractal dimension D_A of the grains using Box-Counting method. From A to D, the box size of a grid increases from 2, 4, 8, to 16 pixels. The shaded areas are mineral grains. The grey boxes are the boxes that are occupied by mineral grains, while the blank boxes are the boxes that are not occupied. One pixel is 0.005 mm.

of GIS technique (Fig. 3A–D). The four data pairs of the box size δ vs. the number N can be plotted as log–log scale. Linear regression can be applied to fit a straight line to the model from which the fractal dimension D_A can be estimated. The results for all five samples show that power-law relationships exist between the box size and the number N and the estimated correlation coefficients are all over 0.997 (Fig. 4a–e) for the five types of images. The fractal dimension D_A of the area is equal to the negative slope of the linear regression line. On the other hand, from the Type 1 to the Type 5, the slope are -1.79, -1.68, -1.69, -1.69, -1.80, and the corresponding D_A varies from 1.79, 1.68, 1.69, 1.69, to 1.80, respectively, with a average 1.73 and a standard deviation 0.0592. For Type 2, 3 and 4 the area fractal dimension D_As are very close to each other; while for other two types, D_As are also close to each other.

THE FRACTAL DIMENSION D_P OF THE PERIMETER

By using Perimeter-Area model, the data of perimeters and areas of the quartz grains identified on Fig. 2C can be plotted on a log–log diagram (Fig. 5b) and

