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Agential Free Choice

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Abstract

The Free Choice effect—whereby $\Diamond(p \text{ OR } q)$ seems to entail both $\Diamond p$ and $\Diamond q$—has traditionally been characterized as a phenomenon affecting the deontic modal 'may'. This paper presents an extension of the semantic account of free choice defended by Fusco (*Philosophers' Imprint*, *15*, 1–27, 2015) to the agentive modal 'can', the 'can' which, intuitively, describes an agent's powers. On this account, free choice is a nonspecific de re phenomenon (Bäuerle 1983; Fodor 1970) that—unlike typical cases—affects disjunction. I begin by sketching a model of inexact ability, which grounds a modal approach to agency (Belnap *Theoria*, *54*, 175–199, 1998; Perloff 2001) in a Williamson (*Mind, 101*, 217–242, 1992; *Erkenntnis*, *79*, 971–999, 2014)-style margin of error. A classical propositional semantics combined with this framework can reflect the intuitions highlighted by Kenny (1976)'s dartboard cases, as well as the counterexamples to simple conditional views recently discussed by Mandelkern et al. (*Philosophical Review*, *126*, 301–343, 2017). In Section 3, I turn to an independently motivated actual-world-sensitive account of disjunction, and show how it extends free choice inferences into an object language for propositional modal logic.

Keywords Natural language modality · Agency · Free choice · Ability · Margin of error

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1 Free Choice, Agency, and the Nonspecific De Re

The Free Choice effect—whereby $\Diamond(\phi \text{ OR } \psi)$ carries a felt entailment to both $\Diamond\phi$ and $\Diamond\psi$—has traditionally been characterized as a phenomenon affecting the deontic modal 'may' [44, 45]. In this paper, I explore how to extend the semantic account of deontic free choice I proposed in [27] to the agentive modal 'can'.

I begin by sketching a model of inexact ability, which grounds a modal approach to agency [5, 6] in a Williamson-style margin of error [80, 81]. A classical propositional semantics combined with this framework can reflect the intuitions highlighted by Kenny dartboard cases [49], as well as the counterexamples to simple conditional views recently discussed by Mandelkern [61]. In §3, I substitute for classical disjunction an independently motivated "de re" or actual-world-sensitive account of disjunction, and show how it extends free choice inferences into an object language for propositional modal logic.

1.1 Agential 'Can'

As I will understand it, agential 'can' is the use of 'can' which describes an agent's abilities or powers, as in (1) and (2):

(1) Otto can clear the 7ft high jump.
$C(\text{J})$

(2) Otto can hit the bullseye at 300 paces.
$C(\text{H})$

Do these modals exhibit the general pattern of free choice, a felt entailment from $\Diamond(p \text{ OR } q)$ to the conjunction of $\Diamond p$ and $\Diamond q$? They do appear to ([29, pg. 108]; [63, fn. 3]). (3a), for example, carries a felt entailment to (3b):[1]

(3) a. Chewy can fly the spaceship to Sirius or Hesperus.
$C(\text{S OR H})$

b. Chewy can fly the spaceship to Sirius, and Chewy can fly the spaceship to Hesperus.
$\Rightarrow C(\text{S}) \wedge C(\text{H})$

In addition, agential modals are a promising testing-ground for an account of free choice that is semantic, in the standard, "compositional" sense of the term. Many of the dynamic and neo-Gricean maneuvers appealed to to explain free choice for other flavors of modality—other natural-language operators whose semantics can be approximated by the modal diamond $\Diamond$—don't, or at least don't obviously, transfer smoothly to the agentive case.

For example, writers since Kamp [44] have looked to the the *performativity* of *deontic* modal talk—the fact that utterances like "you may sit down" can *create* permissions as well as describe them—as the source of the unexpectedly

[1] As Mandelkern et al. do, I focus on *specific* or "one-shot" readings of these ascriptions, given the plausible hypothesis that *generic* ability ascriptions have a distinct GEN operator at LF ([61, §6]; see also [7, 34] and [60].)

strong entailment properties of free choice sentences (that is, sentences of the form $\ulcorner\Diamond(p \text{ OR } q)\urcorner$). However, it is not generally true that agential 'can' is performative in this way. Absent quite magical circumstances, I cannot give Otto the *ability* to jump 7 feet in the air by uttering (1). For another contrast, while conversational moves like the raising-to-salience of skeptical scenarios can arguably expand the domain of epistemically relevant possibilities, it is not clear they cast a similar spell in the agential case: simply saying something like, "for all we really know for sure, Otto was in the last Olympics" is not typically enough to get speakers to accept (1).[2]

Leaving performative views aside, then, the semantics of 'can' I will work with is broadly in the standard Kratzer-Lewis vein, and is grounded, as those approaches are, in the model-theoretic resources of Kripke frames.[3] Contemporary application of this framework to natural language modality anchors itself in the notion of a *modal base* [52, 53]: a set of possible worlds taken to be provided by the context in which a modal expression occurs. In particular, my discussion will presume the contextual availability of a *historical* modal base, which plays a role in representing actuality.

I take it that a commitment of this kind, which pairs the notion of actuality-in-context with what is left open by history, is familiar from the standpoint of rational agency. As Stalnaker [72, pg. 81] describes it:

> a theory of rational action... contains implicitly an intuitive notion of **alternative possible courses of events**... [A] rational agent [...] considers various **alternative possible futures**, knowing that the one to **become actual** depends in part on his choice.

Stalnaker's talk here of a future *becoming actual* fits naturally within a picture of agential choice, according to which what is potentially actual exceeds what is *categorically* actual. This feature will become important below.[4]

1.2 Kenny's Gauntlet

Should the 'can' in (1)–(3) be understood as a (normal) modal operator? An influential negative answer to this question—and a subsequent declaration that "the logic of ability cannot be captured in a modal system"—is due to Anthony Kenny [49, pg. 209]. Intriguingly, for the free choice theorist, Kenny supports his negative answer by recourse to two argument patterns involving natural language disjunction, which I present here as a pair before discussing my take on their connection to free choice.

The first pattern, which I will call the **narrow-scope Kenny objection**, is that disjunctions cannot be introduced under 'can':[5]

(4) a. Otto can touch his toes.
$C(\text{B})$

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[2] In addition, on the linguistics side, recent work by Nouwen [63] has raised the worry that popular implicature-based accounts of free choice—in particular, the influential treatment of Kratzer and Shimoyama [55]—face special obstacles in the agentive case. Nouwen's argument is based on the wide-scope Kenny objection (§1.2 below).

[3] See especially [56, 58], and the papers in [54].

[4] For precedents, see also [6, 77].

[5] Kenny *op. cit.*, pg. 215; his example is inferring 'I can take it or leave it' from 'I can take it'.

b. Otto can touch his toes or hit the bullseye at 500 paces.
$\not\Rightarrow C(\text{B OR H})$

The second pattern, which I will call the **wide-scope Kenny objection**, is that narrow-scope disjunctions under 'can' are not equivalent to their wide-scope counterparts. Suppose that I am faced with an enormous checkered dartboard consisting of small black and red tiles. Kenny argues that (5) might be true:[6]

(5) I can hit red or black.
$C(\text{R OR B})$

Yet it could also be that I can *neither* hit a red tile, *nor* can I hit a black tile: thus, Kenny suggests, (6) is false.

