THERMOELASTIC STATE OF A SEMIINFINITE THERMALLY SENSITIVE THREE-COMPONENT ROD UNDER CONVECTIVE-RADIATIVE HEAT EXCHANGE

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UDC 539.3

We propose a numerical-analytic approach to the determination of the thermoelastic state of a semiinfinite thermally sensitive three-component rod interacting with the ambient medium by convective-radiative heat exchange. The proposed approach is based on the use of the Kirchhoff transformation, generalized functions, Green functions of the linear nonstationary problem of heat conduction for a three-component space, and linear splines. We investigate the influence of thermal sensitivity and heat-exchange parameters on the distributions of temperature, stresses, and displacements.

Keywords: heat conduction, thermal sensitivity, thermoelasticity, layered bodies, convective-radiative heat exchange.

Layered structural elements are extensively used in various branches of industry. Under the conditions of high-temperature action, in order to give a more exact description of their thermoelastic state, it is necessary to use models taking into account the dependence of the physicomechanical characteristics on temperature and the conditions of convective-radiative heat exchange. The numerical and numerical-analytic approaches to the solution of problems of heat conduction and thermoelasticity for one-layer and multilayer thermally sensitive bodies for various methods of heating, including the case of convective-radiative heating, were considered in [1–6].

In what follows, we propose an approach to the solution of quasistatic problems of thermoelasticity for semiinfinite three-component thermally sensitive bodies heated by convective-radiative heat exchange. In this case, we use the Kirchhoff approach, generalized functions, and the Green functions of the linear nonstationary problem of heat conduction for a three-component space represented in the form of functional series and linear splines.

Statement of the Problem

Consider a semiinfinite three-component rod referred to a cylindrical coordinate system r, φ, z (Fig. 1). We assume that the conditions of perfect thermomechanical contact are satisfied on the interfaces $z = z_1 = 0$ and $z = z_2 = h$ and that the surface $r = R$ is thermally insulated and smoothly fixed (radial displacements and tangential stresses are absent). The convective-radiative heat exchange with a medium of temperature t_c occurs through the bounding surface $z = z_3$ and the normal and tangential stresses are absent on this surface.

The initial temperatures of the components are equal to zero. In this body, we determine the nonstationary temperature field and the stresses and displacements induced by this field with regard for the temperature dependences of the physicomechanical characteristics.

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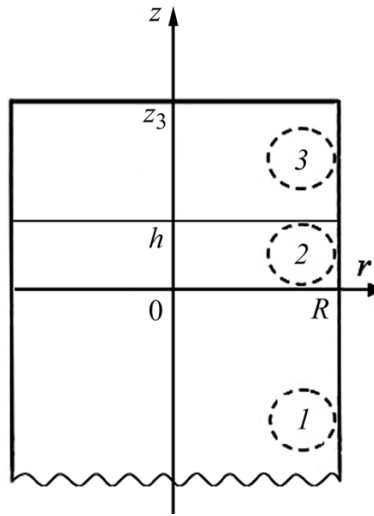
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Fig. 1. Semiinfinite three-component rod.

Solution of the Problem of Heat Conduction

For the determination of the temperature field, we use the heat-conduction equations, the conditions of contact, and the boundary and initial conditions:

$$\frac{\partial}{\partial z}\left[\lambda_t^{(i)}(t_i)\frac{\partial t_i}{\partial z}\right] = c_V^{(i)}(t_i)\frac{\partial t_i}{\partial \tau} \quad (i = \overline{1,3}); \tag{1}$$

$$t_i(z,\tau) = t_{i+1}(z,\tau), \quad \lambda_t^{(i)}\frac{\partial t_i(z,\tau)}{\partial z} = \lambda_t^{(i+1)}\frac{\partial t_{i+1}(z,\tau)}{\partial z} \quad \text{for} \quad z = z_i \quad (i = 1,2); \tag{2}$$

$$\left[\lambda_t^{(3)}\frac{\partial t_3}{\partial z} + \varepsilon\sigma[T(t_3) - T(t_c)] + \alpha(t_3 - t_c)\right]_{z=z_3} = 0, \quad t_1(z,\tau)\big|_{z\to-\infty} = 0, \quad t_i\big|_{\tau=0} = 0, \tag{3}$$

where $t_i(z,\tau)$ is the temperature in the ith layer, $T(\eta) = (\eta + 273)^4$; ε is the emissivity factor; σ is the Stefan–Boltzmann constant; α is the heat-exchange coefficient; the quantities corresponding to the first component $-\infty < z < 0$ are marked by the superscript $i = 1$, the quantities corresponding to the second component $0 < z < z_2$ are marked by $i = 2$ (interlayer), and the quantities corresponding to the third component $z_2 < z < z_3$ are marked by $i = 3$.