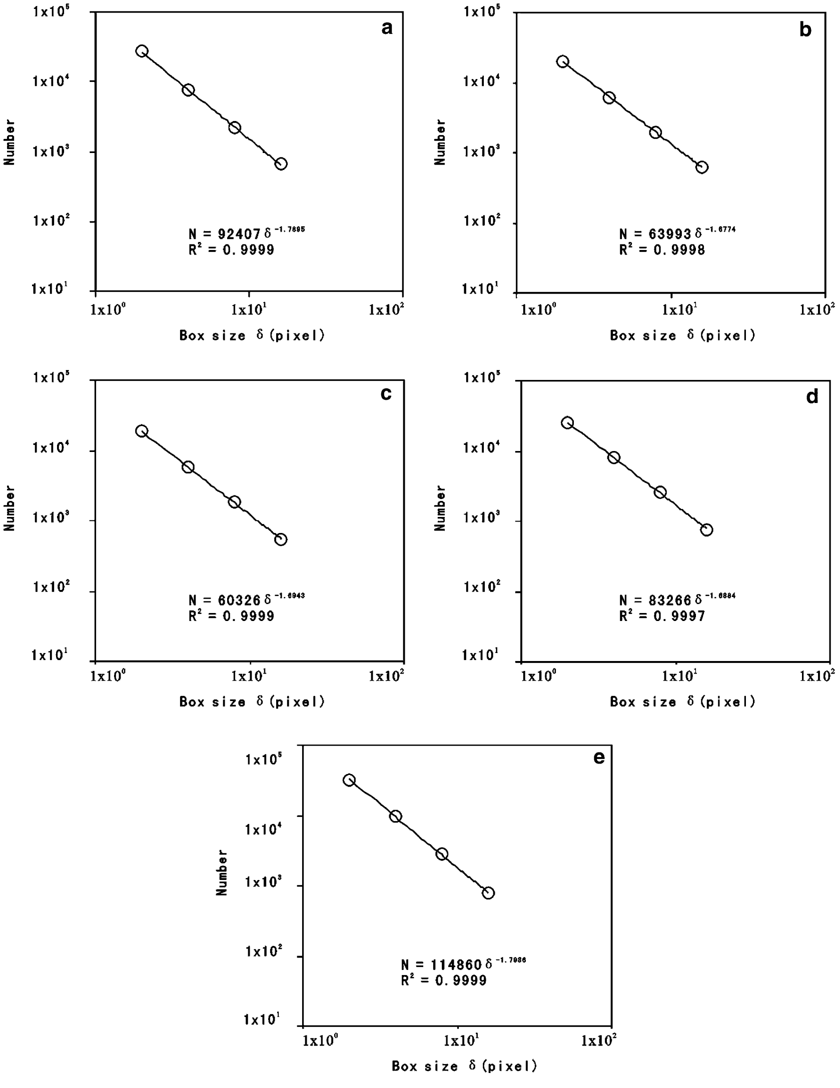

Figure 4. Log–log plot of Number-Size models of quartz grains from mylonites of five types. Diagram a, b, c, d and e are corresponding to mylonites of Type 1, 2, 3, 4 and 5, respectively. The unit of size *δ* is in pixel. One pixel is equal to 0.005 mm.

the exponent D_{AP} of the area-perimeter of quartz grain shape is calculated by mean of linear regression analysis. Using the same way we can obtain the other four D_{AP}s of quartz grains with respect to the other four types of mylonites. The results show that there exist a clear linear relationship between the logarithm area and logarithm perimeter or power law relationship between the area and perimeter as the correlation coefficients R^2 are all greater than 0.93. From Type 1 to Type 5, the slope increases from 0.60, 0.64, 0.69, 0.73 to 0.80, respectively, and the corresponding D_{AP} values increase from 1.20, 1.28, 1.38, 1.46, to 1.60 (Fig. 5a–e), respectively, indicating a stronger and stronger stratification or flattening. On the other hand, as mentioned in previous paragraphs, the fractal dimension D_{A}s of quartz grains from Type 1 to Type 5 are: 1.79, 1.68, 1.69, 1.69, and 1.80. So the matching fractal dimension D_{P} of the perimeter of the grains is 1.07, 1.08, 1.17, 1.23, 1.44, respectively, according to the expression: $D_{\mathrm{P}} = 0.5 D_{\mathrm{AP}} D_{\mathrm{A}}$. As discussed earlier the value of D_{P} can be used as the measure of irregularity of the quartz grains. This result shows that from Type 1 to Type 5, D_{P} increases monotonically indicating that the irregularity of quartz increases gradually with an increase in the degree of deformation progressively. The fact that shape of quartz grains becomes more and more irregular or complex can be observed from Fig. 1A–E.

THE NUMBER-AREA EXPONENT

The Number-Area exponent D in the Cumulative Number-Area model is estimated for quartz grains from the five types of samples using Least Square method from the log–log diagram (Fig. 6a–e). The results show that clear linear relationships exist between the logarithm area and logarithm cumulative number of quartz for all five types as confirmed by the high correlation coefficients $R^2 > 0.98$. It has demonstrated that the power-law model (3) held true for quartz grains from 0.0002 mm^2 to 0.03 mm^2. Suppose the area of original quartz before going through deformation follows a normal distribution and the log–log plot of Cumulative Number-Area should not be linear. But, under the dynamic metamorphism (deformation, prefered alignment of elongate, dynamically recrystallisation of quartz grains), most of the quartz grains become smaller in size and a few may grow up due to recrystalization. This process may cause more irregular shapes and sizes of quartz grains showing power-low distribution, rather than normal distribution. In addition, from Type 1 to Type 5, the Number-Area exponent D of corresponding quartz grains obtained using model (4) increases progressively from 0.50, 0.51, 0.58, and 0.82 to 0.85, indicating few large quartz grains or more small quartz grains. It means that, with an increase in the degree of deformation and recrystallisation, the number of quartz grains increases, but the grain tends to be smaller, due to the facts that more quartz grains breaks into quartz grains with a smaller size, and at the same time a great number of quartz grains were produced by high degree recrystallisation. The conclusion of the size of quartz grains decrease can also be drawn from the statistics shown in Table 2. It shows that from Type 1 to Type 5 (Table 2), the mean size (area) of quartz grains decreases from 0.0125 mm^2 to 0.0016 mm^2 progressively, the standard

