DOI 10.1007/s11003-024-00815-5
Materials Science, Vol. 59, No. 5, March, 2024 (Ukrainian Original Vol. 59, No. 5, September–October, 2023)

DETERMINATION OF THE STRESS STATE OF AN ANISOTROPIC BODY WITH SMOOTH CURVILINEAR INCLUSIONS UNDER LONGITUDINAL SHEAR

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UDC 539.3

The antiplane problem of the theory of elasticity for an elastic piecewise homogeneous anisotropic body was solved using the method of singular integral equations. For one anisotropic inclusion in the orthotropic plane, the constructed system of integral equations of the second kind was solved numerically by the quadrature method. The influence of elastic constants of anisotropic materials of the plane and the inclusion, as well as the shape of the curvilinear inclusion on the shear stress distributions at the interface of the materials was studied.

Keywords: antiplane deformation, anisotropy, inclusions, stress concentration, singular integral equation method.

Introduction

The use of high-strength composite materials in modern technology requires the study of the stress-strain state of such bodies with stress concentrators as holes, inclusions, cracks, and notches. Different types of composite materials can be modeled by a homogeneous or piecewise-homogeneous anisotropic medium. A number of scientific papers [1–10] deal with the problems of the theory of elasticity of anisotropic bodies with holes, cracks, and inclusions. One of the most common methods of solving such problems is the method of complex potentials (CP) written in additional mathematical planes associated with the elastic constants of anisotropic materials [2, 4, 11–17]. Two-dimensional problems of the theory of elasticity for piecewise homogeneous anisotropic bodies were also considered [6, 18–25]. Separate numerical results were obtained for stress distributions along the boundary contour of anisotropic materials of an infinite plane and finite two-dimensional inclusions [2, 6, 18, 19, 22, 25].

The antiplane problem of the theory of elasticity for an infinite elastic anisotropic plane (matrix) with smooth curvilinear anisotropic inclusions under their perfect mechanical contact is considered. The methods of CP and singular integral equations (SIE) were used for anisotropic bodies with cracks, holes, and notches [17, 26–28]. For an orthotropic matrix with one anisotropic inclusion, the SIE system of the second kind is solved numerically. The longitudinal shear stress distributions at the interphase of the matrix materials and the inclusion were determined depending on the elastic solid materials, as well as the shape of the contour of the smooth curvilinear inclusion. The parameters of the problem, which significantly affect the distribution of shear stresses at the boundary contours of anisotropic materials, are established.

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Translated from Fizyko-Khimichna Mekhanika Materialiv, Vol. 59, No. 5, 74–82, September–October, 2023. Original article submitted May 24, 2023.

Some Relations of the Antiplane Problem of the Theory of Elasticity of an Anisotropic Body

Let us consider the longitudinal shear of an anisotropic body in the Cartesian coordinate system (*x*, *y*, z). The components of the vector of elastic displacements can be presented as $u_x = 0,\ u_y = 0,\ u_z = w = w(x, y)$ if the axis of deformation is directed along the axis z. The relationship between non-zero components of deformations $\varepsilon_{yz}, \varepsilon_{xz}$ and stresses τ_{yz}, τ_{xz} was obtained based on Hooke's generalized law [7, 11]. Expressing the general solution of the differential equation of equilibrium in displacements in terms of the analytical function $\varphi_3(z_3)$ of the complex argument $z_3 = x + \mu_3 y$

$$w(x, y) = a_0 \operatorname{Im}\left[\varphi_3(z_3)\right], \tag{1}$$

the relationship for stresses is given in terms of CP $\Phi_3(z_3) = \varphi_3'(z_3)$ as [2, 11]

$$\tau_{yz}(x, y) = \operatorname{Re}\left[\Phi_3(z_3)\right],\ \tau_{xz}(x, y) = -\operatorname{Re}\left[\mu_3 \Phi_3(z_3)\right]. \tag{2}$$

Here $\mu_3 = \alpha_3 + i\gamma_3$ is the complex root of the characteristic equation of the antiplane problem, where $\alpha_3 = \dfrac{a_{45}}{a_{55}},\ \ \gamma_3 = \dfrac{a_0}{a_{55}};\ \ \{a_{55} - a_{45}, a_{44}\} = a_0^2 \{A_{44}, A_{45}, A_{55}\}$ are the elastic constants of anisotropic material, $a_0 = \sqrt{a_{44} a_{55} - a_{45}^2} > 0$.

