DOI 10.1007/s11003-015-9749-4

Materials Science, Vol. 50, No. 4, January, 2015 (Ukrainian Original Vol. 50, No. 4, July–August, 2014)

"TRIDENT" MODEL OF PLASTIC ZONE AT THE END OF A MODE I CRACK APPEARING ON THE NONSMOOTH INTERFACE OF MATERIALS

M. V. Dudyk[1,2] **and Yu. V. Dikhtyarenko**[1]

UDC 539.375

By using the Wiener–Hopf method, within the framework of the "trident" model, we compute a small-scale plastic zone formed under conditions of plane deformation at the corner point of the interface of two different elastoplastic materials in the presence of a mode I crack originating from this point. The indicated zone is modeled by two symmetric lateral lines of discontinuities of tangential displacements and the line of discontinuity of normal-displacement on the continuation of the crack. We deduce analytic expressions for the evaluation of the sizes of the plastic zone and crack opening displacements. On the basis of numerical investigations, we analyze the dependences of parameters of this zone on the opening angle of the interface and the elastic characteristics of the materials.

Keywords: corner point of the interface of different media, mode I crack, "trident" model of plastic zone, crack opening displacement.

The theoretical and experimental investigations demonstrate that the plastic zone formed near pointed stress concentrators in elastoplastic materials has a complex structure and contains a fairly developed plastic domain in which shear strains are predominant and a much smaller plastic process zone adjacent to the tip of the concentrator with high levels of both shear and tensile strains. With regard for these features and symmetry conditions, the "trident" model was proposed in [1] for the description of the plastic zone formed at the end of a mode I crack originating from the corner point of the interface of two media. According to this model, the zone develops in two stages. In the first stage, two narrow lateral plastic strips (modeled by segments of discontinuities of tangential displacements) symmetrically propagate from the crack tip at a certain angle to its initial direction in one of the materials of the joint. If the appearance of these strips does not remove the stress concentration near the crack tip, then, in the next stage, the plastic process zone modeled by a segment of discontinuities of normal displacements can be formed on the continuation of the crack (Fig. 1).

The numerical analysis of the plastic zone performed within the framework of the "trident" model with regard for the two stages of its development is reduced to the solution of four problems (for the slopes of plastic strips β greater and smaller than a half of the opening angle of the interface α) by the Wiener–Hopf method. A similar problem for a plastic strip on the interface of two materials was solved in [2].

Parameters of the Initial Lateral Plastic Zone

Assume that, under the conditions of plane deformation in a piecewise homogeneous isotropic elastoplastic body, a mode I crack originates from a corner point of the interface of materials into the material with Young's modulus E_1 and Poisson's ratio ν_1.

[1] Uman State Pedagogical University, Uman, Ukraine, Ukraine.

[2] Corresponding author; e-mail: dudik_m@hotmail.com.

Translated from Fizyko-Khimichna Mekhanika Materialiv, Vol. 50, No. 4, pp. 41–48, July–August, 2014. Original article submitted October 7, 2013.

 1068-820X/15/5004–0516

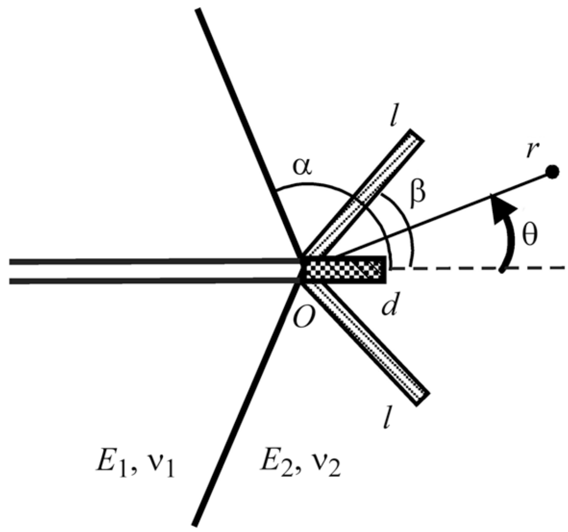

Fig. 1. "Trident" model of plastic zone.

Since the corner point of the interface of two materials playing the role of the origin of mode I crack is a stress concentrator with power singularity [3], this leads to the formation of a plastic zone in its neighborhood. We assume that, in the initial stage of development of the plastic zone, two narrow lateral plastic strips of the same length are formed in the material with elastic constants E_i and ν_i $(i = 1, 2)$. Their length is much smaller than the crack length (here and in what follows, subscript i marks the quantities corresponding to the plastic strips in the ith material). According to the hypothesis of localization [4], we model these strips by two rectilinear segments of discontinuities of tangential displacements inclined at an angle β_i to the direction of crack continuation. The tangential stresses on these segments are equal to the yield limit of the i th material τ_{is}.

