Materials Science, Vol. 42, No. 2, 2006

STRESSED STATE OF A THERMOSENSITIVE PLATE IN A CENTRAL-SYMMETRIC TEMPERATURE FIELD

R. M. Kushnir and V. S. Popovych

UDC 539.3

The problem of thermoelasticity for a thin thermosensitive plate placed in a central-symmetric temperature field is reduced by the perturbation method to a recurrent sequence of boundary-value problems for differential equations with constant coefficients. On this basis, we obtain solutions for the cases of load-free circular washer, infinite plate with circular hole, circular disk, and infinite plate. The stress–strain state of an infinite plate containing a circular hole is investigated.

Thin plates are fairly extensively used as structural elements of contemporary equipment operating in media with high or low temperatures and significant temperature drops both in space and in the course of time. Under the indicated conditions, the engineering practice imposes elevated requirements to the accuracy of determination of the stress–strain state of thin plates. This, can be attained by improving the corresponding mathematical models in which, in particular, it is necessary to take into account the temperature dependence (thermosensitivity) of the characteristics of materials. In [1], the equation of thermoelasticity is deduced for thin thermosensitive plates free of loads by using the Kirchhoff–Love hypothesis under the assumption that Poisson's ratio is constant. Similarly, in [2], the thermoelastic state of a thermosensitive plate is modeled for the case where the lateral surfaces of the plate are uniformly loaded by a constant pressure. Some special problems for thin thermosensitive plates were studied in [3–9].

In the present work, we construct the general solution of the problem of thermoelasticity for a thin load-free plate placed in a central-symmetric temperature field under the assumption that all characteristics of the plate are temperature dependent.

Statement of the Problem

Consider a thin plate whose mechanical (shear modulus G, Poisson's ratio ν, and the coefficient of linear thermal expansion α_t) and thermal (heat-conduction coefficient λ_t and volumetric heat capacity c_υ) characteristics are functions of temperature. The plate is placed in a central-symmetric nonstationary temperature field $t(r, \tau)$, where r is a radial coordinate and τ is time. The initial temperature of the plate at which it is strain-free is constant and equal to t_p $\left(t(r, 0) = t_p\right)$. In view of the symmetry of the problem, only the radial component of displacements $u(r, \tau)$ and two components of the stress tensor are nonzero in a polar coordinate system r, φ [1, 10], namely,

$$\sigma_{rr} = \frac{2G(t)}{1-\nu(t)}\left[\frac{\partial u}{\partial r} + \nu(t)\frac{u}{r} - \overline{\Phi}(t)\right] \quad \text{and} \quad \sigma_{\varphi\varphi} = \frac{2G(t)}{1-\nu(t)}\left[\nu(t)\frac{\partial u}{\partial r} + \frac{u}{r} - \overline{\Phi}(t)\right]. \tag{1}$$

Pidstryhach Institute for Applied Problems in Mechanics and Mathematics, Ukrainian Academy of Sciences, Lviv.

Translated from Fizyko-Khimichna Mekhanika Materialiv, Vol. 42, No. 2, pp. 5–12, March–April, 2006. Original article submitted October 21, 2005.

1068–820X/06/4202–0145

These components satisfy the following balance equation:

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\varphi\varphi}}{r} = 0, \tag{2}$$

where

$$\overline{\Phi}(t) = (1 + \nu(t))\,\Phi(t) \quad \text{and} \quad \Phi(t) = \int_{t_p}^{t} \alpha_t(t)\,dt$$

is thermal strain.

We now represent the shear modulus $G(t)$, Poisson's ratio $\nu(t)$, and the coefficient of linear thermal expansion $\alpha_t(t)$ in the form $\chi(t) = \chi_0\chi^*(t)$, where $\chi_0 = \chi(t)|_{t=t_p}$ is the reference characteristic of the material and $\chi^*(t)$ is a function used to describe the temperature dependence $(\chi^*(t)|_{t=t_p} = 1$, $T = (t - t_p)/t_0$ is a dimensionless increment of temperature, and t_0 is a chosen reference temperature) and introduce the dimensionless coordinate

$$\rho = \frac{r}{l_0},$$

where l_0 is a chosen characteristic size, displacement

$$\bar{u} = \frac{u}{l_0\alpha_{t0}t_0},$$

and stresses

$$\sigma_\rho = \frac{\sigma_{rr}}{2G_0\alpha_{t0}t_0} \quad \text{and} \quad \sigma_\varphi = \frac{\sigma_{\varphi\varphi}}{2G_0\alpha_{t0}t_0}.$$