With the help of relations (2), write formulas for assessing shear stresses $\tau_{nz}(t)$ on some smooth curvilinear contour L with a given normal n, as well as contour stresses n, on the planes orthogonal to the contour L [17]

$$\tau_{nz}(t) = -\operatorname{Re}\left[\frac{\Phi_3(t_3)\,dt_3}{ds}\right] = -\operatorname{Re}\left[\frac{\varphi_3'(t_3)\,dt_3}{ds}\right],\ t \in L,\ t_3 \in L_3; \tag{3}$$

$$\tau_{sz}(t) = \operatorname{Re}\left[\Phi_3(t_3)\left\{\left(\frac{\dfrac{(1 - i\mu_3)\,dt}{ds} - \dfrac{(1 + i\mu_3)\,d\overline{t}}{ds}}{2i}\right)\right\}\right]. \tag{4}$$

Here the contour L_3 in the auxiliary plane of the complex variable z_3 corresponds to the contour L in the plane (x, y); s is the arc abscissa of the point $t \in L$.

Problem Formulation

Consider a two-dimensional problem of the theory of elasticity under antiplane deformation of an anisotropic body S_0 (matrix) with smooth anisotropic inclusions $S_j\left(j = \overline{1, J}\right)$. Assume that there is a perfect mechanical contact between the inclusions and the matrix, that is, when passing through the contours of the inclusions $L_j\left(j = \overline{1, J}\right)$, the normal component of shear stress and displacement $w(x, y)$ are continuous:

$$\tau_{nz}^{+}(t)-\tau_{nz}^{-}(t)=0,\ t\in L_j,\ j=\overline{1,J}; \tag{5}$$

$$w^{+}(t)-w^{-}(t)=0,\ t\in L_j,\ j=\overline{1,J}. \tag{6}$$

Here, the superscripts indicate the limiting values of the corresponding values when approaching the contours L_j from the left (+) or right (–) when bypassing them counterclockwise (Fig. 1).

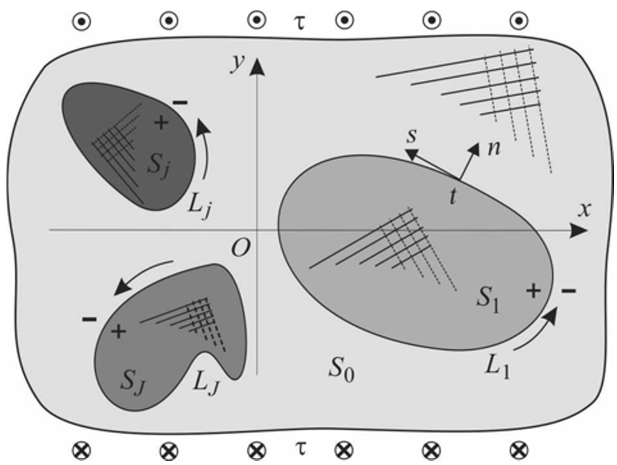

Fig. 1. Longitudinal shear of a piecewise homogeneous anisotropic body.

The matrix S_0 at infinity is under the action of longitudinal shear

$$\tau_{yz}^{\infty}=\tau,\ \ \tau_{xz}^{\infty}=0, \tag{7}$$

which does not limit the generality of the given load, since the matrix is anisotropic.

The problem with boundary conditions (5), and (6) is solved by the SIE method [17, 27, 28]. Under load (7), the stress state of an infinite piecewise homogeneous anisotropic body $S=\bigcup S_j\ \left(j=\overline{0,J}\right)$ is described by relations (2)–(4), written in complex mathematical planes $z_3^j=x+\mu_3^j y\ \left(j=\overline{0,J}\right)$. Here and in the future, the values with the superscript $j=\overline{1,J}$ refer to the jth inclusion S_j ($\mu_3^j=\alpha_3^j+i\gamma_3^j$, $\alpha_3^j=a_{45}^j/a_{55}^j$, $\gamma_3^j=a_j/a_{55}^j$, $a_j=\sqrt{a_{44}^j a_{55}^j-a_{45}^j a_{45}^j}$), and with the index $j=0$ refer to the matrix S_0 and correspond to the elastic constants of anisotropic materials of the inclusions and the matrix.