By using the Kirchhoff transformation

$$\theta_i = \frac{1}{\lambda_{0i}}\int_0^{t_i}\lambda_t^{(i)}(x)\,dx,$$

under the assumptions that the thermal-conductivity coefficients are linear functions of temperature

$$\lambda_t^{(i)}(t_i) = \lambda_{0i}(1 + \beta_i t_i),$$

the coefficients of volumetric heat capacity have the form

$$c_V^{(i)}(t_i) = c_{0i} c_i(t_i) \qquad (i = 1, 2, 3),$$

and the thermal diffusivities a_i are constant within the boundaries of each component

$$\frac{\lambda_t^{(i)}(t_i)}{c_V^{(i)}(t_i)} \approx a_i = \frac{\lambda_{0i}}{c_{0i}},$$

which is typical of numerous materials [7] , we reduce the problem of heat conduction (1)–(3) to a single equation with generalized derivatives with respect to z with boundary and initial conditions [5]:

$$\frac{\partial}{\partial z}\left[\lambda_0(z)\frac{\partial \theta}{\partial z}\right] = c_0(z)\frac{\partial \theta}{\partial \tau} + \sum_{j=1}^{2} \lambda_{0,j+1} F_{j+1}(\tau)\delta'(z - z_j); \tag{4}$$

$$\lambda_{0,3}\frac{\partial \theta_3}{\partial z}\bigg|_{z=z_3} + F_1(\tau) = 0, \qquad \theta_1(z,\tau)\big|_{z\to-\infty} = 0, \qquad \theta_i\big|_{\tau=0} = 0, \tag{5}$$

where

$$F_1(\tau) = \varepsilon\sigma\left[T(\theta_3^*(z_3,\tau)) - T(t_c)\right] + \alpha(\theta_3^*(z_3,\tau) - t_c);$$

$$F_{i+1}(\tau) = \left(1 - \frac{\beta_i}{\beta_{i+1}}\right)\left[\theta_{i+1}(z_i,\tau) - \beta_{i+1}^{-1}(\sqrt{1 + 2\beta_{i+1}\theta_{i+1}(z_i,\tau)} - 1)\right];$$

$$\theta_3^*(\tau) = \beta_3^{-1}(\sqrt{1 + 2\beta_3\overline{\theta}_3(\tau)} - 1); \qquad \overline{\theta}_3(\tau) = \theta_3(z_3,\tau);$$

$$\theta(z,\tau) = \theta_1(z,\tau) + \sum_{k=1}^{2}\left[\theta_{k+1}(z,\tau) - \theta_k(z,\tau)\right]S(z - z_k);$$

$S(z)$ is the Heaviside function; $\delta'(z)$ is the derivative of the Dirac delta-function, and $\lambda_0(z)$ and $c_0(z)$ take the form

$$p_0(z) = p_{0,1} + \sum_{i=1}^{2}(p_{0,i+1} - p_{0,i})S(z - z_i). \tag{6}$$

By using the Green functions $G(z,\zeta,\tau)$ of the linear nonstationary problem of heat conduction for a three-component space [8], we represent the solution of problem (4), (5) in the form

$$\theta(z,\tau) = \sum_{j=1}^{2}\lambda_{0,j+1}\int_0^\tau \frac{\partial G(z,\zeta,\tau-\tau')}{\partial \zeta}\bigg|_{\zeta=z_j+0} F_{j+1}(\tau')\,d\tau'$$

$$- \int_0^\tau\left\{G(z,\zeta,\tau-\tau')F_1(\tau') + \lambda_{0,3}\overline{\theta}_3(\tau')\frac{\partial G(z,\zeta,\tau-\tau')}{\partial z}\right\}\bigg|_{z=z_3} d\tau'. \tag{7}$$

To determine $\theta_{j+1}(z_j, \tau)$ $(j = 1, 2)$ and $\theta_3(z_3, \tau)$ appearing in integrands of relation (7), we approximate [5] the functions $F_i(\tau)$, $\theta_3^*(\tau)$, and $\overline{\theta}_3(\tau)$ by linear splines of the form

$$g(\tau) = s_1^{(1)}(g)\tau + s_1^{(0)}(g) + \sum_{k=1}^{K_\tau - 1} (s_{k+1}^{(1)}(g)\tau + s_{k+1}^{(0)}(g) - s_k^{(1)}(g)\tau - s_k^{(0)}(g))S(\tau - \tau_k), \tag{8}$$

where

$$s_1^{(1)}(\zeta) = \frac{\zeta(\tau_i) - \zeta(\tau_{i-1})}{\Delta\tau_i}; \qquad s_1^{(0)}(\zeta) = \frac{-\zeta(\tau_i)\tau_{i-1} + \zeta(\tau_{i-1})\tau_i}{\Delta\tau_i};$$

$$\Delta\tau_i = \tau_i - \tau_{i-1} \quad (i = \overline{1, K_\tau}),$$

$$0 = \tau_0 < \tau_1 < \tau_2 \ldots < \tau_{K_\tau},$$

and K_τ is the number of nodes in the spline.