We compute the lengths of the lateral plastic strips according to the Wiener–Hopf method by analogy with the problem of plastic strips in the connecting material [5, 6] and arrive at the following expression:

$$l_i = \left(\frac{|C|}{\tau_{is}} \right)^{-1/\lambda} R_i(\beta_i) , \tag{1}$$

where C is a constant characterizing the intensity of external force field (it is specified by the conditions of the problem), λ in the least root, in the interval $(-1, 0)$, of the equation [3]:

$$D_0(-1-x) = 0 , \tag{2}$$

$$\begin{aligned} D_0(p) = {} & 4e^2[\kappa_2 \sin 2p\alpha - p \sin 2\alpha][p^2 \sin^2 \alpha - \sin^2 p(\pi - \alpha)] \\ & + e\Big\{ (1+\kappa_1)(1+\kappa_2) \sin 2p\pi - 4\left[\kappa_2 \sin 2p\alpha - p \sin 2\alpha \right][p^2 \sin^2 \alpha - \sin^2 p(\pi - \alpha)] \\ & - (p \sin 2\alpha + \sin 2p\alpha)[(1+\kappa_1)(1+\kappa_2) - 4(p^2 \sin^2 \alpha + \kappa_1 \sin^2 p(\pi - \alpha))] \Big\} \end{aligned}$$

$$+(p\sin 2\alpha+\sin 2p\alpha)\Big\{(1+\kappa_1)^2-4[p^2\sin^2\alpha+\kappa_1\sin^2 p(\pi-\alpha)]\Big\},$$

$$\kappa_i = 3-4\nu_i, \qquad e = \frac{E_1}{E_2}\frac{1+\nu_2}{1+\nu_1};$$

$$R_i(\theta) = \left[\frac{\sqrt{\pi}\,|F_i(\theta)|\,\Gamma(1+\lambda)I_i(0,\theta)}{2\Gamma(1.5+\lambda)I_i(\lambda,\theta)}\right]^{-1/\lambda},$$

$$F_i(\theta) = \begin{cases} F_1(\theta), \ \alpha\le\theta\le\pi, \\ F_2(\theta), \ 0\le\theta\le\alpha; \end{cases}$$

$$F_1(\theta) = \frac{(2\pi)^\lambda}{4\lambda\varphi_2}\{(\lambda+2)\varphi_1\sin(\lambda+2)(\pi-\theta)-\lambda\varphi_1\sin\lambda(\pi-\theta)$$

$$-\lambda\varphi_2\cos(\lambda+2)(\pi-\theta)+\lambda\varphi_2\cos\lambda(\pi-\theta)\},$$

$$F_2(\theta) = \frac{-(2\pi)^\lambda}{4\lambda\varphi\varphi_2\cos\lambda\alpha}\{[(\lambda+2)\psi_1\varphi_1+\lambda\psi_2\varphi_2]\cos\lambda\alpha\sin(\lambda+2)\theta+\lambda(\psi_3\varphi_1+\psi_4\varphi_2)\sin\lambda\theta\},$$

$$\varphi = -[(\lambda+1)\sin 2\alpha+\sin 2(\lambda+1)\alpha],$$

$$\varphi_1 = \varphi\{(e-1)\lambda\sin(\lambda+2)(\pi-\alpha)-[(e-1)(\lambda+2)-e(1+\kappa_2)+(1+\kappa_1)]\sin\lambda(\pi-\alpha)\}$$

$$-2e(1+\kappa_2)\lambda\cos\pi\lambda\sin(\lambda+1)\alpha\sin\alpha,$$

$$\varphi_2 = \varphi\{[(e-1)(\lambda+2)-e(1+\kappa_2)+(1+\kappa_1)]\cos\lambda(\pi-\alpha)$$

$$-(e-1)(\lambda+2)\cos(\lambda+2)(\pi-\alpha)\}-2e(1+\kappa_2)\sin\pi\lambda(\lambda\sin(\lambda+1)\alpha\sin\alpha+\cos\lambda\alpha),$$

$$\psi_1 = -2\lambda\cos(\pi\lambda-\alpha)\sin\alpha+2\sin(\lambda+2)(\pi-\alpha)\cos\lambda\alpha,$$