The stress potentials for the matrix and inclusions will be searched for in the form of analytical functions

$$\Phi_3^j\left(z_3^j\right)=\Gamma_3^j+\frac{1}{\pi}\int_{L_3^j}\frac{\phi_3'^j\left(t_3^j\right)}{t_3^j-z_3^j}dt_3^j=\Gamma_3^j+\frac{1}{\pi}\int_{L_j}\frac{\phi_3'^j\left(t_3^j\right)}{t_3^j-z_3^j}\frac{dt_3^j}{dt}dt,$$
$$z=x+iy\in S_j,\ j=\overline{0,J}, \tag{8}$$

where $\phi_3'^j\left(t_3^j\right)=\dfrac{d\phi_3^j\left(t_3^j\right)}{dt_3^j}$, $j=\overline{0,J}$; $\phi_3'^0\left(t_3^0\right)=\left\{\phi_{3j}'^0\left(t_{3j}^0\right),\ t_{3j}^0\in L_{3j}^0,\ j=\overline{1,J}\right\}$ are unknown complex functions; $t_3^j=\mathrm{Re}(t)+\mu_3^j\,\mathrm{Im}(t)\in L_3^j,\ t\in L_j$; $L_0=\cup L_j$, $L_3^0=\cup L_{3j}^0$ $\left(j=\overline{1,J}\right)$; contours L_j $\left(j=\overline{1,J}\right)$ in the physical plane $z=x+iy$ correspond to the contours $L_{3j}^0,\ L_3^j$ in mathematical planes $z_3^0,\ z_3^j$ (Fig. 1). Complex constants $\Gamma_3^j=\left(\dfrac{1+i\alpha_3^j}{\gamma_3^j}\right)\tau$ determine the same homogeneous stress states ($\tau_{yz}(z)=\tau,\ \tau_{xz}(z)=0,\ z\in S_j,\ j=\overline{0,J}$) of the matrix ($j=0$) and inclusions ($j=\overline{1,J}$) under loading (7). The corresponding normal and tangential longitudinal shear stresses (3), and (4) on curvilinear smooth contours L_j in anisotropic bodies are determined by simple expressions

$$\tau_{nz}(t)=-\tau\,\mathrm{Re}\left[\frac{dt}{ds}\right],\ \ \tau_{sz}(t)=\tau\,\mathrm{Im}\left[\frac{dt}{ds}\right],\ \ t\in L_j,\ \ j=\overline{1,J},$$

which are independent of elastic constants of materials and remain the same as for isotropic bodies.

Using the Sokhotski–Plemelj formula for Cauchy integrals, we find the limiting values ($z\to t\in L_j$) of the CP of stresses (8) on the contours L_j $(j=\overline{1,J})$ from the sides of the inclusions and the matrix.

$$\Phi_3^{j+}\left(t_3^j\right)=\Gamma_3^j+i\phi_3'^j\left(t_3^j\right)+\frac{1}{\pi}\int_{L_3^j}\frac{\phi_3'^j\left(t_3'^j\right)}{t_3'^j-t_3^j}dt_3'^j,\ \ t_3^j\in L_3^j,\ \ j=\overline{1,J};$$

$$\Phi_3^{0-}\left(t_{3j}^0\right)=\Gamma_3^0-i\phi_{3j}'^0\left(t_{3j}^0\right)+\frac{1}{\pi}\sum_{k=1}^{J}\int_{L_{3k}^0}\frac{\phi_{3k}'^0\left(t_{3k}'^0\right)}{t_{3k}'^0-t_{3j}^0}dt_{3k}'^0,\ \ t_{3j}^0\in L_{3j}^0. \tag{9}$$

