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# Some Ways the Ways the World Could Have Been Can't Be

**Christopher James Masterman**[1,2,3]

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**Abstract**
Let *serious propositional contingentism* (SPC) be the package of views which consists in (i) the thesis that propositions expressed by sentences featuring terms depend, for their existence, on the existence of the referents of those terms, (ii) serious actualism—the view that it is impossible for an object to exemplify a property and not exist—and (iii) contingentism—the view that it is at least possible that some thing might not have been something. SPC is popular and compelling. But what should we say about possible worlds, if we accept SPC? Here, I first show that a natural view of possible worlds, well-represented in the literature, in conjunction with SPC is inadequate. Though I note various alternative ways of thinking about possible worlds in response to the first problem, I then outline a second more general problem—a master argument—which generally shows that any account of possible worlds meeting very minimal requirements will be inconsistent with compelling claims about *mere possibilia* which the serious propositional contingentist should accept.

**Keywords** Possible worlds · Propositional contingentism · Serious actualism · Propositions · Modality

## 1 Introduction

Here are two modal principles. First, it is impossible for an object to have a property, or stand in a relation, and not exist—a thesis often known as serious actualism. Although not uncontroversial, acceptance of serious actualism is widespread: Adams [1, 2], Plantinga [49, 50], Stephanou [61], Stalnaker [60], Williamson [64], Kment [31], and

✉ Christopher James Masterman
christophermasterman96@gmail.com

1 Faculty of Philosophy, University of Cambridge, Raised Faculty Building, Cambridge CB3 9DA, England

2 IFIKK, University of Oslo, PO Box 1020, Blindern 0315, Oslo, Norway

3 Department of Philosophy, University of St Andrews, The Scores, St Andrews KY16 9AL, Scotland

Jacinto [29] each defend some formulation of serious actualism.[1] The second principle: any proposition expressed by a sentence featuring terms ontologically depends on the referents of those terms. For instance, consider

(1) Robert Adams is tall

(1) contains the term 'Robert Adams'. Thus, according to this second principle, if Robert Adams were not to exist, the proposition expressed by (1) would likewise not.[2] Endorsements of this thesis are found, early on, in [53] and an early rigorous exploration of the view is found in [13]. The view is discussed in [7, 34, 38, 62], and [10], and defended at length in [1, 6, 17, 45, 46], and [58]. This second thesis, in conjunction with contingentism—the view that possibly there are things which might not have been something—entails one popular formulation of what is known as *propositional contingentism*. Accordingly, let's call the package of views consisting of contingentism and *both* of these two modal principles, *serious propositional contingentism* (SPC).

This paper is about what we should say about possible worlds if we accept SPC. Possible worlds enjoy a ubiquity in both philosophical and technical discussions of modality; but here I am interested in their use in *philosophical* accounts of modality. According to such accounts, modal-talk is systematically tied up with talk about possible worlds—entities which are genuine ways, or specifications of ways, the world could have been. Standardly, such accounts take it that possibility is just truth at some possible world and necessity is just truth at all possible worlds. Here, I will argue that there are significant issues in reconciling SPC with this kind of connection between worlds, propositions, and modality, as well as other closely related ones. Therefore, I argue, if we accept SPC, we cannot commit to sufficiently strong theses about possible worlds to underwrite philosophical accounts of modality in terms of possible worlds.

Others have come to similar conclusions in the literature. For instance, Fritz [18, 140–1] has argued that certain propositional contingentists are unable to capture generalised quantifier expressions involving possible worlds. More recently, in some of my earlier work, I showed that certain propositional contingentists cannot accept the Leibnizian biconditionals for possibility, i.e., $\Diamond p$ iff there is a world at which $p$ is true, see [35]. My arguments in this paper contribute to the stock of arguments against contingentist possible worlds in three ways. First, in contrast to the results in [18], my arguments rest on more minimal assumptions: they do not hinge on any particular understanding of what a possible world ultimately is—whether a proposition, complex state of affairs, set-theoretic entity, or so on—and appeal only to the truth of some relatively simple modal claims.[3] Second, my arguments target a view which presupposes the popular doctrine of serious actualism—the first modal principle outlined above.

[1] Serious actualism is endorsed, rather than defended, in [8, 30, 34, 37, 38, 53, 62], and [10]. For rarer arguments *against* serious actualism, see [14, 22, 51, 55], and [36]. Note that serious actualism, or as it is increasingly known as 'the being constraint', is trivially true if necessitism holds—the view that necessarily everything necessarily is something. Since Williamson and Jacinto are both necessitists, their arguments in [64] and [29] for serious actualism are *further* arguments for why the view holds independently of necessitism.

[2] Note that a sentence does not feature or contain a term in the relevant sense here, if the term is merely mentioned, e.g., 'The horse called 'Pegasus' is dead' does *not* feature the term 'Pegasus'.

[3] In [18], possible worlds are non-trivial, maximally strong propositions, as in [59].

This is unlike the results presented in [35]. Finally, again in contrast to both the results in [18] and [35], I here show that SPC is inconsistent with logically weaker connections than the standard Leibnizian biconditional for possibility or claims involving generalised quantification over possible worlds.

The general strategy in this paper is two-fold. First, in Section 2, I further discuss SPC and outline a contingentist model theory, adapting the model theory found in [13] and [35]. This allows us to precisely model the behaviour of contingently existing propositions. Second, I explore various ways we may supplement this model theory to model the modal behaviour of different conceptions of possible worlds. To begin, in Section 3.1, I outline a very natural and promising conception of possible worlds if we accept SPC. However, I show in Section 3.2 that such a conception in conjunction with SPC is inadequate, assuming some plausible constraints on adequacy. In Section 4, I discuss several promising ways the contingentist might respond. However, in Section 5, I present a general argument showing that any conception of possible worlds which meets some very minimal requirements will be inconsistent with compelling claims about *mere possibilia* which I show to plausibly follow from SPC.

## 2 Modelling Propositional Contingentism

To get to the heart of the issues that arise for SPC in connection with possible worlds, we first need a perspicuous way of talking about SPC. Here, this is provided by a formal framework—a contingentist model theory, adapted from the model theory found in [13] and [35], within which we can explore how various commitments of SPC fit together and interact with claims about possible worlds. At first, I will outline only how we model contingently existing propositions. Later, I will then look at how we can extend these models to represent different conceptions of possible worlds.

### 2.1 Some Preliminaries

First, we need a perspicuous language to express SPC. For this, let $\mathcal{L}_\Diamond$ be a two-sorted first-order modal language extended with a truth predicate, propositional abstraction operator, and actuality operator.[4] The lexicon of $\mathcal{L}_\Diamond$ is given by the following. First, for each natural number $n$:

- Individual variables: $x_n, y_n, z_n$.
- Propositional variables: $p_n, q_n, r_n$.
- Countably many $n$-place unsorted predicates: $R^n_1, R^n_2, \ldots$.

In addition, the lexicon includes:

[4] I formulate serious propositional contingentism here in a first-order setting. This is in contrast to recent work on propositional contingentism by Fritz [18–21] and Fritz and Goodman [24, 25] in which propositional contingentism is treated as a species of higher-order contingentism and regimented in the language of higher-order relational type theory. It is worth exploring these issues in a first-order setting, since higher-order settings generally and higher-order relational type theory particularly are neither mandated, nor wholly uncontroversial—see [3, 39, 65–66], and [33, 154-156] for criticisms of the latter, and see [41, 47], and [56] for criticisms of the former.

- Logical symbols: $\neg$, $=$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\forall$, $\exists$, $\Diamond$, $\Box$, @, and T.
- Brackets: (, ), [, ].

Notably here, the lexicon contains a logical predicate T for propositional truth and square brackets to allow for propositional abstraction—a means of introducing *terms* denoting the proposition expressed by an enclosed formula of $\mathcal{L}_\Diamond$. That is, in $\mathcal{L}_\Diamond$, the term '$[\phi]$' denotes the proposition expressed by $\phi$, where $\phi$ is any formula of $\mathcal{L}_\Diamond$.

We must be careful in specifying the formation rules for formulae of $\mathcal{L}_\Diamond$. As standard, the notion of a formula of $\mathcal{L}_\Diamond$ is defined using the notion of a term of $\mathcal{L}_\Diamond$. However, since $\mathcal{L}_\Diamond$ allows for propositional abstraction, formulae of $\mathcal{L}_\Diamond$ *themselves* generate complex terms of $\mathcal{L}_\Diamond$, i.e., propositional abstracts. Crucially, we cannot allow complex terms $[\phi]$ in $\mathcal{L}_\Diamond$ to display problematic nested abstraction, where some term $t$ occurring in $\phi$ is the *very same term* as $[\phi]$ itself. To achieve this, we first jointly define the notion of an $n$-level formula and the notion of $n$-level term and, second, define the terms and formulae in $\mathcal{L}_\Diamond$ *simpliciter*.

**Definition 1** All terms of $\mathcal{L}_\Diamond$ are assigned levels $n \geq 0$ such that all variables of $\mathcal{L}_\Diamond$ are 0-terms and, for any $n > 0$, if $\phi$ is an $n$-formula of $\mathcal{L}_\Diamond$, $[\phi]$ is a $n$-term. Here, an $n$-formula of $\mathcal{L}_\Diamond$ is obtained from the following recursive clauses and only contains $m$-terms such that $m < n$, where all and only the propositional variables and abstracts are propositional terms.

(i) $Rt_1...t_{n'}$ is an $n$-formula, for any $n'$-place $R$ and $m$-terms $t_1, ..., t_{n'}$ of any sort.
(ii) $t_1 = t_2$ is an $n$-formula, where $t_1$ and $t_2$ are $m$-terms of the same sort.
(iii) T$t$ is an $n$-formula, where $t$ is a propositional $m$-term.
(iv) If $\phi$ and $\psi$ are any $n$-formulae, then $\neg\phi$, $\Diamond\phi$, $\Box\phi$, $@\phi$, $\phi \dagger \psi$, $\exists v\phi$ and $\forall v\phi$ are $n$-formulae, where $\dagger$ is any binary logical connective of $\mathcal{L}_\Diamond$ and $v$ a variable of any sort.

Any $n$-formula of the first three kinds is atomic and all other $n$-formulae are non-atomic. Now, in what follows, we will speak generally of formulae and terms. This is defined:

**Definition 2** For $n \geq 0$, an $n$-term is a term. For $n > 0$, an $n$-formula is a formula.

Crucially, note that any complex term $[\phi]$ of $\mathcal{L}_\Diamond$ is an $n$-term, for some $n > 0$, and thus by definition, $\phi$ contains only $m$-terms, where $m < n$. Since, $[\phi]$ itself is an $n$-term, it cannot be any term contained in $\phi$. So, no complex term of $\mathcal{L}_\Diamond$ involves the problematic kind of nested abstraction discussed above, nor indeed do any of the extensions of $\mathcal{L}_\Diamond$ utilised in the paper.[5]

## 2.2 Modelling Serious Propositional Contingentism

Here, we are interested in modelling a contingentism which takes certain propositions to be contingent. In particular, the idea is that if proposition $p$ is expressed by a sentence featuring some terms, then $p$ ontologically depends on the existence of the referents of those terms. With the propositional abstraction operation in $\mathcal{L}_\Diamond$, a principle like this is

5 Thanks to an anonymous reviewer for emphasising the need for rigour here.

readily expressible. Following [38, 115], we read off what a proposition ontologically depends on from the syntax of the formula expressing the proposition. This allows us to express this thesis about ontological dependence between propositions and objects in $\mathcal{L}_\Diamond$ as the following scheme. In what follows, $\ulcorner\phi^{t_1,\ldots,t_n}\urcorner$ is schematic in (OD) for any formula of $\mathcal{L}_\Diamond$ which features exactly terms $t_1$, ..., $t_n$, where, generally, any term $t$ features in $\phi$ iff $t$ is either a free variable or a propositional abstract featuring in $\phi$.[6] Here, $\ulcorner \mathrm{E}t\urcorner$ abbreviates $\ulcorner\exists v(v = t)\urcorner$ throughout, where $v$ is a distinct variable from $t$, and $\ulcorner\Box\forall t_1\Box$, ..., $\Box\forall t_n\Box\urcorner$ stands for any sequence of $n$-many quantifiers flanked on both sides by the modal operator.

(OD) $\Box\forall t_1\Box, \ldots, \Box\forall t_n\Box(\mathrm{E}[\phi^{t_1,\ldots,t_n}] \leftrightarrow \bigwedge_{i\leq n} \mathrm{E}t_i)$

Here, $n$-ary conjunction $\bigwedge_{i\leq n}\phi_i$ is defined inductively: for $n = 0$, $\bigwedge_{i\leq n}\phi_i$ is an arbitrary closed tautology $\top$ and when $n > 0$, $\bigwedge_{i\leq n}\phi_i$ is $(\bigwedge_{i\leq(n-1)}\phi_i)\wedge\phi_n$. Thus, if $\phi^{t_1,\ldots,t_n}$ in (OD) features no terms, then $\bigwedge_{i\leq n}\mathrm{E}t_i$ is some closed tautology, ensuring that propositions expressed by $\phi$ featuring no terms are necessarily existent, as we should expect, see [35, 10].[7]

Now, there are many principles *like* (OD) which tie the existence of propositions to objects and it is typical for such principles to be formulated in terms of the notion of a singular proposition—a proposition which is said to be directly about some individuals. However, I here avoid using notions like direct aboutness or singularity for two reasons. First, such notions are often obscure, particularly the notion of direct aboutness, and making them tractable would take us too far away from the aims of this paper, see [26]. Second, and more importantly, making such notions precise will tie them too closely to a specific framework for understanding propositions. Here, I want to present *general* arguments for why propositional contingentism and a possible worlds theory of modality are in tension, not arguments which hinge on this or that way of understanding singularity or direct aboutness.[8]

[6] Note here that a formula $\phi$ features a term only if it is a free variable or a propositional abstract. That is, the notion of a formula $\phi$ featuring a term is more narrowly defined than the notion of formulae *containing* terms utilised in Definition 1. In the latter case, the bound variables of a formulae were also considered *contained* in the formula, e.g., $\exists p\exists q(q = p)$ is a 1-formula. In formulating (OD), only free variables and abstracts are said to be *feature* in $\phi$. In the case of $\phi$ with free variables $t_1$, ..., $t_n$, $\ulcorner\mathrm{E}[\phi]\urcorner$ abbreviates $\ulcorner\exists p(p = [\phi])\urcorner$, where $p$ is distinct from each of $t_1$, ..., $t_n$. This closely follows how I formulate the dependency claim in [35].

