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On the exponential diophantine equation $\left(am^2+1\right)^x+\left(bm^2-1\right)^y=(cm)^z$ with $c \mid m$

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Abstract Let $a,\ b,\ c,\ m$ be positive integers such that $a+b=c^2, 2 \mid a, 2 \nmid c$ and $m>1$. In this paper we prove that if $c \mid m$ and $m>36c^3\log c$, then the equation $(am^2+1)^x+(bm^2-1)^y=(cm)^z$ has only the positive integer solution $(x,\ y,\ z)$=$(1,\ 1,\ 2)$.

Keywords Exponential diophantine equation · Existence of primitive divisor of Lucas and Lehmer numbers · Application of BHV theorem

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1 Introduction

In recent years, many papers investigated pure ternary exponential diophantine equations (see [6–15]).

Let $a,\ b,\ c,\ m$ be positive integers such that

$$a+b=c^2,\ 2 \mid a,\ 2 \nmid c,\ m>1. \tag{1.1}$$

In this paper we discuss the equation

$$\left(am^2+1\right)^x+\left(bm^2-1\right)^y=(cm)^z,\ x,y,z \in \mathbf{N}. \tag{1.2}$$

In 2012, Terai [13] proved that if $(a,\ b,\ c)=(4,\ 5,\ 3)$, then (1.2) has only the solution $(x,\ y,\ z)=(1,\ 1,\ 2)$ under some conditions. Recently, Wang et al. [17] improved that for

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$(a, b, c) = (4, 5, 3)$ and $3 \nmid m$. Then (1.2) has only the solution $(x, y, z) = (1, 1, 2)$. In this paper we prove a general result as follows:

Theorem 1.1 If $c \mid m$ and $m > 36c^3 \log c$, then (1.2) has only the solution $(x, y, z) = (1, 1, 2)$.

As a direct consequence we get:

Corollary 1.2 If $(a, b, c) = (4, 5, 3)$ and $3 \mid m$ and $m > 1068$, then (1.2) has only the solution $(x, y, z) = (1, 1, 2)$.

Thus, combining the result of [17] and our corollary we get that (1.2) is basically solved for $(a, b, c) = (4, 5, 3)$.

2 Preliminaries

For any nonnegative integer n, let F_n and L_n be the n-th Fibonacci and Lucas number, respectively.

Lemma 2.1 ([3]). The equation

$$F_n = X^2, \ n, X \in \mathbf{N}.$$

has only the solutions $(n, X) = (1, 1), (2, 1)$ and $(12, 12)$.

For any positive integer D, let $h(-4D)$ denote the class number of positive binary quadratic forms of discriminant $-4D$.

Lemma 2.2 ([4], Theorems 11.4.3, 12.10.1 and 12.14.3]).

$$h(-4D) < \frac{4}{\pi}\sqrt{D}\log\left(2e\sqrt{D}\right).$$

Let D, D_1, D_2, k be positive integers such that $\min\{D, D_1, D_2\} > 1$, $\gcd(D_1, D_2) = 1$, $2 \nmid k$ and $\gcd(D, k) = \gcd(D_1, D_2, k) = 1$.

Lemma 2.3 ([5], Theorems 1 and 2]). If the equation

$$X^2 + DY^2 = k^z, \ \gcd(X, Y) = 1, \ Z > 0, \ X, Y, Z \in \mathbf{Z} \tag{2.1}$$

has solutions (X, Y, Z), then every solution (X, Y, Z) of (2.1) can be expressed as

$$Z = Z_1 t, \ t \in \mathbf{N},$$
$$X + Y\sqrt{-D} = \lambda_1\left(X_1 + \lambda_2 Y_1\sqrt{-D}\right)^t, \ \lambda_1, \lambda_2 \in \{1, -1\},$$

where X_1, Y_1, Z_1 are positive integers satisfying $X_1^2 + DY_1^2 = k^{z_1}$, $\gcd(X_1, Y_1) = 1$ and $h(-4D) \equiv 0 (\operatorname{mod} Z_1)$.