Substituting expressions for the Green functions in Eq. (7) and taking the corresponding integrals with regard for (8), we get the following relation for the Kirchhoff variables:

$$\theta_i(\overline{z}, \mathrm{Fo}) = \eta_{i,1}\gamma_{i,1}(\overline{z}, \mathrm{Fo}, F_2) + \eta_{i,2}\gamma_{i,2}(\overline{z}, \mathrm{Fo}, F_3)$$

$$- \eta_{i,3}\left[\gamma_{i,3}(\overline{z}, \mathrm{Fo}, F_4) - \frac{a_{i3}^*}{2a_{i0}}\gamma_{i,4}(\overline{z}, \mathrm{Fo}, \theta_3(\overline{z}_3, \mathrm{Fo})) \right], \tag{9}$$

where

$$\gamma_{i,j}(\overline{z}, \mathrm{Fo}, \zeta) = s_1^{(0)}(\zeta)\psi_1^{i,j}(\overline{z}, \mathrm{Fo}) + s_1^{(1)}(\zeta)\psi_2^{i,j}(\overline{z}, \mathrm{Fo})$$

$$+ \sum_{k=1}^{K_\tau - 1} \left[(s_{k+1}^{(0)}(\zeta) - s_k^{(0)}(\zeta))\psi_3^{i,j}(\overline{z}, \mathrm{Fo} - \mathrm{Fo}_k) \right.$$

$$\left. + (s_{k+1}^{(1)}(\zeta) - s_k^{(1)}(\zeta))\psi_4^{i,j}(\overline{z}, \mathrm{Fo} - \mathrm{Fo}_k) \right],$$

$$F_4(\mathrm{Fo}) = \sigma^*(T(\theta^*(\overline{z}_3, \mathrm{Fo})) - T(t_c)) + \mathrm{Bi}(\theta^*(\overline{z}_3, \mathrm{Fo}) - t_c),$$

$$\eta_{1,3} = \frac{2\delta_2\lambda_{0,3}}{\sqrt{a_0}(\delta_1 + \delta_2)(\delta_2 + \delta_3)}, \qquad \eta_{2,3} = \frac{\lambda_{0,3}}{\sqrt{a_0}(\delta_2 + \delta_3)}, \qquad \eta_{3,3} = \frac{\lambda_{0,3}}{2\sqrt{a_0}\delta_3},$$

$$\psi_\rho^{i,j}(\overline{z}, \mathrm{Fo}) = l_\rho^{i,j}(\overline{z}, \mathrm{Fo}) \quad (j = 1, 2),$$

$$\psi_\rho^{i,3}(\overline{z}, \mathrm{Fo}) = \vartheta_{\rho,\chi}^{i,1}(\overline{z}, \mathrm{Fo}), \qquad \psi_\rho^{i,4}(\overline{z}, \mathrm{Fo}) = \vartheta_{\rho,\varphi}^{i,2}(\overline{z}, \mathrm{Fo}) \quad \text{for} \quad \rho = 1, 2;$$

$$\psi_\rho^{i,j}(\overline{z}, \mathrm{Fo} - \mathrm{Fo}_k) = l_\rho^{i,j}(\overline{z}, \mathrm{Fo} - \mathrm{Fo}_k) \quad (j = 1, 2),$$

$$\psi_\rho^{i,3}(\overline{z}, \mathrm{Fo}) = \vartheta_{\rho,\chi}^{i,1}(\overline{z}, \mathrm{Fo} - \mathrm{Fo}_k), \quad \psi_\rho^{i,4}(\overline{z}, \mathrm{Fo}) = \vartheta_{\rho,\varphi}^{i,2}(\overline{z}, \mathrm{Fo} - \mathrm{Fo}_k) \quad \text{for} \quad \rho = 3, 4;$$

$$\vartheta_{\rho,y}^{1,\mathrm{p}}(\overline{z}, \xi) = \sum_{n=0}^{\infty} (v_1 v_2)^n y_{\rho,1}^1(a_{12}^*(1 + 2n)\overline{h} + a_{13}^*(\overline{z}_3 - \overline{h}) - \overline{z}, \xi),$$

