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RESEARCH AND TESTING METHODS

ANALYSIS OF THE ELASTOPLASTIC DEFORMATION OF THE MATERIAL IN THE PROCESS ZONE

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By the method of digital correlation of speckle images, we record the displacements in the vicinity of the crack tip in plane specimens of D16AT alloy. The field of elastoplastic displacements is determined in the vicinity of the crack tip under static loading. By using the distribution of displacements, we compute the levels of strains ε_y on the continuation of the crack. The theoretical results are in good agreement with the experimental data in estimating the length of the plastic zone for the limiting equilibrium state.

For the application of the concepts of fracture mechanics in the engineering practice of evaluation of the residual strength of structures, it is necessary to have efficient procedures aimed at the estimation of the crack resistance of materials (ability of the material to resist the process of crack propagation) [1, 2].

The characteristics of crack resistance of plastic materials in the case where a plastic zone (comparable with the crack length) is formed in the vicinity of the crack tip are computed on the basis of the δ_c-model by using the criterion of critical crack-tip opening displacement and the model of plastic strips [3]. The critical crack opening displacement (δ_c) describes the crack resistance of the material and plays the role of a parameter of the model. To find δ_c, it is necessary to measure the displacements of two points of the opposite crack lips at the crack tip or at a certain distance from the tip [4, 5]. Note that the crack opening displacement is a function of the distance from the tip and the conditions of elastoplastic deformation of the specimen affecting the value of δ_c. The shape and sizes of the plastic zone formed near the crack tip in the process of deformation of the body depend on its mechanical properties, geometric sizes, and stressed state.

The data of measurements of the opening displacements by using standard procedures do not enable one to study the mechanisms of formation of the plastic zone and crack initiation in the process of active loading up to fracture.

The aim of the present work is to develop a method aimed at the investigation of the process of deformation of materials near the crack tips and determine the characteristic sizes of the process zone according to the data of evaluation of elastoplastic displacements based on the correlation processing of the electron speckle images of this zone.

Experimental Procedure

Specimens of a plate were made of a sheet material with lateral or central notches [2]. Fatigue cracks in the specimens were formed according to the methodical recommendations given in [2]. The specimens were stretched in an EU-40 hydraulic machine. The elastoplastic strains on the specimen surface near the crack tip were found by using the following two procedures:

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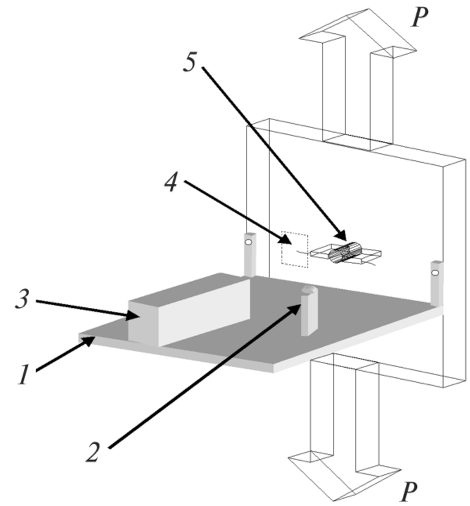

Fig. 1. Schematic diagram of mounting of a camcorder: (1) platform, (2) source of light, (3) camcorder, (4) crack, (5) specimen.

— by measuring the opening displacements of the crack lips at fixed points with a strain gage mounted on the specimen surface according to the well-known method [2];

— with the help of the digital correlation of speckle images [6, 7], i.e., electron photographs of the rough specimen surface; the changes in the surface topography are comparable with the wavelength of light.

The displacements were recorded in the vicinity of a crack tip on the specimen surface with the help of needle legs fastened to the strain gage at a distance (base) of 3 mm. The signals of displacement and force gages were recorded in the fracture diagram by a PDA-1 two-coordinate recorder. Another device recorded a diagram of force vs. the displacement of the crack lips along the line of action of the force.

On the opposite side of the specimen, a support with platform was mounted on two pointed legs in the plane of crack propagation to record the field of elastoplastic strains (Fig. 1). On platform 1, we mounted a source of light 2 and camcorder 3 whose lens was focused to observe a part of the specimen surface 9×12 mm in size in the vicinity of the crack tip.