Lemma 2.4 ([5], Lemma 1]). For a fixed solution (X, Y, Z) of the equation

$$D_1X^2 + D_2Y^2 = k^z, \ \gcd(X, Y) = 1, \ Z > 0, \ X, Y, Z \in \mathbf{Z}, \tag{2.2}$$

there exists a unique positive integer l such that

$$l = D_1\alpha X + D_2\beta Y, \ 0 < l < k,$$

where α, β are integers with $\beta X - \alpha Y = 1$.

The positive integer l defined as in Lemma 2.4 is called the characteristic number of the solution (X, Y, Z) and is denoted by $< X, Y, Z >$.

Lemma 2.5 ([5], Lemma 6]). If $< X, Y, Z >= l$, then $D_1 X \equiv -lY (\mathrm{mod} k)$.

For a fixed positive integer l_0, if (2.2) has a solution (X_0, Y_0, Z_0) with $< X_0, Y_0, Z_0 >= l_0$, then the set of all solutions (X, Y, Z) of (2.2) with $< X, Y, Z >\equiv \pm l_0 (\mathrm{mod}\, k)$ is called a solution class of (2.2) and is denote by $S(l_0)$.

Lemma 2.6 ([5], Theorems 1 and 2]). For any fixed solution class $S(l_0)$ of (2.2), there exists a unique solution $(X_1, Y_1, Z_1) \in S(l_0)$ such that $X_1 > 0$, $Y_1 > 0$ and $Z_1 \le Z$, where Z runs through all solutions $(X, Y, Z) \in S(l_0)$. The solution (X_1, Y_1, Z_1) is called the least solution of $S(l_0)$. Every solution $(X, Y, Z) \in S(l_0)$ can be expressed as

$$Z = Z_1 t,\ 2 \nmid t,\ t \in \mathbf{N},$$
$$X\sqrt{D_1} + Y\sqrt{-D_2} = \lambda_1 \left(X_1\sqrt{D_1} + \lambda_2 Y_1 \sqrt{-D_2}\right)^t,\ \lambda_1, \lambda_2 \in \{1, -1\}.$$

Lemma 2.7 ([2], Theorem 2]). Let (X_1, Y_1, Z_1) be the least solution of $S(l_0)$. If (2.2) has a solution $(X, Y, Z) \in S(l_0)$ satisfying $X > 0$ and $Y = 1$, then $Y_1 = 1$. Further, if $(X, Z) \neq (X_1, Z_1)$, then one of the following conditions is satisfied:

(i) $D_1 X_1^2 = \frac{1}{4}(k^{Z_1} \pm 1),\ D_2 = \frac{1}{4}(3k^{Z_1} \mp 1),\ (X, Z) = (X_1 |D_1 X_1^2 - 3D_2|,\ 3Z_1)$.

(ii) $D_1 X_1^2 = \frac{1}{4} F_{3r+3\varepsilon},\ D_2 = \frac{1}{4} L_{3r},\ k^{Z_1} = F_{3r+\varepsilon},\ (X, Z) = (X_1 |D_1^2 X_1^4 - 10 D_1 D_2 X_1^2 + 5D_2^2|,\ 5Z_1)$, where r is a positive integer, $\varepsilon \in \{1, -1\}$.

Let α, β be algebraic integers. If $\alpha + \beta$ and $\alpha\beta$ are nonzero coprime integers and $\frac{\alpha}{\beta}$ is not a root of unity, then (α, β) is called a Lucas pair. Further, let $A = \alpha + \beta$ and $C = \alpha\beta$. Then we have

$$\alpha = \frac{1}{2}\left(A + \lambda\sqrt{B}\right),\ \beta = \frac{1}{2}\left(A - \lambda\sqrt{B}\right),\ \lambda \in \{1, -1\}, \tag{2.3}$$

where $B = A^2 - 4C$. We call (A, B) the parameters of the Lucas pair (α, β). Two Lucas pairs (α_1, β_1) and (α_2, β_2) are equivalent if $\frac{\alpha_1}{\alpha_2} = \frac{\beta_1}{\beta_2} = \pm 1$. Given a lucas pair (α, β), one defines the corresponding sequence of Lucas numbers by

$$L_n(\alpha, \beta) = \frac{\alpha^n - \beta^n}{\alpha - \beta},\ n = 0, 1, \ldots. \tag{2.4}$$

For equivalent Lucas pairs (α_1, β_1) and (α_2, β_2), we have $L_n(\alpha_1, \beta_1) = \pm L_n(\alpha_2, \beta_2)$ $(n = 0, 1, \ldots)$. A prime p is called a primitive divisor of $L_n(\alpha, \beta)(n > 1)$ if

$$p \mid L_n(\alpha, \beta),\ p \nmid B L_1(\alpha, \beta) \ldots L_{n-1}(\alpha, \beta).$$

A Lucas pair (α, β) such that $L_n(\alpha, \beta)$ has no primitive divisors will be called an n-defective Lucas pair. Further, a positive integer n is called totally non-defective if no Lucas pair is n-defective.