[7] We should be careful to distinguish (OD) from a weaker principle:

(OD′) $\Box\forall t_1, \ldots, \forall t_n\Box(\mathrm{E}[\phi^{t_1,\ldots,t_n}] \leftrightarrow \bigwedge_{i\leq n}\mathrm{E}t_i)$

Crucially, (OD) entails that some propositional abstracts pick out *impossible* propositions, i.e., propositions which do not even possibly exist. This occurs, if $\phi^{t_1,\ldots,t_n}$ features terms $t_i$ and $t_j$ which denote incompossibles. This is as many contingentists expect, see [13, 190], [55, 96], and [22] for discussion. (OD′), in contrast, fails to entail this and thus fails to adequately capture the contingentist's conception of propositions. Thanks to an anonymous reviewer for noting this.

[8] For instance, some understand singular propositions to be those propositions which contain, as constituents, the objects they are about, e.g., the proposition [Robert is tall] contains the *very man* Robert, see [30]. Indeed, Fitch and Nelson [16] *define* singular propositions this way. This obviously presupposes a structuralist view of propositions in which the structure of a proposition closely correlates to the relevant sentential structure. However, structuralism about propositions is neither mandated in understanding a principle like (OD), nor uncontroversial. As such, I avoid formulating the principle connecting propositions and those objects they are 'about' in this way.

The second component of the contingentism explored here—serious actualism—is captured in how we set up the models of the model theory. The full definition of the models follows at the end of this section, but it's worth noting and motivating some distinctive features now. The models extend the standard, variable domain Kripke models for first-order modal logic. As such, they include a set of points $W$, a binary accessibility relation $R$ on $W$, and functions which determine the domains and the extensions of predicates at worlds.[9] To validate serious actualism in the models, predicates are only assigned extensions at points $w \in W$ which are subsets of the domains of those points. The non-propositional domains in the models are defined as standard: in each model there is a function $D_i$ which maps each point in the model $w \in W$ to a non-empty set $D_i(w)$, the non-propositional domain of $w$.

What is distinctive about these models is how we accommodate contingent propositions. Following [13] and [35], we model propositions as pairs of sets of points in the model. The first set of the pair is the set of points at which the proposition is *true*—the truth-set—and the second is the set of points at which the proposition *exists*—the existence set. Each model contains a non-empty set $\mathbb{P}_{\mathfrak{M}}$ of ordered pairs of sets of $W$. For each point $w$, there is a non-empty subset $D_p(w) \subseteq \mathbb{P}_{\mathfrak{M}}$, the propositional domain of $w$. Since the second set of any $\langle \alpha, \beta \rangle \in \mathbb{P}_{\mathfrak{M}}$ is the set of points in which that proposition exists, we define $D_p$ as the function which maps any $w$ to the set of $\langle \alpha, \beta \rangle$ in $\mathbb{P}_{\mathfrak{M}}$ such that $w \in \beta$ and $\alpha \subseteq \beta$. This second condition guarantees that any proposition is only true at a point at which it exists—preserving the truth of serious actualism, as desired. Of course, abstracts must be assigned specific truth-set and existence-set pairs. Accordingly, the proposition $[\phi^{t_1,\ldots,t_n}]$ is modelled as the pair consisting of the truth set of $\phi^{t_1,\ldots,t_n}$, i.e., the set of points at which $\phi^{t_1,\ldots,t_n}$, and existence set of $\phi^{t_1,\ldots,t_n}$, i.e., the set of points at which $t_1$, ..., $t_n$ exist.[10]

It's worth noting that (OD) in conjunction with SPC is inconsistent with a coarse-grained view of propositions—a view that two propositions $p$ and $q$ are identical just in case necessarily $p$ is true if and only if $q$ is true—since propositions can be individuated both in terms of their truth *and* existence conditions, e.g., the propositions $[Fx \wedge \neg Fx]$ and $[Fy \wedge \neg Fy]$, though both true at no points, exist at different points if $x$ and $y$

[9] Throughout this paper I distinguish talk of points of evaluation, or simply points, in the model and talk of possible worlds, or simply worlds. Later, I introduce variables ranging over worlds in the object language and these should be kept apart from those in the metalanguage, i.e., $w$, $v$, $u$. The latter are only *points of evaluation*, whereas the former are interpreted as *genuine* possible worlds, i.e., a special sort of proposition.

[10] Crucially, $\mathbb{P}_{\mathfrak{M}}$ can vary from model to model. If $\mathbb{P}_{\mathfrak{M}}$ were simply $\mathcal{P}(W) \times \mathcal{P}(W)$, rather than a subset of the total set of ordered pairs of sets of $W$, then for any model $\mathfrak{M}$, the proposition $\langle \{w\}, W \rangle$—the proposition which exists at all points, but which is true at one point and one point only, i.e., true at $w$—would exist at $w$, for any $w \in W$. However, according to SPC, the existence of $\langle \{w\}, W \rangle$ at $w$ should not be guaranteed. If $w$ and $u$ differ only over *which* entities exist and generally agree on how many entities there are and how many entities satisfy certain predicates, there should be no proposition which exists in both $w$ and $u$, but which is true in only $w$. Any proposition, in this case, which is true in only $w$ must be capturing a truth about some specific entity which exists in $w$, but not in $u$, and thus must ontologically depend on that entity. As such, it cannot exist at all points. Allowing $\mathbb{P}_{\mathfrak{M}}$ to vary from model to model prevents potentially problematic propositions being guaranteed as features of arbitrary models. Thanks to an anonymous reviewer for pushing me to clarify this.

exist at different points.[11] However, (OD) alone only sets a lower-bound on fineness of grain. Here, I explore a principle like (OD), and SPC generally, by assuming that propositions are moderately finely grained in the sense that they are individuated such that if $t$ and $t'$ exist in all the same worlds and the propositions $[\phi(t)]$ and $[\phi(t')]$ are true in all the same worlds, then $[\phi(t)] = [\phi(t')]$. This is motivated by simplicity and helps to avoid conflating issues for contingentist possible worlds with specific issues for fine-grained propositions.[12]

### 2.3 The Model Theory

Here's the model theory in full detail. We define a general class of models and define truth in a model $\vDash$. Then we define a narrower sub-class of models $\mathbb{M}$ in which (OD) is satisfied by simply stipulating that this claim is valid in all models $\mathfrak{M} \in \mathbb{M}$.

First, the general class of models:

**Definition 3** A model $\mathfrak{M}$ is a tuple, $\langle W, R, \mathbb{P}_{\mathfrak{M}}, D_i, w*, v\rangle$, where $W$ is a non-empty set; $R$ is a binary relation on $W$; $\mathbb{P}_{\mathfrak{M}}$ is a non-empty subset of $\mathcal{P}(W) \times \mathcal{P}(W)$; $D_i$ is a function which maps each $w \in W$ to some non-empty set $D_i(w)$ such that $D_i(w) \cap \mathbb{P}_{\mathfrak{M}} = \varnothing$; and $w* \in W$ is a designated point. We let $D_p$ be the function which maps any $w$ to the set of $\langle \alpha, \beta\rangle$ in $\mathbb{P}_{\mathfrak{M}}$ such that $w \in \beta$ and $\alpha \subseteq \beta$. Letting $D(w)$ be the set $D_i(w) \cup D_p(w)$, the valuation function $v$ assigns to each non-logical $n$-place predicate $R^n$, and world $w \in W$, a set of $n$-tuples, $v(R^n)_w$:

(i) Each $\langle d_1, ..., d_n\rangle \in v(R^n)_w$ is such that $d_1 \in D(w), ..., d_n \in D(w)$

In the nomenclature, we evaluate formulae $\phi \in \mathcal{L}_{\Diamond}$ *in* models, *relative to* worlds, and *under* assignments. We define an assignment function as follows.

**Definition 4** An assignment $a$ is a function which maps each variable to some element $d \in \bigcup_{x \in W} D(x)$. Specifically, for any non-propositional variable, $y$: $a(y) \in \bigcup_{x \in W} D_i(x)$; and, for any propositional variable $p$: $a(p) \in \bigcup_{x \in W} D_p(x)$

Defining a general denotation function, relative to assignment $\delta_a$, and a general relation of truth in a model $\vDash$ is less straightforward than standard model theories for modal languages. As discussed, the denotation of propositional abstracts $[\phi]$ in part depends on truth value of $\phi$ relative to worlds in the model. Thus, again, care needs to be taken to ensure that $\delta_a$ and $\vDash$ are not defined in a problematic way. This is achieved by defining $\delta_a$ and $\vDash$ in stages—those stages being restricted denotation functions and relations of truth in a model for terms and formulae of a certain level, respectively.[13]

[11] This is borne out in the models to follow insofar as $\mathfrak{M} \nvDash \Box\forall p\Box\forall q\Box(\Box(\mathrm{T}p \leftrightarrow \mathrm{T}q) \leftrightarrow p = q)$ holds for any contingentist model $\mathfrak{M}$ such that $\mathfrak{M} \vDash$ OD. Thanks to an anonymous reviewer for noting errors in the original discussion of propositional granularity in the models.

[12] There should be the worry that more fine-grained views than this are incoherent, via Russell-Myhill style arguments, e.g., [54] and [44]. Such a paradox is typically taken to show that it is problematic to assume that propositions display a structure which closely corresponds to sentential structure; or at least it is problematic to take propositions to be individuated as fine-grainedly as a structured conception has them individuated, see [63] and [28] for discussion.

[13] Special thanks to an anonymous reviewing for pushing me to big rigorous on this point.

**Definition 5** Let $\delta^0_a$ be a function defined for all and only the 0-terms $t$ of $\mathcal{L}_\Diamond$ such that $\delta^0_a(t) = a(t)$, where $a$ is an assignment. Let $\vDash_1$ be truth in a model for 1-formulae. $\vDash_1$ is determined by the following principles, where $\pi_1$ is a projection function mapping an ordered pair, $\langle \alpha, \beta \rangle$, to $\alpha$. Clauses for truth functional connectives are as standard, and thus omitted.

(i) $\mathfrak{M}, w, a \vDash_1 Ft_1, ..., t_n$ *iff* $\langle \delta^0_a(t_1), ..., \delta^0_a(t_n)\rangle \in v(F)_w$
(ii) $\mathfrak{M}, w, a \vDash_1 \exists x\phi$ *iff for some* $d \in D_i(w)$: $\mathfrak{M}, w, a[x/d] \vDash_1 \phi$
(iii) $\mathfrak{M}, w, a \vDash_1 \exists p\phi$ *iff for some* $d \in D_p(w)$: $\mathfrak{M}, w, a[p/d] \vDash_1 \phi$
(iv) $\mathfrak{M}, w, a \vDash_1 t_1 = t_2$ *iff* $\delta^0_a(t_1) \in D(w)$ *and* $\delta^0_a(t_1) = \delta^0_a(t_2)$
(v) $\mathfrak{M}, w, a \vDash_1 \Diamond\phi$ *iff for some* $w' \in W$ such that $Rww'$: $\mathfrak{M}, w', a \vDash_1 \phi$
(vi) $\mathfrak{M}, w, a \vDash_1 \Box\phi$ *iff for all* $w' \in W$ such that $Rww'$: $\mathfrak{M}, w', a \vDash_1 \phi$
(vii) $\mathfrak{M}, w, a \vDash_1 \mathrm{T}t$ *iff* $\delta^0_a(t) \in D_p(w)$ and $w \in \pi_1(\delta^0_a(t))$
(viii) $\mathfrak{M}, w, a \vDash_1 @\phi$ *iff* $\mathfrak{M}, w*, a \vDash_1 \phi$

Letting $\ulcorner \delta^n_a(t_1, ..., t_n) \urcorner$ denote the set $\{\delta^n_a(t_1), ..., \delta^n_a(t_n)\}$, we define the $n$-th denotation function relative to an assignment $a$, $\delta^n_a$, and truth in a model for $n$-formula, $\vDash_n$ as follows.

**Definition 6** Let $\delta^n_a$ be a function defined for all and only $m$-terms $t$ of $\mathcal{L}_\Diamond$, where $m \leq n$:

(i) For any $m < n$, $\delta^n_a(t) = \delta^m_a(t)$
(ii) For any $m = n$, where $t$ is some $[\phi^{t_1,...,t_n}]$, $\delta^n_a(t) = \langle es^n_a(\phi^{t_1,...,t_n}), ts^n_a(\phi^{t_1,...,t_n})\rangle$ such that:
 (a) $es^n_a(\phi^{t_1,...,t_n}) = \{w \in W \mid \delta^{n-1}_a(t_1, ..., t_n) \subseteq D(w)\}$
 (b) $ts^n_a(\phi^{t_1,...,t_n}) = \{w \in es^n_a(\phi^{t_1,...,t_n}) \mid \mathfrak{M}, w, a \vDash_n \phi^{t_1,...,t_n}\}$

(ii)(a) defines the existence set of $\phi^{t_1,...,t_n}$, relative to assignment $a$ and (ii)(b) defines the truth set of $\phi^{t_1,...,t_n}$, relative to assignment $a$ and model $\mathfrak{M}$. Let $\vDash_n$ be truth in a model for all and only $m$-formulae, for $m \leq n$. $\vDash_n$ is determined by the principles resulting from replacing $\vDash_1$ with $\vDash_n$ and $\delta^0_a$ with $\delta^{n-1}_a$ throughout 5(i)–(viii).