In the course of experiments, the specimens containing fatigue cracks were subjected to static loading. As a result, we plotted the diagrams of load p vs. the opening displacement Δ_0 at the center of the specimen and force p vs. the opening displacement Δ_c at the crack tip. Simultaneously, in each stage of loading and unloading, we photographed a part of the specimen surface with a camcorder, recorded speckle images, and stored the data in a personal computer for subsequent processing. Speckle images characterize the intensity of light reflected from the specimen surface. The speckle images of loaded $S1$ and unloaded $S2$ specimens were split into square fragments h_{s1} and h_{s1} and the functions of mutual correlation of the corresponding couples of fragments were found according to the formula [7, 8]

$$\hat{C}(k, l) = \frac{1}{m} \sum_{i=0}^{m-1} \sum_{j=0}^{m-1} h_{s1}(i, j) h_{s2}(i+k, j+l), \tag{1}$$

where h_{s1} is a fragment of the image $S1$, $n_1 \times n_2$ is its size, h_{s2} is a fragment of the image $S2$ whose size is also equal to $n_1 \times n_2$, and $k, l = 0, 1, \ldots, m-1$ are integer numbers such that $m \geq n_1 + n_2$.

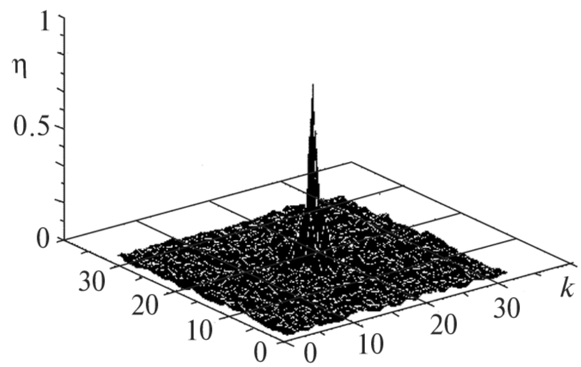

Fig. 2. Correlation peak of a fragment after filtration.

Note that the direct calculation of the function of mutual correlation (1) is quite cumbersome. Therefore, to accelerate computations, we use the operations of direct and inverse rapid Fourier transformations. To this end, we rewrite relation (1) in the form [6]

$$\hat{C}(k, l) = F^{-1}(H_{s1} * H_{s2}^{*}), \tag{2}$$

where $H_{s1}(r, s) = F(h_{s1})$ and $H_{s2}(r, s) = F(h_{s2})$ are the Fourier images of the first and second fragments, respectively, the symbol F denotes the operation of fast Fourier transformation, the symbol F^{-1} denotes the operation of inverse fast Fourier transformation, the symbol $*$ denotes the operation of complex conjugation of a function, and $r, s = 0, 1, \ldots, m-1$ are discrete spatial frequencies of fragments of the image.

To increase the ratio of the maximum peak of correlation of a couple of fragments to the dispersion of the ambient noise, we filtered the common Fourier spectrum of two fragments by using a fractional-order filter [9]. After the procedure of filtration, the correlation signal is described by the formula

$$\hat{C}(k, l) = F^{-1}\left[\frac{H_{s1}H_{s2}^{*}}{\left|H_{s1}H_{s2}^{*}\right|^{\alpha/2}}\right], \tag{3}$$

where the parameter of the filter α takes only real values.

The experimental results obtained in [8] show that, by using a "Helios-44" objective with the lens stop $f_{\#} = 2.8$ for recording speckle images of the surface of metal specimens, the maxim peak/noise ratio can be attained for the parameter of the filter $\alpha = 1.6$. The normed mutual correlation function of a couple of fragments

$$\eta = \frac{\hat{C}(k, l)}{\hat{C}(k, l)_{\max}}$$

computed for this case according to relation (3) is depicted in Fig. 2. The position of the maximum of the correlation function relative to the center of the fragment specifies the relative displacement of the fragments under loading.

Strain gages were mounted on a specimen containing a fatigue crack to measure the crack opening displacements at the center of the specimen and near the crack tip. On the opposite side of the specimen, we mounted a support with a camcorder and a source of light.