Lemma 2.8 ([1], Theorem 1.4]). If $n > 30$, Then n is totally non-defective.

Lemma 2.9 ([16]). Let n satisfy $4 < n \le 30$ and $n \neq 6$. Then, up to equivalence, all parameters of n-defective Lucas pairs are given as follows:

(i) $n = 5$, $(A, B) = (1, 5), (1, -7), (2, -40), (1, -11), (1, -15), (12, -76), (12, -1364)$.

(ii) $n = 7$, $(A, B) = (1, -7), (1, -19)$.

(iii) $n = 8$, $(A, B) = (2, -24), (1, -7)$.

(iv) $n = 10$, $(A, B) = (2, -8), (5, -3), (5, -47)$.

(v) $n = 12$, $(A, B) = (1, 5), (1, -7), (1, -11), (2, -56), (1, -15), (1, -19)$.

(vi) $n \in \{13, 18, 30\}$, $(A, B) = (1, -7)$.

3 Proof of theorem

We now assume that $(x,\ y,\ z)$ is a solution of (1.2) with $(x,\ y,\ z) \neq (1,\ 1,\ 2)$. Since $c \mid m$, we have $cm \mid m^2$, and by (1.2), we get

$$\gcd\left(am^2+1,\ cm\right) = \gcd\left(bm^2-1,\ cm\right) = \gcd\left(am^2+1,\ bm^2-1\right) = 1. \tag{3.1}$$

Since $m > 1$ and $z > 2$, by (1.2), we have $0 \equiv (cm)^z \equiv (am^2+1)^x + (bm^2-1)^y \equiv 1 + (-1)^y (\bmod m^2)$ and

$$2 \nmid y. \tag{3.2}$$

Further, since $z \geq 3$, by (1.2) and (3.2), we get $0 \equiv (cm)^z \equiv (am^2+1)^x + (bm^2-1)^y \equiv (ax+by)m^2 (\bmod m^3)$ and

$$(ax+by) \equiv 0 (\bmod m). \tag{3.3}$$

Notice that $2 \mid a,\ 2 \nmid c$ and $2 \nmid b$ by (1.1). We see from (3.2) and (3.3) that $2 \nmid ax+by$ and

$$2 \nmid m. \tag{3.4}$$

So we have

$$2 \nmid cm,\ 2 \nmid am^2+1,\ 2 \mid bm^2-1. \tag{3.5}$$

We first consider the case of $2 \mid x$. Then, by (3.2), the equation

$$X^2 + (bm^2-1)Y^2 = (cm)^Z,\ \gcd(X,\ Y) = 1,\ Z > 0,\ X, Y, Z \in \mathbf{Z} \tag{3.6}$$

has the solution

$$(X,\ Y,\ Z) = \left((am^2+1)^{\frac{x}{2}},\ (bm^2-1)^{\frac{y-1}{2}},\ z\right). \tag{3.7}$$

By (3.1) and (3.5), applying Lemma 2.3 to (3.6) and (3.7), we have

$$z = Z_1 t,\ t \in \mathbf{N}, \tag{3.8}$$

$$\left(am^2+1\right)^{\frac{x}{2}} + \left(bm^2-1\right)^{\frac{y-1}{2}} \sqrt{1-bm^2} = \lambda_1 \left(X_1 + \lambda_2 Y_1 \sqrt{1-bm^2}\right)^t,$$
$$\lambda_1, \lambda_2 \in \{1, -1\}, \tag{3.9}$$

where $X_1,\ Y_1,\ Z_1$ are positive integers satisfying

$$X_1^2 + \left(bm^2-1\right) Y_1^2 = (cm)^{Z_1},\ \gcd\left(X_1,\ Y_1\right) = 1, \tag{3.10}$$

$$h\left(-4(bm^2-1)\right) \equiv 0 \left(\bmod Z_1\right). \tag{3.11}$$

If $2 \mid t$, let

$$X_2 + Y_2\sqrt{1-bm^2} = \left(X_1 + \lambda_2 Y_1 \sqrt{1-bm^2}\right)^{\frac{t}{2}}. \tag{3.12}$$