(6) I can hit red or I can hit black.
$C(\text{R}) \text{ OR } C(\text{B})$

Kenny's focus on these patterns is strategic, of course: both $\Diamond\phi \vDash \Diamond(\phi \vee \psi)$ and $\Diamond(\phi \vee \psi) \vDash (\Diamond\phi \vee \Diamond\psi)$ are theorems of any normal modal logic.[7]

I'll make two observations about the Kenny objections here, which I hope to fully vindicate in due course. The first is that a semantic account of agential free choice—the felt entailment of e.g. (3b) from (3a)—will naturally provide a solution to the puzzle raised by the narrow-scope Kenny objection. Patently, if we have a semantics on which the FC premise $C(\phi \text{ OR } \psi)$ entails both $C\phi$ and $C\psi$, as in (3a)-(3b), that same premise cannot in turn be implied *by* $C\phi$. For then we would be able to derive the ability to do anything from the ability to do anything else.[8] The *failure* of entailment in (4), which following [27] I will call "(FC$^-$)", is mysterious only from the point of view of semantic theories which do *not* explain the free choice effect. From the point of view of semantic theories which *do* explain the free choice effect, the pattern underlying the narrow-scope Kenny objection is to be expected.

Second, while others have raised doubts about the claim that (6) is stronger than (5), I propose to take it seriously—though *not* by way of taking the wide-scope 'can' in (5) very seriously.[9] The natural feature of agency that the wide-scope Kenny objection highlights is agency's *inexactness*. I may be agentially capable—indeed, in the case of a really enormous black-and-red dartboard, I may be *fated*—to ensure an outcome without being able to ensure any of its more fine-grained sub-outcomes. This captures the reading we naturally give to the modals in (6) when we understand both disjuncts as false. But if this is right, then the wide-scope Kenny objection is just as well illustrated by the failure of the bare disjunction

(7) I will hit red or I will hit black.
R OR B

[6] *op. cit.,* pg. 215-216. For continuity with other cases, my discussion here combines Kenny's "red and black cards" case with his "top and bottom half of the dartboard" case.

[7] That is, the weakest kind of logic that can be modeled in a standard Kripke frame. See e.g. [8], Ch. 1.6, and [78], Ch. 5.

[8] Viz., via $C\phi \vDash C(\phi \text{ OR } \psi) \vDash (C\phi \wedge C\psi) \vDash C\psi$.

[9] See [61], §6.3, which proposes a deflationary response to (6).

Table 1 Data for disjunction and C

	Schema	Example
FC$^-$ *(negative datum)*	$C\phi \not\Rightarrow C(\phi \text{ OR } \psi)$	(4a) $\not\Rightarrow$ (4b)
FC$^+$ *(positive datum)*	$C(\phi \text{ OR } \psi) \Rightarrow C\phi \wedge C\psi$	(3a) $\Rightarrow$ (3b)
I$^-$ *(negative datum)*	$(\phi \text{ OR } \psi) \not\Rightarrow (C\phi) \text{ OR } (C\psi)$	(7) $\not\Rightarrow$ (6)

to entail the disjunction of ability statements in (6);[10] the simplest relevant pattern of *non*-entailment is just:

$$(\text{I}^-) \quad (\phi \text{ OR } \psi) \not\models C\phi \text{ OR } C\psi$$

We can then say—in a case where an agent is not skilled enough to ensure that she hits either color on the dartboard—that the disjunction in (6) is false for the perfectly classical reason that each of its disjuncts is false.

I conclude that the landscape of agential 'can' presents a promising avenue of exploration for a semantic account of free choice. The relevant empirical phenomenon seems to be well-attested, and given Kenny's challenge—especially the fact that a semantic account of free choice is well-suited to providing a simultaneous account of the narrow and wide-scope Kenny objections—success in the project would contribute to a unified view of natural language modal phenomena. Table 1 summarizes the entailment and non-entailment data of this section.

Before proceeding, it is worth commenting on the strength of 'can'.[11] Adding (I$^-$) to (FC$^-$) and (FC$^+$) directs us towards reading of 'can' on which accidental, or *fluky*, success is insufficient for ascriptions of ability. To bring out this interpretation in the context of the open future, we can imagine that I'm about to flip a fair coin. To your surprise, I tell you:

(8) I can flip it so it lands heads or I can flip it so it lands tails.
$C(\text{H}) \text{ OR } C(\text{T})$

(note that the 'or' in (8) is wide-scope, so we can set free choice aside). You say, "oh yeah? which one?" I reply: "I have no idea. Either it'll land heads, in which case I can flip it so it lands heads, or it'll land tails, in which case I can flip it so it lands tails."[12]

[10] In describing (7) as the bare disjunction of R and B, I prescind from the possibility that the 'will' in e.g. 'I will hit red' is itself a modal operator [22, 51]. Even if it is, this seems not to effect the semantics of the wide-scope disjunction in (7) (c.f. the "Will Excluded Middle" principle discussed extensively in [11].)

[11] I am indebted to an anonymous referee for encouraging me to clarify this issue, and to [9] for the relevant framing of the issue.

[12] See [61]'s discussion of a conditional analysis of 'can'-ascriptions (§2, below), paired with a semantics that validates the conditional excluded middle (*op. cit.*, §6.3).

This seems like a strange thing to say. Rather, on the sense of ‘can’ highlighted by (I^-), I should own up to both (9) and (10):

(9) I’m not able to flip the coin so it comes up heads, although of course it *might* come up heads.

(10) I’m not able to flip the coin so it comes up tails, although of course it *might* come up tails.

We will make this intuition more precise below.

Overall, there are two main tasks: first, understanding what it takes to make the claim $C\phi$ true in cases of inexact agency, and second, understanding what gives rise to the free choice reading of sentences like (3a), where disjunction is embedded under the agentive modal. I will go in order: in the next section, I begin by reviewing some recent work in the literature on ‘can’-constructions. Though this work does not provide an account of the data in Table 1, I think it sets us on the right path, by suggesting a *modal* treatment of agency that is nonetheless *transparent* rather than *opaque*. The goal of the discussion is to provide an independently plausible grip on the modal operator C, which can be used as a base from which to evaluate the contribution of disjunction to the free choice patterns (FC^-) and (FC^+). In §3, I spotlight an approach to embedded disjunction from the literature on concealed questions, framing it as a candidate for de re ‘OR’. In §4, I put this disjunction in the scope of C, showing that the interaction of the pair can account for the target patterns.