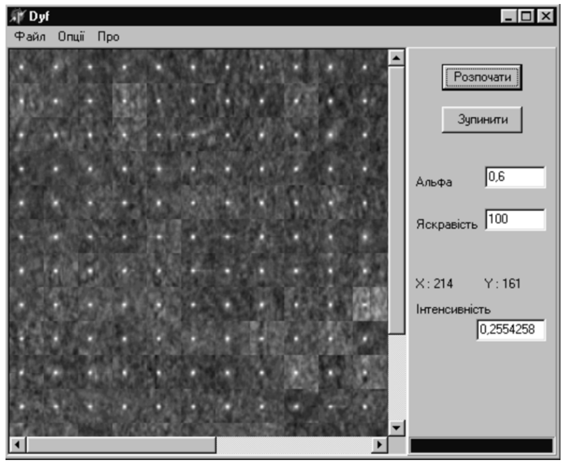

Fig. 3. Output peaks (light points) obtained as a result of the correlation of fragments.

The first image of the specimen surface was recorded in the vicinity of the fatigue crack without loading. The other images were recorded under loading up to the limiting equilibrium state and fracture. In addition, we recorded the speckle images after the complete unloading of the specimen. The onset of crack propagation was determined by using a KM-8 cathetometer.

As a result, we plotted the loading–unloading diagrams and recorded the speckle images of a part of the specimen surface 9×12 mm in size including the process zone prior to the start of the crack.

Analysis of the Experimental Data

The speckle images of the specimen surface near the tip of the fatigue crack accumulated in all stages of loading are processed in a computer by using a specially developed program (written in Delphi) which reads $S1$, $S2$ couples of speckle images, splits these images into fragments h_{s1} and h_{s2} 32×32 pixels in size, and performs the direct and inverse fast Fourier transformations. By using this program, it is possible establish the parameter α necessary for filtration, perform the detailed visual and statistical analysis of each fragment of the image, determine the coordinates of the maximum of the correlation signal inside the fragment with subpixel resolution (by using various algorithms of interpolation), filter and interpolate the accumulated numerical data, etc.

Processing each couple of speckle images by using the program, we obtain the field of correlation peaks of different intensities. The coordinates of the maxima of these peaks, are used to determine the displacement of an element of the deformed surface of the specimen in tension (Fig. 3). The data on changes in the location of the maximum of each correlation peak for the period between two recorded images are used to map the displacements of the specimen surface in the vicinity of the crack tip.

Analysis of the Results

The results of measuring elastoplastic displacements demonstrate that the distribution of strains $\varepsilon_y(x)$ in this zone is nonuniform. By using this distribution obtained for a specimen of D16AT alloy containing a central crack, we measured the levels of strains ε_y and the length of the plastic zone d_p along the x-axis in the vicinity of the crack tip for different gage lengths ($b_1 = 1.28$ mm, $b_2 = 2.56$ mm, and $b_3 = 3.84$ mm) (Fig. 4).

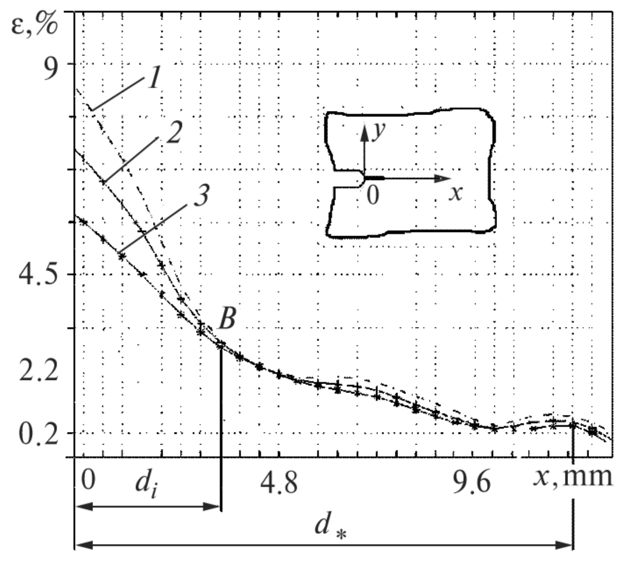

Fig. 4. Distributions of strains along the line of continuation of the crack (x-axis) corresponding to its start for three gage lengths: (1) 1.28 mm, (2) 2.56 mm, and (3) 3.84mm.

It is discovered that the strains in the metal in the vicinity of the crack tip are nonuniform for three gage lengths. At the same time, at a certain distance from the tip of the concentrator, the corresponding levels of strains are, in fact, equal.