As a result, the balance equation takes the form

$$\frac{\partial\sigma_\rho}{\partial\rho} + \frac{\sigma_\rho - \sigma_\varphi}{\rho} = 0 \tag{3}$$

and the formulas for the evaluation of stresses can be rewritten as

$$\sigma_\rho = \frac{G^*(T)}{1 - \nu(T)}\left[\frac{\partial\bar{u}}{\partial\rho} + \nu(T)\frac{\bar{u}}{\rho} - \overline{\Phi}(T)\right] \quad \text{and} \quad \sigma_\varphi = \frac{G^*(T)}{1 - \nu(T)}\left[\nu(T)\frac{\partial\bar{u}}{\partial\rho} + \frac{\bar{u}}{\rho} - \overline{\Phi}(T)\right], \tag{4}$$

where

$$\overline{\Phi}(T) = (1 + \nu(T))\,\Phi(T) \quad \text{and} \quad \Phi(T) = \int_0^T \alpha_t^*(T)\,dT.$$

Substituting relations (4) in Eq. (3), we arrive at the following equation for the dimensionless radial displacement $\bar{u}$:

$$\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\bar{u})\right) = \frac{\partial\bar{\Phi}(T)}{\partial\rho} - \frac{\partial}{\partial\rho}\left[\ln\left(\frac{G^*(T)}{1-\nu(T)}\right)\right]\left[\frac{\partial\bar{u}}{\partial\rho} + \nu(T)\frac{\bar{u}}{\rho} - \bar{\Phi}(T)\right] - \frac{\partial\nu(T)}{\partial\rho}\frac{\bar{u}}{\rho}. \tag{5}$$

For constant Poisson's ratio, this equation coincides with the well-known equation from [1].

To construct the solution of Eq. (5), we use the perturbation method [10, 11]. According to this method, parallel with Eq. (5), we consider the following equation:

$$\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\bar{u})\right) = \frac{\partial\bar{\Phi}(T)}{\partial\rho} + \frac{\partial}{\partial\rho}\left[\ln\left(\frac{G^*(T)}{1-\nu(T)}\right)\right]\bar{\Phi}(T)$$
$$- \varepsilon\left\{\frac{\partial}{\partial\rho}\left[\ln\left(\frac{G^*(T)}{1-\nu(T)}\right)\right]\left(\frac{\partial\bar{u}}{\partial\rho} + \nu(T)\frac{\bar{u}}{\rho}\right) + \frac{\partial\nu(T)}{\partial\rho}\frac{\bar{u}}{\rho}\right\} \tag{6}$$

[for $\varepsilon = 1$, this equation coincides with Eq. (5)].

Note that Eq. (6) linearly depends on the parameter ε. Therefore, its solution is sought in the form of expansion in powers of this parameter:

$$\bar{u} = \sum_{k=0}^{\infty}\varepsilon^k\bar{u}_k. \tag{7}$$

Substituting representation (7) in Eq. (6) and equating the terms with the same powers of ε, we get the following equations for the quantities $\bar{u}_0$ and $\bar{u}_k$ ($k \geq 1$):

$$\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\bar{u}_0)\right) = \frac{\partial\bar{\Phi}(T)}{\partial\rho} + \frac{\partial}{\partial\rho}\left[\ln\left(\frac{G^*(T)}{1-\nu(T)}\right)\right]\bar{\Phi}(T), \tag{8}$$

$$\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\bar{u}_k)\right) = f_{k-1}(\rho), \tag{9}$$

where

$$f_{k-1}(\rho) = -\frac{\partial}{\partial\rho}\left[\ln\left(\frac{G^*(T)}{1-\nu(T)}\right)\right]\left(\frac{\partial\bar{u}_{k-1}}{\partial\rho} + \nu(T)\frac{\bar{u}_{k-1}}{\rho}\right) - \frac{\partial\nu(T)}{\partial\rho}\frac{\bar{u}_{k-1}}{\rho}.$$

