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THERMAL STRESSED STATE OF A DISK IN THE PROCESS OF MULTIPLE BRAKING

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On the basis of the finite-element method, we propose a mathematical model of frictional heating of a disk in the process of multiple braking. We obtain the numerical solution of the axisymmetric initial boundary-value problem of heat conduction and the boundary-value problem of quasistatic thermoelasticity for a disk periodically heated on the friction surface by a heat flow whose intensity is proportional to the specific power of friction. The numerical analyses of temperature and temperature stresses in the process of multiple braking are performed for a disk made of cast iron and a cermet pad.

Keywords: braking, frictional heating, temperature, stresses.

For the determination of the temperature field and thermal stressed state of a disk in the course of single braking, the analytic solutions of axisymmetric thermal problems of heating were constructed in [1–3] for a half space with circular boundary of changes in the boundary conditions on the surface. However, the replacement of a disk of finite thickness by a half space and of the actual heated region determined by the shape and sizes of the brake pad by a circle may lead, under certain conditions, to significant errors in the determination of the temperature mode of braking [4]. This is why it became customary to use numerical procedures based on the finite-element method (FEM) for the solution of the thermal problems of friction in the course of braking [5, 6]. The space-time distributions of axisymmetric nonstationary temperature fields in homogeneous disks were analyzed in [7]. The temperature of the multidisc brakes of tractor wheels was studied for the case where the friction power linearly decreases [8]. The temperature and quasistatic stresses in a metallic disk with friction material deposited on its surface were investigated by the method of finite differences and FEM [9, 10]. The nonstationary temperature field and thermal stressed state of a pad–disk braking system were analyzed by using the ANSYS 8.1 software and the mechanism of initiation of thermal fatigue cracks in the disk was discussed in [11].

All investigations were carried out for the case of one-time braking. However, just high temperatures formed in the process of multiple braking may lead to a decrease in the friction coefficient and the corresponding temperature stresses may result in the formation of cracks on the friction surface of the disk [12]. The evolution and space distribution of the nonstationary temperature field in the disk caused by friction in the course of multiple braking with constant deceleration were studied in [13]. In what follows, we study the influence of the number of brakings on the thermal stressed state of the brake disk.

Statement of the Problems of Heat Conduction and Thermoelasticity

Consider a disk of thickness of 2δ with inner radius r_d and outer radius R_d rotating with a constant angular velocity ω_0. At the initial time $t = 0$, under the action of a pressure p_0, a pad with arc length θ_0, inner radius r_p and outer radius $R_p = R_d$ is pressed to each end face of the disk.

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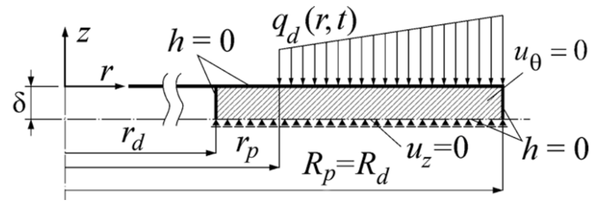

Fig. 1. Schematic diagram of heating of the disk.

Under the action of friction forces, the angular velocity of the disk linearly decreases to zero at the time of stop $t = t_s$, and the kinetic energy transforms into the heat energy in the contact area. When the disk stops, the pads are removed from its surfaces and then the disk speeds up once again to an angular velocity ω_0 for time $t = t_c$ after which the process of braking is repeated. In total, we performed n cycles of braking and acceleration of this kind and the duration of each cycle was $t_{sc} = t_s + t_c$. After the last nth braking, the immobile disk is cooled down for time $t = t_{cn}$. Thus, the entire cycle of heating and cooling of the disk is realized for

$$t_{\text{end}} = nt_{sc} - t_c + t_{cn}.$$

Assume that its free surfaces remain adiabatic (the heat exchange coefficient is $h = 0$) for the entire period $t = t_{\text{end}}$. Here and in what follows, all parameters corresponding to the disk and the pad are denoted by the subscripts d and p, respectively.