The maximum value $\varepsilon_c = 8.6\%$ corresponds to the smallest gage length $b_1 = 1.28$ mm under a load $P = P_c$, where P_c is the force required for the crack start. For the gage length $b_3 = 3.84$ mm, the strain ε_c decreases by 35%. The variations of the level of strains $\varepsilon_y(x)$ along the continuation of the crack from its tip (along the x-axis) for all analyzed gage lengths are similar in shape but have different values. The maximum levels of strains near the crack tip sharply decrease for all gage lengths and become practically equal beginning with a certain point B (Fig. 4). By using the experimentally measured values of strains ε_c at the crack tip, we can find the optimal gage length b for the evaluation of the characteristics of crack resistance of the material (δ_c). For a fixed value of the gage length b, the quantity δ_c is given by the formula

$$\delta_c = b\varepsilon_c. \tag{4}$$

It is also possible to use another approach. If the characteristics ε_c and δ_c are known, then we determine the quantity b ($b = \delta_c / \varepsilon_c$) corresponding to the optimal gage length for the evaluation of the crack-tip opening displacement by the method of digital correlation of speckle images. Thus, for D16AT alloy, we have $\delta_c = 0.12$ mm [11] and the critical strain $\varepsilon_c = 0.08$. According to these data, the optimal gage length b for measuring the opening displacement between the crack lips is equal to 1.5 mm.

In the currently known procedures of measuring the crack opening displacement both at the crack tip [2] and at a certain distance from the tip [4], it is customary to use a gage length of about 3 mm. According to the accumulated experimental data, this introduces noticeable inaccuracies in the measured values of δ_c. In using the method of digital correlation of speckle images, one can choose the gage length for measuring the crack-tip opening displacements depending on the specific physicomechanical characteristics of the material.

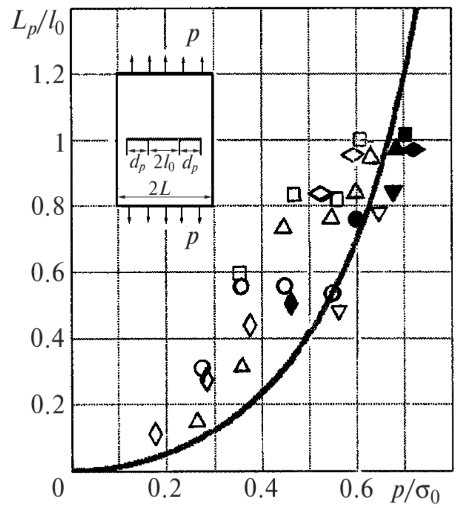

Fig. 5. Dependences of the length of the plastic zone d_p on p/σ_0 for D16AT alloy: the solid line corresponds to the data obtained by using relation (5) and the symbols mark the experimental data obtained by the method of digital correlation of speckle images.

Comparison of the Experimental Data with Theoretical Results

By using the solution of the elastoplastic problem of limiting equilibrium state for a stretched plate with central crack [12], we arrive at the following dependence of the length of the plastic zone d_p on the load p:

$$d_p = l_p - l_0, \qquad l_p = l_0 \sec\left(\frac{\pi p}{2\sigma_0}\right), \tag{5}$$

where l_0 is the initial crack length and σ_0 is the level of stresses in the plastic zone.

To compare the results of investigations, we compute the length of the plastic zone along the x-axis according to the distribution of strains near the crack tip (Fig. 5) depending on the active load p as follows: It is known that the total strain ε contains the elastic ε_e and plastic ε_p components, i.e., $\varepsilon = \varepsilon_e + \varepsilon_p$. According to the standard procedures of evaluation of the mechanical characteristics of materials [13], for the major part of materials, it is customary to assume that the conditional elastic strain $\varepsilon_e \approx 0.05\%$, which corresponds to the proportional stresses σ_p, and the level of plastic strains for stresses whose level is as high as $\sigma_{0.2}$ is equal to $\varepsilon_p \approx 0.2\%$. Since, in the plastic zone, the material is deformed beyond the conditional yield strength $\sigma_{0.2}$, the total strain is approximately equal to $\sim 0.25\%$. Thus, the length of the plastic zone d_p is assumed to be equal to the length of the segment along the x-axis from the crack tip to a point where $\varepsilon = 0.25\%$.

The experimental and theoretical dependences of the length of the plastic zone d_p on p/σ_0 are compared in Fig. 5, where the solid line reproduces the dependence given by relation (5). The analysis of the results reveals satisfactorily agreement between the theory and experiment at the time of crack start for $p = p_*$ and $d_{p_*} = d_*$.