**Definition 7** Let the general denotation function for all terms of $\mathcal{L}_\Diamond$, relative to assignment $a$, $\delta_a$, be $\bigcup_{i=1}^{n} \delta^i_a$ and let the relation of truth in a model, for all formulae of $\mathcal{L}_\Diamond$, $\vDash$, be $\bigcup_{i=1}^{n} \vDash_i$.

Note, for any $n \geq 0$, $\delta^n_a(t) = \delta_a(t)$, where $t$ is an $m$-term, where $m \leq n$ and, for any $n > 0$, $\vDash$ is constrained just as $\vDash_n$. That is, for any model $\mathfrak{M}$, $w \in W$ and assignment $a$, 5(i)–(vii) hold, provided $\vDash_1$ is replaced with $\vDash$ and $\delta^0_a$ with $\delta_a$.

As standard, a formulae $\phi \in \mathcal{L}_\Diamond$ is *valid in a model* $\mathfrak{M}$ just in case $\mathcal{M}, w, a \vDash \phi$, for any $w \in W$ and assignment $a$. A formulae $\phi$ is *valid* just in case $\phi$ is valid in any model $\mathfrak{M}$. Now, crucially Definitions 4–7 are not alone sufficient to guarantee that (OD) is valid. Thus, we stipulate that the relevant models are precisely those ones in which (OD) *is* valid.

**Definition 8** All $\mathfrak{M} \in \mathbb{M}$ satisfy Definition 3 and, for any formulae $\phi^{t_1,...,t_n} \in \mathcal{L}_\Diamond$:

(i) $\mathfrak{M} \vDash \mathrm{E}[\phi^{t_1,\ldots,t_n}] \leftrightarrow \bigwedge_{i \leq n} \mathrm{E}t_i$

Of course, by stipulation (OD) is valid in all $\mathfrak{M} \in \mathbb{M}$. Moreover, given the constraint on the denotation of propositional variables and abstracts, and the valuation function $v$, it's clear that the model theory validates serious actualism. Moreover, we can show that there are models $\mathfrak{M} \in \mathbb{M}$ and thus serious actualism, (OD), and contingentism, as understood using these models, are jointly coherent.[14]

**Proposition 1** *Let's say that* $\mathbb{P}_{\mathfrak{M}}$ *is full if* $\mathbb{P}_{\mathfrak{M}} = \mathcal{P}(W) \times \mathcal{P}(W)$.

(i) *Any* $\mathfrak{M}$ *satisfying Definition 3, where* $\mathbb{P}_{\mathfrak{M}}$ *is full, is an* $\mathfrak{M} \in \mathbb{M}$.
(ii) *For some* $\mathfrak{M} \in \mathbb{M}$, $\mathfrak{M} \vDash \Diamond\exists x \Diamond\neg\exists y(y = x)$

***Proof*** See Appendix.[15] □

We should also note, finally, that propositions in all $\mathfrak{M} \in \mathbb{M}$ are closed under the logical connectives—negation, conjunction, disjunction, and conditional—as we should expect.[16]

## 3 A Natural View of Contingent Worlds

Now that we have a model theory which captures SPC, we can use it to see what we ought to say about possible worlds if we accept SPC. To begin, I will look at one particularly natural way of understanding possible worlds in this context. Ultimately, I'll argue that it is inadequate. However, it is a useful place to start, allowing me to introduce how we make use of the model theory to answer questions about possible worlds, as well as discuss some subtle, preliminary issues with setting up a possible worlds account of modality for serious propositional contingentists.

### 3.1 The Natural View

Let's begin with the basics. Possible worlds are supposed to be the ways, or at least in some sense specifications of the ways, the world, in its totality, could have been.

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14 This result is important: the conception of propositions underlying (OD) is, to some degree, fine-grained and, as noted earlier, such views are notoriously difficult to formulate coherently, an observation going back to [54] and [44]. Moreover, as Kripke [32] observes, with unlimited abstraction and a truth predicate as in $\mathcal{L}_{\Diamond}$, one should worry about paradoxical results arising from predicates $P$ applying uniquely to a propositional abstract such as $[\forall p(Pp \rightarrow \neg \mathrm{T}p)]$.

15 Some proofs are relegated to an Appendix. Proofs of more substantial theorems remain in the main text.

16 This claims comes with a caveat: without propositional functions in the language, we cannot generally express claims such as 'For all $p$, if $p$ exists, then its negation exists'. However, we *can* express such claims for propositions identical to abstracts using schema, e.g., if $[\phi]$ exists, then its negation $[\neg\phi]$ exists. As we should expect, such claims hold in all models $\mathfrak{M} \in \mathbb{M}$. More precisely, for all models $\mathfrak{M} \in \mathbb{M}$ and any $\phi^{t_1,\ldots,t_n}$, $\psi^{t_1^*,\ldots,t_n^*} \in \mathcal{L}_{\Diamond}$, the following hold.

($\neg$) $\mathfrak{M} \vDash \mathrm{E}[\phi^{t_1,\ldots,t_n}] \leftrightarrow \mathrm{E}[\neg\phi^{t_1,\ldots,t_n}]$

($\dagger$) $\mathfrak{M} \vDash (\mathrm{E}[\phi^{t_1,\ldots,t_n}] \wedge \mathrm{E}[\psi^{t_1^*,\ldots,t_n^*}]) \leftrightarrow \mathrm{E}[\phi^{t_1,\ldots,t_n} \dagger \psi^{t_1^*,\ldots,t_n^*}]$, for any two-place Boolean connective $\dagger$

Proofs for ($\neg$) and ($\dagger$) are routine. Thanks to an anonymous reviewer for pushing me on this point.

Possible worlds have two core features. First, they are *possible*. That is, the plurality of possible worlds map out which ways the world *genuinely could* have been like. Second, they are maximal: each possible world informs us how everything, in its totality, could have been. Though not an essential feature, I assume here, as is widespread, that possible worlds are abstract entities. On this conception, then, there exist *many* possible worlds—possible worlds are just some sort of abstract entity playing the right kind of role in a theory of modality.

Typically, propositional contingentists, serious or non-serious alike, take possible worlds to be *themselves* contingent existents.[17] As Stalnaker notes, if we accept that there are object-dependent propositions and that some propositions depend, for their existence, on objects which are themselves contingent, we should conclude that:

> ...if possible worlds are maximal consistent propositions, or maximal consistent sets of propositions, it implies that there are possible worlds (or possible world-states) that exist only contingently [60, 22–23]

Of course, Stalnaker is right here; but this undersells the case for contingently existing possible worlds for the propositional contingentist. A similar conditional would hold, if possible worlds were identified instead with entities like a special sort of state of affairs or complex property. That is, the patterns of contingency exhibited by propositions are often taken to be mirrored by other entities like states of affairs and properties [64, 289]—if John doesn't exist, there should be no state of affairs or complex property involving John; or at least, no such states of affairs or complex properties exist, if no such propositions exist.

In fact, regardless of the kind of entity identified as possible worlds, at an abstract level, possible worlds share enough significant features with propositions for there to simply be a lack of systematicity if we take one, but not the other, to contingently exist. For the contingentist, some propositions are about, or involve, objects in a particularly direct way and thus depend on those objects for their existence. Likewise, possible worlds intuitively involve a variety of individuals in a particularly direct way: I exist in a variety of possible worlds and this does not mean that *some* person, matching my description exists in those worlds. There are a variety of possible worlds which are what they are partly in virtue of their relation to me. What grounds would the contingentist have for thinking that a world in which I exist could *itself* exist in my absence, if *propositions* about me could not exist in my absence?

Precisely how worlds depend, for their existence, on the existence of other objects can be fleshed out in various ways, depending on the precise ways in which we understand their core features, particularly their maximality. Here's one natural way of filling in the details.

**Strong Dependence (SD)** For every world $w$ and for every proposition $p$, either $w$ ontologically depends on $p$ or $w$ ontologically depends on the negation of $p$.

[17] Recently, Kment [31] and Stalnaker [60] defend contingent possible worlds. Early rigorous work on contingent possible worlds can be found in [11, 12], and [13]. In [35], possible worlds are understood as pluralities of propositions, some of which only contingently exist, though their contingency is not discussed at length *per se*.

(SD) is the strongest formulation of a dependence thesis between worlds and propositions which remains plausible. An immediate consequence of (SD), given SPC, is that every possible world ontologically depends on every non-propositional individual. This follows straightforwardly from two facts. First, that $\Box\forall x\exists p(p = [\mathrm{E}x])$, i.e., necessarily, for every individual, there is at least one proposition which ontologically depends on it. Second, that a proposition's ontological dependence is preserved under negation, i.e., if $p$ ontologically depends on $x$, then so too does the negation of $p$.

Though strong, (SD) is not without motivation. For starters, much of what is commonly said about maximality in the literature about possible worlds plausibly entails, given SPC, that they satisfy (SD). This is clearest when possible worlds are taken to be some sort of maximal collection of propositions. For instance, Benjamin Mitchell-Yellin and Michael Nelson [43, 1544] argue that possible worlds, understood as sets of propositions, 'should be maximal in the straightforward sense of including, for every actually existing proposition $p$, either $p$ or its negation'. Since sets depend, for their existence on their members, it is immediate that worlds, on this proposal, satisfy (SD). Indeed, for the *serious* propositional contingentist, the case for (SD) is particularly acute and goes beyond thinking of worlds as some sort of collection. For instance, Stalnaker proposes that possible worlds are *individual* propositions which either entail $p$ or its negation, for any proposition $p$. Yet, if we accept SPC, it's plausible that an arbitrary proposition $p$ depends, for its existence, on the existence of every proposition it entails, provided at least that entailment is a genuine relation between propositions. Thus, given SPC, Stalnaker's proposal plausibly entails (SD).[18]

We can express this conception more precisely in the model theory. To do so, it is natural to represent worlds in the model theory using a special sort of proposition. This conception of possible worlds places some clear constraints on such propositions. Each proposition *qua* a world should be true at one point of evaluation only—each possible world determines one way the world could be. To satisfy (SD) each world which exists at a point in the model must depend, for its existence, on every proposition which exists at that point in the model. This can be captured by modelling worlds as propositions with a particular existence set: the existence set of a world at a point $w$ should contain every point $v$ for which the propositional domain is equal to, or an expansion of, the propositional domain of $w$. That is, letting the set of *expanded or equal* points of $w$ be $e(w) := \{v \in W \mid D_p(w) \subseteq D_p(v)\}$, we define the domain of possible worlds at any point $w$, $D_{\overline{w}}(w)$, as follows.

$$D_{\overline{w}}(w) = \{\langle\alpha, \beta\rangle \in D_p(w) \mid |\alpha| = 1 \wedge \beta = e(w)\} \qquad (D_{\overline{w}})$$

With the domain of possible worlds for any point in the model defined, we can introduce and the define the semantics for quantification over possible worlds in the object language. Let $\mathcal{L}_{\Diamond}^{\overline{w}}$ be the extension of $\mathcal{L}_{\Diamond}$ with world-variables $\overline{w}_n, \bar{u}_n, \bar{v}_n$, for each

[18] More generally, SPC entails (SD) if there are essentially maximal relations, where an essentially maximal relation $R$ is such that (i) for any world $w$ and proposition $p$, $Rwp$ or $Rw{\sim}p$, where ${\sim}p$ is the negation of $p$; and (ii) for any proposition $p$ and world $w$, if $Rwp$, then necessarily, if $w$ exists, $Rwp$. If we interpret $R$ as the relation of *truth at*, then (i) expresses a common view about the maximality of worlds and (ii) expresses a plausible essentialist thesis about worlds and world-relative truth.

natural number $n$, which bind in the usual way to the quantifiers ∃ and ∀.[19] To evaluate such claims in models, let $a^+$ be an assignment function, identical to $a$ defined in Definition 4 except that also, for any world-variable, $\overline{w}$, $a^+(\overline{w}) \in \bigcup_{x \in W} D_{\overline{w}}(x)$. Truth in a model is defined as before, only this time in terms of $a^+$.[20] Crucially, though, we also stipulate that generally:

($\exists\overline{w}$) $\mathfrak{M}, w, a^+ \vDash \exists\overline{w}\phi$ *iff for some* $d \in D_{\overline{w}}(w)$: $\mathfrak{M}, w, a^+[\overline{w}/d] \vDash \phi$.
($\forall\overline{w}$) $\mathfrak{M}, w, a^+ \vDash \forall\overline{w}\phi$ *iff for all* $d \in D_{\overline{w}}(w)$: $\mathfrak{M}, w, a^+[\overline{w}/d] \vDash \phi$.

### 3.2 Are there enough possible worlds?

A conception of worlds which has them satisfying (SD) is, as I have stressed, natural and I have spent some time motivating this. At the very least, it follows from much of what is said in the literature about maximality, if we accept SPC. However, I will now show that surprisingly this conception of worlds in conjunction with SPC is inadequate.