The corresponding components of stresses are given by the formulas

$$\sigma_{\rho k} = \frac{G^*(T)}{1-\nu(T)}\left(\frac{\partial\bar{u}_k}{\partial\rho} + \nu(T)\frac{\bar{u}_k}{\rho} - \bar{\Phi}(T)\delta_{0k}\right),$$

$$\sigma_{\varphi k} = \frac{G^*(T)}{1-\nu(T)}\left(\nu(T)\frac{\partial\bar{u}_k}{\partial\rho} + \frac{\bar{u}_k}{\rho} - \bar{\Phi}(T)\delta_{0k}\right), \tag{10}$$

where δ_{0k} is the Kronecker symbol.

The required displacements and stresses are found as the sums of the components given by relations (8)–(10):

$$\bar{u} = \sum_{k=0}^{\infty} \bar{u}_k, \quad \sigma_\rho = \sum_{k=0}^{\infty} \sigma_{\rho k}, \quad \text{and} \quad \sigma_\varphi = \sum_{k=0}^{\infty} \sigma_{\varphi k}. \tag{11}$$

Circular Washer

By using Eqs. (8), (9) and relations (10), (11), we now determine the stress–strain state of a circular washer with inner and outer radii ρ_0 and ρ_1, respectively, placed in a central-symmetric temperature field. Assume that the ends of the washer are free of loads. This yields

$$\sigma_\rho\big|_{\rho=\rho_0} = 0 \quad \text{and} \quad \sigma_\rho\big|_{\rho=\rho_1} = 0. \tag{12}$$

In this case, the solution of Eq. (8) takes the form

$$\bar{u}_0 = c_{10}\rho + \frac{c_{20}}{\rho} + \frac{1}{\rho} E(\rho) - \frac{\rho}{2} H^{(0)}(\rho) + \frac{1}{2\rho} H^{(2)}(\rho), \tag{13}$$

where

$$E(\rho) = \int_{\rho_0}^{\rho} \left(1 + \ln\left(\frac{G^*(T)}{1-\nu(T)}\right)\right) \bar{\Phi}(T)\rho\, d\rho, \quad H^{(n)}(\rho) = \int_{\rho_0}^{\rho} \ln\left(\frac{G^*(T)}{1-\nu(T)}\right) \frac{\partial \bar{\Phi}(T)}{\partial \rho} \rho^n\, d\rho,$$

and c_{i0} $(i = 1, 2)$ are constants of integration.

The nonzero components of stresses are given by the formulas

$$\begin{aligned} \sigma_{\rho 0} = \frac{G^*(T)}{1-\nu(T)} \Bigg[& (1+\nu(T))c_{10} - \frac{1-\nu(T)}{\rho^2} c_{20} + \ln\left(\frac{G^*(T)}{1-\nu(T)}\right)\bar{\Phi}(T) \\ & - \frac{1-\nu(T)}{\rho^2} E(\rho) - \frac{1+\nu(T)}{2} H^{(0)}(\rho) - \frac{1-\nu(T)}{2\rho^2} H^{(2)}(\rho) \Bigg], \\ \sigma_{\varphi 0} = \frac{G^*(T)}{1-\nu(T)} \Bigg[& (1+\nu(T))\, c_{10} + \frac{1-\nu(T)}{\rho^2} c_{20} - (1-\nu(T))\bar{\Phi}(T) \\ & + \nu(T) \ln\left(\frac{G^*(T)}{1-\nu(T)}\right)\bar{\Phi}(T) + \frac{1-\nu(T)}{\rho^2} E(\rho) - \frac{1+\nu(T)}{2} H^{(0)}(\rho) + \frac{1-\nu(T)}{2\rho^2} H^{(2)}(\rho) \Bigg]. \end{aligned} \tag{14}$$

By using the representations of stresses in the form of expansions similar to (7) and conditions (12) we get the following boundary conditions for all components of stresses $(k \ge 0)$:

$$\sigma_{\rho k}\big|_{\rho=\rho_0} = 0 \quad \text{and} \quad \sigma_{\rho k}\big|_{\rho=\rho_1} = 0. \tag{15}$$