The critical crack opening displacement δ_c is expressed via the length of the plastic zone d_* and the level of stresses σ_0 in the process zone by using the following approximate relation [12]:

$$\delta_c = \frac{8\sigma_0 d_*}{\pi E}. \tag{6}$$

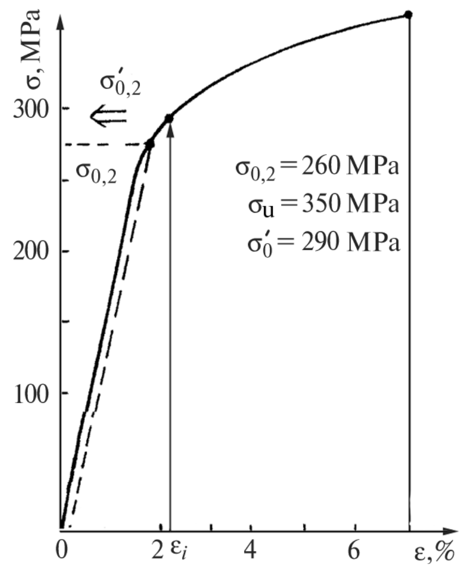

Fig. 6. Tensile stress–strain diagram of D16AT alloy.

Table 1. Critical Crack-Opening Displacements for a Specimen with Central Crack in Tension

l_0, mm	d_*/l_0	d_*, mm	δ_c, mm[1] $\sigma_{0.2}/\sigma'_0$	Experiment, $\delta_c = \varepsilon_c b$, mm	d_i/d_*
12.5	0.96	12.0	0.125/0.112	0.120	0.24
12.6	1.0	12.6	0.131/0.118	0.122	0.22
13.0	0.97	12.6	0.131/0.118	0.125	0.25
14.5	0.73	10.6	0.110/0.099	0.108	0.25
18.1	0.72	13.0	0.135/0.121	0.128	0.22
13.0	0.85	11.0	0.114/0.103	0.110	0.26

Comments: The values of δ_c in the numerator and denominator are computed for $\sigma_0 = \sigma_{0.2}$ and $\sigma_0 = \sigma'_0$, respectively. The results of investigations demonstrate the efficiency of the method of digital correlation of speckle images for the quantitative evaluation of elastoplastic displacements of the material in the vicinity of the crack tip and on the continuation of the crack. This enables one to establish the regularities of formation of the static process zone and determine the deformation characteristics (δ_c and d_*) of the crack resistance of materials.

By using the distribution of strains in the plastic zone (Fig. 4) and dependence (6), we can determine the parameter δ_c. As the level of stresses in the plastic zone σ_0, we either take the parameter $\sigma_{0.2}$ of the material or determine these stresses as follows:

The deformation curve in the plastic zone (Fig. 4) is used to determine the level of strains ε_i at the point B. Further, by using the tensile stress–strain diagram of the material (Fig. 6), we establish the conditional level of stresses σ_0' in the plastic zone, substitute it in relation (6), and compute the value of δ_c.

One can also use the data of direct measurements of the critical strain ε_c for the gage length $b = 1.28$ mm according to the data of digital correlation of speckle images of the specimen surface. The results obtained by using three different approaches are presented in Table 1. The best agreement between the theoretical results and the experimental data is obtained for the opening displacement δ_c in the cases where the stresses σ_0 are determined from the tensile stress–strain diagram (Fig. 6) according to the level of strains ε_i and the data of direct measurements. This illustrates the possibility of realization this approach as a standard procedure of evaluation of the characteristics of crack resistance of thin-sheet materials.

CONCLUSIONS

By the method of digital correlation of speckle images, we determined and evaluated the strains in the zone in front of the crack tip and compared the results with the data of strain-gage measurements. The distribution of elastoplastic displacements in front of the crack on the specimen surface was established. We revealed the fact that the elastoplastic strains ε_y are strongly nonuniform in the vicinity of the crack tip for three gage lengths. The maximum strain $\varepsilon_c = 8.6\%$ in the plane of the crack tip for measurements with a gage length of 1.28 mm is attained in the limiting equilibrium state. At a certain distance from the crack tip, the level of strains is lower by 35% but identical for the analyzed three gage lengths.

We revealed satisfactorily agreement between the results of measurements of the size of the plastic zone by the method of digital correlation of speckle images in the limiting equilibrium state and the theoretical (analytic) results.

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