Int J Mech Mater Des (2020) 16:663–666
https://doi.org/10.1007/s10999-020-09485-y

Comments on "Reflection of plane waves at the initially stressed surface of a fiber-reinforced thermoelastic half space with temperature dependent properties, Int J Mech Mater Des (2019) 15: 159–173"

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Received: 19 June 2019 / Accepted: 23 January 2020 / Published online: 31 January 2020

Abstract Snell's law provides a mathematical basis for the continuation of wave-field at a boundary. In the paper under review, theoretical formulation fails in applying this law correctly. Consequently, the whole reflection procedure goes off-track and the study wanders in incorrect domain. Researchers in the field may note the mathematical discrepancies identifed in this incorrect procedure.

Keywords Reflection · Snell's law · Anisotropic · Dissipative · Propagation · Attenuation

1 Introduction

Deswal et al. (2019) considered to study the reflection phenomenon for the incidence of plane harmonic waves at the plane boundary of initially stressed fiber-reinforced thermoelastic half space. Two different theories of generalized thermoelasticity are applied to compute the velocities, amplitude ratios and energy ratios for various reflected waves. Being thermoelastic, this medium behaves dissipative to the propagation of elastic waves. Then, with anisotropy induced by fiber-reinforcement and initial stress, this medium no longer remains isotropic. Mathematically, the complex coefficients D_j, through the relations (23)–(25) in Deswal et al. (2019), certify the dissipative character of the medium considered. The dependence of these coefficients on the propagation direction (i.e., angle θ) defines the anisotropic propagation. Hence, the study in question deals with the reflection phenomenon at the plane boundary of a dissipative anisotropic semi-infinite medium. Throughout this text, the numbers in parentheses identify the equations or relations, as specified in Deswal et al. (2019).

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2 Anisotropic attenuated propagation

The expressions (22) identify the time harmonic wave-field restricted to x–y plane of the considered medium. This medium being orthotropic, such a restriction demands three orthogonal symmetry planes to coincide with the coordinate planes in Euclidean space. Then, in the x–y plane, propagation is considered along a direction making angle θ with the x-direction. Roots of the equation (26), a cubic in V^2, defines the propagation of three bulk waves in considered medium. In this equation, the complex coefficients $(A,\ B,\ C)$ involve the angle θ, which implies the velocities (V_1, V_2, V_3) as functions of propagation direction. Hence, for any chosen θ, a complex $V_j(\theta)$ is the velocity of the corresponding wave for propagation along the chosen direction. Then, with complex V, the frequency $\omega = kV$ cannot be real unless k is a complex number (real multiple of conjugate to V), which should imply the attenuated wave-field throughout the x–y plane. Consequently, with complex k, the phase $k(-x\cos\theta + y\sin\theta) - \omega t$ in expression (22) becomes complex. With real θ, this phase defines same direction for propagation as well attenuation vector, i.e., homogeneous propagation of attenuated wave in considered medium. This represents only a special case of the inhomogeneous propagation, which is generally considered in any dissipative medium (Borcherdt 1982).

3 Reflection

A harmonic plane wave (qP_1) is incident at the plane surface $x = 0$, making an angle θ_0 with the x-direction. For this θ_0, the cubic equation (26) is solved into three complex velocities, $V_j, (j = 1, 2, 3)$. The direction dependence in anisotropy demands to consider the orientation of the angle θ_0 as well. Then, $V_0 = V_1(\theta_0)$ defines the velocity of qP_1 wave incident along the direction of θ_0 (to be measured anti-clockwise from positive y-direction in figure 1). Similarly, for the incidence of qP_2 (or, qP_3) wave along the direction of θ_0, $V_2(\theta_0)$ (or, $V_3(\theta_0)$) will denote the velocity of incident wave. It may be noticed that the coefficients D_j, and hence (A, B, C) in (26), change with the angle θ. The presence of $'\sin\theta'$ in the expressions for D_j implies different values for $V_1(\theta_1)$ and $V_1(\theta_0) = V_0(\theta_0)$, even when $\theta_1 = -\theta_0$. That means, only $\theta_1 = \theta_0$ (in magnitude as well as orientation) ensures that the velocity $V_1(\theta_1)$ of any reflected qP_1 wave is same as the velocity $V_0(\theta_0)$ of incident qP_1 wave. Unfortunately, with orientation, this equality can only be possible for normal incidence only. Even in magnitudes, this equality between angles of incidence and reflection cannot be assumed but needed to be derived from Snell's law. In other words, the 'angle of reflection = angle of incidence' rule does not hold for any reflection at the boundary of anisotropic medium. Consequently, the velocity of reflected wave will be different from the velocity of incident wave, even when the type of reflected wave is same as that of the incident wave.