$$\vartheta_{\rho,y}^{2,\mathrm{p}}(\overline{z}, \xi) = y_{\rho,2}^1(a_{23}^*(\overline{z}_3 - \overline{h}) + \overline{h} - \overline{z}, \xi)$$

$$+ \sum_{n=1}^{\infty} (v_1 v_2)^n y_{\rho,2}^1(a_{23}^*(\overline{z}_3 - \overline{h}) + 2n\overline{h} - \overline{z} + \overline{h}, \xi)$$

$$- v_1 \sum_{n=0}^{\infty} (v_1 v_2)^n y_{\rho,2}^1(a_{23}^*(\overline{z}_3 - \overline{h}) + (1 + 2n)\overline{h} + \overline{z}, \xi),$$

$$\vartheta_{\rho,y}^{3,\mathrm{p}}(\overline{z}, \xi) = y_{\rho,3}^1(\overline{z}_3 - \overline{z}, \xi) + (-1)^{\delta_{\mathrm{p}2}} v_2 y_{\rho,3}^0(2\overline{h} - \overline{z}_3 - \overline{z}, \xi)$$

$$+ v_2 \sum_{n=1}^{\infty} (v_1 v_2)^n y_{\rho,3}^1(2n\overline{h}a_{32}^* + \overline{z}_3 + \overline{z} - 2\overline{h}, \xi)$$

$$- v_1 \sum_{n=0}^{\infty} (v_1 v_2)^n y_{\rho,3}^1(2(1 + n)\overline{h}a_{32}^* + \overline{z}_3 + \overline{z} - 2\overline{h}, \xi),$$

$$\chi_{\rho,i}^{\rho'}(\zeta, \xi) = (S(\xi))^{\delta_{\mathrm{p}3}} \left(2\frac{\sqrt{\xi}}{\sqrt{\pi}} \exp\left[\frac{-\zeta^2}{4a_{i0}\xi}\right] - \frac{\zeta}{\sqrt{a_i}}\left((-1)^{\rho'+1} - \mathrm{erf}\left(\frac{\zeta}{2a_{i0}^*\sqrt{\xi}}\right)\right)\right) \quad \text{for} \quad \rho = 1, 3;$$

$$\chi_{2,i}^{\rho'}(\zeta, \mathrm{Fo}) = \frac{\sqrt{\mathrm{Fo}}}{\sqrt{\pi}} \exp\left[\frac{-\zeta^2}{4a_{i0}\mathrm{Fo}}\right]\left(\frac{4}{3}\mathrm{Fo} + \frac{\zeta^2}{3a_{i0}}\right)$$

$$- \frac{\zeta}{2\sqrt{a_{i0}}}\left(\frac{\zeta^2}{3a_{i0}} + 2\mathrm{Fo}\right)\left((-1)^{\rho'+1} - \mathrm{erf}\left(\frac{\zeta}{2a_{i0}^*\sqrt{\mathrm{Fo}}}\right)\right),$$

$$\chi_{4,i}^{\rho'}(\zeta, \mathrm{Fo} - \mathrm{Fo}_k) = S(\mathrm{Fo} - \mathrm{Fo}_k)\left(\frac{\sqrt{\mathrm{Fo} - \mathrm{Fo}_k}}{\sqrt{\pi}} \exp\left[\frac{-\zeta^2}{4a_{i0}(\mathrm{Fo} - \mathrm{Fo}_k)}\right]\right.$$

$$\times\left(-\frac{2}{3}(\mathrm{Fo} - \mathrm{Fo}_k)^2 + \frac{\zeta^2}{3a_{i0}} + 2\mathrm{Fo}\right)$$

$$\left. - \frac{\zeta}{2\sqrt{a_{i0}}}\left(\frac{\zeta^2}{3a_{i0}} + 2\mathrm{Fo}\right)\left((-1)^{\rho'+1} - \mathrm{erf}\left(\frac{\zeta}{2a_{i0}^*\sqrt{\mathrm{Fo} - \mathrm{Fo}_k}}\right)\right)\right),$$

where

$$\mathrm{Fo} = \frac{a_0 \tau}{l_0^2}; \qquad \mathrm{Fo}_k = \frac{a_0 \tau_k}{l_0^2}; \qquad \mathrm{Bi} = \frac{\alpha l_0}{\lambda_{0,3}}; \qquad \sigma^* = \frac{\varepsilon \sigma l_0}{\lambda_{0,3}};$$