$$\psi_2 = -2\lambda\sin(\pi\lambda-\alpha)\sin\alpha+2\cos\lambda\pi-2\cos(\lambda+2)(\pi-\alpha)\cos\lambda\alpha,$$

$$\psi_3 = -\varphi\cos\lambda(\pi-\alpha)-2\sin\pi\lambda(\lambda\sin(\lambda+1)\alpha\sin\alpha+\cos\lambda\alpha),$$

$$\psi_4 = -\varphi\sin\lambda(\pi-\alpha)+2\lambda\cos\pi\lambda\sin(\lambda+1)\alpha\sin\alpha;$$

$$I_i(x,\theta) = \exp\left[\frac{x+1}{\pi}\int_0^{\infty}\frac{\ln G_i(it,\theta)}{t^2+(x+1)^2}dt\right],$$

$$G_i(p,\theta) = \frac{D_i(p,\theta)\cos p\pi}{D_0(p)\sin p\pi},$$

$$D_1(p,\theta) = 4(e-1)(\Delta_1\Delta_4-\Delta_2\Delta_5)[e(1+\kappa_2)\sin 2p\alpha-(e-1)\Delta_6]$$

$$-4(e-1)(1+\kappa_1)\Delta_6[\sin 2p(\theta-\alpha)\Delta_1+\sin^2 p(\theta-\alpha)\Delta_5]$$

$$+e(1+\kappa_1)(1+\kappa_2)[4\sin(p-1)\theta\sin(p+1)\theta\Delta_1+(\Delta_6-\Delta_3)\Delta_5]-(1+\kappa_1)^2\Delta_5\Delta_6,$$

$$D_2(p,\theta) = e(1+\kappa_1)(1+\kappa_2)[\Delta_3(\Delta_4-\Delta_5)+4(\Delta_1-\Delta_2-\Delta_7)\sin(p-1)\theta\sin(p+1)\theta]$$

$$+[4\Delta_2\sin(p-1)\theta\sin(p+1)\theta-\Delta_3\Delta_4][(1+\kappa_1)^2+4(e-1)(1+\kappa_1)\sin^2 p(\pi-\alpha)$$

$$-4(e-1)^2\Delta_7]+4e^2(1+\kappa_2)^2\Delta_7\sin(p-1)\theta\sin(p+1)\theta+4(e-1)e(1+\kappa_2)\Delta_7\Delta_8,$$

$$\Delta_1 = p^2\sin^2\theta-\sin^2 p(\pi-\theta),\quad \Delta_2 = p^2\sin^2(\theta-\alpha)-\sin^2 p(\theta-\alpha),$$

$$\Delta_3 = p\sin 2\theta+\sin 2p\theta,\quad \Delta_4 = p\sin 2(\theta-\alpha)+\sin 2p(\theta-\alpha),$$

$$\Delta_5 = p\sin 2\theta-\sin 2p(\pi-\theta),\quad \Delta_6 = p\sin 2\alpha+\sin 2p\alpha,$$

$$\Delta_7 = p^2\sin^2\alpha-\sin^2 p(\pi-\alpha),$$

$$\Delta_8 = p\sin 2\beta\sin 2p(\alpha-\theta)-2\sin^2 p(\alpha-\theta)\cos 2\theta+2\sin p(\alpha+\theta)\sin p(\alpha-\theta),$$

and $\Gamma(p)$ is the Euler gamma-function.

As a criterion for choosing the direction of the propagation of plastic strips, it is customary to use the condition of maximum of the rate of energy dissipation [7], which yields the following conditions:

$$\frac{dW_i}{dt} = \frac{8\tau_{is}^2(1-\nu_i^2)}{\pi E_i(2+\lambda)}\left(\frac{|C|\sqrt{\pi}\Gamma(\lambda+1)}{2\tau_{is}\Gamma(1.5+\lambda)}\right)^{-2/\lambda} w_i(\beta_i)\frac{1}{C}\frac{dC}{dt} \to \max,$$

$$w_i(\theta) = \left[\frac{|F_i(\theta)|\,I_i(0,\theta)^{1+\lambda}}{I_i(\lambda,\theta)}\right]^{-2/\lambda}. \tag{3}$$

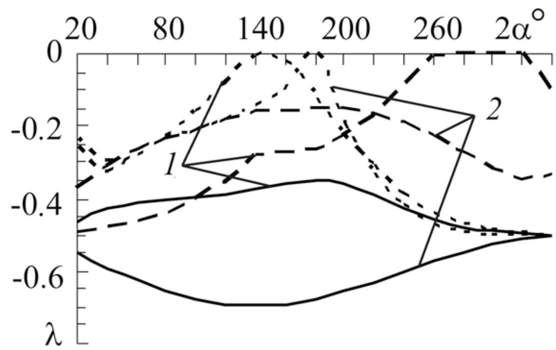

Fig. 2. Dependences of the stress-singularity indices on the opening angle of the interface of the media 2α $(\nu_1 = \nu_2 = 0.25)$: (1) $E_1/E_2 = 0.2$, (2) $E_1/E_2 = 5$.