Based on relations (1), (3), (9) from the boundary conditions at each of the contours of the inclusions L_j (5) and the conditions differentiated along the arc abscissas s_j (6)

$$\frac{dw^+(t)}{ds_j}-\frac{dw^-(t)}{ds_j}=0,\ \ s_j=s_j(t),\ \ t\in L_j,\ \ j=\overline{1,J} \tag{10}$$

a system $2J$ of real SIEs of the 2nd kind is obtained

$$A_{3j}\,\mathrm{Im}\left[\phi_{3j}'^0\left(s_j\right)\right]+\frac{1}{2\pi}\sum_{k=1}^{J}\int_{L_k}\mathrm{Re}\left[D_{11}\left(s_j,s_k'\right)\phi_{3k}'^0\left(s_k'\right)+D_{12}\left(s_j,s_k'\right)\overline{\phi_{3k}'^0\left(s'\right)_k}\right]ds_k'=0, \tag{11}$$

$$A_{3j}\,\mathrm{Re}\left[\phi_{3j}'^0\left(s_j\right)\right]+\frac{1}{2\pi}\sum_{k=1}^{J}\int_{L_k}\mathrm{Im}\left[D_{21}\left(s_j,s_k'\right)\phi_{3k}'^0\left(s_k'\right)+D_{22}\left(s_j,s_k'\right)\overline{\phi_{3k}'^0\left(s_k'\right)}\right]ds_k'=f_2\left(s_j\right),$$

where $s_j,\ s_k'$ are arc abscissas of the points $t\in L_j,\ t'\in L_k$ $\left(j,k=\overline{1,J}\right)$,

$$D_{11}\left(s_j,s'\right)_k = D^0_{jk} - A_{3j}D^j_{jk};\quad D_{12}\left(s_j,s'_k\right) = -B_{3j}D^j_{jk};$$
$$D_{21}\left(s_j,s'_k\right) = A_{3j}D^j_{jk} - \left(A_{3j}-B_{3j}\right)D^0_{jk};\quad D_{22}\left(s_j,s'_k\right) = B_{3j}D^j_{jk};$$
$$D^0_{jk} = \frac{\dfrac{dt^0_{3j}}{ds_j}}{t'^0_{3k} - t^0_{3j}};\quad D^j_{jk} = \frac{\dfrac{dt^j_3}{ds_j}}{t'^k_3 - t^j_3};\quad f_2\left(s_j\right) = \frac{1}{2}\mathrm{Im}\left\{a_{0j}\Gamma^0_3\frac{dt^0_{3j}}{ds_j} - \Gamma^j_3\frac{dt^j_3}{ds_j}\right\}. \tag{12}$$

Here, taking into account the boundary conditions (5) and (10) the dependences between the unknown complex functions $\phi'^j_3\left(s_j\right) = A_{3j}\phi'^0_{3j}\left(s_j\right) + B_{3j}\overline{\phi'^0_{3j}\left(s_j\right)}$ [6, 18], where $\phi'^0_{3j}\left(s_j\right) = \dfrac{\phi'^0_{3j}(t^0_{3j})dt^0_{3j}}{ds_j}$, $\phi'^j_3\left(s_j\right) = \dfrac{\phi'^j_3(t^j_3)dt^j_3}{ds_j}$, $A_{3j} = \dfrac{1+a_{0j}}{2}$, $B_{3j} = \dfrac{1-a_{0j}}{2}$, $a_{0j} = \dfrac{a_0}{a_j}$ were used. For homogeneous stress states of the inclusions and the matrix, there are the following dependencies $\mathrm{Re}\left[\dfrac{\Gamma^j_3 dt^j_3}{ds_j}\right] = \mathrm{Re}\left[\dfrac{\Gamma^0_3 dt^0_{3j}}{ds_j}\right]$, $t^j_3 \in L^j_3$, $t^0_{3j} \in L^0_{3j}$. For a piecewise homogeneous isotropic body ($\alpha^j_3 = 0, \gamma^j_3 = 1, j = \overline{0,J}$), the well-known SIE system is obtained [29] from equations (11).

One Anisotropic Inclusion

The SIE system (11) was numerically solved by the quadrature method [17] in the presence of one anisotropic inclusion S_1 $\left(J=1,\ L_0 = L_1 = L\right)$ in the matrix. We considered an orthotropic matrix S_0 (with the main axes of orthotropy along the axes *Ox*, *Oy*), for which $\mu^0_3 = i\gamma^0_3$, $\alpha^0_3 = 0$, $\gamma^0_3 = \sqrt{\dfrac{G^0_{x3}}{G^0_{y3}}}$, $a_0 = \dfrac{1}{\sqrt{G^0_{x3}G^0_{y3}}}$, and an orthotropic inclusion S_1 (with axes of orthotropy inclined to the axes *Ox*, *Oy* at an angle β), for which