3.1 Snell's Law

The identical horizontal slowness for all the waves at the plane surface $x = 0$ defines the Snell's law. That means, an identical $'k\sin\theta'$ for incident as well as all the reflected waves. Hence, the complex relations $k_0\sin\theta_0 = k_1\sin\theta_1 = k_2\sin\theta_2 = k_3\sin\theta_3$ define a generalised Snell's law. Else, the relations (29) make Snell's law with complex (anisotropic) velocities $V_j = V_j(\theta_j), (j = 0, 1, 2, 3)$. Hence, the relation $\frac{\sin\theta_0}{V_0(\theta_0)} = \frac{\sin\theta_r}{V_r(\theta_r)}$ should provide the propagation characteristics (propagation direction, attenuation direction, complex velocity) of each reflected wave (identified with a value of $r = 1, 2, 3$). Note that $V_0(\theta_0)$ can be extracted from the cubic equation (26), for fixed (known) incidence angle θ_0. Similarly, the velocity $V_r(\theta_r)$ of any reflected wave can come from (26), only when θ_r is known. Then, $V_r(\theta_0)$ cannot be considered as the velocities of reflected waves (except, for incidence along some particular symmetry directions).

3.2 Reflected wave: propagation characteristics

Consider the incidence of qP_1 wave with velocity $V_0 = V_1(\theta_0)$ along the incident angle θ_0. For the propagation characteristics of reflected qP_1 wave in x–y plane, it requires to solve $\frac{\sin\theta_0}{V_1(\theta_0)} = \frac{\sin\theta_1}{V_1(\theta_1)}$ for θ_1. But, through (26), this θ_1 is implicit in the calculation of $V_1(\theta_1)$, as complex velocity of reflected qP_1 wave. Unfortunately, presence of anisotropy blocks the isotropy bye-pass of $\theta_1 = \theta_0$. This requires solving the complex irrational equation $'\sin\theta_0 V_1(\theta_1) - V_1(\theta_0)\sin\theta_1 = 0'$ for θ_1, as a complex function of θ_0. Similarly, for any other reflected wave along the direction of θ_r ($r = 2$ or 3) in x–y plane, one needs to solve the relation $\frac{\sin\theta_0}{V_1(\theta_0)} = \frac{\sin\theta_r}{V_r(\theta_r)}$ for angle θ_r. Consequently, the direction (i.e., complex angle θ_r) and the complex velocity V_r of any reflected wave vary with the incident angle of qP_1 wave.

Finally, the reflection phenomenon demands to solve, in turn, $\frac{\sin\theta_0}{V_0(\theta_0)} = \frac{\sin\theta_r}{V_r(\theta_r)}, (r = 1, 2, 3)$, for θ_r. Obviously, for incident qP_1 wave, we have $V_0(\theta_0) = V_1(\theta_0)$. On starting with a fixed real θ_0 (assuming the incidence of homogeneous wave), the equation (26) gets three complex velocities $V_j(\theta_0), (j = 1, 2, 3)$. Incident wave is fixed through a value for j (e.g., $j = 1$ for qP_1 wave). Now, with known θ_0 (chosen) and complex velocity $V_1(\theta_0)$ of the incident qP_1 wave from (26), the relation $\frac{\sin\theta_0}{V_0(\theta_0)} = \frac{\sin\theta_r}{V_r(\theta_r)}$ (from Snell’s law) defines a complex irrational equation, $\sin\theta_0 V_r(\theta_r) - \sin\theta_r V_0(\theta_0) = 0$, in complex unknown θ_r.