In view of the axial symmetry of the thermal load, we consider the process of heating and cooling of the disk with thickness δ in a cylindrical coordinate system $r\theta z$ (Fig. 1). The intensity of the heat flow q_d directed along the normal from the friction surface into the disk is proportional to the specific friction power q [14]:

$$q_d(r,t) = \gamma\eta q(r,t), \quad q(r,t) = fp_0 p^*(t) r\omega_0\omega^*(t), \quad r_p \le r \le R_p, \quad 0 < t \le t_{\text{end}}, \tag{1}$$

$$p(t) = \begin{cases} 1, & i \cdot t_{sc} \le t < i \cdot t_{sc} + t_s, \\ 0, & i \cdot t_{sc} + t_s \le t < (i+1)t_{sc} \wedge (n-1) \cdot t_{sc} - t_c \le t \le t_{\text{end}}, \quad i = 0,1,\ldots,n-1, \end{cases} \tag{2}$$

$$\omega^*(t) = \begin{cases} 1 - \dfrac{t - i \cdot t_{sc}}{t_s}, & i \cdot t_{sc} \le t < i \cdot t_{sc} + t_s, \quad i = 0,1,\ldots,n-1, \\ \dfrac{t - (i \cdot t_{sc} + t_s)}{t_c}, & it_{sc} + t_s \le t < (i+1)t_{sc}, \quad i = 0,1,\ldots,n-2, \\ 0, & (n-1)t_{sc} - t_c \le t \le t_{\text{end}}, \end{cases} \tag{3}$$

$\gamma = \theta_0/(2\pi)$ is the coefficient of overlapping of the pad and the disk [15], $\eta = K_d\sqrt{k_p}/(K_d\sqrt{k_p} + K_p\sqrt{k_d})$ is the coefficient of distribution of heat flows between the disk and the pad [16]; $K_{d,p}$ and $k_{d,p}$ are, respectively, the heat-conduction coefficient and thermal diffusivity of the materials of the disk and the pad, and f is the friction coefficient.

We determine the distribution of the nonstationary axisymmetric temperature field $T(r,z,t)$ in the disk from the solution of the following initial-boundary-value problem of heat conduction (Fig. 1):

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2} = \frac{1}{k_d}\frac{\partial T}{\partial t}, \quad r_d \le r \le R_d, \quad -\delta < z < 0, \quad 0 < t < t_{\text{end}}, \tag{4}$$

$$K_d \left.\frac{\partial T}{\partial z}\right|_{z=0} = \begin{cases} q_d(r,t), & r_p \le r \le R_p,\ 0 < t \le t_{\text{end}}, \\ 0, & r_d \le r \le r_p, \quad t > 0, \end{cases} \tag{5}$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=-\delta} = 0, \quad r_d \le r \le R_d,\ \ t > 0, \tag{6}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=r_d} = \left.\frac{\partial T}{\partial r}\right|_{r=R_d} = 0, \quad -\delta \le z \le 0, \quad t > 0, \tag{7}$$

$$T(r,z,0) = T_0, \quad r_d \le r \le R_d, \quad -\delta \le z \le 0, \tag{8}$$

where $T_0 = 20°\text{C}$ is the initial temperature of the disk and the function $q_d(r,t)$ is given by relations (1)–(3).

Given the temperature field, we determine the components of the stress tensor σ_r, σ_θ, σ_z, and σ_{rz} from the solution of the boundary-value problem of quasistatic thermoelasticity (Fig. 1)

$$(1-2\nu_d)\nabla^2 \boldsymbol{u}+\nabla \operatorname{div} \boldsymbol{u} = 2\alpha_d(1+\nu_d)\nabla T, \quad r_d \le r \le R_d, \quad -\delta < z < 0, \quad 0 \le t \le t_{\text{end}}, \tag{9}$$

$$\sigma_z(r,0,t) = \sigma_{rz}(r,0,t) = 0, \quad r_d \le r \le R_d, \quad 0 \le t \le t_{\text{end}}, \tag{10}$$

$$u_z(r,-\delta,t) = 0, \quad \sigma_{rz}(r,-\delta,t) = 0, \quad r_d \le r \le R_d, \quad 0 < t < t_{\text{end}}, \tag{11}$$

$$\sigma_r(r_d,z,t) = 0, \quad \sigma_{rz}(r_d,z,t) = 0, \quad -\delta \le z \le 0, \quad 0 < t < t_{\text{end}}, \tag{12}$$

$$\sigma_r(R_d,z,t) = 0, \quad \sigma_{rz}(R_d,z,t) = 0, \quad -\delta \le z \le 0, \quad 0 < t < t_{\text{end}}, \tag{13}$$

where $\boldsymbol{u} = \{u_r, u_z\}$ is the vector of displacements, ν_d is Poisson's ratio, α_d is the coefficient of linear thermal expansion of the material of the disk, and ∇ is a Hamiltonian operator in a cylindrical coordinate system. The stresses and displacements in Eq. (9) and the boundary conditions (10)–(13) are related by the Duhamel–Neumann relations [17].