2 Doing Things, Opaquely and Transparently

What makes it true that an agent can ϕ? A recent paper by Mandelkern, Schultheis, and Boylan [61] begins with the following appealing thought: what it means to say that α can ϕ is that if α tries to ϕ, she will succeed.[13] Mandelkern et al. situate this “conditional analysis” within an intensional semantics which employs a Kratzerian modal base. Against this domain of quantification, the initial thought is that $\ulcorner\alpha$ can $\phi\urcorner$ is true just in case the conditional

(11) if α tried to ϕ, then α would ϕ.

is true.[14]

However, Mandelkern et al. suggest that the first-pass conditional analysis fails in cases where agents have mistaken beliefs, like this one:

Elevator. John is in a 10-story building with an elevator whose buttons are, unbeknownst to him, incorrectly wired. If he presses the button marked

[13] The authors trace this proposal back to David Hume [42] and G. E. Moore [62].

[14] The analyses of the conditional under consideration in Mandelkern et al.’s discussion include [56] and [70, 71], both of which, like Kratzer [52, 53], require a ranking or ordering on some set of possible worlds. In this paper I prescind from most of the debate regarding what grounds membership in this set, and how (and whether) its members are ranked or selected.

'basement', the elevator will go to the first floor. If he presses the button marked '1st floor', the elevator will go to the basement. [61, §4.1]

In *Elevator*, schema (11) is false for both actions: in light of the crossed wiring, any attempt by John to go to the first floor *or* the basement is doomed to (one-shot) frustration. Nonetheless, the authors take a "hard line". They maintain that (12) and (13) are true:

(12) John can take the elevator to the 1st floor.
(13) John can take the elevator to the basement.

This hard line reading is difficult to square with a flatfooted account of rational agency, since in *Elevator*, John might *desire* to go to the basement, *believe he can* go to the basement, and still wind up *not* going to the basement. Yet it seems unquestionably correct as well, insofar as there is clearly a sense in which (12)-(13) are true in *Elevator*, while a sentence like (14)

(14) John can take the elevator to Budapest.

is false.

Putting it quantificationally, there is *something* John can do in *Elevator* which would make the prejacent of (12) true. But since there is no tunnel from the elevator shaft to Budapest, it's false that there is something he can do in *Elevator* would make the prejacent of (14) true.

Taking the quantificational intuition seriously ("*something* he can do ..."), I propose to reconcile the conflicting intuitions regarding (12)-(13) in cases like *Elevator* by means of a broadly Fregean distinction, between intensional and extensional readings of the claim that an agent brings about, or *realizes*, some outcome. According to the intensional reading of "realizes", how an agent conceives of her actions matters to what she realizes. According to the extensional reading, it is not. This allows to analyze extensional realizing—the factive analogue of trying—as a modal operator which permits quantifying in.[15]

Hypothesis 1 α realizes$_{ex}$ ϕ iff $\exists\mathcal{B}$: α realizes$_{op}$ $\mathcal{B}$, and $\mathcal{B}$ is a mode of presentation of the proposition $V(\phi)$.

The claim that extensional realizing can be analyzed in terms of opaque realizing is akin to the Davidson/Anscombe claim that intentional action can be analyzed as action under *some* description ([2, §46-47]; [17]).[16] Perhaps the description, $\mathcal{B}$, under

[15] For example, in the style of [46].

[16] Some philosophers will take it as axiomatic that only *propositional attitudes* can, strictly speaking, be read transparently or opaquely: to say, "Oedipus married his mother, but not transparently", for example, is at best to speak sloppily. There are two things to say in response. First, intensional transitive verbs, like "seek" and "worship", seem to admit of this distinction directly. Second, even if the claim is true, realization$_{ex}$ can be identified more strongly with (factive) trying—which has a claim to be a propositional attitude—than with intentional doing. I also take it that the present semantic project need make no assumptions about conceptual priority relations between realizing$_{ex}$ and realizing$_{op}$, any more than a semantics for "knows" needs to take a stand on conceptual priority relations between knowledge by acquaintance and knowledge by description.

which an agent opaquely realizes an outcome, is a description uniquely suited for explaining the causal facilitation of $V(\phi)$ by α's intentional states; perhaps it is even a sentence in the language of thought ([25]). I won't take a stand on that here. However, since we are in pursuit of a *modal* analysis of realization$_{ex}$, I do want to assume that the *presentatum* of $\mathcal{B}$ can be characterized as a coarse-grained proposition $V(\phi)$—indeed, as some coarse-grained proposition compatible with the modal base.

In *Elevator*, for example, it's true that John can go to the basement because it is historically possible for him to extensionally realize the (coarse-grained) proposition that he is *in* the basement.[17] At a context c, the target truth-conditions of the "hard line reading" of (13), then, are simply

(9') $\exists w \in h_c$: John realizes$_{ex}$ [$\lambda w'$: John is in the basement w'] in w.

This is equivalent by Hypothesis 1 to:

(9") $\exists w \in h_c$: $\exists \mathcal{B}$: John realizes$_{op}$ $\mathcal{B}$ in w, and $\mathcal{B}$ is a mode of presentation of [$\lambda w'$: John is in the basement in w'].

Given John's false beliefs in *Elevator*, any world w in the modal base h_c in which John extensionally realizes this outcome is one in which $\mathcal{B}$ is witnessed by a "heterogenous" mode of presentation—one which, in *Elevator*, might correspond to the mentalese command he would translate as *going to the first floor*.

Once opaque realization is disentangled from extensional realization, the latter can be treated as a factive operator, in the sense that a proposition p is realized$_{ex}$ in some world w only if p is in fact true in w.

2.1 The Granularity of Acts

The disambiguation between extensional and opaque realization brings the former close to the causal conception of agency in [6], while leaving the latter closer to the *luminous* attitude theorized about by various philosophers of action [10, 35, 64]. But why distinguish at all? Crucially, focusing on realization in the extensional sense allows the proposition to which α is realize$_{ex}$-related in a world w to characterize the *fineness of grain* of the outcome α has brought about in w, rather than the (arguably, *hyperintensional*) mode of presentation under which she—perhaps due to lack of worldly knowledge—conceives of what she is realizing.[18]

As I suggested earlier, this framework is a natural fit for the intuitions brought to bear by Kenny's dartboard cases. A transparent domain of quantification h_c

[17] For a precedent for this idea, see esp. [39, pg. 606], and the distinction between action types and tokens in [40].

[18] Schwarz [67, §1] presents this example of a hyperintensional distinction between 'can'-statements:

(15) Cyril can recite the first 10 digits of π.

(16) Cyril can recite the numerals 'three', 'one', 'four', 'one'...

As Schwarz notes, if Cyril does not know the first 10 digits of π, there is a natural urge to say that (15) is false while (16) is true, though any world in which the prejacent of the latter is true is a world in which the prejacent of the former is true.

is the kind of modal background, for example, against which Frege puzzles distinctive of rational agency take shape.[19] Here is Davidson, glossing one such puzzle:

> I am asked to explain [...] my shooting of the bank president (d), for the victim was that distinguished gentleman. My excuse is that I shot the escaping murderer (e), and surprising and unpleasant as it is, my shooting the escaping murderer and my shooting of the bank president were one and the same action ($e = d$), since the bank president and the escaping murderer were one and the same person. [18, pgs. 109-110]

It matters not at all to Davidson's example that it is *metaphysically* or *epistemically* contingent that the same person is both the bank president and the escaping murderer. For the identity of the two *abilities*—the ability to shoot the bank president and the ability to shoot the escaping murderer—mere actual-future equivalence suffices: the agent could not *actually* have done one without *actually* doing the other. Likewise in (3), if Chewy's piloting skills are precise enough to get the spaceship to Sirius, and precise enough to get the spaceship to Hesperus, it follows that

(17) Chewy can fly the Falcon to Phosphorus.