Applying conditions (15) for $k = 0$ to relations (14), we find

$$c_{10} = \frac{\nu^- b_1^* \rho_1^2 - \nu_1^- b_0 \rho_0^2}{\nu^- \nu_1^+ \rho_1^2 - \nu^+ \nu_1^- \rho_0^2} \quad \text{and} \quad c_{20} = \frac{(\nu^+ b_1^* - \nu_1^+ b_0)\rho_0^2 \rho_1^2}{\nu^- \nu_1^+ \rho_1^2 - \nu^+ \nu_1^- \rho_0^2}, \tag{16}$$

where

$$\nu^{\pm} = 1 \pm \nu(T)\big|_{\rho=\rho_0}, \quad \nu_1^{\pm} = 1 \pm \nu(T)\big|_{\rho=\rho_1}, \quad b_i = -\left[\ln\left(\frac{G^*(T)}{1-\nu(T)}\right)\overline{\Phi}(T)\right]_{\rho=\rho_i} \quad (i = 0, 1),$$

$$b_1^* = b_1 + \frac{\nu_1^-}{\rho_1^2} E(\rho_1) + \frac{\nu_1^+}{2} H^{(0)}(\rho_1) + \frac{\nu_1^-}{2\rho_1^2} H^{(2)}(\rho).$$

We now solve Eq. (9) and obtain the kth $(k \geq 1)$ component of radial displacements in the form

$$\bar{u}_k = c_{1k}\rho + \frac{c_{2k}}{\rho} + \frac{\rho}{2} H_{k-1}^{(0)}(\rho) - \frac{1}{2\rho} H_{k-1}^{(2)}(\rho), \tag{17}$$

where

$$H_{k-1}^{(0)}(\rho) = \int_{\rho_0}^{\rho} f_{k-1}(\rho)\rho^n \, d\rho.$$

The corresponding components of stresses are given by formulas (10)

$$\begin{aligned} \sigma_{\rho k} &= \frac{G^*(T)}{1-\nu(T)}\left[(1+\nu(T))c_{1k} - \frac{1-\nu(T)}{\rho^2}c_{2k} + \frac{1+\nu(T)}{2}H_{k-1}^{(0)}(\rho) + \frac{1-\nu(T)}{2\rho^2}H_{k-1}^{(2)}(\rho)\right], \\ \sigma_{\varphi k} &= \frac{G^*(T)}{1-\nu(T)}\left[(1+\nu(T))c_{1k} + \frac{1-\nu(T)}{\rho^2}c_{2k} + \frac{1+\nu(T)}{2}H_{k-1}^{(0)}(\rho) - \frac{1-\nu(T)}{2\rho^2}H_{k-1}^{(2)}(\rho)\right] \end{aligned} \tag{18}$$

with the following constants of integration determined from conditions (15) for $k \geq 1$:

$$c_{1k} = \frac{\nu^- b_k \rho_1^2}{\nu^- \nu_1^+ \rho_1^2 - \nu^+ \nu_1^- \rho_0^2} \quad \text{and} \quad c_{2k} = \frac{\nu^+ b_k \rho_0^2 \rho_1^2}{\nu^- \nu_1^+ \rho_1^2 - \nu^+ \nu_1^- \rho_0^2}, \tag{19}$$

where

$$b_k = -\left[\frac{\nu_1^+}{2} H_{k-1}^{(0)}(\rho) + \frac{\nu_1^-}{2\rho_1^2} H_{k-1}^{(2)}(\rho)\right].$$

The strains e_ρ and e_φ are computed as follows:

$$e_\rho = \sum_{k=0}^{\infty} e_{\rho k} \quad \text{and} \quad e_\varphi = \sum_{k=0}^{\infty} e_{\varphi k}, \tag{20}$$

where

$$e_{\rho 0} = \frac{\partial \bar{u}_0}{\partial \rho} = c_{10} - \frac{c_{20}}{\rho^2} + \left(1 + \ln\left(\frac{G^*(T)}{1-\nu(T)}\right)\right)\bar{\Phi}(T) - \frac{1}{\rho^2}E(\rho) - \frac{1}{2}H^{(0)}(\rho) - \frac{1}{2\rho^2}H^{(2)}(\rho),$$

$$e_{\varphi 0} = \frac{\bar{u}_0}{\rho}, \quad e_{\rho k} = \frac{\partial \bar{u}_k}{\partial \rho} = c_{1k} - \frac{c_{2k}}{\rho^2} + \frac{1}{2}H_{k-1}^{(0)}(\rho) + \frac{1}{2\rho^2}H_{k-1}^{(2)}(\rho), \quad e_{\varphi k} = \frac{\bar{u}_k}{\rho}.$$