By Lemma 2.3, X_2 and Y_2 are integers satisfying

$$X_2^2 + \left(bm^2-1\right) Y_2^2 = (cm)^{\frac{Z_1 t}{2}} = (cm)^{\frac{z}{2}},\ \gcd\left(X_2,\ Y_2\right) = 1. \tag{3.13}$$

Substitute (3.12) into (3.9), we have $(am^2+1)^{\frac{x}{2}} + (bm^2-1)^{\frac{y-1}{2}}\sqrt{1-bm^2} = \lambda_1(X_2 + Y_2\sqrt{1-bm^2})^2$ and

$$\left(bm^2-1\right)^{\frac{y-1}{2}} = 2\lambda_1 X_2 Y_2. \tag{3.14}$$

By (3.1) and (3.13), we get $\gcd\left(X_2,\ bm^2-1\right)=1$. Therefore, we see from (3.14) that

$$|X_2|=1,\ |Y_2|=\frac{1}{2}(bm^2-1)^{\frac{y-1}{2}}. \tag{3.15}$$

Substitute (3.15) into (3.13), we get

$$1+\frac{1}{4}(bm^2-1)^y=(cm)^{\frac{z}{2}}. \tag{3.16}$$

Since $z>2$, we have $\frac{z}{2}\geq 2$. By (3.2) and (3.16), we get $0\equiv (cm)^{\frac{z}{2}}\equiv 1+\frac{1}{4}(bm^2-1)^y\equiv 1-\frac{1}{4}\equiv\frac{3}{4}(\bmod m^2)$ and $m^2\mid 3$, a contradiction. So we have

$$2\nmid t. \tag{3.17}$$

Let

$$\alpha=X_1+Y_1\sqrt{1-bm^2},\ \beta=X_1-Y_1\sqrt{1-bm^2}. \tag{3.18}$$

By (3.10) and (3.18), we have $\alpha+\beta=2X_1,\ \alpha-\beta=2Y_1\sqrt{1-bm^2},\ \alpha\beta=(cm)^{Z_1}$ and $\frac{\alpha}{\beta}$ satisfies $(cm)^{Z_1}(\frac{\alpha}{\beta})^2-2\left(X_1^2-(bm^2-1)Y_1^2\right)(\frac{\alpha}{\beta})+(cm)^{Z_1}=0$. It implies that $(\alpha,\ \beta)$ is a Lucas pair with parameters

$$(A,\ B)=\left(2X_1,\ -4(bm^2-1)Y_1^2\right). \tag{3.19}$$

Further, let $L_n(\alpha,\ \beta)$ $(n=0,\ 1,\ldots)$ be the corresponding Lucas numbers. By (2.3), (3.9) and (3.18), we have

$$(bm^2-1)^{\frac{y-1}{2}}=Y_1\left|\frac{\alpha^t-\beta^t}{\alpha-\beta}\right|=Y_1|L_t(\alpha,\ \beta)|. \tag{3.20}$$

We see from (3.19) and (3.20) that the Lucas number $L_t(\alpha,\ \beta)$ has no primitive divisors. Therefore, by Lemmas 2.8 and 2.9, we get from (3.17) and (3.19) that

$$t\leq 3. \tag{3.21}$$

By (3.8), (3.11) and (3.21), we have

$$z\leq 3h\left(-4(bm^2-1)\right). \tag{3.22}$$

Applying Lemma 2.2 to (3.22), we get

$$z<\frac{12}{\pi}\sqrt{bm^2-1}\log\left(2e\sqrt{bm^2-1}\right). \tag{3.23}$$

Further, since $b<a+b=c^2$, by (3.23), we have

$$z<\frac{12}{\pi}cm\log(2ecm). \tag{3.24}$$

On the other hand, since $2\mid x$, if $z=3$, then $(cm)^3>(am^2+1)^x\geq(am^2+1)^2>a^2m^4$, whence we get $c^3>a^2m>m>36c^3\log c$, a contradiction. It implies that $z\geq 4$ and $0\equiv(cm)^z\equiv(am^2+1)^x+(bm^2-1)^y\equiv(ax+by)m^2(\bmod m^4)$, whence we obtain $ax+by\equiv 0(\bmod m^2)$ and