Our starting point is a definition of adequacy for a conception of possible worlds. Here, I follow Menzel and Zalta [42], as I did in [35], and tie the adequacy of a conception of worlds to the Leibnizian biconditional for possibility. Loosely put:

(LP) $\Diamond\phi$ if and only if there exists an accessible possible world at which [$\phi$] is true.

We say that a conception of possible worlds is adequate only if there are enough possible worlds, according to this conception, such that (LP) holds generally and necessarily. (LP) holds necessarily if it remains true under arbitrary iterations of the necessity operator taking widest scope. (LP) holds generally, relative to a language, if it remains true substituting in any well-formed formula in that language for $\phi$. (Later, in Section 4, I discuss this account of adequacy given in terms of (LP) in more detail.) In what follows, I show that if (SD) holds, then (LP) holds necessarily and generally for $\mathcal{L}_{\Diamond}^{\overline{w}}$ only if necessitism, the negation of contingentism, is true. Therefore, an account of worlds satisfying (SD) in conjunction with SPC is inadequate.

We establish this by establishing what must hold in models which validate (LP) generally. First, we need to fix a way of formulating (LP) in the object language—we need to make explicit claims about *truth at a world* and *accessibility*. Formally, this is straightforward. However, before getting into the details there are some subtle issues about world-relative truth and SPC which we need to discuss at the outset. To

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19 Officially, $\mathcal{L}_{\Diamond}^{\overline{w}}$ is defined in precisely the same way as $\mathcal{L}_{\Diamond}$ in Definition 1, only with $\mathcal{L}_{\Diamond}^{\overline{w}}$ there is an extra stock of variables, treated as 0-terms, for possible worlds, i.e., $\overline{w}$, $\overline{v}$, $\overline{u}$. The full definition of $\mathcal{L}_{\Diamond}^{\overline{w}}$ is omitted: the details are unsurprising and nothing is gained by spelling out such a definition in full detail.

20 Of course, $\vDash$ is given in Definition 5–7 in terms of $\delta_a$, with $\delta_a$ being defined in terms of the series $\delta_a^0, \delta_a^1, ..., \delta_a^n$, where $\delta_a^0(t) = a(t)$, for any 0-term. Evaluating formulae of $\mathcal{L}_{\Diamond}^{\overline{w}}$ thus requires a relation of truth in a model defined similarly in terms of the series of denotation functions $\delta_{a^+}^0, \delta_{a^+}^1, ..., \delta_{a^+}^n$, where $\delta_{a^+}^0(t) = a^+(t)$. Such a definition is a routine extension of the kind of definition in Definitions 5–7, so I omit the full details. This does mean that, strictly speaking, here we utilise a notion of truth in a model distinct from that defined in Definitions 5–7. However, I refrain from further complicating the formalism to reflect this, since it will always be clear which notion of truth in a model is at play, indicated by the type of assignment function utilised.

begin, though it is natural to think that a proposition is true relative to a world just in case it *would* be true, were that world actual, this cannot be right if we accept SPC.[21] Possible worlds, on this understanding, tell us everything which *would* be true, were the world that way. This means that if a proposition is possible, on this understanding, it is, therefore, possibly true. However, the following result shows this idea to be inconsistent with SPC. Let $\mathbb{M}^\tau \subset \mathbb{M}$ be the class of models $\mathfrak{M}$ for which $\mathfrak{M} \vDash \Diamond\phi \rightarrow \Diamond \mathrm{T}[\phi]$, for any formula $\phi \in \mathcal{L}^{\overline{w}}_{\Diamond}$.

**Proposition 2** *Any* $\mathfrak{M} \in \mathbb{M}^\tau$*:* (i) $\mathfrak{M} \vDash \Box\forall x \Box \exists y(y = x)$ *and* (ii) $\mathfrak{M} \vDash \Box\forall p \Box \exists q(q = p)$.

***Proof*** See Appendix. □

Thus, if $\Diamond\phi$ is true only if $\Diamond \mathrm{T}[\phi]$ is true, then, given SPC, necessarily everything, propositional or non-propositional, is necessarily identical to something, i.e., necessitism is true.

This observation has prompted many to introduce a distinction between two sorts of world-relative truth—truth in, and truth at, a world.[22] Kit Fine [14, 163] draws the distinction 'in terms of perspective': with truth *at* a world we 'stand outside a world and compare the proposition with what goes on in the world in order to ascertain whether it is true', whereas with truth *in* a world, we 'must first enter with the proposition in the world before ascertaining its truth'. The thought is that we should evaluate propositions relative to worlds from the perspective of the actual world and thus resist thinking that what is important is whether the proposition *would* be true, were that world actual. As Robert Adams [1, 19] phrases it, we must evaluate what goes on at worlds in a way which '... [denies], then, that ⌜It is possible that $p$⌝ always implies that the proposition that-$p$ could have been true', accepting that some propositions will be true at worlds at which they *themselves* do not exist.

Here, we can model truth at a world using the relationship between formulae and points in the model. First, we extend $\mathcal{L}^{\overline{w}}_{\Diamond}$ to $\mathcal{L}^{\overline{w}+}_{\Diamond}$ which includes a new logical connective ⌜$\rhd$⌝ in the lexicon and we say that ⌜$t_{\overline{w}} \rhd [\phi]$⌝ is a formula of $\mathcal{L}^{\overline{w}+}_{\Diamond}$, for formula $\phi \in \mathcal{L}^{\overline{w}}_{\Diamond}$ if $t_{\overline{w}}$ is a world term. This is read as ⌜$[\phi]$ is true *at* $t_{\overline{w}}$⌝. Then we say the following, for every $\mathfrak{M}$, $w$, $a^+$, where $\pi^*_1$ is a projection function which takes a singleton as an argument and returns the sole member as the value.

$$\mathfrak{M}, w, a^+ \vDash \overline{w} \rhd [\phi] \textit{ iff } \mathfrak{M}, \pi^*_1(\delta_a(\overline{w})), a \vDash \phi \qquad (\rhd)$$

Intuitively, the idea is simple. A proposition $[\phi]$ is true at a world $\overline{w}$ just in case the formulae $\phi$ is true relative to the point of evaluation $\pi^*_1(\delta_a(\overline{w}))$, i.e., the only point relative to which the world $\overline{w}$ is true. That is, we understand truth at a world in terms of

[21] This idea is endorsed, most prominently, in (Plantinga, [48, 45–46], [50, 342]) and [52, 48–9]; but we also find it endorsed, in passing, in [4, 358] and [5, 53].

[22] This distinction is drawn and defended in [14, 163], [1, 20–32], [8], [37, 350–60], [38, 136–42], [10, 62], and [58].

whether the *formulae* holds at $w$ independently of whether it expresses a proposition at $w$.[23]

To express (LP) and show that (LP), SPC, and (SD) are jointly inconsistent, we also need to express the notion of accessibility between genuine possible worlds. Presently, it is simplest to define the accessibility of a proposition or world as its possible truth. Thus, (LP) is expressible in $\mathcal{L}_{\Diamond}^{\overline{w}+}$ as the following:

$$\Diamond\phi \leftrightarrow \exists\overline{w}(\Diamond \mathrm{T}\overline{w} \wedge \overline{w} \rhd [\phi]) \tag{LP}$$

Now we can state the crucial result.

**Theorem 1** *Any* $\mathfrak{M} \in \mathbb{M}$*: if* $\mathfrak{M} \vDash$ (LP)*, for any formula* $\phi \in \mathcal{L}_{\Diamond}^{\overline{w}}$, $\mathfrak{M} \vDash \Box\forall p\Box\exists q(q = p)$.

***Proof*** Suppose (i) $\mathfrak{M} \vDash \Diamond\phi \leftrightarrow \exists\overline{w}(\Diamond \mathrm{T}\overline{w} \wedge \overline{w} \rhd [\phi])$, for any $\phi \in \mathcal{L}_{\Diamond}^{\overline{w}}$ and (ii) $\mathfrak{M} \nvDash \Box\forall p\Box\exists q(q = p)$, for arbitrary $\mathfrak{M}$. If (ii), then: (iii) $\mathfrak{M}, w, a^{+} \vDash \Diamond\exists p\Diamond\forall q\neg(q = p)$, for some $w \in W$ and $a^{+}$. (iii) iff some $v \in W$ is such that $Rwv$: $\mathfrak{M}, v, a^{+} \vDash \exists p\Diamond\forall q\neg(q = p)$. In turn, this holds iff (iv) $\mathfrak{M}, v, a^{+}[x/d] \vDash \Diamond\forall q\neg(q = p)$, for some $d \in D_p(v)$. Given our supposition of (i), it follows that, if (iv) holds, then (v) $\mathfrak{M}, v, a^{+}[x/d] \vDash \exists\overline{w}(\Diamond \mathrm{T}\overline{w} \wedge \overline{w} \rhd [\forall p\neg(p = q)])$. Now, (v) is true only if there is some $d' \in D_{\overline{w}}(v)$ and $\mathfrak{M}, \pi_1^{*}(d'), a^{+}[x/d] \vDash \forall q\neg(q = p)$. Thus, (v) is only true if there is some $d' \in D_{\overline{w}}(v)$ such that $D_p(v) \nsubseteq D_p(\pi_1^{*}(d'))$. However, for any $\langle\alpha, \beta\rangle \in D_{\overline{w}}(v)$, $\beta = e(v)$, where $e(v) = \{w \in W \mid D_p(v) \subseteq D_p(w)\}$. Thus, there is *no* such $d' \in D_{\overline{w}}(v)$. Consequently, (i) is true only if (ii) is false. Thus $\mathfrak{M} \vDash \Box\forall p\Box\exists q(q = p)$ and this suffices for our result. □

Thus, we cannot accept SPC as well as (LP), as a general and necessary truth, if possible worlds satisfy (SD).

## 4 Alternative Views of Contingent Worlds

The result in the last section show very clearly what the serious propositional contingentist should *not* say about possible worlds: assuming SPC, (LP) as a general and necessary truth is inconsistent with (SD). But what are the alternatives?

On the face of it, there are two options for the contingentist. One option is to preserve the account of worlds discussed in the last section, and thus (SD), but loosen the constraints on an adequate theory of worlds. The second option is to reject the conception of worlds discussed in the last section and articulate alternative ways of understanding possible worlds. Let's take these options in turn.

There are limits to the first option. Although the question of what general constraints worlds must meet to be adequate is rarely addressed, one central theoretical role

23 That is, $[\phi]$ can be true at some world $\overline{w}$ such that $\pi_1^{*}(\delta_a(\overline{w})) = u$ even if $\mathfrak{M}, u, a \vDash \neg\mathrm{E}[\phi]$. If $\mathfrak{M}, u, a \vDash \neg\mathrm{E}t_1$ and $\pi_1^{*}(\delta_a(\overline{w})) = u$, then $\overline{w} \rhd [\neg Rt_1 ... t_n]$, where $R$ is an arbitrary $n$-place predicate. This is line with how theories of possible worlds incorporating truth at a world are standardly formulated, see [1, 23] and [38, 131], reflecting the fact that if $t_1$ fails to exist at $\overline{w}$, it must be true at $\overline{w}$ that $t_1$ fails to satisfy any predicates.

possible worlds play in accounts of modality which is rarely, if ever, questioned is that they satisfy the Leibnizian biconditionals. At an abstract level of description, possible worlds accounts of modality hold that, at the very least, for every possibility *there exists something* of some metaphysical importance to witness that possibility. Of course, that *something*, i.e, a world, must also be possible and maximal in some sense. But regardless of these details, if the contingentist is to vindicate the actual purpose to which possible worlds are put, they must not opt for any alternative account of adequacy which does not preserve the Leibnizian biconditionals, or some closely related principle.

As such, some alternative measures of adequacy are too weak. For instance, one might hold that an account of worlds is adequate just if it entails the following.

$$\Diamond\phi \leftrightarrow \Diamond\exists\overline{w}(\Diamond\mathrm{T}\overline{w} \wedge \overline{w} \rhd [\phi]) \qquad (\mathrm{LP}^{\Diamond})$$

(LP$^{\Diamond}$) is far from a trivial claim and, importantly, it is not ruled out by Theorem 1. However, it cannot *alone* suffice as a good measure of adequacy. Loosening the constraint to only require the merely possible existence of a world for every possibility takes us too far from the widespread assumptions about the role of possible worlds in our theorising about modality. The question for the contingentist here is not whether they can define some notion of a 'possible world' which is tied to possibility in some way—the question for the contingentist is whether they can define some *adequate* notion of a possible world. This question is about whether the contingentist can define a notion of possible world which can play the actual theoretical role worlds are taken to play and central to this role are substantive claims like the Leibnizian biconditionals—or, as Menzel and Zalta [42] call them 'The Fundamental Theorems of World Theory'.

Similar worries apply to the idea that the contingentist loosen the constraints and only require the Leibnizian biconditionals to hold for possibilities *which express existent propositions*. That is, we might take the satisfaction of the following scheme, or some variation of the following, as sufficient for an adequate theory of possible worlds.

$$\mathrm{E}[\phi] \rightarrow \big(\Diamond\phi \leftrightarrow \exists\overline{w}(\Diamond\mathrm{T}\overline{w} \wedge \overline{w} \rhd [\phi])\big) \qquad (\mathrm{LP}^{E})$$

The immediate worry here is that there is no guarantee that a theory entailing (LP$^{E}$), but no stronger claim, would be sufficiently comprehensive. That is, (LP$^{E}$) is consistent with there being possibilities $\Diamond\phi$ and no corresponding possible worlds, e.g., in cases where $\neg\mathrm{E}[\phi]$. To endorse (LP$^{E}$) as part of a *comprehensive* theory restrictions must be placed on the extent of possibility. However, the kinds of required restrictions on possibility are problematic. The most natural restriction would involve endorsing some scheme like (i) $\Box(\Diamond\phi \rightarrow \mathrm{E}[\phi])$, i.e., only claims expressing propositions can be possible. However, (i) is problematic. In S5—the most plausible logic for metaphysical modality—(i), in conjunction with (OD), entails $\Diamond\phi^{t_1,\ldots,t_n} \rightarrow \Box(\mathrm{E}t_1 \wedge, \ldots, \wedge \mathrm{E}t_n)$. Thus, to endorse (i), the serious propositional contingentist must problematically assume that there are only *de re* possibilities about necessary existents.[24]

[24] Thanks to anonymous referee for raising both (LP$^{\Diamond}$) and (LP$^{E}$) as potential alternatives to (LP).