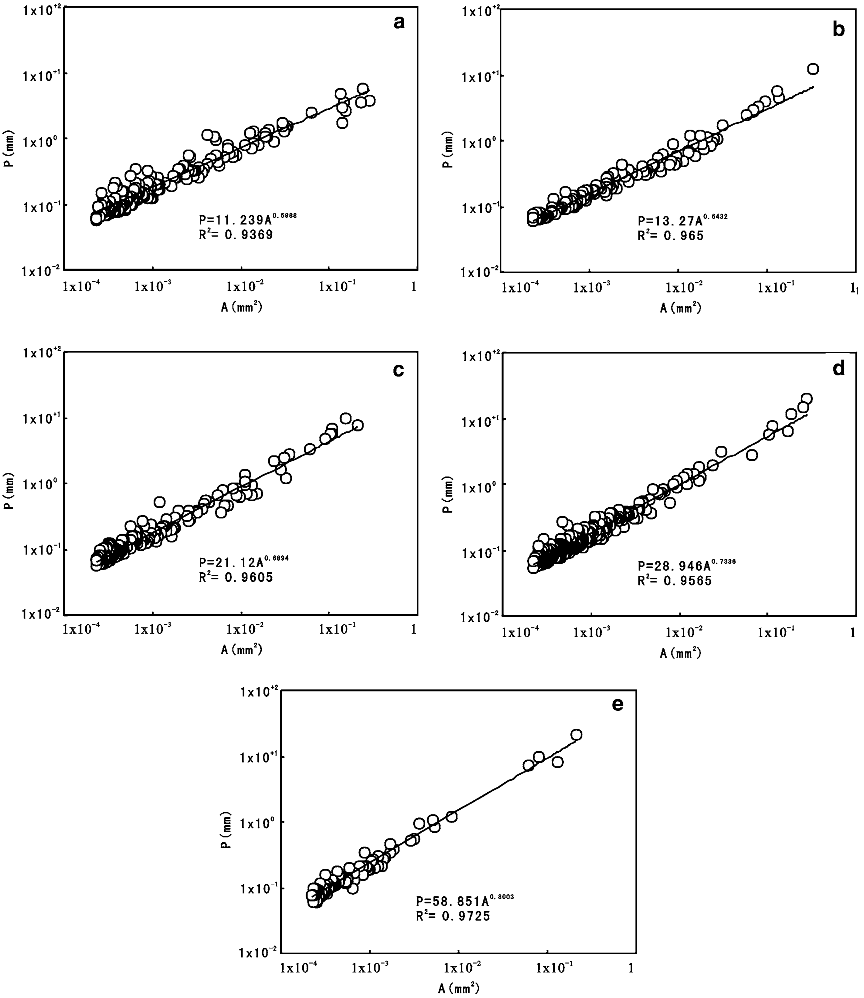

Figure 5. Log–log plot of area-perimeter of quartz grains from mylonites of five types. Diagram a, b, c, d and e are corresponding to mylonites of Type 1, 2, 3, 4 and 5, respectively.

deviations from 0.0414 mm^2 to 0.0067 mm^2, but the maximum grains stay relatively unchanged.

Characterization of Anisotropic Scaling Properties Using GSI Technique

To further the study of anisotropic scaling properties of quartz in mylonites with different degree of deformation, we utilize the generalized scale invariance