$$ax+by\geq m^2. \tag{3.25}$$

Since $m>36c^3\log c$, by (1.2), we have

$$z>x\frac{\log(am^2+1)}{\log(cm)}>x,\ z>y\frac{\log(bm^2-1)}{\log(cm)}>y. \tag{3.26}$$

Hence, by (3.25) and (3.26), we get

$$c^2z = (a+b)z > ax + by \geq m^2. \tag{3.27}$$

The combination of (3.24) and (3.27), we have

$$m < \frac{12}{\pi}c^3 \log(2ecm). \tag{3.28}$$

But, since $m > 36c^3 \log c$, (3.28) is false. Thus, (1.2) has no solutions $(x,\ y,\ z)$ with $2 \mid x$.

Finally, we consider the case of $2 \nmid x$. Then, by (1.2) and (3.2), the equation

$$\left(am^2+1\right)X^2 + (bm^2-1)Y^2 = (cm)^Z,\ \gcd(X,\ Y) = 1,\ Z > 0,\ X, Y, Z \in \mathbf{Z} \tag{3.29}$$

has the solution

$$(X,\ Y,\ Z) = \left((am^2+1)^{\frac{x-1}{2}},\ (bm^2-1)^{\frac{y-1}{2}},\ z\right). \tag{3.30}$$

Let $l = \langle (am^2+1)^{\frac{x-1}{2}},\ (bm^2-1)^{\frac{y-1}{2}},\ z\rangle$. Since $cm \mid m^2$, by Lemma 2.5, l satisfies

$$am^2+1 \equiv (am^2+1)^{\frac{x+1}{2}} \equiv -l(bm^2-1)^{\frac{y-1}{2}} \equiv (-1)^{\frac{y+1}{2}} l (\mathrm{mod}\, cm). \tag{3.31}$$

On the other hand, since $(x,\ y,\ z) \neq (1,\ 1,\ 2)$, (3.29) has an other solution

$$(X,\ Y,\ Z) \neq (1,\ 1,\ 2). \tag{3.32}$$

Let $l_0 = \langle 1,\ 1,\ 2\rangle$. Then we have

$$am^2+1 \equiv -l_0 (\mathrm{mod}\, cm). \tag{3.33}$$

Obviously, since $z \geq 2$ for any solution $(x,\ y,\ z)$ of (3.29), the least solution of $S(l_0)$ is

$$(X_1,\ Y_1,\ Z_1) = (1,\ 1,\ 2). \tag{3.34}$$

Compare (3.31) and (3.33), we have $l \equiv \pm l_0 (\mathrm{mod}\, cm)$. It implies that the solution (3.30) belongs to $S(l_0)$. Therefore, using Lemma 2.6, we get from (3.30) and (3.32) that

$$\begin{aligned} z &= 2t,\ 2 \nmid t,\ t \in \mathbf{N}, \\ &\quad (am^2+1)^{\frac{x-1}{2}}\sqrt{am^2+1} + (bm^2-1)^{\frac{y-1}{2}}\sqrt{1-bm^2} \\ &= \lambda_1(\sqrt{am^2+1} + \lambda_2\sqrt{1-bm^2})^t,\ \lambda_1, \lambda_2 \in \{1, -1\}. \end{aligned} \tag{3.35}$$

By (3.35), we have

$$(bm^2-1)^{\frac{y-1}{2}} = \lambda_1\lambda_2 \sum_{i=0}^{\frac{t-1}{2}} \binom{t}{2i+1} (am^2+1)^{\frac{t-1}{2}-i}(1-bm^2)^i. \tag{3.36}$$