We also determine the crack-tip opening displacement caused by the formation of plastic strips, which is expressed via the jump of tangential displacements in the strip determined in the course of solution of the problem:

$$\delta_i^{pl} = 2\langle u_r(0,\beta_i)\rangle \sin\beta_i = -\frac{16(1-\nu_i^2)\tau_{is}l_i}{\pi E_i I_i(0,\beta_i)\sqrt{G_i(0,\beta_i)}}\frac{\lambda}{1+\lambda}\sin\beta_i\,. \tag{4}$$

The solution of the problem is used to find the principal terms of the expansions of stresses in asymptotic series in the vicinity of the crack tip after the formation of the plastic zone. To do this, we use the roots λ_i $(-1 < \lambda_i < 0)$ of the equation

$$D_i(-x-1,\beta_i) = 0\,. \tag{5}$$

The difference between the roots of Eqs. (2) and (5) reveals the transformation of the stress-strain state in the neighborhood of the corner point caused by the formation of the initial plastic zone. In Fig. 2, we present the results of numerical analysis of the dependences of the stress-singularity indices on the opening angle of the interface of the media 2α in the absence of the plastic zone (λ, solid curves) and after the formation of the plastic zone in the first (λ_1, dotted lines) and second (λ_2, dashed lines) materials. This analysis shows that, for some opening angles, stress concentration disappears as a result of the formation of the plastic strip $(\lambda_i = 0)$. At the same time, for the other angles, we observe its weakening after the formation of the plastic zone $(\lambda_i > \lambda)$, except the case of propagation of the plastic strip in the second material where $E_1/E_2 < 1$ and $\alpha < 45°$.

Parameters of the Plastic Process Zone on the Continuation of the Crack

In the presence of stress concentration in the neighborhood of the corner point, the plastic zone develops as the load increases not only due to the increase in the geometric sizes of plastic strips but also as a result of the formation of a new strip. Due to symmetry conditions, it is assumed that a secondary plastic strip is formed on the crack continuation (plastic process zone). Its length d_i is much smaller that the length of initial plastic strips. Modeling this narrow plastic process zone by the segment of discontinuities of normal displacements, where the normal stresses are equal to a given constant of the second material σ_2, we find its length by analogy with [2]:

$$d_i = \left(\frac{\tau_{is}}{\sigma_2 \tilde{X}_i}\right)^{-1/\lambda_i} l_i \tag{6}$$

$$\tilde{X}_i = \frac{\tilde{I}_i(\lambda_i) I_i(0,\beta_i)}{\tilde{I}_i(0) I_i(\lambda_i,\beta_i) |S_i(-1-\lambda_i,\beta_i)|}, \quad \tilde{I}_i(x) = \exp\left[\frac{x+1}{\pi}\int_0^\infty \frac{\ln \tilde{G}_i(it)}{t^2+(x+1)^2}dt\right],$$

$$\tilde{G}_1(p) = -\frac{2D_3(p,\beta_1)\cos p\pi}{D_1(p,\beta_1)\sin p\pi}, \quad \tilde{G}_2(p) = -\frac{2D_4(p,\beta_2)\cos p\pi}{D_2(p,\beta_2)\sin p\pi},$$

$$D_3(p,\theta) = 2(1+\kappa_1)e(1+\kappa_2)(\Delta_1\Delta_{24} - \Delta_5\Delta_{26}) - (1+\kappa_1)^2\Delta_5\Delta_{25}$$

$$+ 4(e-1)(1+\kappa_1)(\Delta_1 \sin 2p(\alpha-\theta) - \Delta_5 \sin^2 p(\alpha-\theta))\Delta_{25}$$

$$+ (\Delta_1\Delta_4 - \Delta_2\Delta_5)[e^2(1+\kappa_2)^2 - 4(e-1)e(1+\kappa_2)\sin^2 p\alpha - 4(e-1)^2\Delta_{25}],$$