For the mechanical characteristics independent of temperature, the formulas presented above turn into the following well-known relations [12, 13]:

$$\bar{u} = \frac{1}{\rho_1^2 - \rho_0^2}\left((1-\nu)\rho + (1+\nu)\frac{\rho_0^2}{\rho}\right)\mathrm{I}(\rho_1) + \frac{1+\nu}{\rho}\mathrm{I}(\rho)$$

$$\sigma_\rho = \frac{1+\nu}{\rho_1^2 - \rho_0^2}\left(1 - \frac{\rho_0^2}{\rho}\right)\mathrm{I}(\rho_1) - \frac{1+\nu}{\rho^2}\mathrm{I}(\rho),$$

$$\sigma_\varphi = \frac{1+\nu}{\rho_1^2 - \rho_0^2}\left(1 + \frac{\rho_0^2}{\rho}\right)\mathrm{I}(\rho_1) + (1+\nu)\left(\frac{1}{\rho^2}\mathrm{I}(\rho) - T\right), \tag{21}$$

$$e_\rho = \frac{\partial \bar{u}}{\partial \rho} = \frac{1}{\rho_1^2 - \rho_0^2}\left(1 - \nu - (1+\nu)\frac{\rho_0^2}{\rho}\right)\mathrm{I}(\rho_1) + (1+\nu)\left(T - \frac{1}{\rho^2}\mathrm{I}(\rho)\right), \quad e_\varphi = \frac{\bar{u}}{\rho},$$

where

$$\mathrm{I}(\rho) = \int_{\rho_0}^{\rho} \rho T(\rho)\, d\rho.$$

Infinite Plate Containing a Circular Hole

Assume that a hole of radius $\rho = \rho_0$ in an infinite plate is free of loads and the temperature of the plate $t(\rho, \tau)$ approaches its initial value t_p as $\rho \to \infty$.

For this plate, the components of displacements have the form (13) and (18) and the stresses corresponding to these displacements are given by relations (14) and (18). The corresponding constants of integration can be found from relations (16) and (19) by passing to the limit as ρ_1 tends to infinity. As a result of this limit transition, we get

$$c_{10} = \frac{1}{2}H^{(0)}(\infty), \quad c_{20} = \frac{\rho_0^2}{\nu^-}\left(\frac{\nu^+}{2}H^{(0)}(\infty) - b_0\right),$$
$$c_{1k} = -\frac{1}{2}H_{k-1}^{(0)}(\infty), \quad c_{2k} = -\frac{\nu^+\rho_0^2}{2\nu^-}H_{k-1}^{(0)}(\rho). \tag{22}$$

For these constants of integration, the stresses σ_ρ and σ_φ disappear at infinity. The strains are computed by using relations (20), where the constants of integration are specified by relations (22).

For constant mechanical characteristics, the components of the stress–strain state in the plate with circular hole take the form

$$\bar{u} = \frac{1+\nu}{\rho}\mathrm{I}(\rho), \quad \sigma_\rho = -\frac{\bar{u}}{\rho}, \quad \sigma_\varphi = -(\sigma_\rho + (1+\nu)T), \quad e_\rho = -\sigma_\varphi, \quad e_\varphi = -\sigma_\rho. \tag{23}$$

Circular Plate (Disk)

Assume that the end $\rho = \rho_1$ of a circular plate is free of loads. For a central-symmetric temperature field $t(\rho, \tau)$, we find the components of displacements by using relations (13) and (17) and the components of stresses by using relations (14) and (18) with

$$c_{10} = \frac{b_1^*}{\nu_1^+}, \quad c_{20} = 0, \quad c_{1k} = \frac{b_k}{\nu_1^+}, \quad \text{and} \quad c_{2k} = 0.$$

In this case, the lower limit in the integrals $E(\rho)$, $H^{(n)}(\rho)$, and $H_{k-1}^{(n)}(\rho)$ is equal to zero. The corresponding strains are computed according to relations (20), where the constants of integration take the values presented above.