$$\mu^1_3 = \alpha^1_3 + i\gamma^1_3,\quad \alpha^1_3 = \frac{\left(\left(\gamma^1_{30}\right)^2 - 1\right)\sin\beta\cos\beta}{\left(\gamma^1_{30}\right)^2\sin^2\beta + \cos^2\beta},\quad \gamma^1_3 = \frac{\gamma^1_{30}}{\left(\gamma^1_{30}\right)^2\sin^2\beta + \cos^2\beta}, \tag{13}$$

where $\gamma^1_{30} = \sqrt{\dfrac{G^1_{13}}{G^1_{23}}}$, $a_1 = \dfrac{1}{\sqrt{G^1_{13}G^1_{23}}}$. Here G^0_{x3}, G^0_{y3}; G^1_{13}, G^1_{23} are shear moduli along the orthotropy axes of the matrix and inclusion materials, respectively. Having set the inclusion L contour in the physical plane *xOy* in the parametric form $t = \omega(\eta)$, $t' = \omega(\xi)$; $\eta, \xi \in [0;\, 2\pi]$, the problem is reduced to solving two real SIEs of the second kind (11) ($J = 1$).

Based on the limiting CP values (9) for one inclusion

$$\Phi_3^{1+}(t_1)=\Gamma_3^1+i\frac{|\omega'(\eta)|}{\omega'_1(\eta)}\{u_1(\eta)+ia_{01}u_2(\eta)\}+\frac{1}{\pi}\int_0^{2\pi}\frac{\{u_1(\xi)+ia_{01}u_2(\xi)\}}{\omega_1(\xi)-\omega_1(\eta)}|\omega'(\xi)|d\xi\ ,$$

$$\Phi_3^{0-}(t_0)=\Gamma_3^0-i\frac{|\omega'(\eta)|}{\omega'_0(\eta)}\{u_1(\eta)+iu_2(\eta)\}+\frac{1}{\pi}\int_0^{2\pi}\frac{\{u_1(\xi)+iu_2(\xi)\}}{\omega_0(\xi)-\omega_0(\eta)}|\omega'(\xi)|d\xi,\ \eta\in[0;\,2\pi], \tag{14}$$

contact distributions $\tau_{nz}(t)=\tau_{nz}^{+}(t)=\tau_{nz}^{-}(t)$ (3), (5), and contour (4) shear stresses from the sides of inclusion $\tau_{sz}^{+}(t)$ and matrix are defined:

$$\tau_{sz}^{+}(t)=\mathrm{Re}\left[\Phi_3^{1+}(t_1)f_s(t,\mu_3^1)\right],\ \ \tau_{sz}^{-}(t)=\mathrm{Re}\left[\Phi_3^{0-}(t_0)f_s(t,\mu_3^0)\right]. \tag{15}$$

Here $f_s(t,\mu)=\dfrac{(1-i\mu)\omega'(\eta)-(1+i\mu)\overline{\omega'(\eta)}}{2i|\omega'(\eta)|}$, $\omega'_k(\eta)=\dfrac{d\omega_k(\eta)}{d\eta}$, $k=0;1$,

$\omega_0(\eta)=\mathrm{Re}[\omega(\eta)]+\mu_3^0\,\mathrm{Im}[\omega(\eta)]$, $\omega_1(\eta)=\mathrm{Re}[\omega(\eta)]+\mu_3^1\,\mathrm{Im}[\omega(\eta)]$, $u_1(\eta)=\mathrm{Re}\left[\phi_{31}'^{0}(s)\right]=$; $=\mathrm{Re}\left[\dfrac{\phi_{31}'^{0}(t_{31}^0)dt_{31}^0}{ds}\right]$; $u_2(\eta)=\mathrm{Im}\left[\phi_{31}'^{0}(s)\right]$, where s is the arc abscissa of the point $t=\omega(\eta)\in L$; $a_{01}=\dfrac{a_0}{a_1}$; $\Gamma_3^0=\tau$, $\Gamma_3^1=\left(1+\dfrac{i\alpha_3^1}{\gamma_3^1}\right)\tau$. Calculation is done for the inclusion contour of an elliptical shape

$$t=\omega(\eta)=a\cos(\eta)+ib\sin(\eta),\ \ \eta\in[0;\,2\pi]. \tag{16}$$

and some values of the elastic constants of the matrix and inclusion materials.