$$D_4(p,\theta) = 2(1+\kappa_1)e(1+\kappa_2)[(\Delta_1 - \Delta_2 - \Delta_7)\Delta_3 + (\Delta_4 - \Delta_5)\Delta_{23}]$$

$$+ 2[\Delta_2\Delta_3 - \Delta_4\Delta_{23}]\{(1+\kappa_1)^2 + 4(e-1)(1+\kappa_1)\sin^2 p(\pi-\alpha) - 4(e-1)^2\Delta_7\}$$

$$+ 2e^2(1+\kappa_2)^2\Delta_3\Delta_7 - 4(e-1)e(1+\kappa_2)\Delta_7(2\Delta_3\sin^2 p(\alpha-\theta) - 2\Delta_{23}\sin 2p(\alpha-\theta)),$$

$$S_i(p,\theta) = \frac{f_i(p,\theta)}{D_i'(p,\theta)}\frac{\lambda}{\lambda_i(\lambda_i-\lambda)}, \quad D_i'(p,\theta) = \frac{\partial D_i(p,\theta)}{\partial p},$$

$$f_1(p,\theta) = \{2(e-1)[2\Delta_1\Delta_9 + \Delta_5(p\sin\alpha\,\Delta_{11} + \sin p\alpha\,\Delta_{12}) + 2\Delta_{10}(p\sin\alpha\,\Delta_{13} + \sin p\alpha\,\Delta_{14})]$$

$$+ (1+\kappa_1)(\Delta_5\Delta_{15} + 2\Delta_{10}\Delta_{16}) - e(1+\kappa_2)(\Delta_5\Delta_{17} + 2\Delta_{10}\Delta_{18} + 2\Delta_1\sin(p+1)\beta)\}(1+\kappa_1),$$

$$f_2(p,\theta) = e(1+\kappa_1)(1+\kappa_2)[\sin(p+1)\theta(\Delta_2 - \Delta_1 + \Delta_7) + p\sin\theta\,\Delta_{19} - \sin p\theta\,\Delta_{20}]$$

$$+ 2e(1+\kappa_2)(e-1)\Delta_7\Delta_{21} - e^2(1+\kappa_2)^2\Delta_7\sin(p+1)\theta$$

$$- (p-1)[(1+\kappa_1)^2 - 4(e-1)^2\Delta_7 + 4(1+\kappa_1)(e-1)\sin^2 p(\pi-\alpha)]\Delta_{22},$$

$$\Delta_9 = p\sin\alpha\cos[(p+1)(\alpha-\theta) + p\alpha] + \sin p\alpha\cos[p(\alpha-\theta) - \theta],$$

$$\Delta_{10} = p\sin^2\theta - \sin^2 p(\pi-\theta),$$

$$\Delta_{11} = p\sin(\alpha-\theta)\cos p(2\alpha-\theta) - \sin p(\alpha-\theta)\cos(p\alpha+\alpha-\theta),$$

$$\Delta_{12} = p\sin(\alpha-\theta)\cos[p(\alpha-\theta)-\alpha] - \sin p(\alpha-\theta)\cos\theta,$$

$$\Delta_{13} = p\sin(\alpha-\theta)\sin p(2\alpha-\theta) + \sin p(\alpha-\theta)\sin(p\alpha+\alpha-\theta),$$

$$\Delta_{14} = p\sin(\alpha-\theta)\sin[p(\alpha-\theta)-\alpha] - \sin p(\alpha-\theta)\sin\theta,$$

$$\Delta_{15} = p\sin\alpha\sin(p\theta+\alpha-\theta) - \sin p\alpha\sin(p(\alpha-\theta)+\theta),$$

$$\Delta_{16} = p\sin\alpha\cos(p\theta+\alpha-\theta) + \sin p\alpha\cos(p(\alpha-\theta)+\theta),$$

$$\Delta_{17} = p\sin(\alpha-\theta)\sin(p\theta+\alpha) - \sin p(\alpha-\theta)\sin(p\alpha+\theta),$$

$$\Delta_{18} = p\sin(\alpha-\theta)\cos(p\theta+\alpha) + \sin p(\alpha-\theta)\cos(p\alpha+\theta),$$

$$\Delta_{19} = p\sin\alpha\sin(p\theta+2\theta-\alpha) - \sin p(\pi-\alpha)\sin p(\pi+\alpha-\theta),$$