(LP) is not, however, the logically weakest constraint which can be plausibly interpreted as requiring that the Leibnizian biconditionals hold. A particularly promising constraint is the idea that for any possibility $\Diamond\phi$, there is a corresponding *proposition* which *could* be a world at which $[\phi]$ is true. This suggests the following strategy for the serious propositional contingentist. First, letting $\ulcorner\mathcal{W}(p, [\phi])\urcorner$ abbreviate $\ulcorner\Diamond\exists\overline{w}(\overline{w} = p \wedge p \rhd [\phi])\urcorner$, the contingentist formulates the following constraint on worlds. For every metaphysical possibility there be is some accessible *proposition* which *could be* a world and which witnesses that possibility. That is:

$$\Diamond\phi \leftrightarrow \exists p(\Diamond \mathrm{T} p \wedge \mathcal{W}(p, [\phi])) \tag{LP$^-$}$$

The contingentist then holds that for a conception of worlds to be adequate, we only require that (LP$^-$) hold generally and necessarily.

This response is promising for a number of reasons. First, Theorem 1 doesn't rule out (LP$^-$) holding generally and necessarily, assuming SPC. Second, this approach, unlike (LP$^E$), does not involve a restriction on which kinds of possibility are witnessed by the theory of worlds, or which purported possibilities are genuine possibilities. Third, the approach of requiring (LP$^-$) does not involve any problematic loosening of the constraints on worlds, unlike (LP$^\Diamond$). Requiring (LP$^-$) to hold generally and necessarily still involves requiring that any possibility be witnessed by entities which exist—for any possibility, there is some special proposition witnessing that possibility—but it avoids the issues raised in the last section by not requiring that such propositions are important in that they *actually* qualify as worlds, but only possibly qualify as worlds.[25]

This, I take it, represents the most promising instance of the first kind of option for the contingentist—loosening the requirements on adequate worlds to respond to the issues raised by Theorem 1. What about the second option—how could the contingentist explicitly reject the conception of worlds discussed in the last section? One alternative is to define worlds as unique propositions: propositions which are true just in case the world is one particular way. This is the approach taken by Fine and Prior [11], and discussed in [9] and [23]. The thought is that what we strictly speaking need for world-like propositions are possibly true propositions which are maximal, specifying how the world could have been up to uniqueness. In the model-theoretic setting, this would motivate taking quantification over possible worlds to be quantification over propositions with singleton truth-sets, nothing more or less. That is, we discard $D_{\overline{w}}$ in understanding world-quantification. Instead, we say:

$$D_{\mathcal{U}}(w) = \{\langle\alpha, \beta\rangle \in D_p(w) \mid |\alpha| = 1\} \tag{$\mathcal{U}$}$$

25 In fact, we can equivalently think of the requirement that (LP$^-$) holds generally and necessarily as the requirement that (LP) hold, but for a conception of possible worlds in which a proposition qualifies as a world if it is possibly true and *possibly* maximal. If we define $\ulcorner\mathrm{W}(p)\urcorner := \ulcorner\Diamond\exists\overline{w}(\overline{w} = p \wedge \overline{w} \rhd [\phi])\urcorner$, then (LP$^-$) can be re-formulated as $\Diamond\phi \leftrightarrow \exists p\mathrm{W}(p, [\phi])$. Thus, requiring (LP$^-$) can be plausibly interpreted as requiring that *some formulation* of the Leibnizian biconditional holds. Note, given that $\Diamond$ takes a wide-scope in (LP$^\Diamond$), the same kind of reformulating cannot be done for (LP$^\Diamond$). This makes clear why we should be interested in whether the contingentist can secure (LP$^-$), but not in whether they can secure (LP$^\Diamond$).

Then, letting $a^{\mathcal{U}}$ be an assignment function, identical to $a$ defined in Definition 4, except that also, for any world-variable $\overline{w}$, $a^{\mathcal{U}}(\overline{w}) \in \bigcup_{x \in W} D_{\mathcal{U}}(x)$, truth in a model is defined as before, only this time in terms of $a^{\mathcal{U}}$.[26] Moreover, we stipulate that for any $\mathfrak{M}, w, a^{\mathcal{U}}$:

$$\mathfrak{M}, w, a^{\mathcal{U}} \vDash \exists \overline{w} \phi \quad \textit{iff} \quad \textit{for some } d \in D_{\mathcal{U}}(w) : \mathfrak{M}, w, a^{\mathcal{U}}[\overline{w}/d] \vDash \phi. \qquad (\exists^{\mathcal{U}})$$

$$\mathfrak{M}, w, a^{\mathcal{U}} \vDash \forall \overline{w} \phi \quad \textit{iff} \quad \textit{for all } d \in D_{\mathcal{U}}(w) : \mathfrak{M}, w, a^{\mathcal{U}}[\overline{w}/d] \vDash \phi. \qquad (\forall^{\mathcal{U}})$$

Of course, the heart of why this is an *alternative* to the conception of worlds discussed in the last section is that, on the face of it, not all unique propositions satisfy (SD). For instance, consider a two-point model $\mathfrak{M}_{\mathcal{U}} \in \mathbb{M}$, where $W = \{1, 2\}$, $R21$, $D_i(1) = \{3\}$, $D_i(2) = \{4\}$, $v(F)_1 = \{3\}$, and $v(F)_2 = \varnothing$. For simplicity, we assume $\mathbb{P}_{\mathfrak{M}}$ is full. Now, consider the proposition $[\exists x Fx]$. Under any assignment $a$, $\delta_a([\exists x Fx]) = \langle\{1\}, \{1, 2\}\rangle$ and thus $\delta_a([\exists x Fx]) \in D_{\mathcal{U}}(1) \cap D_{\mathcal{U}}(2)$.[27] $[\exists x Fx]$ is, in other words, a necessarily existent, unique proposition. However, it fails to satisfy (SD), since, for any $w \in W$:

$$\mathfrak{M}_{\mathcal{U}}, w, a[p/\delta_a([\exists x Fx])] \vDash \exists x \Diamond(\mathrm{E}p \wedge \neg \mathrm{E}x)$$

Immediately, then, this suggests that understanding worlds as unique propositions is promising. Again, consider $\mathfrak{M}_{\mathcal{U}}$. Since $R21$ and $\mathfrak{M}_{\mathcal{U}}, 1 \vDash \exists x Fx$, it follows that $\mathfrak{M}_{\mathcal{U}}, 2 \vDash \Diamond \exists x Fx$. However, $D_i(1) \cap D_i(2) = \varnothing$, and so there is no $d \in D_{\overline{w}}(2)$ such that $\pi_1^*(d) = 1$. This means that, for any $a^+$:

$$\mathfrak{M}_{\mathcal{U}}, 2, a^+ \nvDash \exists \overline{w}(\Diamond \mathrm{T}\overline{w} \wedge \overline{w} \rhd [\exists x Fx])$$

However, under any $a^{\mathcal{U}}$, $\delta_{a^{\mathcal{U}}}([\exists x Fx]) = \langle\{1\}, \{1, 2\}\rangle$. Thus, given $(\exists^{\mathcal{U}})$:

$$\mathfrak{M}, 2, a^{\mathcal{U}} \vDash \exists \overline{w}(\Diamond \mathrm{T}\overline{w} \wedge \overline{w} \rhd [\exists x Fx])$$

One further, equally radical option for the serious propositional contingentist, is to weaken the constraints on truth sets. It is well-observed that for views like SPC, a certain level of granularity of modal space must be given up. That is, for view like SPC, we have good reason for thinking that there are distinct ways the world could have been which are in a significant sense indistinguishable. For instance, Kit Fine considers the following case.

> Suppose there is some radioactive material in the actual world $w$ that just happens not to emit any particles from a certain time on but that might have emitted two particles of the same type at that time. These two particles, call them $\alpha$ and $\beta$, are presumably merely possible; they are not identical to any actual particles. And it is plausible to suppose that there is no actualistically acceptable

[26] Again, *strictly speaking* we require a new notion of truth in a model defined in terms of $\delta_{a^{\mathcal{U}}}$, with $\delta_{a^{\mathcal{U}}}$ being defined in terms of the series $\delta^0_{a^{\mathcal{U}}}, \delta^1_{a^{\mathcal{U}}}, \ldots, \delta^n_{a^{\mathcal{U}}}$, where $\delta^0_{a^{\mathcal{U}}}(t) = a(t)$, for any 0-term. I omit the full details here, as they are routine, and don't complicate the formalism to reflect this difference, see fn. 20.

[27] Since $\ulcorner \exists x Fx \urcorner$ doesn't contain any world-variables, we can use here, without any loss of generality, the basic assignment function $a$ given in Definition 4.

> means by which they might be distinguished. Of course, there is a possible world $w_1$ in which $\alpha$ is distinguished by one trajectory and $\beta$ another. But if there is such a world, then there is presumably another world $w_2$ just like it in which the trajectories are interchanged ... Thus we will be as unable to distinguish between the worlds as we are to distinguish between the particles themselves. [15, 217]

For Fine, this case spells trouble for any actualist understanding possibilist discourse—that is to say, any attempt to understand talk of what does not exist, but might have, whilst maintaining that everything actually exists. However, this case is relevant to SPC. For the serious propositional contingentist, there actually are no propositions directly about $\alpha$ or $\beta$, since neither actually exist. As such, there are no propositions to distinguish between worlds which differ only over which of $\alpha$ and $\beta$ play a certain qualitative role. This kind of case promptly motivates including propositions with non-singleton truth-sets as possible worlds. If there are two ways the world could have been, $w_1$ and $w_2$, which cannot be distinguished using propositions which actually exist, we should not then require that there is a possible world which can distinguish between $w_1$ and $w_2$. After all, if we accept SPC, there is no contentful distinction which can actually be made between the two.

This last option is a sketch at best. One difficulty in fleshing out this proposal is finding new constraints for possible worlds which get the balance right. We don't want to trivialise the class of possible worlds and allow any proposition to be a world. On the other hand, we want to allow for propositions to function as possible worlds without arbitrarily restricting the number of indistinguishable ways the world could have been. I don't propose to flesh this out any further, or any other proposal for that matter, since in what follows I want to present a general argument for why each and every one of these proposals, including any potential variations on these promising proposals, will fail to deliver an adequate possible worlds theory of modality, if we accept SPC.

## 5 Master Argument against Possible Worlds

At an abstract level, each of these promising proposals above attempted in different ways to modify how we understand the maximality of possible worlds. The second proposal of understanding worlds as unique propositions can be seen as rejecting the idea that a maximal proposition must ontologically depend a maximal class of propositions. The third proposal of understanding worlds as non-unique propositions can be seen as rejecting the idea that maximal propositions must determine a unique way the world could have been. Although the first proposal of requiring less of adequate theories of possible worlds does not on the face of it involve a change in maximality, by understanding world-quantification as modalised propositional quantification, this proposal in a sense allows the serious propositional contingentist to use non-maximal, though possibly maximal, propositions as possible worlds. The problem for each of these proposals is that they imply a deeply problematic principle for the serious propositional contingentist, one which is inconsistent with a compelling claim about *mere possibilia* plausibly entailed by SPC. Indeed, as I will argue, *any* theory of possible worlds will imply this deeply problematic principle, or an equally problematic analogous one, if it is adequate. For this reason, I dub the following argument the 'Master Argument'.

### 5.1 The Master Argument

The crux of the master argument is that accepting SPC entails accepting two claims about *mere possibilia* which are themselves jointly inconsistent with a minimal claim about propositions and possibility which is itself very plausibly entailed by any adequate theory of possible worlds. First, I outline this minimal claim about propositions and possibility and show why it very plausibly follows from any adequate theory of possible worlds. Second, I outline the two claims about *mere possibilia* and show why they follow from accepting SPC. Then, finally, I show that the two claims about *mere possibilia* are inconsistent with this minimal claim.

The minimal claim about propositions and possibility is that the following holds generally and necessarily for $\mathcal{L}_{\Diamond}$:

$$\Diamond\phi \rightarrow \exists p(\Diamond \mathrm{T}p \wedge \Box(\mathrm{T}p \rightarrow \phi)) \quad \text{(LPP)}$$

That is, if it is possible that $\phi$, then there exists a possibly true proposition $p$ and the truth of $p$ necessitates $\phi$ being the case. Just as with the standard biconditional for possibility, (LP), we say that (LPP) holds generally for $\mathcal{L}_{\Diamond}$ if (LPP) remains true under any substitution of formulae $\phi \in \mathcal{L}_{\Diamond}$. This claim is indeed minimal. Note that the claim here is only that (LPP) holds generally for $\mathcal{L}_{\Diamond}$—that is, the formal language defined in Section 2 which does not even feature quantification over possible worlds.