δ_{ij} is the Kronecker symbol; l_0 is a linear size; a_0 is a quantity with dimensionality of thermal diffusivity;

$$a_{ij}^* = \sqrt{\frac{a_i}{a_j}}; \qquad \delta_i = \frac{\lambda_i}{\sqrt{a_i}}; \qquad v_1 = \frac{\delta_1 - \delta_2}{\delta_1 + \delta_2}; \qquad v_2 = \frac{\delta_3 - \delta_2}{\delta_3 + \delta_2};$$

$$\overline{z} = \frac{z}{l_0}; \qquad \overline{z}_i = \frac{z_i}{l_0}; \qquad \overline{h} = \frac{h}{l_0}; \qquad a_{ij} = \frac{a_i}{a_j};$$

the functions $\chi_{\rho,i}^{\rho'}(\zeta,\xi)$ or $\varphi_{\rho,i}^{\rho'}(\zeta,\xi)$ correspond to the functions $y_{\rho,i}^{\rho'}(\zeta,\xi)$ if the subscript "y" in $\vartheta_{\rho,y}^{i,p}$ takes the values φ and χ, respectively, and $\varphi(\zeta,\xi)$ and the remaining notation coincide with the notation presented in [6].

Setting $\overline{z} = \overline{z}_j + 0$ $(j = 1,2)$, $\overline{z} = \overline{z}_3$, and $\mathrm{Fo} = \mathrm{Fo}_k$ $(k = \overline{1, K_\tau})$ in relations (9) for $\theta_i(\overline{z}, \mathrm{Fo})$ $(i = 2, 3)$, we arrive at the recurrent system of three nonlinear algebraic equations for $\theta_2(\overline{z}_1, \mathrm{Fo}_k)$, $\theta_3(\overline{z}_2, \mathrm{Fo}_k)$, and $\theta_3(\overline{z}_3, \mathrm{Fo}_k)$. As a result of the solution of this system, we obtain expressions for the Kirchhoff variables and determine the temperature field $t_i(\overline{z}, \mathrm{Fo})$ from the relation

$$t_i(\overline{z}, \mathrm{Fo}) = \beta_i^{-1}(\sqrt{1 + 2\beta_i \theta_i(\overline{z}, \mathrm{Fo})} - 1).$$

Solution of the Problem of Thermoelasticity

In the body, the radial and circumferential stresses are not equal to zero [5, 9]

$$\sigma_{rr} = \sigma_{\varphi\varphi} = \sigma_0(\overline{z}, \mathrm{Fo}) = -\frac{\overline{E}(\overline{z}, \mathrm{Fo})}{1 - \overline{\upsilon}(\overline{z}, \mathrm{Fo})} \overline{\Phi}(\overline{z}, \mathrm{Fo}). \qquad (10)$$

Moreover, the radial displacements are absent and the axial displacements are given by the formula

$$w(\overline{z}, \mathrm{Fo}) = \int_{\overline{z}_3}^{\overline{z}} \frac{1 + \overline{\upsilon}(z, \mathrm{Fo})}{1 - \overline{\upsilon}(z, \mathrm{Fo})} \overline{\Phi}(\overline{z}, \mathrm{Fo})\, dz. \qquad (11)$$

Here, the functions $\overline{E}(\overline{z}, \mathrm{Fo})$, $\overline{\upsilon}(\overline{z}, \mathrm{Fo})$, and $\overline{\Phi}(\overline{z}, \mathrm{Fo})$, which have the form (6), coincide, within the boundaries of the ith component with the moduli of elasticity $E_i(t_i)$, Poisson's ratios $\upsilon_i(t_i)$, and thermal strains

$$\Phi_i(t_i) = \int_0^{t_i(\overline{z}, \mathrm{Fo})} \alpha_{ti}(\eta)\, d\eta,$$