$$\Delta_{20} = p\sin\alpha\sin(\alpha-\theta) - \sin p(\pi-\alpha)\sin[p(\pi+\alpha-2\theta)-\theta],$$

$$\Delta_{21} = (p-1)\sin\theta\cos p(2\alpha-\theta) + \sin(p+1)\theta,$$

$$\Delta_{22} = p\sin\alpha\sin(\alpha-\theta)\sin p\theta - \sin p\alpha\sin p(\alpha-\theta)\sin\theta,$$

$$\Delta_{23} = p^2\sin^2\theta - \sin^2 p\theta, \qquad \Delta_{24} = p\sin\alpha\cos(\alpha-2\theta) + \sin p\alpha\cos p(\alpha-2\theta),$$

$$\Delta_{25} = p^2\sin^2\alpha - \sin^2 p\alpha, \qquad \Delta_{26} = p^2\sin\alpha\cos\theta\sin(\theta-\alpha) - \sin p\alpha\cos p\theta\sin p(\theta-\alpha).$$

The solution of the problem enables us to determine the additional contribution to the crack-tip opening displacement caused by the formation of the plastic process zone modeled by the jump of normal displacements given by the formula

$$\delta_i^{pf} = -u_\theta(0,\pi) + u_\theta(0,-\pi) = -\frac{64\pi\sigma_2 e(1+\kappa_2)(1-\nu_1^2)J_i\sqrt{\tilde{G}_i(0)}d_i\lambda_i}{E_1\tilde{J}_i\tilde{I}_i(0)(1+\lambda_i)}, \tag{7}$$

$$J_1 = g_1[e(1+\kappa_2)g_2 - (1+\kappa_1)g_3], \qquad J_2 = g_4[e(1+\kappa_2)g_5 - (1+\kappa_1)g_2],$$

$$g_1 = (\pi-\beta)\cos\beta - \sin\beta, \qquad g_2 = \cos\alpha\sin(\alpha-\beta) + (\alpha-\beta)\cos\beta,$$

$$g_3 = \sin\alpha\cos(\alpha-\beta)+\alpha\cos\beta, \qquad g_4 = \sin\beta+\beta\cos\beta,$$

$$g_5 = \sin\alpha\cos(\alpha-\beta)-(\pi-\alpha)\cos\beta;$$

$$\tilde{J}_1 = \lim_{p\to 0} p^{-3}D_3(p), \quad \tilde{J}_2 = \lim_{p\to 0} p^{-3}D_4(p).$$

The total crack opening displacement δ_i is determined as the sum of opening displacements caused by the formation of the lateral plastic zone (4) and the plastic process zone (7): $\delta_i = \delta_i^{pl} + \delta_i^{pf}$. Equating the crack opening displacement to its critical value, we can find the ultimate load corresponding to the onset of crack propagation.

After the formation of the plastic process zone, the stress field near the corner point is characterized by the roots of the equations $D_3(-1-x,\beta_1)=0$ or $D_4(-1-x,\beta_2)=0$ specifying the stress-singularity index for $-1<\operatorname{Re}x<0$. The numerical results show that these equations have no roots in the strip $-1<\operatorname{Re}x<0$ in broad ranges of the elastic parameters of the joined materials and opening angles of the interface. Hence, the stress concentration disappears at the corner point.

Analysis of the Accumulated Results

As follows from relations (1) and (6), plastic strips nonlinearly increase with the external load appearing in the expressions for their lengths via the factor C. Moreover, the lower the yield limit of the material σ_i, the greater the length of the lateral plastic zone and the smaller the constant σ_2, the greater the length of plastic process zone. Formula (6) also implies that the sizes of plastic strips synchronously increase with the load.

If only one material in the analyzed piecewise homogeneous body is elastoplastic, then the lateral plastic strips propagate in it in the directions determined from the corresponding condition (3). However, if both materials of the joint are elastoplastic, then it is necessary to additionally compare the maximal rates of the dissipation of energy in the strips in each material. The ratios of these rates and the lengths of plastic strips are as follows:

$$\frac{dW_1/dt}{dW_2/dt} = \left(\frac{\tau_{2s}}{\tau_{1s}}\right)^{-2(\lambda+1)/\lambda} Z, \qquad \frac{l_1}{l_2} = \left(\frac{\tau_{2s}}{\tau_{1s}}\right)^{-1/\lambda} X, \tag{8}$$

$$Z = \frac{1-\nu_1}{e(1-\nu_2)}\left[\frac{|F_1(\beta_1)|\, I_1(0,\beta_1)^{1+\lambda} I_2(\lambda,\beta_2)}{|F_2(\beta_2)|\, I_2(0,\beta_2)^{1+\lambda} I_1(\lambda,\beta_1)}\right]^{-2/\lambda},$$

$$X = \left[\frac{|F_1(\beta_1)|\, I_1(0,\beta_1) I_2(\lambda,\beta_2)}{|F_2(\beta_2)|\, I_2(0,\beta_2) I_1(\lambda,\beta_1)}\right]^{-1/\lambda}.$$