Unsurprisingly, if any of the approaches discussed in Section 4 are adequate, (LPP) holds generally for $\mathcal{L}_{\Diamond}$. First, if for every possibility $\Diamond\phi$, there is a unique possibly true proposition which necessitates $\phi$, then (LPP) holds generally, since, for any $\mathfrak{M} \in \mathbb{M}$:

For any $a^{\mathcal{U}}$: if $\mathfrak{M}, a^{\mathcal{U}} \vDash$ (LP), for any $\phi \in \mathcal{L}_{\Diamond}^{\overline{w}}$, then $\mathfrak{M}, a^{\mathcal{U}} \vDash$ (LPP), for any $\phi \in \mathcal{L}_{\Diamond}$

The above follows from the semantic clause for $\triangleright$ and the definition of the assignment function $a^{\mathcal{U}}$. Second, if for every $\Diamond\phi$, there is a proposition which could have been a world at which $[\phi]$ is true, then (LPP) holds generally, since, for any $\mathfrak{M} \in \mathbb{M}$:

For any $a^{+}$: if $\mathfrak{M}, a^{+} \vDash$ (LP$^{-}$), for any $\phi \in \mathcal{L}_{\Diamond}^{\overline{w}}$, then $\mathfrak{M}, a^{+} \vDash$ (LPP), for any $\phi \in \mathcal{L}_{\Diamond}^{\overline{w}}$

The above follows, again, from the semantic clause for $\triangleright$ and the definition of the assignment function $a^{+}$. Third, allowing worlds to have non-singleton truth sets containing indistinguishable points in the model still means that if (LP) or (LP$^{-}$) are general and necessary truths, then so is (LPP)—if these latter claims hold, then, for every possibility, there are possibly true propositions the truth of which necessitate $\phi$.

In fact, there's good reason to think that regardless of how we fill in the details here, if we have an adequate theory of modality in terms of possible worlds, (LPP) must hold generally. This is certainly the case if possible worlds are to be ultimately understood as propositions. After all, (LPP) requires only minimally that for any possibility, there is a corresponding possibly true proposition, the truth of which necessitates that possibility. However, broadly there are two good reasons for thinking that we can generalise even further than this and think that (LPP), or at least an analogous claim to (LPP), must hold if any theory of possible worlds is adequate.

First, if we have some adequate conception of possible worlds, then we should expect (LPP) to follow, since for every possibility $\Diamond\phi$, there should be possibly true propositions *about that conception of possible worlds* which necessitate $\phi$. For instance, suppose we follow Plantinga [48] and take possible worlds to be possible and maximally inclusive state of affairs—entities similar to propositions but which are not true or false, but rather obtain or fail to obtain. This can be spelled out as follows.

**SOA-Worlds:** Let O be a primitive one-place predicate, applying to states of affairs $s, s', s''$, ..., and understood as 'obtains'. Further, we say that for every state of affairs $s$, there is the complement of $s$, $\bar{s}$—the state of affairs which obtains if and only if $s$ fails to obtain. A *world* is a state of affairs $s$ such that:

(i) it possibly obtains, i.e., $\Diamond\, \mathrm{O}s$; and
(ii) it includes, for every state of affairs $s'$, either $s'$ or $\bar{s}'$, i.e., $\forall s'(\Box(\mathrm{O}s \rightarrow \mathrm{O}s') \vee \Box(\mathrm{O}s \rightarrow \mathrm{O}\bar{s}'))$

Letting $[\phi]^S$ be the state of affairs such that $\phi$, we say that $\phi$ is true at a world $s$ if $[\phi]^S$ is included in $s$. Now, if such an account were adequate, it would follow that

$$\text{If } \Diamond\phi, \text{ then there is a world } s \text{ which includes } [\phi]^S$$

(LPP) promptly follows, since for every possibility $\Diamond\phi$, there is a world $s$ such that:

$$\Diamond \mathrm{T}[\mathrm{O}s] \wedge \Box(\mathrm{T}[\mathrm{O}s] \rightarrow \phi)$$

Of course, a similar argument can be given for (LPP) from any account of worlds, provided they are adequate and are understood along similar lines: any adequate account of possible worlds furnishes us with possibly true propositions about the relevant worlds of that theory and those propositions necessitate $\phi$, for every possibility $\Diamond\phi$.

Beyond this, there is a broader philosophical reason for thinking that, for any adequate philosophical theory of worlds, at least some analogous claim *like* (LPP) follows. Again, consider a theory of possible worlds which takes them to be states of affairs. If such a theory is adequate, it follows that:

$$\Diamond\phi \rightarrow \exists s(\Diamond \mathrm{O}s \wedge \Box(\mathrm{O}s \rightarrow \phi)) \qquad (\mathrm{LPP}^S)$$

Of course, the problem I present below which stems from the fact that SPC entails various claims about *mere possibilia* and *propositions* does not strictly speaking entail that we should reject (LPP$^S$). However, as I noted earlier, we should expect the same patterns of contingency in other intensional entities such as states of affairs, complex properties, sets of propositions, and so on, as we find in propositions—there would simply be a lack of systematicity if propositions were taken to be contingent but states of affairs and complex properties, including those which analogously *involve* particular objects, were not taken to be contingent. Thus, if we accept SPC, we should accept a series of claims about *mere possibilia* and, e.g., state of affairs analogous to those which I outline below about *mere possibilia* and propositions. So, in principle, the issues raised by the master argument presented below should arise analogously regardless of whether we take possible worlds to be propositions or some other, closely related, entity.

This, so far, has shown the importance of a claim like (LPP) holding generally, if we want to endorse an adequate account of modality in terms of possible worlds. The crux of the master argument is that (LPP), or any analogous claim about relevant entities other than propositions, is inconsistent with what we should say about *mere possibilia* if we accept SPC. To begin, here's the intuitive shape of the problem, sticking to formulating matters in terms of propositions. On the face of it, if we accept SPC, we ought to accept that there might have been things which do not actually exist. Moreover, we should accept that the propositions which actually exist often under-determine the nature of those possible nonactual things. In particular, at least for some properties $F$, whilst there may be actual propositions which necessitate the truth of there being *something* which doesn't actually exist and which is $F$, there shouldn't in general be propositions which necessitate the truth of there being *something in particular* which is nonactual, and which is $F$. For instance, it's possible that there might have been, say, two electrons which do not actually exist. Considering only the actual propositions, we should take it that no such proposition necessitates one of those two electrons being a certain way and the other one not being a certain way. After all, if we accept SPC, then there is simply no content to saying that one of the electrons *in particular* is a certain way, whilst the other one *in particular* is not—no such electrons *in particular* actually exist for there to be such propositions.[28] Yet, we can show that, if we accept (LPP), then even in highly problematic cases, we must accept that there do exist such actual propositions.

Let's make this into a more precise problem. Let $\ulcorner \mathcal{H}x \urcorner$ stand for the claim that $\ulcorner x$ is part of some hydrogen atom$\urcorner$ and let $x^e$, $y^e$, $z^e$ be variables which range over electrons only. So, for instance, we understand $\ulcorner \forall x^e Fx^e \urcorner$ as the claim that all electrons are $F$. For our purposes here, this kind of quantification can be understood as a syntactic abbreviation—we say that $\ulcorner \forall x^e Fx^e \urcorner$ abbreviates $\ulcorner \forall x(\mathcal{E}x \rightarrow Fx) \urcorner$ and $\ulcorner \exists x^e Fx^e \urcorner$ abbreviates $\ulcorner \exists x(\mathcal{E}x \wedge Fx) \urcorner$, where $\ulcorner \mathcal{E}x \urcorner$ is understood as $\ulcorner x$ is an electron$\urcorner$. It will also be convenient to have $\ulcorner \Lambda(x, y) \urcorner$ abbreviate $\ulcorner @\mathrm{E}x \vee @\mathrm{E}y \urcorner$, i.e., that either $x$ or $y$ is actual. Now, with this in mind, the heart of the master argument is that if we accept SPC, we should accept the following two claims.

(P$^@$) $@\Diamond\exists x^e \exists y^e \big(\neg\Lambda(x^e, y^e) \wedge \mathcal{H}x^e \wedge \neg\mathcal{H}y^e\big)$

(*To be read*: Actually, it is possible that there are two non-actual electrons $x^e$ and $y^e$ and $x^e$ is part of a hydrogen atom, and yet $y^e$ is not.)

(N$^@$) $\Box\forall x^e \forall y^e \Big(\neg\Lambda(x^e, y^e) \rightarrow @\forall p(\Box(\mathrm{T}p \rightarrow \mathcal{H}x^e) \rightarrow \Box(\mathrm{T}p \rightarrow \mathcal{H}y^e))\Big)$

28 Compare, for instance, Stalnaker's discussion of similar cases in [60, 18–19], as well as the discussion in [18].

(*To be read*: Necessarily, for any electrons, $x^e$ and $y^e$, if $x^e$ and $y^e$ do not actually exist, then for every actual proposition $p$, $p$ necessitates $x^e$ being part of some hydrogen atom only if $p$ necessitates $y^e$ likewise.) ($P^@$) requires little in the way of motivation. If we accept SPC, we should accept ($P^@$), since we readily accept that there might have been two nonactual electrons and it is perfectly plausible that such electrons differ over whether they are a part of a hydrogen atom. ($P^@$) is a claim about *mere possibilia*, though to be clear such entities are handled here in an ontologically hygienic way: we talk about such mere possibilia using modalised quantification and do not smuggle in any illegitimate direct reference to the non-actual entities.

($N^@$), on the other hand, is a more complicated claim. To better understand why we should accept ($N^@$), if we accept SPC, we should first pause to clarify two matters. First, ($N^@$) is about electrons—paradigmatically mereologically simple objects. Thus, the serious propositional contingentist cannot appeal to actually existing, uniquely determining *parts* of the relevant non-actuals to motivate rejecting ($N^@$), as discussed in [64, 21].[29] Second, it is crucial that the predicate $\ulcorner\mathcal{H}\urcorner$ is interpreted as being part of *some* hydrogen atom. This means that ($N^@$) involves only a qualitative predicate—some electron satisfies $\mathcal{H}$ if it is part of *some* hydrogen atom. Of course, $\mathcal{H}$ is not unique in that a claim like ($N^@$) holds true of it: many qualitative properties applicable to some mereologically simple entity would suit our purposes here. The important point is solely that insofar as $\mathcal{H}$ is qualitative, there ought to be no actual propositions which can distinguish between one particular non-actual entity satisfying $\mathcal{H}$ and another particular non-actual entity not satisfying $\mathcal{H}$.

Here are two arguments for why SPC implies ($N^@$). First, consider the contraposition of ($N^@$), applying the law of actuality in which $\neg@\phi \leftrightarrow @\neg\phi$, for any $\phi \in \mathcal{L}_\Diamond$:

$$(\overleftarrow{\mathrm{N}}^{@})\quad \Box\forall x^e\forall y^e\Big(@\exists p(\Box(\mathrm{T}p \rightarrow \mathcal{H}x^e) \wedge \neg\Box(\mathrm{T}p \rightarrow \mathcal{H}y^e)) \rightarrow \Lambda(x^e, y^e)\Big)$$

(*To be read*: Necessarily, for any electrons $x^e$ and $y^e$, if there actually is a proposition $p$ which necessitates $x^e$ being part of a hydrogen atom but does not necessitate $y^e$ being so, then either $x^e$ actually exists or $y^e$ actually exists.) Now, suppose it's possible that there are some individuals and that there actually exists a proposition such that necessarily, if that proposition is true, $\mathcal{H}x^e$, yet it is not the case that necessarily if that proposition is true, $\mathcal{H}y^e$. This means that, using only actual propositions, we are able to distinguish between $x^e$ and $y^e$. For such $x^e$ and $y^e$ to be distinguishable in this sense, is for there to actually be propositions the truth of which draw a difference *particularly* between $x^e$ and $y^e$. If we accept SPC, then there only is an actual proposition distinguishing between two electrons $x^e$ and $y^e$ like this if at least one of $x^e$ and $y^e$ actually exist, i.e., $\Lambda(x^e, y^e)$.

Second, consider what we must accept, if we *reject* ($N^@$):

$$(\neg\mathrm{N}^{@})\quad \Diamond\exists x^e\exists y^e\Big(\neg\Lambda(x^e, y^e) \wedge @\exists p(\Box(\mathrm{T}p \rightarrow \mathcal{H}x^e) \wedge \neg\Box(\mathrm{T}p \rightarrow \mathcal{H}y^e))\Big)$$

[29] The kind of case discussed here is like the following. Suppose a knife *handle* $h$ and two knife blades $b_1$ and $b_2$ exist at $w$. Were $h$ attached to $b_1$, a knife $k_1$ would exist. Were the same handle $h$ attached to $b_2$, a second distinct knife $k_2$ would exist. Suppose that there is a way the world could have been $v_1$ in which $k_1$ exists, and $v_2$ in which $k_2$ exists—that is, both are ways the world could be in which the knives are assembled. Although neither $k_1$ nor $k_2$ exist at $w$, there is still a proposition at $w$ which distinguishes $v_1$ and $v_2$: the proposition that $b_1$ and $h_1$ are assembled.