For constant mechanical characteristics, the stress–strain state of the disk is described by the formulas

$$\bar{u} = (1-\nu)\frac{\rho}{\rho_1^2}\mathrm{I}(\rho_1) + \frac{1+\nu}{\rho}\mathrm{I}(\rho), \quad \sigma_\rho = (1+\nu)\left(\frac{\mathrm{I}(\rho_1)}{\rho_1^2} - \frac{\mathrm{I}(\rho)}{\rho^2}\right),$$
$$\sigma_\varphi = (1+\nu)\left(\frac{\mathrm{I}(\rho_1)}{\rho_1^2} + \frac{\mathrm{I}(\rho)}{\rho^2} - T\right), \quad e_\rho = \frac{1-\nu}{\rho_1^2}\mathrm{I}(\rho_1) + (1+\nu)\left(T - \frac{\mathrm{I}(\rho)}{\rho^2}\right), \quad e_\varphi = \frac{\bar{u}}{\rho}, \tag{24}$$

where the lower limit in the integral $\mathrm{I}(\rho)$ is equal to zero.

Infinite Plate

Consider an infinite plate placed in a central-symmetric temperature field approaching the initial temperature of the plate t_p at infinity. The components of displacements and stresses in this plate are given by relations (13), (17), (14), and (18) with the following constants of integration:

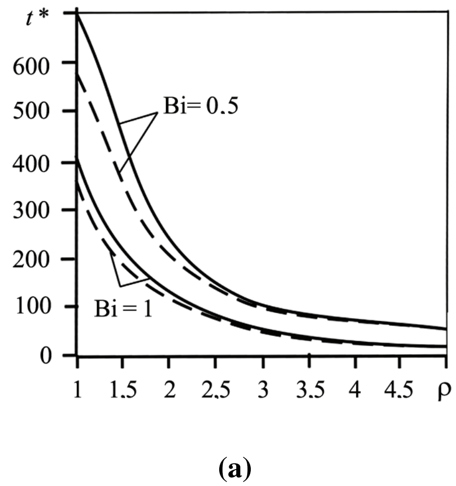

Fig. 1. Radial distributions of the increment of temperature (a) and displacements (b) for the temperature-dependent (solid lines) and temperature-independent (equal to the reference values; dashed lines) characteristics of the material.

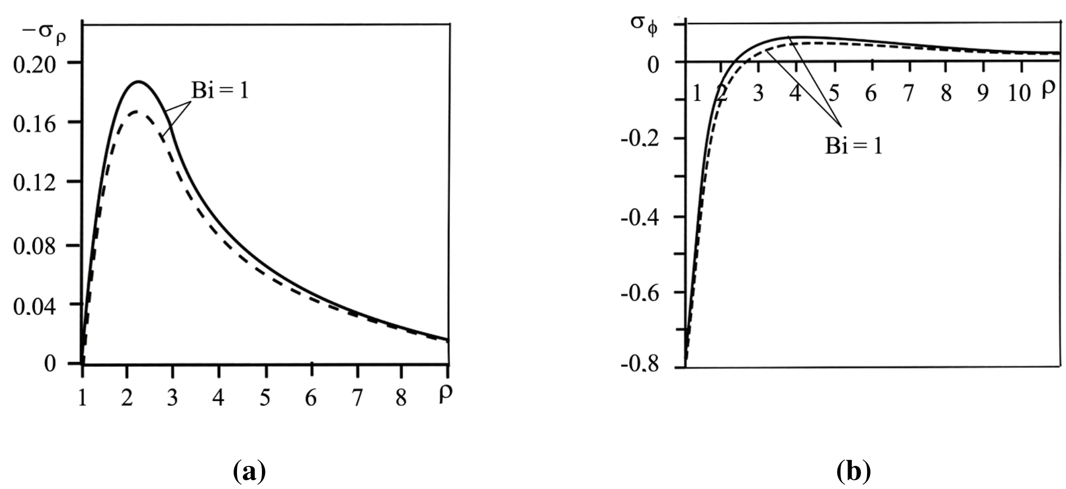

Fig. 2. Radial distributions of the radial (a) and hoop (b) stresses. See also the explanation in Fig. 1.

$$c_{10} = \frac{1}{2}H^{(0)}(\infty), \quad c_{20} = 0, \quad c_{1k} = -\frac{1}{2}H^{(0)}_{k-1}(\infty), \quad \text{and} \quad c_{2k} = 0.$$

The lower limits in the integrals $E(\rho)$, $H^{(n)}(\rho)$, and $H^{(n)}_{k-1}(\rho)$ are equal to zero. The strains e_ρ and e_φ can be found from relations (20) with the constants of integration specified above.