In Table 1, we present the results of numerical analysis of the dependences of the parameters of plastic zones on the opening angle of the interface of the media 2α for the ratios of Young's moduli of the materials $E_1/E_2 = 0.2$ and $E_1/E_2 = 5$, equal Poisson's ratios $(\nu_1 = \nu_2 = 0.25)$, and equal yield limits of the joined materials $(\tau_{1s} = \tau_{2s})$.

Table 1. Parameters of the Process Zone within the Framework of the "Trident" Model

$2\alpha°$	$E_1/E_2 = 0.2$						$E_1/E_2 = 5$					
	$\beta_1°$	$\beta_2°$	X	Z	$\tilde{X}_1$	$\tilde{X}_2$	$\beta_1°$	$\beta_2°$	X	Z	$\tilde{X}_1$	$\tilde{X}_2$
20	99.1	10	7.336	78.09	3.869	0.169	69.4	10	5.649	6.045	0.233	0.134
40	110.6	20	3.635	23.09	10.591	0.284	61.5	20	1.984	1.345	0.275	0.226
60	114.5	30	4.960	44.68	6.302	0.259	30	30	1.009	1	0.296	0.296
80	115.3	40	16.797	504.2	2.152	0.149	40	40	1.009	1	0.344	0.344
100	115.6	40.5	341.66	$4.1 \cdot 10^{5}$	0.861	0.042	50	50	1.009	1	0.367	0.366
120	116.1	60	115.86	$2.2 \cdot 10^{4}$	0.274	0.061	60	60	1.009	1	0.358	0.357
140	116.8	70	12.67	254.1	–	0.122	70	67.1	1.060	0.996	0.305	0.323
160	117.5	80	4.159	26.20	–	0.118	80	70.6	1.091	0.958	0.191	0.284
180	117.6	65.1	0.854	3.121	0.121	0.317	90	73.9	1.063	0.877	–	0.232
200	115.9	61.2	0.168	0.128	0.135	0.439	100	77.0	0.984	0.747	0.169	0.185
220	110	61.6	0.053	0.011	0.106	0.448	110	80.0	0.861	0.574	0.258	0.165
240	120	63.9	0.024	$2.1 \cdot 10^{-3}$	0.079	0.264	120	83.0	0.707	0.387	0.277	0.184
260	130	66.7	0.011	$4.9 \cdot 10^{-4}$	0.052	–	130	85.8	0.547	0.228	0.253	0.241
280	140	69.5	$4.7 \cdot 10^{-3}$	$9.0 \cdot 10^{-5}$	0.032	–	140	87.6	0.397	0.116	0.212	0.329
300	150	72.1	$1.6 \cdot 10^{-3}$	$1.1 \cdot 10^{-5}$	0.017	–	150	87.4	0.248	0.045	0.164	0.418
320	160	74.1	$3.2 \cdot 10^{-4}$	$5.5 \cdot 10^{-7}$	$7.3 \cdot 10^{-3}$	–	160	83.5	0.096	$7.1 \cdot 10^{-3}$	0.105	0.469
340	170	75.4	$2.2 \cdot 10^{-5}$	$3.8 \cdot 10^{-9}$	$1.8 \cdot 10^{-3}$	0.109	170	78.2	0.010	$8.9 \cdot 10^{-5}$	0.036	0.458

It is easy to see that if $E_1 < E_2$ and the opening angles of the interface of two media are $2\alpha \le 180°$, then $dW_1/dt > dW_2/dt$ $(Z > 1)$. According to the accepted energy criterion, this leads to the formation of two symmetric lateral plastic strips in the first material. At the same time, if $180° < 2\alpha < 360°$, then $dW_1/dt < dW_2/dt$ and, hence, the formation of two lateral strips in the second material is predominant. If $E_1 > E_2$, and the opening angles $2\alpha \le 40°$, then $dW_1/dt > dW_2/dt$, i.e., two symmetric plastic strips are formed in the first material. For opening angles from $2\alpha \approx 60°$ up to a certain value $2\alpha_1 \simeq 120°$, which can be found from the condition $dW_1/dt = dW_2/dt$, the plastic strip propagates along the interface of the media either in the first or in the second material $(\beta_1 = \beta_2 = \alpha)$ depending on their yield limits. For the angles $2\alpha > 120°$ and $E_1 > E_2$, we have $dW_1/dt < dW_2/dt$ and, hence, we observe the formation of two symmetric lateral strips in the second material.

