Materials Science, Vol. 41, No. 3, 2005

PROPAGATION OF CRACKS IN METALS UNDER THE ACTION OF HYDROGEN AND LONG-TERM STATIC LOADING

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UDC 669.778

We propose a computational model of crack growth caused by the action of hydrogen and long-term static loads. The model is based on the energy criterion of fracture of materials. As a result, we deduce the expression for the crack growth rate as a function of the load, size of the initial crack, and physicochemical and strength characteristics of the material. The theoretical curve reveals satisfactory agreement with the experimental data.

The existing models of crack growth in metals under the action of static loads and hydrogen [1–3] are, as a rule, too complicated for application in the engineering practice.

In what follows, we propose a computational model of hydrogen-assisted crack growth based on the energy criterion of fracture of materials generalized to the case of influence of hydrogen.

Generalization of the Energy Criterion of Propagation of Macrocracks in Metal Bodies Under the Action of Hydrogen-Containing Media and Long-Term Static Loads

We consider an isotropic elastoplastic body Ω with boundary Γ weakened by a planar macrocrack S_0 with smooth contour L_0 subjected to the action of uniformly distributed forces P perpendicular to the plane of the crack and a hydrogen-containing medium. Our aim is to determine the subcritical crack growth rate.

According to the law of conservation of energy [4–7], the sum of the work $\dot{A}$ of the forces P per unit time and the thermal energy $\dot{Q}$ conveyed to the body per unit time is equal to the rate of growth of the sum of the kinetic energy K, internal energy W, and fracture energy Π of the body as the area of the crack increases by S:

$$\dot{A} + \dot{Q} = \dot{K} + \dot{W} + \dot{\Pi}, \tag{1}$$

where the overdots denote the total derivatives of the corresponding quantities with respect to time t. The area of the crack S changes with time and

$$\frac{\partial S(t)}{\partial t} \geq 0.$$

The quantities A, Q, K, W, and Π are functions of S and t. Hence, Eq. (1) takes the form

$$\frac{\partial(A + Q - K - W - \Pi)}{\partial S}\frac{\partial S}{\partial t} + \frac{\partial(A + Q - K - W - \Pi)}{\partial t} = 0, \tag{2}$$

whence we get the following law of changes in the crack area:

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Translated from Fizyko-Khimichna Mekhanika Materialiv, Vol. 41, No. 3, pp. 29–33, May–June, 2005. Original article submitted January 31, 2005.

$$\frac{\partial S}{\partial t} = -\frac{\partial(A+Q-K-W-\Pi)}{\partial t}\left[\frac{\partial(A+Q-K-W-\Pi)}{\partial S}\right]^{-1}, \tag{3}$$

where

$$\frac{\partial A}{\partial S} = \int_\Gamma \sigma_{ij} n_j \frac{\partial u_i}{\partial S}\, d\Gamma + \int_\Omega \rho F_i \frac{\partial u_i}{\partial S}\, dV, \qquad \frac{\partial Q}{\partial S} = \int_\Gamma \frac{\partial q_i}{\partial S} n_i \, d\Gamma,$$

$$\frac{\partial \Pi}{\partial S} = 2\int_S \gamma_0 \, dS, \qquad \frac{\partial K}{\partial S} = 0.5\frac{\partial\left(\int_\Omega \rho u_i u_i \, dV\right)}{\partial S}, \qquad \frac{\partial W}{\partial t} = \frac{d\int_\Omega W_0 \, dV}{dt},$$

q_i and F_i are the components of the vectors of heat flow and bulk forces, n_j are the components of the unit vector of outer normal to the surface Γ, ρ is density, γ_0 is the energy consumption per unit area of the newly formed crack surface, σ_{ij} and u_i are, respectively, the components of the stress tensor and vector of displacements on the surface of the body Γ (including cracks), and

$$\frac{dW_0}{dt} = \sigma_{ij}\frac{d\varepsilon_{ij}}{dt} + q_{i,i}$$

is the volume density of internal energy.