If the mechanical characteristics of the plate are independent of temperature, then the components of its stress–strain state are computed according to relations (23) in which the lower limit of the integral $I(\rho)$ is equal to zero.

Numerical Investigations

By using relations (11) and (20), where the terms of expansions of displacements are given by (13) and (17), the corresponding components of stresses by (14) and (18), and the constants of integration by (22), we perform the numerical analysis of dimensionless displacements, stresses, and strains induced in a load-free steel thin plate containing a circular hole $\rho_0 = r_0/\delta = 1$, where δ is the half thickness of the plate, by a temperature field caused by a continuously acting heat source of constant power Q located at the origin of coordinates (the temperature dependences of the characteristics are described by the following functions [14]:

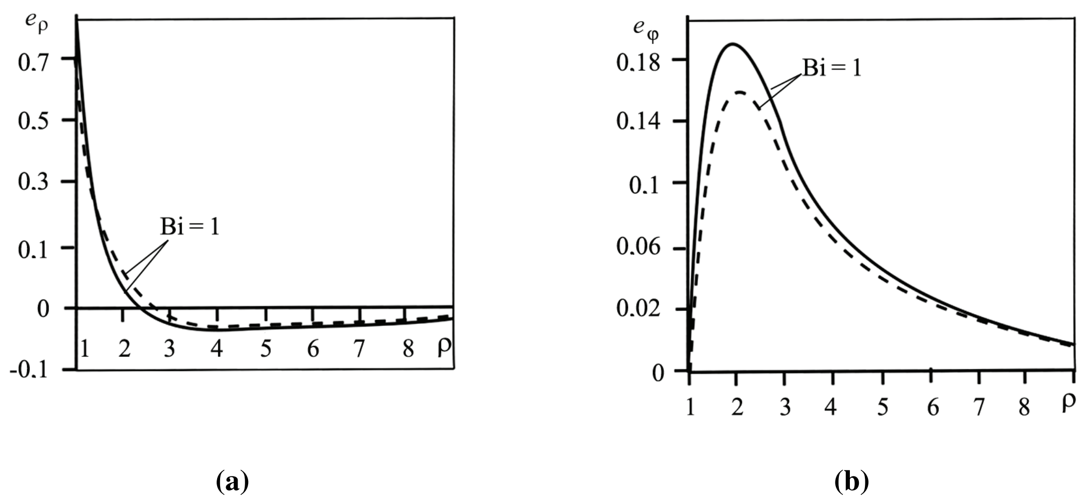

Fig. 3. Radial distributions of the radial (a) and hoop (b) strains. See also the explanation in Fig. 1.

$$\lambda_t^*(T) = 1 + kT, \quad \alpha_t^*(T) = 1 + k_\alpha T, \quad G^*(T) = 1 - k_{G1}T - k_{G2}T^2, \quad \text{and} \quad \nu(T) = \nu_0(1 - k_\nu T),$$

where $k = -0.12$, $k_\alpha = -0.31$, $k_{G1} = 0.19$, $k_{G2} = 0.09$, $\nu_0 = 0.4$, and $k_\nu = 0.08$). The increment of temperature of the plate $t_* = t - t_p$ (determined in [10]) is given by the formula

$$t_* = \frac{\sqrt{1 + 2k\theta} - 1}{k}, \tag{25}$$

where

$$\theta = Q_0 \mathrm{K}_0\left(\rho\sqrt{(1+\kappa)(\mathrm{Bi}_1 + \mathrm{Bi}_2)}\right), \qquad Q_0 = \frac{Q}{2\pi\lambda_{t0}\delta}, \qquad \mathrm{Bi}_i = \frac{\alpha_i\delta}{\lambda_{t0}}$$