...even if we stipulate that Chewy does not know Hesperus is Phosphorus, believes Phosphorus does not exist, and would in fact withhold assent from (17).

With these distinctions in mind, we introduce a toy language $\mathcal{L}$ which captures these "hard line", or extensional, intuitions about agentive modals. This language contains a propositional fragment with clauses for atomics and conjunction that are fully classical and world-bound. In addition, it contains a historical modality operator $\blacklozenge$, which quantifies existentially over worlds in the historical modal base, h. I introduce 'C', for agentive *can*, by way of a normal modal operator '$\boxdot$' for world-bound extensional realization. $\boxdot$ is underwritten by a Kripke-style accessibility relation $R \subseteq W \times W$ of the usual kind: we say that wRv if world v is compatible with everything the agent's powers are able to necessitate in world w.[20] We can then define

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[19]Following [57], I take it that the truth of a sentence ϕ at a context c is the truth of ϕ relative to the index of the context, and use the convention of subscripting a 'c' on a parameter which is initialized by c. Hence 'h_c' is the (historical) modal base of the context of utterance. See also [47, pg. 522] on the sentential truth relative to a pair consisting of the context and the "circumstance of the context."

[20]$\boxdot$ is thus comparable to the Chellas $stit$ operator described in [39]. Horty and Belnap adopt the name following [12].

Table 2 Toy Language $\mathcal{L}$, with consequence

Propositional fragment:			
	a is true at $\langle h, w\rangle$	iff	w is an a-world
	$\pi_1 \wedge \pi_2$ is true at $\langle h, w\rangle$	iff	π_1 is true at $\langle h, w\rangle$ and π_2 is true at $\langle h, w\rangle$
Historical modality:			
	$\blacklozenge\pi$ is true at $\langle h, w\rangle$	iff	$\exists v \in h$: π is true at $\langle h, v\rangle$
Agentive modality:			
	$\boxdot\pi$ is true at $\langle h, w\rangle$	iff	$\forall w' \in h$: if wRw', then π is true at $\langle h, w'\rangle$.
	$C\pi$ is true at $\langle h, w\rangle$	iff	$\exists v \in h$ such that: (i) π is true at $\langle h, v\rangle$ and (ii) $\forall w' \in h$: if vRw', then π is true at $\langle h, w'\rangle$.
Interdefinition (given factivity):			
	$C\pi$	$:=$	$\blacklozenge(\pi \wedge \boxdot\pi)$
Consequence:			
	$\Gamma \vDash \pi$	iff	$(\forall h$: $(\forall w' \in h$: γ_i is true at $\langle h, w'\rangle$ for all $\gamma_i \in \Gamma)) \rightarrow (\forall h$: $(\forall w' \in h$: π is true at $\langle h, w'\rangle))$

$C(\pi)$ as $\blacklozenge(\pi \wedge \boxdot\pi)$: the operator C tracks the relation of *possibly* (relative to a historical modal base, h) being realized$_{ex}$.[21]

2.2 Agenda

Because $\mathcal{L}$ does not yet contain disjunction, the toy semantics of Table 2 cannot even *express*, much less underwrite, the patterns in Table 1. However, we can see the beginning of a sketch of (I$^-$)—our version of the wide-scope Kenny objection—which I repeat below:

(I$^-$) $(\phi$ OR $\psi) \nvDash C\phi$ OR $C\psi$

[21] The truth-conditions proposed here look different from those proposed by Mandelkern et al., but are equivalent given the following assumptions, not endorsed by Mandelkern et al. themselves: (i) tryings are successful: S tries A iff S As [64]; (ii) the set of acts S can perform in $\langle c, w\rangle$ is the set of cells realizable$_{ex}$ in some $w \in h_c$ (viz., $\mathcal{A}_{S,c,w} = \{p \in \wp(W) : \exists w \in h_c \text{ s.t. } p = \{v : wRv\}\}$); and (iii) the conditional '$\Box\!\!\rightarrow$' in Mandelkern et al.'s semantic entry is interpreted as strict implication over h_c (viz., $f_c(\phi, w) = \{v \in h_c : [\![\phi]\!]^{c,w} = 1\}$). Then, where $\boxdot$ is the modal operator that expresses realization$_{ex}$:

$$\begin{aligned} \exists A \in \mathcal{A}_{S,c,w} &: S \text{ tries to } A \;\Box\!\!\rightarrow \phi \text{ iff} \\ \exists A \in \mathcal{A}_{S,c,w} &: S A\text{s} \;\Box\!\!\rightarrow \phi \text{ iff} \\ \exists w \in h_c &: \{v : wRv\} \;\Box\!\!\rightarrow \phi \text{ iff} \\ \exists w \in h_c &: \langle h_c, w\rangle \vDash \boxdot\phi \text{ iff (by Hypothesis 1):} \\ & \text{for arbitrary } w' \in h\text{: } \langle h, w'\rangle \vDash \blacklozenge(\phi \wedge \boxdot\phi). \end{aligned}$$

NB however that (iii) in particular is not—except in degenerate cases—compatible with the Conditional Excluded Middle [74], a principle Mandelkern et al. endorse elsewhere in their paper (*op. cit.*, §6.3).

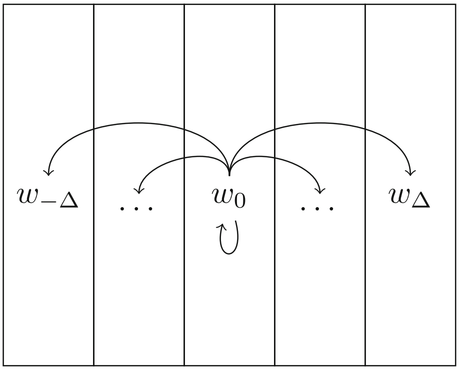


Fig. 1 A dartboard with skinny, $\mathbb{Z}$-numbered tiles. Where i is the tile aimed for in w, j is the tile hit in v, and Δ is the agent's margin of error, wRv iff $|i - j| \leq \Delta$

Suppose, as I shall prove, that disjunction is always classical—it is equivalent to the Boolean $\vee$ of propositional logic—whenever it has wide scope with respect to any modal operators. Then (I$^-$) is equivalent to

(I'$^-$) $(\phi \vee \psi) \nvDash C\phi \vee C\psi$

It is easy to give a countermodel illustrating (I'$^-$) with $\mathcal{L}$ interpreted on a standard Kripke frame. To do so, I will use an adaptation of Kenny's dartboard, which will be of service in future sections, as well as in the Appendix.