(*To be read:* It is possible that there are two electrons $x^e$ and $y^e$ which do not actually exist and actually there exists a proposition $p$ which necessitates $x^e$ being $\mathcal{H}$ but which fails to necessitate $y^e$ being $\mathcal{H}$.) In other words, to accept ($\neg N^@$) is to accept that there is an actual proposition which is able to distinguish between the two non-actual electrons. However, given SPC, it is obscure how some actual proposition necessitates $x^e$ being $\mathcal{H}$ and yet fails to necessitate $y^e$ being $\mathcal{H}$. To see this difficulty clearly, contrast ($\neg N^@$) with the following, more acceptable claims for the serious propositional contingentist.

$$\text{(N1)}\ \Diamond\exists x^e \exists y^e \Big(\neg\Lambda(x^e, y^e) \wedge \exists p(\Box(\mathrm{T}p \rightarrow \mathcal{H}x^e) \wedge \neg\Box(\mathrm{T}p \rightarrow \mathcal{H}y^e))\Big)$$

(*To be read:* It is *both* possible that there are two non-actual electrons $x^e$ and $y^e$ and that there is a proposition $p$ which necessitates $x^e$ being $\mathcal{H}$ but which fails to necessitate $y^e$ being $\mathcal{H}$.)

$$\text{(N2)}\ \Diamond\exists x^e \exists y^e \Big(\neg\Lambda(x^e, y^e) \wedge @\exists p(\Box(\mathrm{T}p \rightarrow \exists x^{e*}\exists y^{e*}(\neg\Lambda(x^{e*}, y^{e*}) \wedge \mathcal{H}x^{e*} \wedge \neg\mathcal{H}y^{e*}))\Big)$$

(*To be read:* It is possible that there are two non-actual electrons $x^e$ and $y^e$ and actually there exists a proposition $p$ which necessitates there being *some* two non-actual electrons $x^{e*}$ and $y^{e*}$ such that $\mathcal{H}x^{e*}$ and $\neg\mathcal{H}y^{e*}$.) Now, (N1) is acceptable: it is consistent with SPC that there might both have been two electrons $x^e$ and $y^e$ as well as some proposition which necessitates $x^e$ being part of a hydrogen atom, but which does not necessitate $y^e$ being part of a hydrogen atom. In this case, the proposition would be something like $[\mathcal{H}(e)]$, where $e$ is some electron which *would* exist if $[\exists x^e \neg @\mathrm{E}x^e]$ were true. Of course, $[\mathcal{H}(e)]$ does not actually exist, but (N1) only requires that a proposition like this *would* exist, *were* some non-actual electrons to exist. Likewise with (N2): it is consistent with SPC that there might have been two electrons which do not actually exist and there actually is a proposition such that, necessarily, it is true only if there exists *some* two non-actual electrons, one of which satisfies $\mathcal{H}$ and the other which doesn't satisfy $\mathcal{H}$. Rather trivially, the proposition $[\exists x^{e*}\exists y^{e*}(\neg\Lambda(x^{e*}, y^{e*}) \wedge \mathcal{H}x^{e*} \wedge \neg\mathcal{H}y^{e*})]$ would suit.

However, the same is not true of ($\neg N^@$). We can frame the difference clearly in terms of possible worlds in the following way. ($\neg N^@$) requires that there is a possible world $w$ at which there are some things $x^e$ and $y^e$ which do not exist at the actual world $w^*$ and there is at least one proposition at $w^*$ which necessitates $x^e$ being a part of a hydrogen atom and yet does not necessitate $y^e$ being part of a hydrogen atom. However, there actually being the propositional resources required to satisfy ($\neg N^@$) is simply antithetical to SPC: such propositions, if they actually exist at all, depend, for their existence, on $x^e$ and $y^e$.

Of course, such propositions would be available at the actual world and ($N^@$) would be false, if there are qualitative essences of individuals—properties which are both qualitative and which are exemplified uniquely by a particular individual, if it exists. If such properties existed, there would be a unique way of specifying that $x^e$ *rather than* $y^e$ was part of a hydrogen atom. Now, it is doubtful that there are no qualitative essences whatsoever. As Menzel [40], notes, for instance, the number two necessarily exemplifies the property of being the smallest prime number. The property of being the smallest prime number is plausibly a qualitative property and thus can function as to pick out uniquely the number two necessarily. However, that being said, such examples are special cases and accepting SPC means we should reject the thesis that there are qualitative essences for all individuals, including contingent ones. First, there are broad worries about the legitimacy of qualitative essences in any propositional contingentist framework—it is under-explained how properties are able to 'lock on' to the objects which they are the essences for in the absence of those objects.[30] Second, there is another worry that allowing necessarily existent essences for all individuals undermines much of the motivation for SPC, or propositional contingentism more broadly [60]. If propositions about necessary, purely qualitative essences which are able to 'lock on' to individual $i$ specifically can exist in the absence of $i$, then why should we dismiss the legitimacy of meaningful talk about $i$ in particular, in the absence of $i$.

In summary, then, we have good reason to think that both ($N^@$) and ($P^@$) follow from SPC. Now, the problem is that ($P^@$) and ($N^@$) are inconsistent with (LPP). More precisely, ($P^@$) and ($N^@$) are inconsistent with (LPP) holding generally for $\mathcal{L}_\Diamond$ in all models with a reflexive accessibility relation $\mathfrak{M}^{Ref}$.

**Theorem 2** *For any $\mathfrak{M}^{Ref}$, if, for any $\phi \in \mathcal{L}_\Diamond$, $\mathfrak{M}^{Ref} \vDash$ LPP, then $\mathfrak{M}^{Ref} \nvDash \mathrm{N}^@ \wedge \mathrm{P}^@$*

***Proof*** Suppose, for *reductio*, that $\mathfrak{M}^{Ref} \vDash$ LPP, for any $\phi \in \mathcal{L}_\Diamond$, and $\mathfrak{M}^{Ref} \vDash \mathrm{N}^@ \wedge \mathrm{P}^@$, for arbitrary $\mathfrak{M}^{Ref}$. If $\mathfrak{M}^{Ref} \vDash \mathrm{P}^@$, then, for arbitrary $w \in W$ and $a$:

$$\mathfrak{M}^{Ref}, w, a \vDash @\Diamond\exists x^e \exists y^e \big(\neg\Lambda(x^e, y^e) \wedge \mathcal{H}x^e \wedge \neg\mathcal{H}y^e\big) \tag{i}$$

(i) entails, for some $v \in W$ such that $Rw^*v$, and some $d, d' \in D_i(v)$:

$$\mathfrak{M}^{Ref}, v, a[x^e/d, y^e/d'] \vDash \neg\Lambda(x^e, y^e) \wedge \mathcal{H}x^e \wedge \neg\mathcal{H}y^e$$

Thus, $\mathfrak{M}^{Ref}, v, a[x^e/d, y^e/d'] \vDash \mathcal{H}x^e \wedge \neg\mathcal{H}y^e$. Since $Rw^*v$, it follows that:

$$\mathfrak{M}^{Ref}, w*, a[x^e/d, y^e/d'] \vDash \Diamond\big(\mathcal{H}x^e \wedge \neg\mathcal{H}y^e\big) \tag{ii}$$

Thus:

$$\mathfrak{M}^{Ref}, v, a[x^e/d, y^e/d'] \vDash @\Diamond\big(\mathcal{H}x^e \wedge \neg\mathcal{H}y^e\big) \tag{iii}$$

It follows from $\mathfrak{M}^{Ref} \vDash$ LPP, for any $\phi \in \mathcal{L}_\Diamond$, letting $\phi := \mathcal{H}x^e \wedge \neg\mathcal{H}y^e$ that:

$$\mathfrak{M}^{Ref} \vDash \Diamond\Big(\mathcal{H}x^e \wedge \neg\mathcal{H}y^e\Big) \rightarrow \exists p\Big(\Diamond\mathrm{T}p \wedge \Box(\mathrm{T}p \rightarrow (\mathcal{H}x^e \wedge \neg\mathcal{H}y^e))\Big)$$

And thus, given that if $\mathfrak{M}^{Ref} \vDash \phi \rightarrow \psi$, then $\mathfrak{M}^{Ref} \vDash @\phi \rightarrow @\psi$, for any $\phi, \psi \in \mathcal{L}_\Diamond$:

$$\mathfrak{M}^{Ref}, v, a[x^e/d, y^e/d'] \vDash @\Diamond\Big(\mathcal{H}x^e \wedge \neg\mathcal{H}y^e\Big) \rightarrow @\exists p\Big(\Diamond\mathrm{T}p \wedge \Box(\mathrm{T}p \rightarrow (\mathcal{H}x^e \wedge \neg\mathcal{H}y^e)\Big)$$

[30] See [27, 64], and [57] for discussion of Williamson's argument against propositional contingentists making use of such qualitative haecceities.

Therefore, from (iii) and the above:

$$\mathfrak{M}^{Ref}, v, a[x^e/d, y^e/d'] \vDash @\exists p\Big(\Diamond \mathrm{T}p \wedge \Box(\mathrm{T}p \rightarrow (\mathcal{H}x^e \wedge \neg \mathcal{H}y^e)\Big) \tag{iv}$$

From our supposition that $\mathfrak{M}^{Ref} \vDash \mathrm{N}^@$ and $\mathfrak{M}^{Ref}, w, a[x^e/d, y^e/d'] \vDash \neg \Lambda(x^e, y^e)$ that:

$$\mathfrak{M}^{Ref}, v, a[x^e/d, y^e/d'] \vDash @\forall p\Big(\Box(\mathrm{T}p \rightarrow \mathcal{H}x^e) \rightarrow \Box(\mathrm{T}p \rightarrow \mathcal{H}y^e)\Big) \tag{v}$$

From (iv) and (v), it then follows:

$$\mathfrak{M}^{Ref}, v, a[x^e/d, y^e/d'] \vDash @\exists p\Big(\Diamond \mathrm{T}p \wedge \Box(\mathrm{T}p \rightarrow (\mathcal{H}y^e \wedge \neg \mathcal{H}y^e))\Big) \tag{vi}$$

However, if $\mathfrak{M}^{Ref}, v, a[x^e/d, y^e/d'] \vDash @\exists p\Big(\Diamond \mathrm{T}p \wedge \Box(\mathrm{T}p \rightarrow (\mathcal{H}y^e \wedge \neg \mathcal{H}y^e))\Big)$, then for some $u \in W$: $\mathfrak{M}^{Ref}, u, a[x^e/d, y^e/d'] \vDash \mathcal{H}y^e \wedge \neg \mathcal{H}y^e$. Thus, $a(y^e) \in v(\mathcal{H})_u$ and $a(y^e) \notin v(\mathcal{H})_u$. Contradiction. Thus, if $\mathfrak{M}^{Ref} \vDash$ LPP, then $\mathfrak{M}^{Ref} \nvDash \mathrm{N}^@ \wedge \mathrm{P}^@$. Since $\mathfrak{M}^{Ref}$, $w$ and $a$ are arbitrary, this suffices for our result. □

In short, then, if we accept SPC, we should accept both ($\mathrm{P}^@$) and ($\mathrm{N}^@$). However, the above result shows that (LPP) holding for any $\phi \in \mathcal{L}_\Diamond$ is jointly inconsistent with ($\mathrm{P}^@$) and ($\mathrm{N}^@$) if the accessibility relation is reflexive. As I argued, (LPP) follows from any adequate theory of possible worlds which treats them as some kind of proposition. Moreover, were we to have any adequate theory of possible worlds, an analogous claim to (LPP) formulated in terms of the relevant entity should hold. Given the close parallel between propositions and entities like states of affairs, properties, or sets of propositions, analogous claims to ($\mathrm{P}^@$) and ($\mathrm{N}^@$), formulated in terms of the relevant entity, should hold, given SPC. Generally, then, this master argument casts serious doubt on the adequacy of any theory of possible worlds, given SPC.

## 6 Concluding Remarks

In Section 2, I outlined a contingentist model theory which captured serious propositional contingentism. I then argued in Section 3 that a natural conception of possible worlds, if we accept SPC, fails to be adequate. I then outlined, in Section 4, three promising alternative approaches to possible worlds in response to problem raised in Section 3. However, in Section 5, I presented what I dubbed the master argument against any adequate theory of possible worlds, if we accept SPC: any adequate theory of possible worlds is inconsistent with certain claims about *mere possibilia*, plausibly entailed by SPC.

## Appendix

Here, I prove minor technical results underpinning the arguments in this paper.

**Proposition 1** *Let's say that* $\mathbb{P}_{\mathfrak{M}}$ *is full if* $\mathbb{P}_{\mathfrak{M}} = \mathcal{P}(W) \times \mathcal{P}(W)$.

(i) *Any* $\mathfrak{M}$ *satisfying Definition 3, where* $\mathbb{P}_{\mathfrak{M}}$ *is full, is an* $\mathfrak{M} \in \mathbb{M}$.
(ii) *For some* $\mathfrak{M} \in \mathbb{M}$, $\mathfrak{M} \vDash \Diamond\exists x \Diamond\neg\exists y(y = x)$.