Further, since $2 \mid bm^2 - 1$ and $2 \nmid (am^2+1)t$, we see from (3.36) that $y = 1$ and $(bm^2 - 1)^{\frac{y-1}{2}} = 1$. It implies that (3.30) is a solution of $S(l_0)$ satisfying $X > 0,\ Y = 1$ and $(X,\ Z) \neq (X_1,\ Z_1) = (1,\ 2)$. Therefore, by Lemma 2.7, we get either

$$am^2+1 = (am^2+1)X_1^2 = \frac{1}{4}\left((cm)^2 \pm 1\right) \tag{3.37}$$

or

$$(cm)^2 = (cm)^{Z_1} = F_{3r+\varepsilon}. \tag{3.38}$$

When (3.37) holds, since $c \mid m$, we have $1 \equiv am^2+1 \equiv \frac{1}{4}\left((cm)^2 \pm 1\right) \equiv \pm\frac{1}{4}(\mathrm{mod}\, c^2)$. But, since $c^2 \geq 9$, it is impossible. On the other hand, since $cm > 1$ and $2 \nmid cm$, by Lemma

2.1, (3.38) is false. Thus, (1.2) has only the solution $(x,\ y,\ z) = (1,\ 1,\ 2)$ with $2 \nmid x$. the theorem is proved.

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References

1. Y. Bilu, G. Hanrot, P. M. Voutier (with an appendix by M. Mignotte), Existence of primitive divisors of Lucas and Lehmer numbers. J. Reine Angew. Math. **539**, 75–122 (2001)
2. Y. Bugeaud, T.N. Shorey, On the number of solutions of the generalized Ramanujan–Nagell equation. J. Reine Angew. Math. **539**, 55–74 (2001)
3. J.H.E. Cohn, On square Fibonacci numbers. J. Lond. Math. Soc. **39**(4), 537–540 (1964)
4. L.K. Hua, *Introduction to Number Theory* (Springer, Berlin, 1982)
5. M.H. Le, Some exponential diophantine equations I: the equation $D_1x^2 - D_2y^2 = \lambda k^z$. J. Number. Theory **55**(2), 209–221 (1995)
6. M.H. Le, A. Togbé, H.L. Zhu, On a pure ternary exponential diophantine equation. Publ. Math. Debr. **85**(3–4), 395–411 (2014)
7. F. Luca, On the system of diophantine equation $a^2 + b^2 = (m^2 + 1)^r$ and $a^x + b^y = (m^2 + 1)^z$, Acta Arith. **153**(4), 373–392 (2012)
8. T. Miyazaki, Generalizations of classical results on Jeśmanowicz' conjecture concerning Pythagorean triples. J. Number Theory **133**(2), 583–589 (2013)
9. T. Miyazaki, A note on the article by F. Luca "On the system of diophantine equations $a^2+b^2 = (m^2+1)^r$ and $a^x + b^y = (m^2 + 1)^z$". Acta Arith. **164**(1), 31–42 (2014)
10. T. Miyazaki, N. Terai, on the exponential diophantine equation $(m^2 + 1)^x + (cm^2 - 1)^y = (am)^z$, Bull. Aust. Math. Soc. **90**(1), 9–19 (2014)
11. T. Miyazaki, A. Togbé, The diophantine equation $(2am - 1)^x + (2m)^y = (2am + 1)^z$, Int. J. Number Theory **8**(8), 2035–2044 (2012)
12. T. Miyazaki, P.Z. Yuan, D.Y. Wu, Generalizations of classical results on Jeśmanowicz' conjecture concerning Pythagorean triples II, J. Number Theory **141**(1), 184–201 (2014)
13. N. Terai, On the exponential diophantine equation $(4m^2 + 1)^x + (5m^2 - 1)^y = (3m)^z$, Int. J. Algbra **6**(21–24), 1135–1146 (2012)
14. N. Terai, On Jeśmanowicz' conjecture concerning Pythagorean triples. J. Number Theory **141**(2), 316–323 (2014)
15. H. Yang, R.Q. Fu, A note on Jeśmanowicz' conjecture concerning primitive Pythagorean triples. J. Number Theory **156**(1), 183–194 (2015)
16. P.M. Voutier, Primitive divisors of Lucas and Lehmer sequences. Math. Comp. **64**, 869–888 (1995)
17. J.P. Wang, T.T. Wang, W.P. Zhang, A note on the exponential diophantine equation $(4m^2 + 1)^x + (5m^2 - 1)^y = (3m)^z$, Colloq. Math. **139**(1), 121–126 (2015)