Suppose I am facing a dartboard consisting of a series of skinny tiles, numbered according to the integers $\mathbb{Z}$. At my prior context—as I prepare to throw the dart—an accessibility relation between tiles i and j is determined by a margin of error Δ that reflects my level of skill: it is, intuitively, the maximum possible distance between what I aim for and what I get (Fig. 1). [22]

Why does (I'$^-$) fail? Let E be the proposition that the dart lands on some even number, and O the proposition that it lands on some odd number. If my margin of error Δ is 1 or greater, then in any world w where I aim for an even number $n \in E$, success in hitting E is merely an accident with respect to my powers: for all my margin of error could *guarantee*, the dart could just as well have landed on some neighboring O-tile instead. And likewise with E and O reversed. Hence, while either E or O is true in every world in h_c, $\boxdot E$—(I realized$_{ex}$ E)—and $\boxdot O$—(I realized$_{ex}$ O)—are false in every world in h_c. It follows that $\blacklozenge \boxdot E$ and $\blacklozenge \boxdot O$ are false. Thus while the wide-scope disjunction $(E \vee O)$ is true at every point of evaluation, the disjunction $C(E) \vee C(O)$ is *false* at every point of evaluation. Moreover, it is false for the reason Kenny suggests sentences like (6) are false—agency is inexact.

3 Disjunction

The preceding section's account of wide-scope disjunction was a warm-up to free choice itself. It still remains to us to characterize the positive entailment properties

[22] Assuming adjustment, that is, for heterogenous modes of presentation.

of embedded disjunction—that is, to characterize its behavior in (FC^-) and (FC^+). To get a grip on it, I will, on the next page, briefly review the nonspecific de re (or "third") reading of *indefinites*. I then turn to the question of how a clausal *disjunction* could give rise to a nonspecific, actuality-sensitive reading, taking a page from the literature on embedded wh-questions.

3.1 What is the Nonspecific De Re?

The nonspecific de re reading of quantificational expressions is a reading not accounted for by the traditional de dicto/de re distinction, where the latter is framed as a binary ambiguity of scope. Consider (18).

(18) Mary wants <u>a friend of mine</u> to win.
[23, pg. 83]

On the binary picture, the indefinite noun phrase 'a friend of mine' in (18) is read de re when it takes wide scope with respect to the intensional verb 'wants' at LF; otherwise, it takes narrow scope, and is thus read de dicto.

(19) a. de dicto:
Mary wants [λw_1 a friend-of-mine$_{w_1}$ win$_{w_1}$]
b. de re:
[a friend-of-mine] [λx. Mary wants [λw_1 x win $_{w_1}$]]

According to the standard de re reading (19b), the predicate 'friend of mine' in (18) is ultimately evaluated with respect to whatever parameter or parameters represent the actual world.

Since [24], however, it has been observed that there is an available interpretation for a sentence like (18) that is not equivalent to either of the alternatives in (19). Mary might, for example, have a belief that is de dicto in the sense that there is no particular person whom she wants to win—yet be truly described with (18) despite the fact that, as in the de re case, 'friend of mine' is evaluated transparently.

Heim and von Fintel gloss the target interpretation thus:

> To bring out this rather exotic reading, imagine [that] Mary looks at the ten contestants [in the race] and says *I hope one of the three on the right wins - they are so shaggy - I like shaggy people*. She doesn't know that those [three] are my friends. But I could still report her hope as in [(18)]. [23, pg. 79-80]

The textbook treatment of the nonspecific de re von Fintel & Heim go on to offer involves postulating world-variables in the syntax. On this formulation, w_0 is a variable dedicated to the sentence's world of evaluation.[23]

(20) a. de dicto:
λw_0 Mary wants$_{w_0}$ [λw_1 a friend-of-mine$_{w_1}$ win$_{w_1}$]

[23] So as not to leave this variable free in the syntax of the clause, Heim & von Fintel introduce a w_0-indexed variable binder at the top of the sentence (op cit., pgs. 80, 83). More recent work on such readings, such as [50], uses situation variables instead of world variables.

b. de re:
λw_0 [a friend-of-mine$_{w_0}$] [λx Mary wants$_{w_0}$ [λw_1 x win$_{w_1}$]]
c. non-specific de re:
λw_0 Mary wants$_{w_0}$ [λw_1 a friend-of-mine$_{w_0}$ win$_{w_1}$]

[23, pg. 83]

The key syntactic fact for securing the nonspecific de re reading at LF is that the world-variable in the predicate 'a friend of mine' is coindexed with w_0, rather than being bound by the λw_1-binder introduced under 'wants'. As a result, the two aspects of the traditional de re reading—actual-world relativity and wide-scope existential force—are separated, with the predicate 'friend of mine' in (20c) retaining its sensitivity to the actual world.

3.2 de re 'or'

The intuition I want to pursue is that free choice-triggering readings of modally embedded disjunctions, such as (3a) and (4b), are nonspecific de re readings. The task of understanding how a nonspecific de re reading could be given to sentential *disjunction*, rather than an indefinite NP like 'a friend of mine', involves understanding how disjunction could, even under intensional operators, display semantic sensitivity to the actual world.

Evidence for disjunction displaying such sensitivity can be found in the behavior of 'or' scoped under 'whether' + intensional verb combinations, as in (21)-(22):

(21) Al knows whether [Eve had [an apple] or [a pear]].
K_A (whether [A] OR [P])
(22) Dr. Jones will tell us whether [the test was [negative] or [positive]].
T_J (whether [N] OR [P])

The classic analysis of these 'whether…or'-ascriptions comes from the cluster of work in [48], [38], [59], and [32]. These authors take as their starting point that the semantic contribution of a 'whether p OR q'-constituent must be filled in in a way which preserves the familiar intensional analysis of the verbs *know (that)* and *tell (that)* in e.g. (23)-(24):

(23) Al knows that [Eve had a pear].
K_A(P)
(24) Dr. Jones will tell us that [the test was negative].
T_J(N)

The account initially presumes that the disjuncts p and q—for example, *Eve had an apple* and *Eve had a pear*, in (21)—are mutually exclusive and jointly exhaustive, a simplifying assumption I will also adopt until §4.1.[24] It centers around the following idea: given the truth of p, $\ulcorner\alpha\ v$s whether p OR $q\urcorner$ is *equivalent* to the nondisjunctive attitude ascription $\ulcorner\alpha\ v$s that $p\urcorner$.[25] For example: if Eve had an apple, then Al knows *whether* Eve had an apple or a pear just in case Al knows *that* Eve had an apple.[26]

There is thus a dependency between which of p and q is true in the actual world, and the proposition expressed by the embedded disjunction. Groenedijk & Stokhof and Lewis treat this by offering a semantic entry for 'whether p or q' that is *two-dimensional*, in the sense of [16] and [47].