respectively; here, $\alpha_{ti}(t_i)$ are the coefficients of linear thermal expansion.
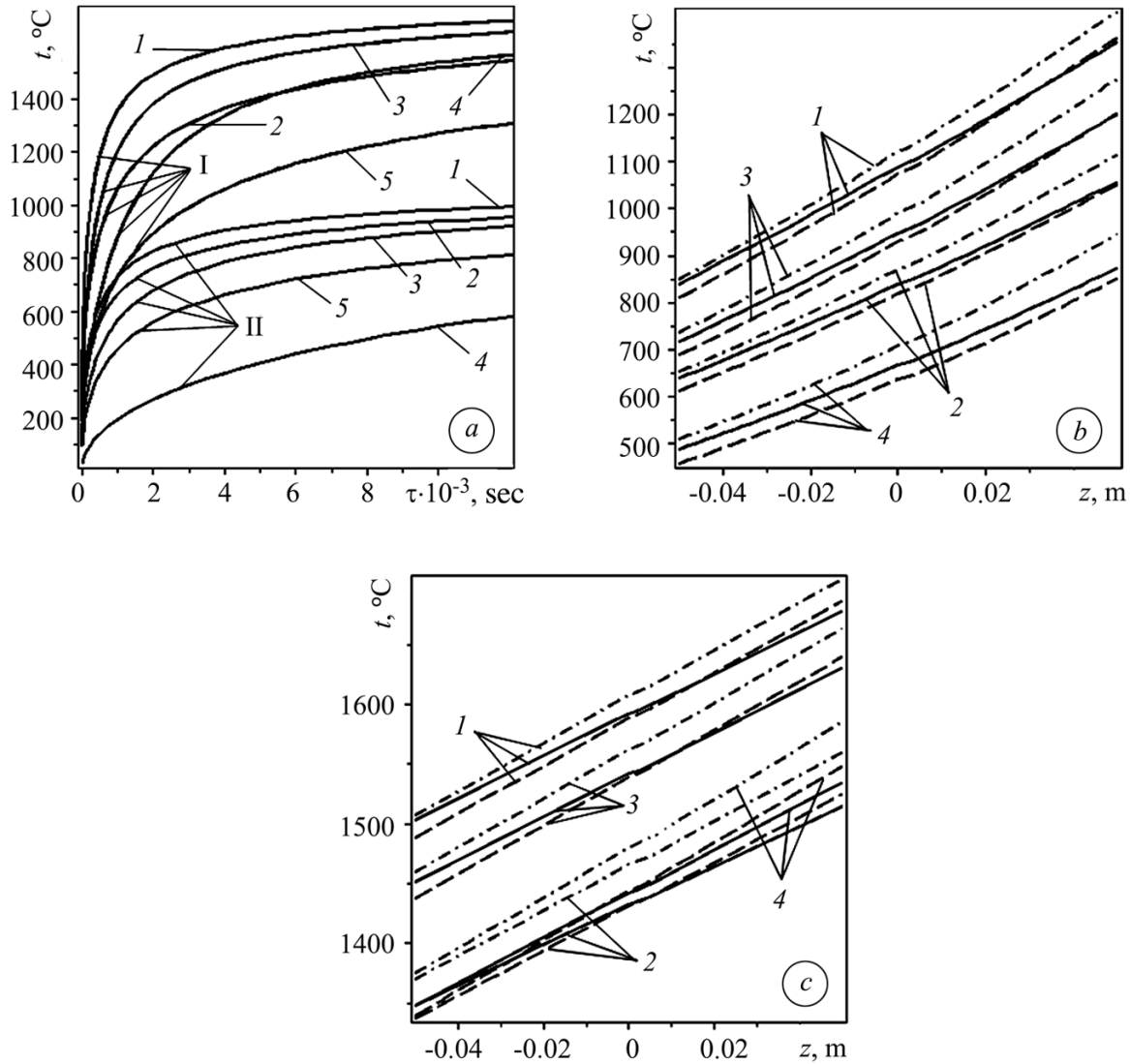
Fig. 2. Dependences of temperature on time on the bounding surface $z = z_3$ (a) and on the coordinate for $\tau = 10^3$ sec (b) and $\tau = 10^4$ sec (c) and different parameters of heat exchange [(1) $\alpha = 400$ W/(m$^2 \cdot$°C), $\varepsilon = 0.3$; (2) $\alpha = 400$, $\varepsilon = 0$; (3) $\alpha = 200$, $\varepsilon = 0.3$; (4) $\alpha = 0$, $\varepsilon = 0.3$; (5) $\alpha = 200$, $\varepsilon = 0$]: (a) for $t_c = 1800$°C (I) and $t_c = 1100$°C (II); (b, c) for $t_c = 1800$°C.