***Proof*** First, (i). Consider arbitrary $\mathfrak{M} = \langle W, R, \mathbb{P}_{\mathfrak{M}}, D_i, w^*, v\rangle$, where $\mathbb{P}_{\mathfrak{M}} = \mathcal{P}(W) \times \mathcal{P}(W)$. Suppose that Definition 3 is satisfied. $\mathfrak{M} \in \mathbb{M}$ iff $\mathfrak{M} \vDash \mathrm{E}[\phi^{t_1,\ldots,t_n}] \leftrightarrow \bigwedge_{i \leq n} \mathrm{E}t_i$, for any $\phi^{t_1,\ldots,t_n} \in \mathcal{L}_{\Diamond}$. $\mathfrak{M} \vDash \mathrm{E}[\phi^{t_1,\ldots,t_n}] \leftrightarrow \bigwedge_{i \leq n} \mathrm{E}t_i$ iff, any $w \in W$ and $a$: $\mathfrak{M}, w, a \vDash \mathrm{E}[\phi^{t_1,\ldots,t_n}]$ iff $\mathfrak{M}, w, a \vDash \bigwedge_{i \leq n} \mathrm{E}t_i$, for any $\phi^{t_1,\ldots,t_n} \in \mathcal{L}_{\Diamond}$. First, the left-to-right direction:

$$\begin{aligned}\mathfrak{M}, w, a \vDash \mathrm{E}[\phi^{t_1,\ldots,t_n}] \; &only\,if\;\; \delta_a([\phi^{t_1,\ldots,t_n}]) \in D_p(w)\\ &only\,if\;\; \langle\alpha, \beta\rangle : w \in \beta, \;\text{ where } \delta_a([\phi^{t_1,\ldots,t_n}]) = \langle\alpha, \beta\rangle\\ &only\,if\;\; \mathfrak{M}, w, a \vDash \bigwedge_{i \leq n} \mathrm{E}t_i\end{aligned}$$

Second, the right-to-left direction. If $\mathbb{P}_{\mathfrak{M}} = \mathcal{P}(W) \times \mathcal{P}(W)$, then every every $\langle\alpha, \beta\rangle$ such that $\alpha \subseteq \beta$ and $w \in \beta$ is in $\mathbb{P}_{\mathfrak{M}}$, for every $w \in W$. Thus, for any $\phi^{t_1,\ldots,t_n} \in \mathcal{L}_{\Diamond}$, there is a $\langle\alpha, \beta\rangle \in \mathbb{P}_{\mathfrak{M}}$ such that $\delta_a([\phi^{t_1,\ldots,t_n}]) = \langle\alpha, \beta\rangle$. Given the constraints on $D_p$, it follows that $\mathfrak{M}, w, a \vDash \mathrm{E}[\phi^{t_1,\ldots,t_n}]$ iff $\mathfrak{M}, w, a \vDash \bigwedge_{i \leq n} \mathrm{E}t_i$, for any $\phi^{t_1,\ldots,t_n}$. Second, (ii). Consider $\mathfrak{M} = \langle W, R, \mathbb{P}_{\mathfrak{M}}, D_i, w^*, v\rangle$, where $W = \{1, 2\}$, for any $w, w' \in W$, $Rww'$, $\mathbb{P}_{\mathfrak{M}}$ is full, $D_i(1) = \{3\}$ and $D_i(2) = \{4\}$, $v(F)_1 = \{3\}$, $v(F)_2 = \varnothing$, $v(G)_1 = \varnothing$, and $v(G)_2 = \{4\}$. By inspection, $v(F)_w \subset D(w)$ and $v(G)_w \subset D(w)$, for any $w \in W$ and so $\mathfrak{M}$ satisfies Definition 3. Since $\mathbb{P}_{\mathfrak{M}}$ is full, $\mathfrak{M} \vDash \mathrm{E}[\phi^{t_1,\ldots,t_n}] \leftrightarrow \bigwedge_{i \leq n} \mathrm{E}t_i$, for any $\phi^{t_1,\ldots,t_n} \in \mathcal{L}_{\Diamond}$. Since $D_i(1) \neq D_i(2)$ and $Rww'$, for any $w, w' \in W$, it follows that, for some $w \in W$ and $a$: $\mathfrak{M}, w, a \vDash \Diamond\exists x \Diamond\neg\exists y(y = x)$. □

**Proposition 2** *Any* $\mathfrak{M} \in \mathbb{M}^{\tau}$*:* (i) $\mathfrak{M} \vDash \Box\forall x \Box\exists y(y = x)$ (ii) $\mathfrak{M} \vDash \Box\forall p \Box\exists q(q = p)$.

***Proof*** Suppose $\mathfrak{M}$ is some arbitrary $\mathfrak{M} \in \mathbb{M}^{\tau}$ and that $\mathfrak{M}, w, a \vDash \Diamond\neg\mathrm{E}x$, for arbitrary $w$ and $a$. Given that $\mathfrak{M} \vDash \Diamond\phi \rightarrow \Diamond\mathrm{T}[\phi]$, it follows that $\mathfrak{M}, w, a \vDash \Diamond\mathrm{T}[\neg\mathrm{E}x]$. Now, given that $\mathbb{M}^{\tau} \subset \mathbb{M}$, $\mathfrak{M}, w, a \vDash \Box(\mathrm{T}[\neg\mathrm{E}x] \rightarrow \mathrm{E}x)$. Thus, if $\mathfrak{M}, w, a \vDash \Diamond\neg\mathrm{E}x$, then $\mathfrak{M}, w, a \vDash \Diamond(\mathrm{E}x \wedge \neg\mathrm{E}x)$. Thus: $\mathfrak{M}, w, a \vDash \Box\mathrm{E}x$. Now, this just means: $\mathfrak{M}, w, a \vDash \Box\exists y(y = x)$, for arbitrary $w$. Given no specific variable played a role: $\mathfrak{M}, w, a[x/d] \vDash \Box\exists y(y = x)$, for any $d \in D_i(w)$. Thus: $\mathfrak{M}, w, a \vDash \forall x \Box\exists y(y = x)$. Since $w$, as well as $a$, was arbitrary: $\mathfrak{M} \vDash \Box\forall x \Box\exists y(y = x)$. The same reasoning can be given for $\mathfrak{M} \vDash \Box\forall p \Box\exists q(q = p)$, modulo the changes because of the changes in the sort of variable. □

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## References

1. Adams, R. M. (1981). Actualism and Thisness. *Synthese, 49*, 3–41.
2. Adams, R. M. (1986). Time and Thisness. *Midwest Studies in Philosophy, 11*, 315–329.
3. Bealer, G. (1994). Property Theory: The Type-Free Approach v. The Church Approach. *Journal of Philosophical Logic, 23*, 139–171.
4. Bergmann, M. (1996). A New Argument from Actualism to Serious Actualism. *Noûs, 30*, 356–359.
5. Bricker, P. (2006). Absolute Actuality and the Plurality of Worlds. *Philosophical Perspectives, 20*, 41–76.
6. Cartwright, R. (1997). On Singular Propositions. *Canadian Journal of Philosophy, 27*, 67–83.
7. David, M. (2009). Defending Existentialism? In M. E. Reicher (Ed.), *States of Affairs* (pp. 167–20). Ontos Verlag.
8. Deutsch, H. (1990). Contingency and Modal Logic. *Philosophical Studies, 60*, 89–102.
9. Dorr, C., Hawthorne, J., & Yli-Vakkuri, J. (2021). *The Bounds of Possibility: Puzzles of Modal Variation*. OUP.
10. Einheuser, I. (2012). Inner and Outer Truth. *Philosophers' Imprint*12.
11. Fine, K. (1977a). Prior on the Construction of PossibleWorlds and Instants. In K. Fine, & A. Prior (Eds.), *Worlds, Times and Selves* (pp. 116–161).
12. Fine, K. (1977). Properties, Propositions and Sets. *Journal of Philosophical Logic, 6*, 135–191.
13. Fine, K. (1980). First-order Modal Theories. II: Propositions. *Studia Logica, 39*, 159–202.
14. Fine, K. (1985). Plantinga on the Reduction of Possibilist Discourse. In J. E. Tomberlin & P. van Inwagen (Eds.), *Profiles: Alvin Plantinga* (pp. 145–186). Springer.
15. Fine, K. (2005). The Problem of Possibilia. In K. Fine (Ed.), *Modality and Tense: Philosophical Papers* (pp. 214–232). Oxford University Press.
16. Fitch, G., & Nelson, M. (2018). Singular Propositions. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy*. Spring 2018. Metaphysics Research Lab, Stanford University.
17. Fitch, G. (1996). In Defense of Aristotelian Actualism. *Philosophical Perspectives, 10*, 53–71.
18. Fritz, P. (2016). *Propositional Contingentism. Review of Symbolic Logic, 9*, 123–142.
19. Fritz, P. (2017). Logics for Propositional Contingentism. *Review of Symbolic Logic, 10*, 203–236.
20. Fritz, P. (2018). Higher-Order Contingentism, Part 2: Patterns of Indistinguishability. *Journal of Philosophical Logic, 47*, 407–418.
21. Fritz, P. (2018). Higher-Order Contingentism, Part 3: Expressive Limitations. *Journal of Philosophical Logic, 47*, 649–671.
22. Fritz, Peter. (2023). Being Somehow Without (Possibly) Being Something. *Mind, 132*, 348–371.
23. Fritz, P. (2023b). *The Foundations of Modality: From Propositions to Possible Worlds*. Oxford University Press.
24. Fritz, P., & Goodman, J. (2016). Higher-order Contingentism, Part 1: Closure and Generation. *Journal of Philosophical Logic, 45*, 645–695.

25. Fritz, P., & Goodman, J. (2017). Counterfactuals and Propositional Contingentism. *Review of Symbolic Logic, 10*, 509–529.
26. Glick, E. N. (2018). What is a Singular Proposition? *Mind, 127*, 1027–1067.
27. Goodman, J. (2016). *An Argument For Necessitism. Philosophical Perspectives, 30*, 160–182.
28. Goodman, J. (2017). Reality is Not Structured. *Analysis, 77*, 43–53.
29. Jacinto, B. (2019). Serious Actualism and Higher-Order Predication. *Journal of Philosophical Logic, 48*, 471–499.
30. King, J. (2007). *The Nature and Structure of Content*. Oxford University Press.
31. Kment, B. (2014). *Modality and Explanatory Reasoning*. Oxford University Press.
32. Kripke, S. (1976). Outline of a Theory of Truth. *The Journal of Philosophy, 72*, 690–716.
33. Linnebo, Ø. (2006). Sets, Properties, and Unrestricted Quantification. In G. Uzquiano & A. Rayo (Eds.), *Absolute Generality.* Oxford University Press.
34. Loptson, P. (1996). Prior, Plantinga, Haecceity, and the Possible. In B. J. Copeland (Ed.), *Logic and Reality: Essays on the Legacy of Arthur Prior* (pp. 419–435). Oxford University Press.
35. Masterman, C. J. (2022). Propositional Contingentism and Possible Worlds. *Synthese, 200*, 1–34.
36. Masterman, C. J. (2024). Serious Actualism and Nonexistence. *Australasian Journal of Philosophy*.
37. Menzel, C. (1991). The True Modal Logic. *Journal of Philosophical Logic, 20*, 331–374.
38. Menzel, C. (1993). Singular Propositions and Modal Logic. *Philosophical Topics, 21*, 113–148.
39. Menzel, C. (1993). The Proper Treatment of Predication in Fine-Grained Intensional Logic. *Philosophical Perspectives, 7*, 61–87.
40. Menzel, C. (2022). Actualism. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy*. Summer 2022. Metaphysics Research Lab, Stanford University.
41. Menzel, C. (forthcoming). Pure Logic and Higher-order Metaphysics. In P. Fritz, & N. Jones (Eds.), *Higher-order Metaphysics*. Oxford University Press.
42. Menzel, C., & Zalta, E. (2014). The Fundamental Theorem of World Theory. *Journal of Philosophical Logic, 43*, 333–363.
43. Mitchell-Yellin, B., & Nelson, M. (2016). S5 for Aristotelian Actualists. *Philosophical Studies, 173*, 1537–1569.
44. Myhill, J. (1958). Problems arising in the formalization of intensional logic. *Logique et Analyse, 1*, 74–83.
45. Nelson, M. (2009). The Contingency of Existence. In S. Newlands & L. M. Jorgensen (Eds.), *Metaphysics and the Good: Themes from the Philosophy of Robert Adams* (pp. 95–155). Oxford University Press.
46. Nelson, M. (2013). Contingently existing propositions. *Canadian Journal of Philosophy, 43*, 776–803.
47. Pickel, B. (forthcoming). Against Second-Order Primitivism. In P. Fritz, & N. K. Jones (Eds.), *Higher-order Metaphysics*. Oxford University Press.
48. Plantinga, A. (1974). *The Nature of Necessity*. Oxford University Press.
49. Plantinga, A. (1983). On Existentialism. *Philosophical Studies, 44*, 1–20.
50. Plantinga, A. (1985). Reply to Critics. In J. E. Tomberlin & P. van Inwagen (Eds.), *Profiles: Alvin Plantinga* (pp. 313–396).
51. Pollock, J. L. (1985). Plantinga On Possible Worlds. In J. E. Tomberlin & P. van Inwagen (Eds.), *Alvin Plantinga* (pp. 121–144). Springer.
52. Prior, A. N. (1957). *Time and Modality*. Oxford University Press.
53. Prior, A. N. (1967). *Past*. Present and Future: Oxford University Press.
54. Russell, B. (1937). *The Principles of Mathematics*. 2nd Ed. Allen & Unwin.
55. Salmon, N. (1987). Existence. *Philosophical Perspectives, 1*, 49–108.
56. Sider, T. (ms). Higher-order Metametaphysics.
57. Skiba, L. (2021). Higher-Order Metaphysics. Philosophy. *Compass, 16*, 1–11.
58. Speaks, J. (2012). On Possibly Nonexistent Propositions. *Philosophy and Phenomenological Research, 85*, 528–562.
59. Stalnaker, R. (1976). Propositions. In A. F. MacKay & D. D. Merrill (Eds.), *Issues in the Philosophy of Language: Proceedings of the 1972 Colloquium in Philosophy* (pp. 79–91). Yale University Press.
60. Stalnaker, R. (2012). *Mere Possibilities: Metaphysical Foundations of Modal Semantics*. Princeton University Press.
61. Stephanou, Y. (2007). Serious Actualism. *The Philosophical Review, 116*, 219–250.
62. Turner, J. (2005). Strong and Weak Possibility. *Philosophical Studies, 125*, 191–217.

63. Uzquiano, G. (2015). A Neglected Resolution of Russell's Paradox of Propositions. *Review of Symbolic Logic, 8*, 328–344.
64. Williamson, T. (2013). *Modal Logic as Metaphysics*. Oxford University Press.