To see this in action, suppose in (22) that *tell* can be modeled as a normal modal operator, which relates a world w to a world v just in case what was told in w is true in v. (The semantic value of *tell*—or *what is told*—is thus akin to Kaplan's technical notion of *what is said*.[27]) Then in working out, at some historically possible world w, what follows from the fact that Jones promised to tell us whether (N or P), w will play the role of *world-as-actual*—determining the propositional identity of what Jones is committed to telling us—even though the semantic value of *tell* itself shifts the world of evaluation, v, under its characteristic accessibility relation.

Where '$\boxplus$' is a Hintikka operator analyzing an arbitrary intensional verb v, a semantics for 'whether'-ascriptions will thus have (at least) two world parameters, a world-as-actual and a world of evaluation:

$$\overbrace{w,}^{\text{world-as-actual}} \qquad \underbrace{v}_{\text{world of evaluation}} \qquad \vDash \boxplus\psi$$

For a working implementation of this machinery for (21)-(24), let $ans_w(.)$ be a world-parameterized function in $\mathcal{L} \times \mathcal{L} \rightarrow \mathcal{L}$ which takes a pair of sentences p and q and outputs only the sentence in the pair which is true at w, the world-as-actual. $ans_w(.)$ thus provides the *true-in-w answer* to the question, "p, or q?"[28] I will write this as:

[24] My simplifying assumption is that p and q are mutually exclusive and jointly exhaustive with respect to the relevant *modal base*, h. Since Lewis and Groenedijk & Stokhof do not work explicitly with a modal base, their simplifying assumption is stated in a different, but clearly homologous, way. See esp. Lewis *op. cit.*, pg. 52, and Groenedijk & Stokhof *op. cit.*, pg. 184, where they postulate that it is a presupposition (in the technical sense) of sentences like (21)-(22) that "exactly one of the [embedded] alternatives is the case." It is worth noting that some linguists have also argued that disjunctions are *obligatorily* interpreted as mutually exclusive; see, for example, recent literature on Hurford's Constraint (esp. [69] and [13].) For the controversy over mutual exclusivity and joint exhaustiveness—that is, partitionality—with respect to the treatment of *wh*-semantic values more generally, see, in the "for" camp, [33, 41, 43] and [31]; in the "against" camp, see [14, 15], and the subsequent tradition in Inquisitive Semantics. Naturally I take no stand on the matter of wh-semantic values more generally, or on the strong construal of Hurford's Constraint, as the pertinent data falls far outside the scope of the present project.

[25] [32, pg. 176]; [48, pg. 7].

[26] See esp. [32, pg. 180]; [59, pg. 51].

[27] See [47, pg. 19].

[28] This "answerhood operator" is thus related to the more sophisticated answerhood operators in the linguistics literature: see, *inter alia*, [20, 21, 37]. The gloss I give here exploits the simplifying assumptions that (i) the inputs p and q are atomic, and (ii) mutually exclusive and jointly exhaustive. Hence exactly one of $\{p, q\}$ is true in w.

(ans_w-or) $h, w, v \vDash (wh[[p] \text{ OR}_w [q]])$ iff $h, w, v \vDash ans_w(p, q)$

Given these standard Hintikka-style entries for *know* and *tell*:

(25) $h, w, v \vDash \ulcorner\alpha$ knows $\phi\urcorner$ iff $\forall v' \in K_w^\alpha$: $h, w, v' \vDash \phi$.
(26) $h, w, v \vDash \ulcorner\alpha$ tells $(\beta)\ \phi\urcorner$ iff $\forall v' \in T_w^\alpha$: $h, w, v' \vDash \phi$.

(ans_w-or) will validate the equivalence of (23) and (21) and the equivalence of (24) and (22) under uniform atomic substitution.[29]

4 OR$_w$ Under 'can': Free Choice

I would like to use (ans_w-or), on the treatment sketched above, as a model for nonspecific de re disjunction. I'll call this Hypothesis 2-Rough:

Hypothesis 2-Rough $h, w, v \vDash (p \text{ OR}_w\ q)$ iff $h, w, v \vDash ans_w(p, q)$

According to Hypothesis 2-Rough, $(p$ OR $q)$, on its nonspecific de re reading, contributes to modal environments a propositional concept which means something like: the proposition that p or the proposition that q, whichever is actually true—according to whichever local or global parameter tracks the actuality in the overall formal system. This disjunction is thus actuality-sensitive even in embedded environments.

With this hypothesis on the table, we are in a position to return to agentive modality, looking in particular at the strong modal entailment pattern (FC$^+$), featuring disjunction under the modal 'can':

(FC$^+$) $C(\phi \text{ OR}_w\ \psi) \Rightarrow C\phi \wedge C\psi$

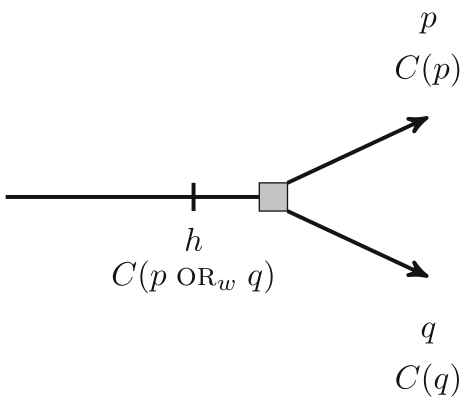
Earlier, I argued for an account of 'can' according to which $C\phi$ is true at h_c just in case it is historically possible in h_c for the agent to (extensionally) realize the proposition expressed by ϕ. But since $(p \text{ OR}_w\ q)$ expresses the proposition that p or the proposition that q, whichever one is true in the relevant world-as-actual, the particular claim being made about realization$_{ex}$ is tied, in a historical modal base, to what transpires at different worlds in the modal base. We can think of these as diverging histories.

In histories where it is (actually) true that p (upper branch of Fig. 2), the claim that it is historically possible for the agent to realize $(p \text{ OR}_w\ q)$ is equivalent (by

[29] We use the notation $h_c \rhd \phi$ for the (postsemantic) truth of ϕ at c. Since truth at an open-future context c is akin to global truth [77], we hold that $h_c \rhd \phi$ iff $\forall w \in h_c : h, w, w \vDash \phi$. Now, suppose that the actually true proposition in $\{p, q\}$ at context c is p: hence, $\forall w \in h_c : ans_w(p, q) = p$. Then

$$\forall w \in h_c : h_c, w, w \vDash \alpha \text{ knows } (wh[p \text{ OR } q]) \text{ iff (by (25))}$$
$$\forall w \in h_c : \forall v \in K_w^\alpha : h_c, w, v \vDash (wh[p \text{ OR } q]) \text{ iff (by } (ans_w\text{-or}))$$
$$\forall w \in h_c : \forall v \in K_w^\alpha : h_c, w, v \vDash ans_w(p, q)\ p \text{ is true in } c)$$
$$\forall w \in h_c : \forall v \in K_w^\alpha : h_c, w, v \vDash p \text{ iff (by (25) again, postsemantic truth)}$$
$$h_c \rhd \alpha \text{ knows that } p.$$

Fig. 2 A prior occurrence of $C(p \text{ OR}_w\ q)$, with future contingents p and q



Hypothesis 2-Rough) to the claim that it is historically possible for the agent to realize the more specific outcome, p. In histories where it is instead (actually) true that q (lower branch of Fig. 2), the claim that it is historically possible for the agent to realize $(p \text{ OR}_w\ q)$ is equivalent (by Hypothesis 2-Rough) to the claim that it is historically possible for the agent to realize the more specific outcome, q.