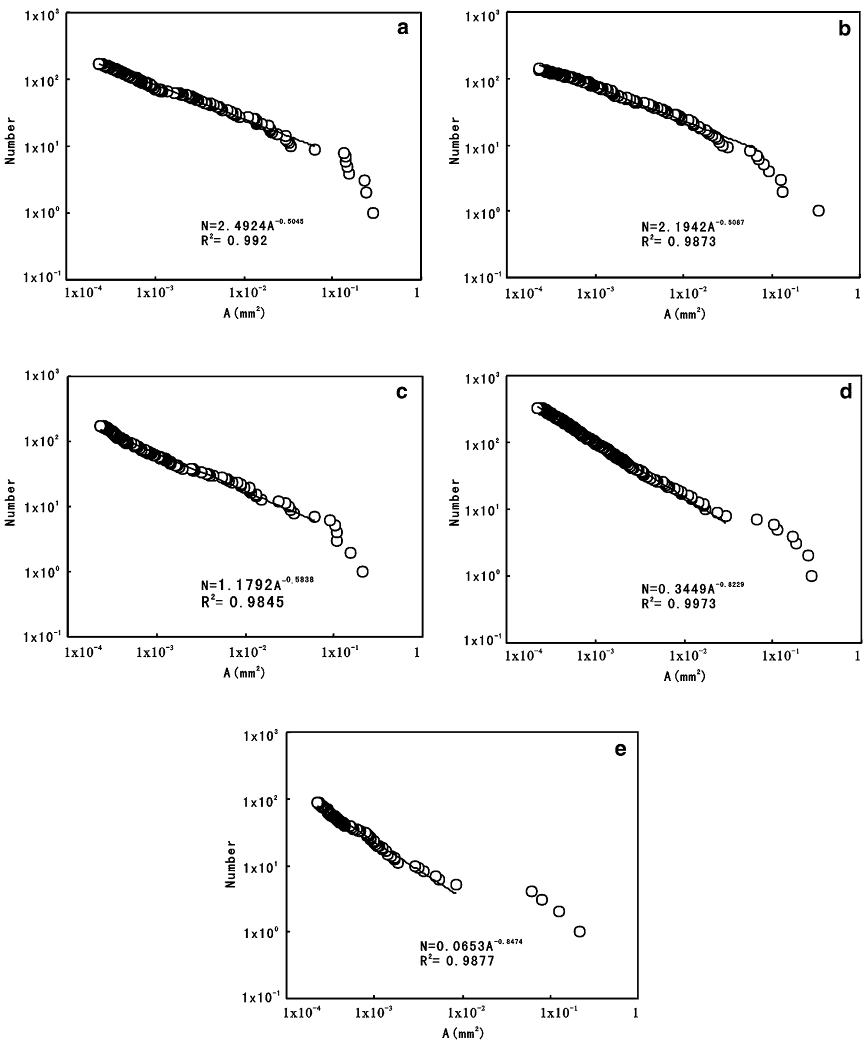

Figure 6. Log–log plot of cumulative number – area of quartz grains from mylonites of five types. Diagram a, b, c, d, and e are corresponding to mylonites of Type 1, 2, 3, 4 and 5, respectively. Several points on the right are not included as they are the large unbroken grains. Note that area range of quartz is from 0.0002 mm^2 to 0.3 mm^2.

(GSI) technique developed by Lovejoy and Schertzer (1991). In the formulation of GSI four parameters (c, d, e, f) were involved to characterize the generalized anisotropic scale invariance by means of scale invariant generator. The parameter *d* is a measure of overall contraction, which determines how the volumes change

Table 2. The Statistics of Quartz Grains' Size in Area (mm^2)

Type	Mean	Standard deviation	Minimum	Maximum	Confidence intervals −95%	Confidence intervals + 95%
1	0.0125	0.0414	0.0002	0.0644	0.0002	0.0242
2	0.0043	0.0080	0.0002	0.0583	0.0026	0.0216
3	0.0033	0.0099	0.0002	0.0914	0.0003	0.0134
4	0.0017	0.0048	0.0002	0.0662	0.0003	0.0064
5	0.0016	0.0067	0.0002	0.0616	0.0002	0.0035