Orthotropic Matrix and Elastic Isotropic Inclusion

The influence of the level of orthotropy of the matrix material (mechanical parameter $\gamma_3^0=\sqrt{\dfrac{G_{13}^0}{G_{23}^0}}$) on the distributions of contact (3) and contour (15) stresses significantly depends on the parameter $a_{01}=\dfrac{a_0}{a_1}=\dfrac{\sqrt{G_{13}^1G_{23}^1}}{\sqrt{G_{13}^0G_{23}^0}}$, which characterizes the total relative rigidity (for longitudinal shear along the *Oz* axis) of the inclusion material relative to the matrix material (Fig. 2). An orthotropic matrix ($\gamma_3^0\in[0{,}25;\,4]$) with an isotropic ($\gamma_3^1=1$, $\alpha_3^1=0$, $G_{13}^1=G_{23}^1=G_3^1$) circular ($\dfrac{b}{a}=1$) inclusion is considered. A change in the parameter a_{01} from 0.25 to 4 leads to a 4-fold increase in stresses $\tau_{nz}(\eta)$ and $\tau_{sz}^{+}(\eta)$ (Figs. 2*a*–*d*), and the relative contour stresses

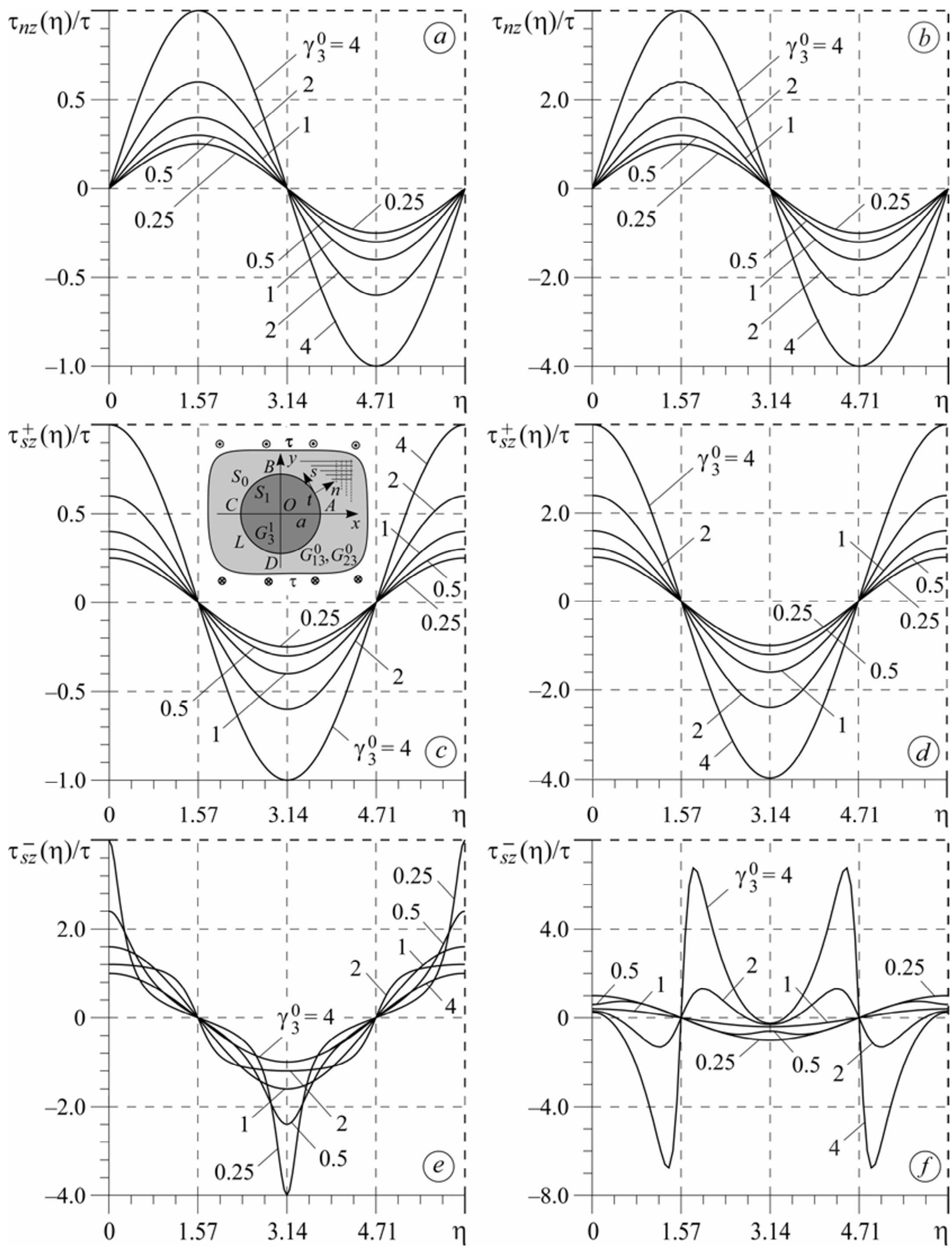

Fig. 2. Influence of the parameter γ_3^0 on the relative stress distributions (*a*, *b*), $\tau_{nz}(\eta)/\tau$, (*c*, *d*) $\tau_{sz}^{+}(\eta)/\tau$, (*e*, *f*) $\tau_{sz}^{-}(\eta)/\tau$ for a flexible (*a*, *c*, *e*) (a_{01} = 0.25) and (*b*, *d*, *f*) rigid (a_{01} = 4) isotropic circular inclusion.