Assuming that both p and q are live *historical* possibilities, then, and that what is historically possible at a later time is historically possible at any preceding time, no prior ability ascription of the form $C(p \text{ OR}_w\ q)$ can be satisfied at a modal base h unless both $C(p)$ and $C(q)$ are also satisfied at h. This follows from Hypotheses 1-2-Rough (proof in Appendix, Fact 2), and is, I propose, what gives rise to the strong entailment pattern (FC^+). This pattern makes contact with Kenny's intuitions about the granularity of acts because the semantic value of $(p \text{ OR}_w\ q)$ is a proper subset of the Boolean join of p and q.

For a case study, we return to the infinitely long, $\mathbb{Z}$-tiled dartboard. Again, the classical disjunction *even or odd* $(E \vee O)$ is settled-true at h_c. However, there is a reading of (27):

(27) I can hit even *or* odd.
$C(E \text{ OR}_w\ O)$

on which it makes quite a boast.[30] To show this, we think again in terms of a constant margin of error Δ and the two possible witnesses for the disjunction, E and O. On the proposed view, (27) means something like this: in any world where the dart lands on an E-number—call this world w_E—the agent's margin of error must be narrow enough to *guarantee* the w_E-actual witness in $\{E, O\}$: that is, to guarantee E itself. But for that to be true, the agent's aim must, in some E-landing world $w_{E'}$, be precise enough to prevent the dart landing on *any* non-E number, even the on E''s immediate

[30] I here mark the pitch accent which many linguists suggest sharpens the free choice reading. Thanks to Chris Barker for discussion.

46	47	48	49	50	51	52	53	54

Fig. 3 Composite and prime (shaded)

neighbors $E' \pm 1$.[31] Likewise for the other historically possible disjunct, O: in any world w_O where the dart lands on an odd number, the claim being made by (27) is that the speaker's margin of error is narrow enough to guarantee the w_O-actual witness in $\{E, O\}$: that is, to guarantee O itself. Hence (27) is not settled-true at h_c unless the margin of error that determines the R relation for realization$_{ex}$ is uniformly *zero*—a margin small enough to keep α's dart within the range of *one* tile (Appendix, Fact 3).

(27) thus makes a considerably stronger claim about ability than a sentence like

(28) I can hit positive [P] or negative [N].
$C(P \text{ OR}_w\ N)$

While the the agent's margin of error for the truth of (27) must be zero, the agent's margin of error for (28) can be as large as any finite number $k \in \mathbb{N}$: to ensure P (positive), she need only aim for tile $(k+1)$, and to ensure N (negative), she need only aim for $-(k+1)$.

Finally, consider a lopsided case, in which one disjunct is sparsely distributed compared to the other.

(29) I can hit composite [K] or prime [M] .
$C(K \text{ OR}_w\ M)$

which, for concreteness, we can study on this region of the dartboard (Fig. 3, primes shaded).

(29) illustrates (I$^-$), the blocking of disjunction introduction under C. It is not valid to infer $C(K \text{ OR}_w\ M)$ from $C(K)$: an agent can have a margin of error of up to 2 while still ensuring the premise—such an agent can, for example, aim at tile 50—but the presumptive conclusion requires a margin of zero. This case is also presented in detail in the Appendix.

The failure of entailment in (I$^-$) follows the precedent set in the deontic case by Ross's puzzle and Deontic FC$^-$:

(Ross)	$\text{OUGHT}\ \phi \nvDash \text{OUGHT}\ (\phi\ \text{OR}_w\ \psi)$
(Deontic FC$^-$)	$\text{MAY}\ \phi \nvDash \text{MAY}\ (\phi\ \text{OR}_w\ \psi)$

[31] It is possible that $w_E \neq w_{E'}$: w_E may be a world in a modal base relative to which the agent has the *ability* to guarantee E, but chooses not to (for example, because she throws the dart with her eyes closed).

As has been widely observed since [66], a recipe for generating instances of (Ross) and (Deontic FC$^-$) is to begin with some ϕ which is obligatory (or permissible), and disjoin, under the modal operator, some ψ which is *impermissible*.[32] Hence Ross's original:

(30) You ought to post the letter. $\nvDash$ You ought to post the letter or burn it.

and its 'may' analogue:

(31) You may post the letter. $\nvDash$ You may post the letter or burn it.

As (29) illustrates, things are similar in the agentive case illustrated by (29): instead of adding a disjunct that is impermissible, one simply disjoins some ψ which is *more difficult* for the agent than ϕ.

(32) I can hit composite. $\nvDash$ I can hit composite or prime.
(33) Eve can do a double axel. $\nvDash$ Eve can do a double or a triple axel.
(34) You can take the cigarette. $\nvDash$ You can take it or leave it. [49, pg. 215]

4.1 Classicality and indeterminacy

Hypotheses 1-2-Rough illustrate the basic interaction between a margin-of-error sensitive 'can' and an actuality-sensitive 'or'. In this section, I briefly adapt and generalize Hypothesis 2-Rough to suit the needs of a fuller modal logic, and make a few remarks on the nature of that logic. Two lacunae need addressing. The first is that the function $ans_w(\cdot)$, which underwrites the semantics for disjunction, is not complete for the domain of pairs of possible disjuncts. It is undefined in cases where *both* p and q are true, as well as cases in which *neither* is true. The second shortcoming—constraining our response to the first—is that we must make good on the promise of extensional classicality, the claim that OR is equivalent to classical disjunction outside of intensional environments. It is this feature of de re disjunction which secures compatibility with the gloss I offered in §1 for (I$^-$)—the claim that $(p \text{ OR}_w\ q) \nvDash Cp \text{ OR}_w\ Cq$, —as well as safeguarding other known features of propositional logic.

Return, therefore, to the function $ans_w(\phi, \psi)$, previously defined as taking a pair of exclusive and exhaustive sentences and returning the true-at-w sentence in the pair—the true answer to the question, "ϕ, or ψ?". For free choice sentences that *do* feature exclusive and exhaustive disjuncts—like the dartboard sentences—truth-in-the-actual-world is what intuitively breaks the symmetry between the candidate witnesses ϕ and ψ for the disjunction $\ulcorner \phi \text{ OR } \psi \urcorner$. There is nothing to break *that* symmetry in the both- and neither-cases. A default view, which I will pursue here, is that *nothing* does so.[33]

[32] This connection between (Ross), (Deontic FC$^-$), and Free Choice effect is noted, *inter alia*, by [79, pg. 466] and [27].