Solution of the Problems and Numerical Analysis

For the numerical analysis, we used the known input data [13]. We studied the changes in temperature and thermal stresses for a disk made of ChNMKh cast iron after ten events of braking ($n = 10$). For this disk, we have $r_d = 66$ mm, $R_d = 113.5$ mm, $\delta = 5.5$ mm, $K_d = 51\ \text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$, $k_p = 14.4\cdot 10^{-6}\,\text{m}^2\cdot\text{sec}^{-1}$, $\alpha_d = 0.108\cdot 10^{-6}\ \text{K}^{-1}$, and $\nu_d = 0.29$.

The sizes and thermal properties of the FMK-11 cermet pad are as follows: $r_p = 76.5$ mm, $R_p = 113.5$ mm, $\theta_0 = 64.5°$, $K_p = 34.4\ \text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$, and $k_p = 14.6\cdot 10^{-6}\,\text{m}^2\cdot\text{sec}^{-1}$. It was also assumed that $p_0 = 1.47$ MPa,

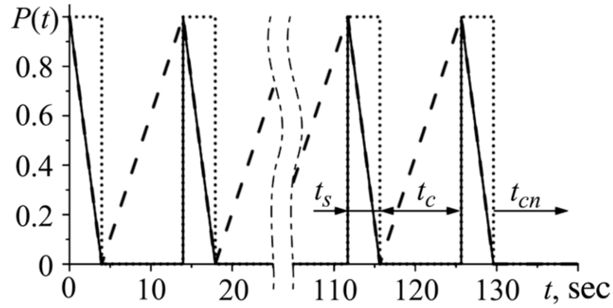

Fig. 2. Plots of the dimensionless functions $P(t)$: the dotted lines correspond to $P(t) = p^*(t)$, the dashed lines correspond to $P(t) = \omega^*(t)$, and the solid lines correspond to $P(t) = p^*(t)\omega^*(t)$.

$\omega_0 = 88.46\,\text{sec}^{-1}$, $t_s = 3.96$ sec, $t_c = 10$ sec, $t_{cn} = 300$ sec, and $f = 0.5$. The time of the entire braking process is determined as follows: $t_{\text{end}} = 10\cdot(3.96 + 10) - 10 + 300 = 429.6$ sec $= 7.16$ min. According to the accepted scheme of heating and cooling of the disk, we plotted the dimensionless functions that describe the variations of pressure $p^*(t)$ (2) and angular velocity $\omega^*(t)$ (3) with time (Fig. 2). In this figure, we also illustrate the time dependence of the product $p^*(t)\,\omega^*(t)$ that describes the time profile of the intensity of heat flow $q_d(r,t)$ (1).

The initial-boundary-value problem of heat conduction (1)–(7) and the boundary problem of thermoelasticity (8)–(12) were successively solved by the FEM in the environment of the MSC.Patran software package with the use of the MSC.Nastran computational module [18]. The computational model consisted of 33,193 nodes and 65,243 CTRIAX6 axisymmetric triangular elements. The density of these elements was made higher near the region of frictional heating characterized by high gradients of temperature and thermal stresses. The minimum size of elements was 0.02 mm and the maximum size was equal to 0.2 mm.

The solution of the thermal problem of friction (1)–(7) was sought with time steps $\Delta t = 0.001$ sec in the course of braking and $\Delta t = 0.01$ sec in the course of acceleration of the disk. The obtained values of the temperature field at the nodes of the space mesh at each time step were used as initial values for the determination of temperature stresses from the solution of the boundary-value problem of thermoelasticity (9)–(13). The numerical calculations required significant amounts of computer time because, in the case of recording the values of temperature field only at every tenth time step, to find the components of the stress tensor, it is necessary to solve 7860 boundary-value problems of thermoelasticity of the form (9)–(13) in order to reach the time t_{end}. This is why, for the automation of the process of reading of the values of temperature at the nodes of the finite-element mesh at each time step and their subsequent application as input parameters for the solution of the problem of thermoelasticity, the MSC.Nastran module was additionally equipped with a special program in the Python language [19]. Another author's program in the same language made it possible to perform simultaneous calculations for four time steps. One more program made it possible to the process results of calculations in individual files in each time step.