Numerical Investigations

We tested the proposed procedure for different parameters of heat exchange in the case where the physicomechanical characteristics of the first and third components correspond to niobium $(\alpha_{ti}(t_i) = (6.186 + 0.00236 t_i) \cdot 10^{-6}$°C^{-1}; $\lambda_t^{(i)} = 53.17(1 + 0.226 \cdot 10^{-3} t_i)$ W/(m$^2 \cdot$°C); $a_i = 23.9 \cdot 10^{-6}$ m^2/sec; $E_i(t_i) = (100 - 918 \cdot 10^{-5} t_i - 411 \cdot 10^{-8} t_i^2) \cdot 10^9$ N/m^2, $\upsilon_i = 0,33)$, and the physicomechanical characteristics of the interlayer correspond to platinum $(E_2(t_2) = (168 - 338 \cdot 10^{-4} t_2) \cdot 10^9$ N/m^2; $\alpha_{t2}(t_2) = (8.865 + 0.00278 t_2) \cdot 10^{-6}$ °C^{-1}; $a_2 = 24.4 \cdot 10^{-6}$ m^2/sec; $\lambda_t^{(2)} = 71.301(1 + 0.207 \cdot 10^{-3} t_2)$ W/(m$^2 \cdot$°C), $\upsilon_2 = 0.35)$ for $h = 5 \cdot 10^{-3}$m and $z_3 = 10h$.
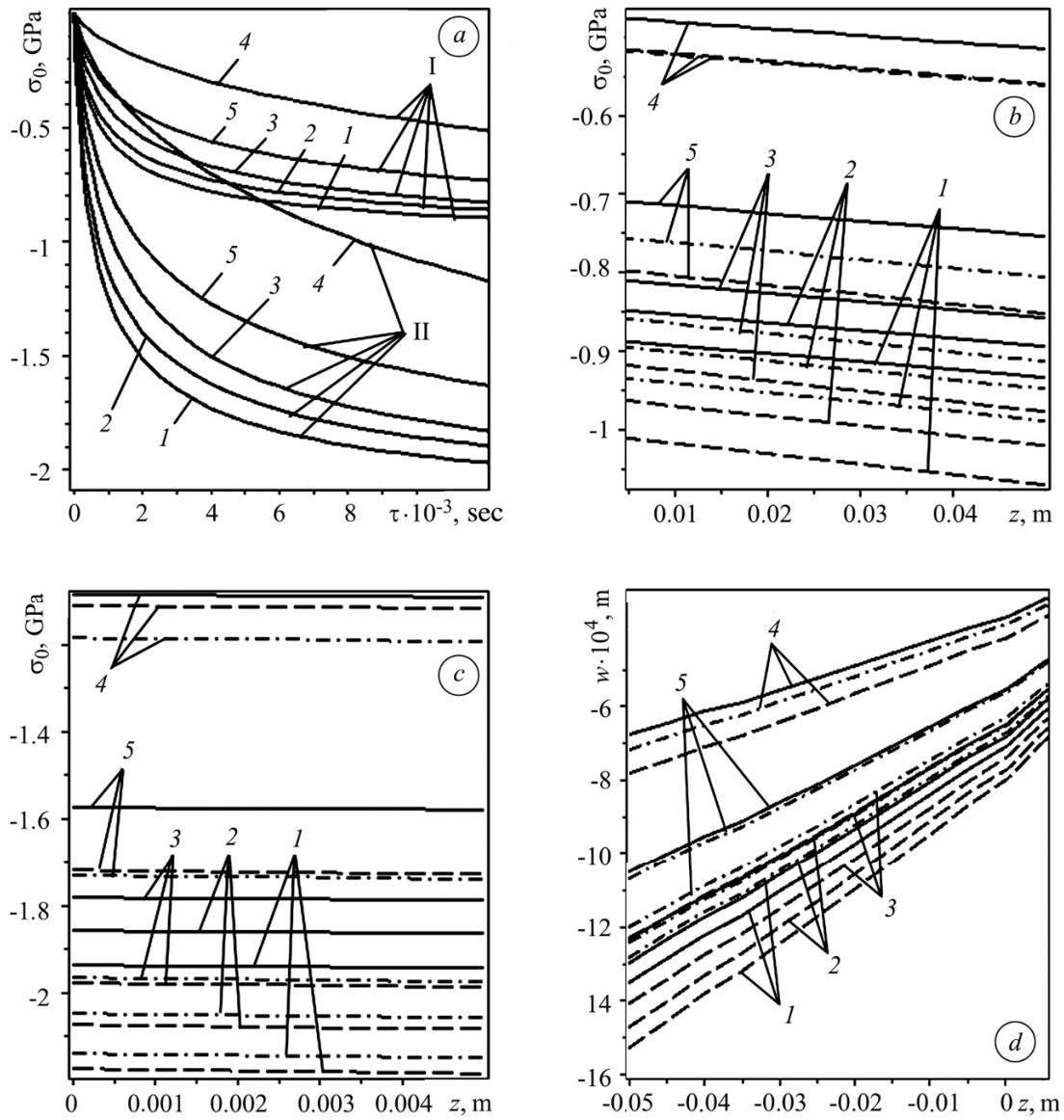
Fig. 3. Dependences of the stresses on time on the surfaces $z = z_1 - 0$ (I) and $z = z_1 + 0$ (II) (a), on the coordinate within the third
component (b) and in the interlayer (c); and of the displacements (d) on the coordinate for $\tau = 10^4$ sec, $t_c = 1100°C$, and dif-
ferent parameters of heat exchange [(1) $\alpha = 400$ W/(m$^2\cdot°$C), $\varepsilon = 0.3$; (2) $\alpha = 400$, $\varepsilon = 0$; (3) $\alpha = 200$, $\varepsilon = 0.3$; (4) $\alpha = 0$,
$\varepsilon = 0.3$; (5) $\alpha = 200$, $\varepsilon = 0$].