with scale which in the current case setting as 1 implying without volume lose during the deformation. c is a measure of the relative scaling of the two coordinate axes. When c equals to zero, the scaling of the two coordinate axes x and y are equal implying isotropic scaling. If c is large than zero, the relative scaling of x against y becomes stronger implying horizontally stratified deformation whereas c is much smaller than zero, the relative scaling of y against x becomes stronger implying vertically stratified deformation. f is a measure of stratification along diagonal to the axes x and y. e is a measure of rotation. These four parameters can be used to characterize the anisotropic scaling transformation. By the combination of these four parameters, a variety of patterns can be processed for characterizing a variety of anisotropic and statistical characteristics. With the anisotropic scale transformation one can calculate the anisotropic scaling exponent s which is related to the exponent of power-law distribution of power energy density $E(k) \propto k^{-\beta}$ as $s = \beta + 1$ (Lovejoy and Schertzer, 1991). The smaller the value of β, the more random the field; the larger the value of β, the more persistence the field. $\beta = 2$ corresponds to Brownian walk and $\beta = 0$ corresponds to white noise. The results obtained from the five types of samples are shown in Table 3. From the results in Table 3 one can see that the value of e and f are fluctuating without a clear trend but the values of c and s monotonically change in ascending and descending orders, respectively. The increase of c value (absolute value) indicates that the degree of stratification increases as the increase of deformation intensity. The decreasing trend of s value implies that the degree of random of patterns increase as the increase of deformation intensity. More high frequency signals are created by the deformation which once again indicates that the deformation changes the variability of the quartz grains in the mylonites. More detailed explanation about the GSI technique can be found in Lovejoy and Schertzer (1991).

CONCLUSIONS AND DISCUSSION

The deformation of mylonite causes changes of grain shape and grain size of quartz. The quantitative modeling introduced in the paper has demonstrated

Table 3. The Statistics of Estimated Parameters

Mylonite type	e	C	f	S
Type 1	0.006	0.201	0.012	3.700
Type 2	−0.019	0.175	−0.012	4.117
Type 3	−0.020	0.297	−0.095	3.375
Type 4	−0.018	−0.500	0.030	3.348
Type 5	0.099	−1.528	−0.115	2.609
Average	0.010	−0.271	−0.036	3.430
Std	0.051	0.771	0.065	0.554

that with an increase in the degree of deformation the shape tends to become more irregular and stratified, and the size tends to decrease with randomness increase due to deformation and fragmentation. Perimeter-Area model is shown effective for quantifying the change of irregularity of quartz grains of mylonites with different degree deformation. GSI technique has been demonstrated capable of characterizing the general stratification and anisotropic scaling of the images of mylonites.

For five types of different mylonites at the Stack of Glebcoul in the northern Assynt in Cambrian Pipe Rock quartzite within the foreland of the Moine Thrust Zone, the area-perimeter exponent (D_{AP}) increases from 1.20, 1.28, 1.38, 1.46, to 1.60, respectively, reflecting the increase of stratification or flattening of quartz grains in two-dimension; the box-counting fractal dimension (D_A) are 1.79, 1.68, 1.69, 1.69, and 1.80, respectively, indicating a general conservation of the quartz volume during the deformation process. The fractal dimension D_P of the perimeter of the quartz grains increases from 1.07, 1.08, 1.17, 1.23, to 1.44, respectively, indicating an increase in the irregularity of quartz grains. This can be explained by the fact that deformation and elongation of original glabular quartz, as well as recrystalization producing new quartz in a solid state in a constrained space over a long period of time due to strain. On the other hand, Cumulative-Number-Area model can be applied to quantify the change in grain size to reveal the linear relationship between cumulative number and size of mineral grains, rather than the normal distribution. The Number-Area exponent increases from 0.50, 0.51, 0.58, and 0.82 to 0.85, respectively, implying the less large grains or more small grains, and minerals tend to become smaller grains during the metamorphic process. This may be due to breaking down of original glabular quartz grains, as well as recrystallization, which leads to a great number of new quartz with samller size. The estimation of parameters by GSI technique douable proved the above changes with systematic increase of value of stratification parameter (c) and decrease of the anisotropic exponent (s).

The ratio of major axis to minor axis of deformed minerals has been commonly used as an index characterizing the degree of deformation of grains by many previous geologists. The higher the ratio, the greater the degree of deformation. However, for irregular mineral grains the ratio can not be easily measured so it is impossible to use this ratio to quantify the irregularity of the shape. In addition, the ratio of major and minor axis may be different when measured with different size of grains. Therefore, measuring the shape ratio sometimes is not applicable in complex grain shapes. The models used in the current paper can overcome the above shortcomings. Therefore, the fractal dimensions obtained from these models can be used as new indexes for quantifying the degree of deformation of mylonites. Similarly these models can be used for other purposes in petrology study.