Since the maximum temperature is attained on the working surface of the disk ($z = 0$), we analyzed the time dependences of temperature and stresses on this surface for four values of the radial variable: on the surface of the disk $r = r_d$ (free of heating), on the inner boundary $r = r_p$, on the outer boundary $r = R_p$ inside the contact zone, and on the mean radius of this area $r = r_m = 0.5(r_p + R_p)$.

As the number of the events of braking increases, the temperature of the disk surface becomes higher (Fig. 3a). However, the character of its elevation is different for the chosen values of the radial variable. The temperature of the free surface increases with time almost monotonically and attains the steady-state value $Ta = 582.9$°C after the tenth braking. In the zone of contact of the disk with the pad, the temperature fluctuates,

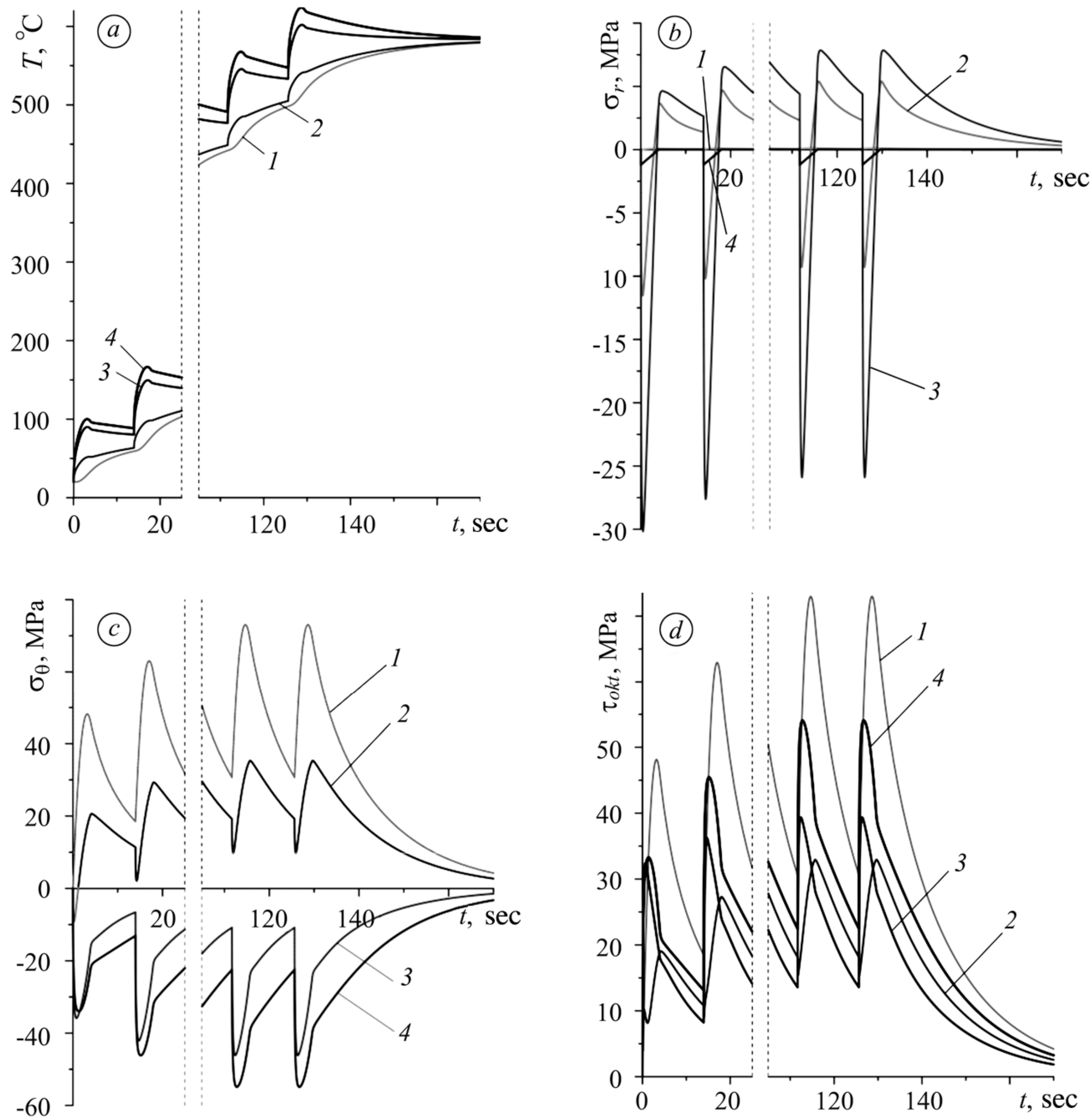

Fig. 3. Time dependences of temperature T (a) and radial σ_r (b), circumferential σ_θ (c), and octahedral tangential τ_{okt} (d) temperature stresses on the surface of the disk $z = 0$ for different values of the radial variable: (1) $r = r_d$; (2) r_p; (3) r_m; (4) R_p.

namely, it increases in the course of each braking and decreases down to a certain value (somewhat higher than the initial value) as the disk accelerates. The temperature $T_{\max} = 624°\mathrm{C}$ is attained during the last braking. As soon as the disk stops, the temperature of its working surface gradually takes its initial value T_0. The oscillating character of the behavior of temperature as a function of time is especially well visible on the mean radius and on the outer boundary of the contact zone.

The variations of temperature in the course of multiple braking are responsible for a similar character of changes in the temperature stresses as functions of time. On the inner boundary $r = r_d$ of the friction surface of the disk, the radial stresses σ_r are absent (Fig. 3b). At the beginning of each of the ten events of braking, the radial stresses σ_r are compressive within the zone of contact of the disk with the pad and its maximum absolute value $|\sigma_r| = 30.2$ MPa is attained on the mean radius of the disk $r = r_m$ during the first braking. In each braking, it changes its sign and attains the maximum value $\sigma_r = 30.2$ MPa for $r = r_m$ when the disk stops after the tenth braking. The increase in the number of the events of braking insignificantly affects the extreme values of the radial stresses.