To choose the step of the grid of splines in relations (9), we compared the distributions of temperature
at fixed points for different time intervals depending on K_τ. It was shown that, for times $\tau \leq \tau^*$, where
$\tau^* = 12 \cdot 10^3$ sec is a time close for the time of reaching the stationary mode, it suffices to restrict ourselves
to $K_\tau = 10$ because an increase in K_τ (a decrease in the step of the grid) exerts almost no influence on the ac-
curacy of calculations.

The results of investigations in the form of plots are presented in Figs. 2–3 (the solid lines correspond to
temperature-dependent physicomechanical characteristics, whereas the dotted and dashed lines correspond
to constant physicomechanical characteristics measured at 700 and 0°C, respectively).

Under the conditions of convective-radiative heat exchange (Fig. 2), the influence of heat losses is weaker than in the presence solely of convective heat exchange, which becomes more pronounced for higher temperatures of the medium. Thus, the increase in the heat-transfer coefficient under the conditions of convective-radiative heat exchange causes an increase in temperature at $t_c = 1100°C$ and $t_c = 1800°C$ by up to 12 and 8%, respectively, whereas under the conditions of convective heat exchange (in the absence of radiation), the corresponding increase constitutes 17 and 20%, respectively. Thus, if the radiative heat exchange is neglected, then the temperature is underestimated by up to 25% at $t_c = 1800°C$ and up to 15% at $t_c = 1100°C$. The difference between the temperatures computed with and without regard for the convective heat exchange decreases as the temperature of the medium increases. Thus, in particular, it can attain 55% for $t_c = 1100°C$ and 20% for $t_c = 1800°C$.

Within the boundaries of individual regions, the difference between the temperatures (Figs. 2b, c) obtained for temperature-dependent and constant characteristics taken at 0°C is smaller (for some values of time) than in the case of characteristics taken at 700°C; at the same time, for the other values of time, the indicated difference is larger. The temperatures computed with and without regard for thermal sensitivity may differ by 10%.

The character of the behavior of stresses on the surfaces $z = h$ and $z = z_3$ is the same as on the surface $z = 0$ (Fig. 3a). The stresses on interfaces are discontinuous. At the same time, they remain almost constant over the thickness of the interlayer and almost identical on the surfaces of the first and third layer on the boundary of the interlayer. If the effect of radiation or heat transfer is neglected, then the absolute values of stresses are underestimated by up to 15 and 40%, their jumps on the interfaces are underestimated by up to 13 and 50%, respectively, and the displacements are underestimated by up to 11 and 38%.

The difference between the stresses obtained with and without regard for thermal sensitivity (Fig. 3b, c) can be as high as 17%. For fixed times, this difference for the characteristics measured at 0°C in the first and third components can be smaller than for the characteristics measured at 700°C, whereas in the interlayer this difference is larger. Moreover, as in the case of temperature, within the boundaries of individual domains, this difference for characteristics measured at 0°C can be smaller, at certain times, than for the characteristics measured at 700°C but can become larger for the other times. The difference between the displacements with and without regard for thermal sensitivity (Fig. 3d) for the characteristics measured at 700°C is always larger than for the characteristics measured at 0°C and can be as high as 15%.

CONCLUSIONS

We propose and test a numerical-analytic approach to the solution of quasistatic problems of thermoelasticity for a semiinfinite three-component thermally sensitive rod with regard for the convective-radiative heat exchange. By using the Kirchhoff transformation, generalized functions, and Green functions in the form of functional series and linear splines, we reduce the corresponding problem of heat conduction to the solution of a recursive system of three nonlinear algebraic equations for the values of the Kirchhoff variable on the interfaces and on the bounding surface at the nodes of the spline grid. The expressions for the radial and circumferential stresses, and axial displacements are obtained. The influence of thermal sensitivity and the parameters of radiative and convective heat exchange on the distributions of temperature, stresses, and displacements are investigated.

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