ACKNOWLEDGMENTS

This research was financially supported by a NSERC Discovery Research Grant (ERC-OGP0183993) awarded to Dr. Cheng and the Open Fund by the State Key Lab of Geo-Processes and Mineral Resources of China. Dr. R.D. Law, a professor of Virginia Tech, is appreciated for kindly providing the images of mylonites for supporting this research.

REFERENCES

Cheng, Q., 1994, Multifractal modeling and spatial analysis with GIS: Gold potential estimation in the Mitchell-Sulphurets area, northwestern British Columbia: Unpublished doctoral dissertation, The University of Ottawa, 268 p.

Cheng, Q., 1995, The perimeter-area fractal model and its application to geology: Math. Geol., v. 27, no. 1, p. 69–82.

Cheng, Q., Russell, H., Sharpe, D., Kenny, F., and Qin, Q., 2001, GIS-based statistical and fractal/multifractal analysis of surface stream patterns in the Oak Ridges Moraine: Comput. & Geosci., v. 27, no. 5, p. 513–526.

Elliott, D., 1976, The motion of thrust sheets: Jour. Geophys. Res., v. 81, p. 949–963.

Elliott, D., and Johnson, M. R. W., 1980, The structural evolution of the northern part of the Moine thrust zone: Trans. R. Soc. Edinburgh Earth Sci., v. 71, p. 69–96.

ESRI, 2004, Using ArcGIS Spatial Analysis. ESRI, New York, p 63.

Goodchild, M. F., 1988, Lake on fractal surfaces: A null hypothesis for lake-rich landscapes: Math. Geol., v. 20, no. 6, p. 615–630.

Gulbin, Y. L., and Evangulova, E. B., 2003, Morphometry of quartz aggregates in granites: Fractal images reference to nucleation and growth processes: Math. Geol., v. 35, no. 7. p. 819–833.

Law, R. D., Casey, M., and Knipe, R. J., 1986, Kinemetic and tectonic significance of microstructures and crystallographic fabrics within quartz mylonites from the Assynt and Eriboll regions of the Moine Thrust zone, NW Scotland. Transactions of the Royal Society of Edinburgh: Earth Sci., v. 77, p. 99–125.

Law, R. D., and Potts, G. J., 1987, The Tarskavaig Nappe of Skye, northwest Scotland: A re-examination of the fabrics and their kinematic significance: Geol. Mag., v. 124, p. 231–248.

Lovejoy, S., 1982, Area-perimeter relation for rain and cloud areas: Science, v. 216, no. 4542, p. 185–187.

Lovejoy, S., and Schertzer, D., 1991, Multifractal analysis techniques and the rain and cloud fields from 10–3 to 106 m, *in* Schertzer, D., and Lovejoy, S., eds., Nonlinear Variability in Geophysics: Kluwer, Dordrecht, 318 p.

Mandelbrot, B. B., 1977, Fractals: form, chance, and dimension: Freeman, San Francisco, 365 p.

Mandelbrot, B. B., 1983, The fractal geometry of nature (updated and augmented edition): Freeman, New York, 468 p.

Mandelbrot, B. B., Passoja, D. E., and Paullay, A. J., 1984, Fractal character of fracture surfaces of metals: Nature, v. 308, no. 5961, p. 721–722.

Weathers, M. S., Bird, J. M., Cooper, R. F., and Kohlstedt, D. C., 1979, Differential stress determined from deformation induced microstructures of the Moine thrust zone: Jour. Geophys. Res., v. 84, p. 7459–7509.

Zhang, Z., Mao, H., and Cheng, Q., 2001, Fractal geometry of element distribution on mineral surface: Math. Geol., v. 33, no. 2, p. 217–228.