For $\tau_{1s} \ne \tau_{2s}$, the direction of propagation of the strips according to condition (8) also depends on the ratio of the yield limits of the materials. For the joint of identical materials, the slope of the plastic strips is equal to 75.8°, which is close to the value ≈ 72° obtained in [8]. Their difference is explained by the use of the condition of maximum of the strip lengths in [8]. The comparison of the ratios of strip lengths (X) and the rates of energy dissipation (Z) in different materials shows that the conditions of their maximum for some parameters of the joint ($E_1/E_2 < 1$, $\alpha \simeq 90°$ and $E_1/E_2 > 1$ $\alpha \simeq 70°\text{–}90°$) may lead to different conclusions concerning the orientation of the strips.

The sizes of the plastic process zone are presented in Table 1 in the form of the factors $\tilde{X}_i$ specifying the ratio of the lengths of the plastic process zone and the lateral plastic zone in the corresponding material (the missing values correspond to vanishing of the stress concentration in the stage of formation of the initial plastic strips). Note that the condition $d_i << l_i$ accepted in the formulation of the problem requires the validity of the inequality $\tilde{X}_i >> \tau_{is}/\sigma_2$, which can be violated for small values of $\tilde{X}_i$.

CONCLUSIONS

Under the conditions of plane deformation, within the framework of the "trident" model, we perform the numerical analysis of a small-scale plastic zone at the end of a mode I crack originating from the corner point of interface of two different elastoplastic materials. We reveal the possibility of prediction of the direction of propagation of lateral plastic strips. It is shown that the formation of the plastic process zone on the continuation of the crack after propagation of the initial lateral plastic strips removes stress concentration at the crack tip in a broad range of parameters of the investigated joint. Finally, we determine the crack-tip opening displacements, which enables one to find the ultimate loads.

The authors are grateful to Prof. L. A. Kipnis for his valuable advice and comments.

REFERENCES

1. A. A. Kaminskii, L. A. Kipnis, and V. A. Kolmakova, "Numerical analysis of the plastic zone at the end of a crack within the framework of the 'trident' model," *Prikl. Mekh*, **33**, No. 5, 70–76 (1997).
2. A. A. Kaminskii, L. A. Kipnis, and M. V. Dudik, "On the initial development of the process zone near the end of a crack appearing on the interface of different media," *Prikl. Mekh.*, **40**, No. 2, 74–81 (2004).
3. A. A. Kaminskii, M. V. Dudik, V. N. Dyakon, et al., "On the behavior of stresses near the end of a crack appearing on the interface of different media," in: *Theoretical and Applied Mechanics* [in Russian], Issue 32 (2001), pp. 103–108.

4. V. V. Panasyuk and M. P. Savruk, "Model for plasticity bands in elastoplastic failure mechanics," *Fiz.-Khim. Mekh. Mater.,* **28**, No. 1, 49–68 (1992); ***English translation:*** *Mater. Sci.,* **28**, No. 1, 41–57 (1992).
5. M. V. Dudik, L. A. Kipnis, and A. V. Pavlenko, "Calculation of plastic slip lines at the end of a crack appearing on the interface of different media," *Prikl. Mekh.,* **38**, No. 2, 90–95 (2002).
6. A. O. Kamins'kyi, M. V. Dudyk, and L. A. Kipnis, "Stresses near crack tips on the boundary of two media in the presence of plastic strips," *Fiz.-Khim. Mekh. Mater.,* **37**, No. 3, 71–76 (2001); ***English translation:*** *Mater. Sci.,* **37**, No. 3, 447–455 (2001).
7. G. P. Cherepanov, "General theory of failure," *Fiz.-Khim. Mekh. Mater.,* **22**, No. 1, 36–44 (1986); ***English translation:*** *Mater. Sci.,* **22**, No. 1, 32–39 (1986).
8. G. P. Cherepanov, "Plastic discontinuity lines at the end of a crack," *Prikl. Mat. Mekh.,* **40**, No. 4, 720–728 (1976).