The level of macroscopic strains formed in steels in the process of hydrogenation depends on the concentration of hydrogen [3]. In the process zone, the dependence of the components of strains ε_{ij} on the concentration of hydrogen $C_H(t)$ can be regarded as linear for low concentrations. The total strain ε_{ij} is represented as the sum of two components: the strain caused by external fields e_{ij} (e.g., the action of forces and heating) and the strain caused by the presence of hydrogen atoms in the lattice:

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl} + \delta_{ij}(\alpha_T \Delta T + B C_H), \tag{4}$$

where B is a parameter characterizing the dependence of the level of strain in the material on the concentration of hydrogen, S_{ijkl} are stress functions, and α_T is the coefficient of thermal expansion.

By using relation (4), we obtain

$$\frac{dW}{dt} = \int_\Omega \left[\sigma_{ij}\left(\frac{\partial e_{ij}}{\partial S} + B\delta_{ij}\frac{\partial C_H}{\partial t}\right) + q_{i,i}\right] dV. \tag{5}$$

According to the law of conservation of energy for the immobile crack, the time derivative in relation (3) takes the form

$$\frac{\partial(A+Q-K-W-\Pi)}{\partial t} = \int_\Omega B\sigma_{ij}\delta_{ij}\frac{\partial C_H}{\partial t}\, dV. \tag{6}$$

Under the assumption that the values of released thermal and kinetic energy are small, we rewrite the equation for the hydrogen-assisted crack growth rate (3) in the form

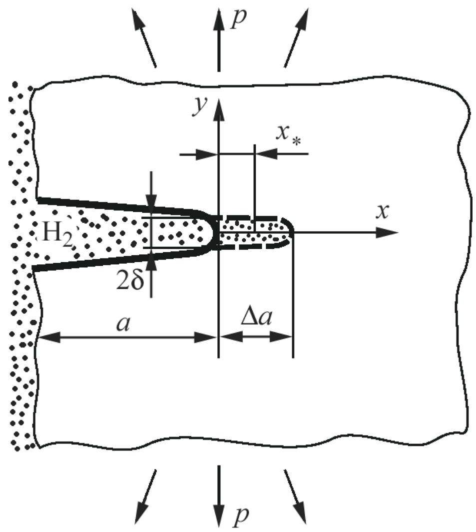


Fig. 1. Schematic diagram of a strained body containing a macrocrack in hydrogen.

$$\frac{\partial S}{\partial t} = \frac{\int_{\Omega} B\sigma_{ij}\delta_{ij}\frac{\partial C_{\mathrm{H}}}{\partial t}dV}{\int_{\Gamma}\sigma_{ij}n_j\frac{\partial u_i}{\partial S}d\Gamma - \int_{\Omega}\sigma_{ij}\frac{\partial e_{ij}}{\partial S}dV - 2\int_S \gamma_0\, dS}. \tag{7}$$

Plate Containing a Rectilinear Crack

Consider an elastoplastic isotropic body containing a macrocrack of length a subjected to the action of tensile stresses P and a hydrogen-containing medium (Fig. 1). The concentration of hydrogen at the crack tip is equal to C_S.

According to the experimental data [3], the hydrogen-assisted growth of the macrocrack is jumpwise. This is connected with the fact that pileups of dislocations are formed near the crack tip. This leads to the accumulation of hydrogen and, hence, to the formation of microcracks. Finally, the main crack propagates as a result of merging with a microcrack. It stops at a certain distance from the original position of its tip, where the level of saturation of the metal with hydrogen is insufficiently high. At the tip of the newly formed crack, we also observe the formation of a process zone, which undergoes hydrogenation, etc. The mean duration $\Delta t = t_*$ of the cycle of hydrogen accumulation and formation of microcracks is equal to the period of jumps which, together with the mean length of the jumps $\Delta a = x_*$, determines the macroscopic growth rate of the main crack:

$$v = \frac{x_*}{t_*}.$$

Thus, we can write the equation of balance of rates (7) in the form [8]

$$\frac{\partial a}{\partial t} = \frac{B\sigma_{\mathrm{y}}}{\gamma_0 - \gamma_p}\int_0^{a_p}\left.\frac{\partial C_{\mathrm{H}}(x,t)}{\partial t}\right|_{t=t_*}dx, \tag{8}$$

where a_p is the length of the plastic zone in the vicinity of the crack tip, $\gamma_p = \sigma_y \delta$ is the specific energy of dissipation of plastic strains under static loading, δ is the crack-tip opening displacement, and σ_y is the yield limit of the material.