Within the entire period of ten events of braking and acceleration of the disk, the circumferential stresses σ_θ on the surface $z = 0$ are tensile for $r = r_d$ and $r = r_p$ and compressive for $r = r_m$ and $r = R_p$ (Fig. 3c). At the beginning of each braking, these stresses monotonically increase with time, attain their maximum value when the disk stops, and decrease in the course of cooling and acceleration. The maximum level of tensile circumferential stresses is attained on the inner boundary of the disk $r = r_d$.

According to the boundary condition (10), the stresses $\sigma_z = \sigma_{rz} = 0$ on the friction surface of the disk $z = 0$. Therefore, to determined the general stressed state of this surface in the course of multiple braking, we used the octahedral tangential stresses $\tau_{okt} = \sqrt{[(\sigma_r - \sigma_\theta)^2 + \sigma_r^2 + \sigma_\theta^2]/6}$ [20]. The fluctuating character of changes in this quantity in the course of multiple braking is preserved on the entire surface of the disk (Fig. 3d). Its maximum value 72.97 MPa is attained on the inner boundary of the disk at the end of the last braking cycle for $t = 128.54$ sec.

CONCLUSIONS

We study the distributions of temperature and thermal stressed state of the disk in the course of multiple braking. For this purpose, we first formulate an axisymmetric initial-boundary-value problem of heat conduction for a disk periodically heated in an annular region on the friction surface by a frictional heat flow and then a quasistatic boundary-value problem of thermoelasticity for a disk with known temperature field. The numerical solution of these problems was obtained by the FEM with the help of the MSC.Patran/Nastran software. The calculations were performed for the tenfold process of braking of a “cermet pad–cast-iron disk” couple. At the beginning of each individual event of braking, the temperature of the friction surface of the disk monotonically increases to the maximum value and then decreases. The temperature maximum increases with each next braking. The most (least) heated is a part of the friction surface located near the outer (inner) boundary of the disk.

The variations of temperature in the course of each braking determine the evolution of temperature stresses. Near the inner boundary of the friction surface of the disk, we observe the formation of significant (≈ 70 MPa) tensile circumferential stresses and the effect of these stresses is especially pronounced for the octahedral tangential stresses. This part of the friction surface proves to be especially dangerous. This conclusion agrees with the experimental data presented in [12], where it was shown that the thermal cracking of the working surface of the disk in the radial direction is observed near its inner boundary.