There may not be any standard method for solving a complex irrational equation for its one or all roots. Moreover, getting a real root (θ_r) of the complex equation, $\sin\theta_0 V_r(\theta_r) - \sin\theta_r V_0(\theta_0) = 0$, may not be less than a magic. Then, for complex values of θ_r and $V_r(\theta_r)$ from (26), the corresponding wave is represented by a complex slowness vector ($\mathbf{p} = \hat{N}/V$; complex $\hat{N}$ such that $\hat{N}\cdot\hat{N} = 1$). Consequently, each reflected wave should propagate as an inhomogeneous wave, having different directions for propagation and attenuation vectors. Then, the figure 1, in Deswal et al. (2019), cannot be a correct exhibition of the reflection phenomenon in the considered anisotropic dissipative medium.

4 Consequences

With incorrect directions as well as velocities of all the reflected waves, the whole reflection process in Deswal et al. (2019) goes off-track. The incorrect coefficients (b_{ij} and Y_i) in (34) yield incorrect amplitude ratios (Z_j) as well as energy partition (E_i). Each of the complex Z_j is resolved to define amplitude ratio ($|Z_j|$) and phase shift ($\arg Z_j$) for the corresponding reflected wave. Thus, the plots in figures 1 to 7 are exhibiting the incorrect and incomplete attributes of reflected waves. Further, these complex Z_i yield the complex energy ratios (32)–(35) for different reflected waves. In fact, the energy partition at a boundary is represented through the real energy fluxes in the direction normal to the plane boundary (Achenbach 1973). That means, the plots of $|E_i|, (i = 1,2,3)$ in figure 8 show an incorrect partition of incident energy among the reflected waves at the boundary.

The phase velocities of all the reflected waves vary with the direction of incident wave as well. Hence, the various plots in figures 9 and 10 do not carry any meaning unless the incident direction is specified. Moreover, through the incorrect interpretation of Snell’s law, the complex velocities $V_j, (j = 1,2,3)$, cannot be accurate. Further, for any reflected wave in dissipative medium, magnitude of complex velocity keeps no physical significance. Rather, a complex V is used to define the phase velocity $[1/\Re(\mathfrak{B}^{-})]$ and attenuation coefficient $[-\Im(\mathfrak{B})/\Re(\mathfrak{B})]$.

5 Remarks

The way-out lies in solving the equations of motion in terms of vertical slowness for known horizontal slowness. This starts with writing the wave-field expression (22) in complex slowness vector, say $\mathbf{p} = (p_1, p_2, q)$, instead of wave velocity V. The resulting Christoffel system yields a homogeneous system of three equations in components of slowness vector. This system is solved into an algebraic equation of degree six in vertical slowness (q), while coeffcients being the functions of horizontal slowness (p_1, p_2).

An incident (homogeneous) wave is considered with known (chosen) direction of propagation towards the boundary. This direction enables to calculate the velocity of incident wave from (26) and hence to define the horizontal slowness (p_1, p_2). This horizontal slowness, identical for all the waves at a boundary (cf., Snell’s law), is used to calculate the coefficients of the sixth-degree algebraic equation in q. The six roots of this equation define six values for vertical slowness (q). Out of these six values, only three corresponds to the waves traveling away from the boundary, i.e., three reflected waves. Now, each vertical slowness value combines, in turn, with the common horizontal slowness to define the slowness vectors for three reflected waves. For a reflected wave, the corresponding slowness vector, $\mathbf{p} = (p_1, p_2, q)$, is resolved as $\mathbf{p} = \mathbf{P} + \imath\mathbf{A}$ to define the propagation vector ($\mathbf{P}$), attenuation vector ($\mathbf{A}$) and hence inhomogeneity (deviation of $\mathbf{A}$ from $\mathbf{P}$). A relation $\mathbf{p}\cdot\mathbf{p} = \frac{1}{V^2}$ gets the complex velocity for this reflected wave, resulting from the considered homogeneous incident wave.

For inhomogeneous (general case) incident wave, there is different procedure to calculate the slowness vector of incident wave, which provides the common horizontal slowness to define Snell’s law. The relevant procedures may be found in Cerveny and Psencik (2005) or Sharma (2008). A more recent study (Sharma 2015) on solving Snell’s law at the boundaries of various elastic media can also be helpful.

Funding This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Compliance with ethical standards

Conflict of interest The author declares that have no conflict of interest.

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