We can represent the specific fracture energy of the material in the form $\gamma_0 = \sigma_y \delta_c$, where δ_c is the critical crack-tip opening displacement. As a result, we arrive at the following equation for the growth rate of a hydrogen-assisted crack:

$$\frac{\partial a}{\partial t} = \frac{B}{\delta_c - \delta} \int_0^{a_p} \left. \frac{\partial C_H(x, t)}{\partial t} \right|_{t=t_*} dx. \tag{9}$$

The concentration of hydrogen in the process zone is given by the formula [3]

$$C(x, t) = \frac{2C_S \sqrt{\tau} \exp[2m\xi - (4\tau)^{-1} \xi^2]}{\xi \sqrt{\pi} \exp m\xi + 2\sqrt{\tau} \exp(-0.25\xi^2 \tau^{-1})}, \tag{10}$$

where

$$\xi = \frac{x}{2\delta}, \quad \tau = \frac{Dt}{(2\delta)^2}, \quad m = \frac{0.9\sigma_y V_H}{RT},$$

D is the coefficient of hydrogen diffusion in the metal, V_H is the partial molar volume of hydrogen in the metal, and R is the universal gas constant.

It is now necessary to find the mean length of jump of a crack x_*. In [9], it is shown that the deformation parameters are especially sensitive to the influence of hydrogen. As a rule, they rapidly decrease as the concentration of hydrogen in the metal increases. Therefore, we can use the deformation criterion to determine the conditions of local fracture. According to this criterion, a crack gets an increment of length x_* for time t_* provided that the following equality is true for a given load at the point $x = x_*$:

$$\varepsilon_H(x_*, p) = \varepsilon_c(p) - BC(x_*, t_*), \tag{11}$$

where $\varepsilon_c(p)$ is the fracture strain of the material in the absence of hydrogen.

The levels of strains in the process zone (both in the hydrogenated and nonhydrogenated materials) are determined via the crack opening displacements, i.e., we assume that

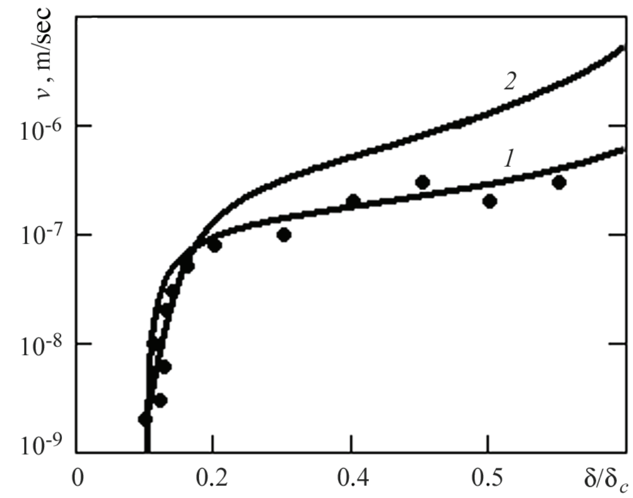
$$\varepsilon_H(x_*, p) = \frac{\delta}{h}, \quad \varepsilon_c(p) = \frac{\delta_c}{h}, \tag{12}$$

where h is a proportionality factor.

Further, in view of relations (11) and (12), we arrive at the following condition for finding x_*:

$$1 - \lambda = \alpha C\left(x_*, \frac{x_*}{v}\right), \tag{13}$$

where



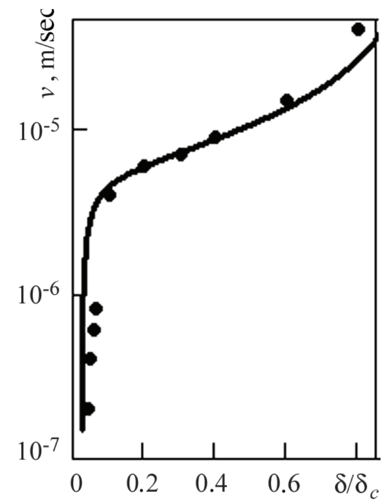


Fig. 2. Dependences of the crack growth rate of on the crack opening displacement: (a) 4147 steel, (b) 4340 steel {curve 1 corresponds to the results of calculations according to relation (15), curve (2) is plotted according to the results taken from [3], and dark symbols correspond to the experimental data}.