are the Biot criteria characterizing the intensity of heat exchange from the surfaces $z = \pm\delta$, α_i are the coefficients of heat exchange from these surfaces, $\rho = r/\delta$, δ is a characteristic size, $\mathrm{K}_0(\circ)$ is a modified Bessel function of order zero [15], and, for all ρ, the quantity κ is determined from the equation $t_*(\theta) = (1 + \kappa)\theta$. Moreover, we take $t_p = 293°\mathrm{K}$, $t_0 = 673°\mathrm{K}$, $Q_0 = 900$, and $\mathrm{Bi} = \mathrm{Bi}_1 + \mathrm{Bi}_2 = 0.5;\ 1$. The results of numerical investigations are presented in Figs. 1–3.

CONCLUSIONS

The difference between the levels of stresses and strains computed for the plates with constant (equal to reference values at the initial temperature) and temperature-dependent characteristics is $\approx 15\%$. The difference between the corresponding values of displacements is $\approx 23\%$. This means that the thermosensitivity of material should be taken into account in the determination of its stress–strain state. The necessity of doing this was, in particular, emphasized by V. V. Panasyuk in his survey [16].

REFERENCES

1. Ya. S. Podstrigach and Yu. M. Kolyano, *Nonstationary Temperature Fields and Stresses in Thin Plates* [in Russian], Naukova Dumka, Kiev (1972).
2. S. Tang, "Thermal stresses in temperature-dependent isotropic plates," *J. Spacecr. Rock.,* **5**, 987–990 (1968).
3. S. Tang, "Natural vibration of isotropic plates with temperature-dependent properties," *AIIA J.,* **7**, 725–727 (1969).
4. M. Newman and M. Forray, "Thermal stresses and deflections in thin plates with temperature-dependent elastic moduli," *J. Aerospace Sci.,* **29**, 372–373 (1962).
5. J. Minardi, "Effect of vibrations in the coefficient of thermal expansion upon thermal stress," *AIIA J.,* **4**, 542–544 (1966).
6. I. S. Alvarez and A. R. Cooper, Jr., "Thermal stress distribution for a thin plate with temperature variation through the thickness," *J. Appl. Mech.,* **42**, 513–514 (1975).
7. B. K. Ganguly, K. R. McKinney, and D. P. H. Hasselman, "Thermal-stress analysis of flat plate with temperature-dependent thermal conductivity," *J. Amer. Ceram. Soc.,* **58**, 455–456 (1975).
8. I. A. Ismail and J. L. Nowinski, "Thermoelastic problem for shells of revolution exhibiting temperature dependent properties," *Appl. Sci. Res.,* **A14**, 211–232 (1965).
9. N. Kamiya, E. Kameyama, and A. Fukui, "Axisymmetric finite deflections of heated flat plates exhibiting temperature-dependent mechanical and thermal properties," *Res. Rep. Fac. Eng. Mie Univ.,* **5**, 31–48 (1980).
10. V. S. Popovych, *Models and Methods for the Numerical Analysis of the Thermal Stressed State of Thermosensitive Structural Elements Under the Conditions of Complex Heat Exchange* [in Ukrainian], Author's Abstract of the Doctoral Degree Thesis (Engineering), Luts'k (2005).
11. V. A. Lomakin, *Theory of Elasticity of Inhomogeneous Bodies* [in Russian], Izd. MGU, Moscow (1976).
12. S. P. Timoshenko and J. Goodier, *Theory of Elasticity,* McGraw-Hill, New York (1970).
13. V. M. Vigak, *Optimal Control Over the Temperature Conditions* [in Russian], Naukova Dumka, Kiev (1979).
14. V. E. Zinov'ev, *Thermal Properties of Metals at High Temperatures. Handbook* [in Russian], Metallurgiya, Moscow (1989).
15. M. Abramowitz and I. A. Stegun (editors), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables,* National Bureau of Standards, Math. Appl. Ser., **55**, Washington (1964).
16. V. V. Panasyuk, "Fracture mechanics and strength of materials: Achievements and prospects," *Fiz.-Khim. Mekh. Mater.,* **40**, No. 3, 5–18 (2004).