from the matrix $\tau_{sz}^{-}(\eta)$ side also undergo significant qualitative changes (Figs. 2*e*, *f*). The obtained maximum values of contact stresses $\tau_{nz}(\eta)$ for a piecewise isotropic body (isotropic matrix with an elliptical inclusion) agree well with those known [6].

Orthotropic Matrix and Anisotropic Inclusion

The influence of the inclination angle β of the orthotropy axes of the inclusion S_1 material to the axes *Ox*, *Oy* on the stress distributions at the interphase of the materials was studied (Fig. 3). Such an inclusion for angles $\beta \neq \{0; \pi/2\}$ in the introduced Cartesian coordinate system *xOy* (Fig. 3*d*) is described by the model of an anisotropic body with mechanical parameters (13). The elliptic inclusion ($b/a = 0.25$) with the ratio of shear moduli along the axes of material orthotropy $G_{13}^1 / G_{23}^1 = 1/8$ in a relatively compliant orthotropic matrix ($a_{01} = 8\sqrt{2}$; $\gamma_3^0 = 2$) is considered. The rotation of the axes of orthotropy of the inclusion material significantly changes the contour stress distributions from the inclusion side (Fig. 3*b*). It almost does not affect the contact (Fig. 3*a*) and contour stresses from the matrix side (Fig. 3*c*).