Table 1. Chemical Composition of Steels (wt. %)

Type of steel	C	Mn	P	S	Si	Cr	Mo
4147	0.47	0.98	0.012	0.011	0.26	0.99	0.18
4340	0.36	0.76	0.01	0.25	0.25	0.99	0.18

$$\lambda = \frac{\delta}{\delta_c} \quad \text{and} \quad \alpha = \frac{Bh}{\delta_c}.$$

After necessary mathematical transformations, relations (10) and (13) yield the following formula for the length of an elementary jump of the crack:

$$x_* \approx \frac{8\alpha C_S D\delta_c\lambda(1-\lambda) + 4D\alpha^2 C_S^2\delta_c\lambda + 4D\delta_c\lambda(1-\lambda)^2}{8\alpha^2 C_S^2 Dm + 8\alpha C_S Dm(1-\lambda) + \delta_c\lambda\pi\upsilon(1-\lambda)^2}. \tag{14}$$

Finally, in view of relations (8), (10), and (14), we arrive at the following expression for the crack growth rate:

$$\upsilon = \frac{8B\alpha C_S^2 Dm\delta_c\lambda(\alpha C_S + (1-\lambda))(\exp(ma_p/(\delta_c\lambda)) - 1)}{32\alpha C_S m^2\delta_c\lambda(1-\lambda)(\alpha C_S + (1-\lambda)) - BC_S\delta_c^2\lambda^2\pi(1-\lambda)^2(\exp(ma_p/(\delta_c\lambda)) - 1)}$$

$$-\frac{8D\delta_c\lambda m(1-\lambda)\left(2\alpha C_S(1-\lambda)+(1-\lambda)^2+\alpha^2C_S^2\right)}{32\alpha C_S m^2\delta_c\lambda(1-\lambda)\left(\alpha C_S+(1-\lambda)\right)-BC_S\delta_c^2\lambda^2\pi(1-\lambda)^2\left(\exp\left(ma_p/(\delta_c\lambda)\right)-1\right)}. \quad (15)$$

To use this relation, one must know the quantities δ_c, α, B, D, C_S, and m for the analyzed metal–hydrogen system. They can be experimentally found according to the dependences of the crack growth rate in the hydrogenated metal on the load applied to the material near the crack tip and determined by the quantity K_I (or δ).

Verification of the Model

To check the proposed model, we used the experimental data obtained for 4147 [10] and 4340 [11] steels whose chemical compositions are presented in Table 1. Note that curve 2 in Fig. 2a is plotted according to the model proposed by Panasyuk, Andreikiv, and Obukhivs'kyi [3] in which, unlike the analyzed model, it is assumed that the critical situation in the process zone is formed at a distance $x_* = 2\alpha\delta$ from the crack tip, where $0 < \alpha < 1$. For 4147 steel, we have

$$\delta_c = 0.89\cdot10^{-5}\text{ m},\quad D = 10^{-11}\text{ m}^2/\text{sec},\quad m = 0.6528,\quad \sigma_y = 869\text{ MPa},\quad \text{and}\quad K_{Ic} = 160\text{ MPa}\sqrt{\text{m}}.$$

The quantities B and α are determined according to the experimental data by the least-squares method. As a result, we get $BC_S = 3.56$ and $\alpha C_S = 1.48$.

The comparison of curves 1 and 2 in Fig. 2a demonstrates that the additional condition imposed in finding the length of elementary jumps of the crack leads to better agreement between the calculated curves and the experimental data. For 4340 steel, we have

$$\delta_c = 58.33\ \mu\text{m},\quad D = 10^{-10}\text{ m}^2/\text{sec},\quad \sigma_y = 1240\text{ MPa},\quad K_{Ic} = 154\text{ MPa}\sqrt{\text{m}},\quad m = 0.906,\quad \text{and}\quad BC_S = 1.9.$$

The analysis of the results presented in Fig. 2 shows that the proposed model of growth of a macrocrack formed in a hydrogenated metal in tension is in satisfactory agreement with the experimental data and, hence, can be used for the prediction of the serviceability of structural elements under the indicated conditions.

The present work was supported by the Ukrainian Scientific and Technological Center (Grant No. 1628).

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