REFERENCES

1. O. O. Evtushenko, E. H. Ivanyk, and N. V. Gorbachova, “Temperature and stresses in the vicinity of friction contacts of microasperities in the process of braking,” *Fiz.-Khim. Mekh. Mater.,* **32**, No. 6, 72–80 (1996); ***English translation:*** *Mater. Sci.,* **32**, No. 6, 714–723 (1996).
2. A. Yevtushenko and E. Ivanyk, “Determination of temperatures for sliding contact with applications for braking systems,” *Wear,* **206**, No. 1–2, 53–59 (1997).
3. A. A. Evtushenko, E. G. Ivanik, and N. V. Gorbacheva, “Influence of convective cooling on the temperature and stressed state in braking,” *Tren. Iznos,* **18**, No. 5, 578–586 (1997).

4. O. Jewtuszenko (editor), *Analityczne i Numeryczne Modelowanie Procesu Nieustalonej Generacji Ciepła w Elementach Tarciowych Układów Hamulcowych,* Oficyna Wydawnicza Politechniki Białostockiej, Białystok (2014).
5. A. A. Yevtushenko and P. Grzes, "FEM-modeling of the frictional heating phenomenon in the pad/disc tribosystem (a review)," *Numer. Heat Transfer, Part A: Appl.,* **58**, No. 3, 207–226 (2010).
6. A. A. Yevtushenko, P. Grzes, and A. Adamowicz, "Numerical analysis of temperature stresses in disk brakes and clutches (a review)," *Numer. Heat Transfer, Part A: Appl.,* **67**, No. 2, 170–188 (2015).
7. F. Talati and S. Jalalifar, "Analysis of heat conduction in a disk brake system," *Heat Mass Transf.,* **45**, No. 8, 1047–1059 (2009).
8. V. P. Sergienko, M. Yu. Tseluev, V. I. Kolesnikov, A. P. Sychev, V. A. Savonchik, V. I. Yanuchkovskii, "Investigation of thermal conditions for the friction pairs of multidisc brakes," *Tren. Iznos,* **34**, No. 6, 555–564 (2013).
9. P. Zagrodzki, "Numerical analysis of temperature fields and temperature stresses in the friction discs of a multidisc wet clutch," *Wear,* **101**, No. 3, 255–271 (1985).
10. P. Zagrodzki, "Analysis of thermomechanical phenomena in multidisc clutches and brakes," *Wear,* **140**, No. 2, 291–308 (1990).
11. C. H. Gao, J. M. Huang, X. Z. Lin, and X. S. Tang, "Stress analysis of thermal fatigue fracture of brake disks based on thermomechanical coupling," *J. Tribol. Trans. ASME,* **129**, No. 3, 536–543 (2007).
12. H. Sakamoto and K. Hirakawa "Fracture analysis and material improvement of brake discs," *JSME Int. J. Ser. A. Solid Mech. Mater. Eng.,* **48**, No. 4, 458–464 (2005).
13. A. Adamowicz and P. Grzes, "Influence of convective cooling on a disc brake temperature distribution during repetitive braking," *Appl. Therm. Eng.,* **31**, No. 14, 2177–2185 (2011).
14. O. V. Pereverzeva, and V. A. Balakin, "The choice of thermal schemes and boundary conditions in the numerical analyses of nonstationary temperature fields in high-speed and heavy-duty friction units," *Tren. Iznos,* **14**, No. 3, 487–497 (1993).
15. V. Balakin and V. Sergienko, *Thermal Analysis of Brakes and Friction Units* [in Russian], Belyi Institute for Mechanics of Metal–Polymer Systems, Belarus National Academy of Sciences, Gomel' (1999).
16. A. A. Yevtushenko and P. Grzes, "Finite-element analysis of heat partition in a pad/disc brake system," *Numer. Heat Transfer, Part A: Appl.,* **59**, No. 7, 522–542 (2010).
17. A. D. Kovalenko, *Foundations of Thermoelasticity* [in Russian], Naukova Dumka, Kiev (1970).
18. MSC Nastran, *Thermal Analysis User's Guide,* MSC Nastran Beginner (2002).
19. M. Luts, *Programming in Python* [in Russian], Simvol-Plyus, St. Petersburg (2002).
20. W. Nowacki, *Teoria Sprężystości,* PWN, Warszawa (1970).