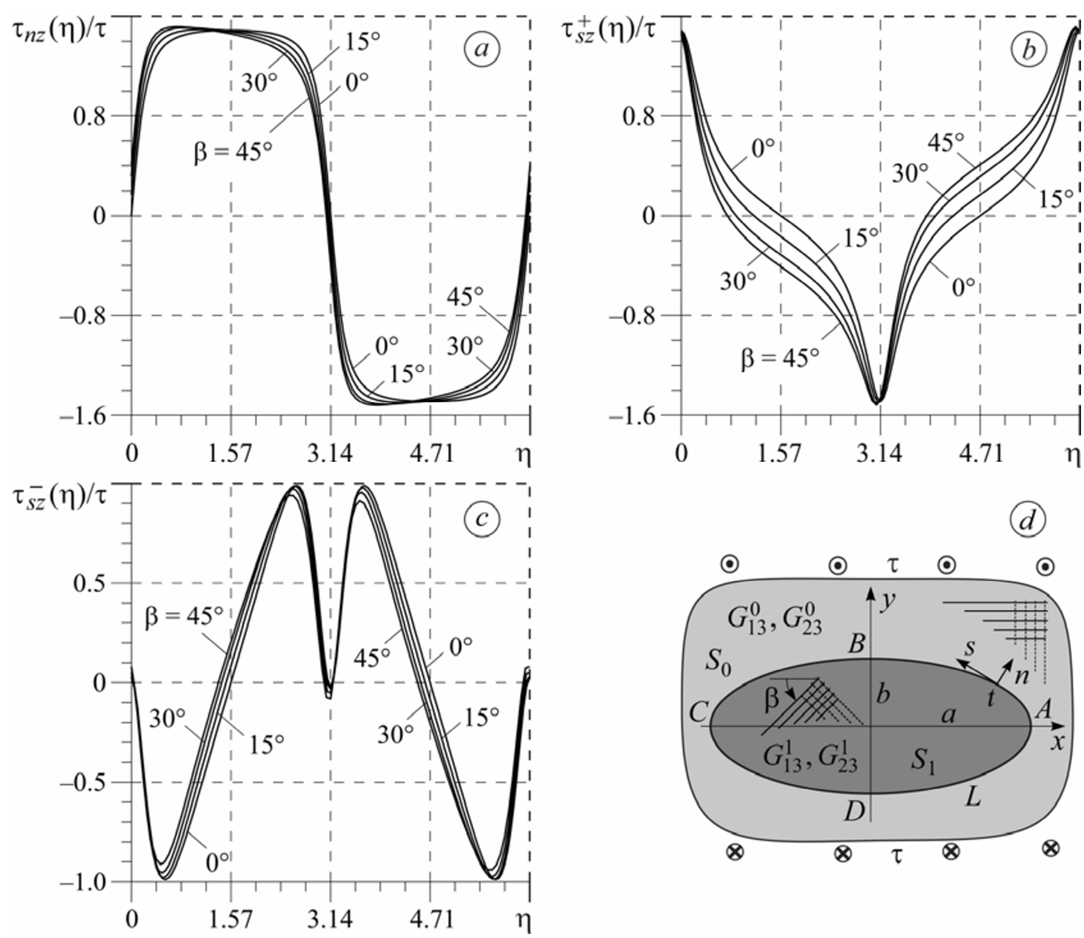

Fig. 3. Influence of the angle β of the orthotropy axes of the elliptical inclusion material (*b*/*a* = 0.25) on the relative stress distributions (*a*) $\tau_{nz}(\eta)/\tau$; (*b*) $\tau_{sz}^{+}(\eta)/\tau$; (*c*) ($\tau_{sz}^{-}(\eta)/\tau$, for $G_{13}^1 / G_{23}^1 = 1/8$ and compliance matrix ($a_{01} = 8\sqrt{2}$; $\gamma_3^0 = 2$); *d* is the scheme of the problem.

For the given load (7) for different geometric and mechanical parameters of the problem (b/a, β, γ_3^0, γ_3^1, a_0/a_1), the stress $\tau_{xz}(z)$, $\tau_{yz}(z)$ inside the elliptical anisotropic inclusion ($z \in S_1$) was calculated and the homogeneity of its stress state was found, which is consistent with the known analytical result [24].

Constant stresses $\tau_{xz} = \text{const}$, $\tau_{yz} = \text{const}$ in an elliptical inclusion, depend significantly on the parameters of the problem discussed above.

CONCLUSIONS

The antiplane problem of the theory of elasticity for an infinite anisotropic body with a finite number of smooth curvilinear anisotropic inclusions is considered. Using the methods of complex potentials and singular integral equations, the problem was reduced to solving systems of real integral equations of the second kind. For an orthotropic plane with one anisotropic inclusion, the constructed system of equations was solved numerically by the method of quadratures. The distributions of shear contact and contour stress at the materials interphase both from the side of the plane and the side of the inclusion were determined. The influence of these stresses on elastic constants of orthotropic materials of the plane and the inclusion of the material interphase of elliptical shape was studied. The problem parameters are established, which essentially affect the distribution of shear stresses at the inclusion contour.

Author contributions. All authors have contributed equally to the work, authors also read and approved the final manuscript.

Conflict of interest. Author M. P. Savruk is an Editorial Board Member of Materials Science (translated from Fizyko-Khimichna Mekhanika Materialiv). The paper was handled by another Editorial Board Member and has undergone a rigorous peer review process. Author M. P. Savruk was not involved in the journal's peer review or decisions related to this manuscript.

Funding. The partial financial support was received from the National Academy of Sciences of Ukraine under the R&D project (Registration No. 0123U100936).

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