[33] This view is attractively simple, but it has empirical motivation as well. It helps account for the well-known "Exclusivity" intuition associated with free choice, to the effect that $\Diamond(p \text{ OR } q) \vDash \Diamond p \wedge \Diamond q$ but *not* by way of entailing the stronger $\Diamond(p \wedge q)$; see [26] and [28] for discussion.

	p	q
w_1	T	T
w_2	T	F
w_3	F	T
w_4	F	F

	w_1	w_2	w_3	w_4
w_1	T	T	T	F
w_2	T	T	F	F
w_3	T	F	T	F
w_4	T	T	T	F

Fig. 4 Truth-tables for p and q (left); 2D matrix for $\phi = \ulcorner p \text{ OR}_w\ q \urcorner$ (right)

Hence, define $Ans_w(\phi, \psi)$ as a function that takes an arbitrary pair of sentences in $\mathcal{L}$ and outputs the *set* containing the true-in-w sentences, if there are any, and the whole set, otherwise. Then tie disjunction's semantic value to existential quantification over that set:

Hypothesis 2-Final $h, w, v \vDash (\phi \text{ OR}_w\ \psi)$ iff $\exists\pi \in Ans_w(\phi, \psi)$ s.t. $h, w, v \vDash \pi$

Where $\pi \in Ans_w(\phi, \psi)$ just in case $\pi \in \{\phi, \psi\}$ and π is true at $\langle h, w, w\rangle$.

In the simple two-dimensional framework of Groenendijk & Stokhof and Lewis, Hypothesis 2-Final generates the Stalankerian matrix [73, pg. 81] in Fig. 4 for the four world-types associated with a truth table for disjuncts p and q.

In these Stalnaker matrices, positions along the y-axis represent worlds in their role as world-as-actual; fixing such a world, one can read the proposition expressed by a sentence ϕ along the horizontal.

We can relate Hypothesis 2-Final to classical disjunction by following the traditional view of consequence in two-dimensional semantics: we hold that interpretation tracks diagonal consequence in unembedded environments, but that in the scope of modals, interpretation is shunted off the diagonal.[34] Then the shaded cells of the matrix in Fig. 4 illustrate the following

Fact (Classicality) For any ϕ, ψ in the nonmodal fragment of $\mathcal{L}$: $(\phi \text{ OR}_w\ \psi) \dashv\vDash (\phi \vee \psi)$.[35]

see Appendix, Lemma 2.

This Fact answers the question of whether the present treatment of disjunction scuttles classical logic. It does not. In the wider lexicon, however, the validity of the classical disjunction introduction- and elimination-rules is *limited*. An illicit example of "off-diagonal" disjunction introduction is precisely the "in-scope" derivation of "I can hit composite *or prime*" from "I can hit composite".[36] This respects the pattern

[34] See, for example, the corresponding notion of validity in [47, pg. 547], and the notion of *real world validity* in [19]. For off-diagonal interpretation in the scope of modals, see e.g. Kaplan pg. 545, clause 8.

[35] See Appendix, Theorem 1.

[36] In more detail: C is an upward-entailing modal operator, which generally preserves the direction of consequence: if $\phi \vDash \psi$, then $C(\phi) \vDash C(\psi)$. However, disjunction introduction is not a valid rule at off-diagonal points, and embedding under C moves interpretation to such points. Where $\vDash_d$ is diagonal consequence of the kind highlighted in Fig. 4, we have: $\phi \vDash_d (\phi \text{ OR } \psi)$ but $C(\phi) \nvDash_d C(\phi \text{ OR } \psi)$. (For more on two-dimensional and diagonal consequence, see Appendix.)

suggested by the data: disjunction behaves familiarly in unembedded environments, but can give rise to free choice readings under modals. In the richer world-variables framework typically used to frame the nonspecific de re, the analogue of this result is that so long as each clause is capped with a λw_0-abstractor over the actual world (as in Heim & von Fintel's (20a)-(20c)), the nonspecific de re reading can only occur under intensional operators.

5 Conclusion

In this paper, I sketched a semantic account of agential free choice. I began, in §2, by laying out a modal account of agential *can*, which incorporated both the transparency suggested by [61]'s "crossed wires" cases and the sensitivity to fineness of grain foregrounded by [49]'s dartboard. In §3, I turned to disjunction, tapping a two-dimensional analysis of 'or' to sketch how such disjunction might give rise to nonspecific de re readings under such an operator, while remaining resolutely classical outside of modal environments. This approach moves in step with analyses in linguistics of indefinites whose quantificational force depends on their embedding environments, and gives rise to a well-behaved two-dimensional modal logic in the vein of [47], a fuller exploration of which I leave for elsewhere.

The natural question to raise in closing is whether the account offered here is one on which 'or' is lexically ambiguous. Identifying the phenomenon with the nonspecific de re does not straightforwardly settle the issue, as the nonspecific de re itself combines lexical and structural aspects.

In the simple, two-dimensional system under study here, one way to paraphrase the Fact illustrated in Fig. 4 is that, where '†' is Stalnaker's dagger,[37]

$$h, w, v \vDash (p \vee q) \text{ iff } h, w, v \vDash \dagger(p \text{ OR}_w\ q)$$

Thus if we follow [57], for example, in holding that diagonalization can be freely applied without syntactic triggers, there will be no need to say that 'or' is lexically ambiguous in order to recover its Boolean interpretation even *within* embedded environments.[38] Disjunction can *always* have the entry in Hypothesis 2-Final. It will simply have differing interpretations depending on whether, and how, it is bound by further operators.

Finally, it is worth briefly discussing a different observation which has traditionally been taken to show that modal free choice inferences are pragmatic, rather than semantic, in nature: the fact that they "disappear" under negation [1]. [76], following [3], calls the following very appealing schema "Double Prohibition" (DPr):

$$\text{(DPr)} \quad \neg C(\phi \text{ OR } \psi) \Rightarrow \neg C\phi \wedge \neg C\psi$$

[37] Viz., the operator defined by the clause $h, w, v \vDash \dagger\phi$ iff $h, w, w \vDash \phi$. See Stalnaker *op. cit.* pg. 82, [68, pg. 81] and subsequent literature.

[38] See Lewis *op. cit.*, pg. 94.

This is illustrated, for example, in the transition from (35a) to (35b):

(35) a. Otto cannot clear the high jump or hit the bullseye.
$\neg C(\text{H OR B})$
b. Otto cannot clear the high jump, and Otto cannot hit the bullseye.
$\neg C(\text{H}) \wedge \neg C(\text{B})$

Whereas the Free Choice transition between e.g. $C(\text{H OR B})$ and $C(\text{H}) \wedge C(\text{B})$ is not valid in any normal modal logic, (DPr) *is* valid in *every* such modal logic.

This has seemed to many like evidence against a semantic treatment of free choice generally: the (modal+disjunction) combination $\ulcorner C(\phi \text{ OR } \psi)\urcorner$ reverts to the behavior predicted by classical semantics when it is embedded under negation.[39]

The situation here, though, is not as simple as it seems. Throughout this paper, I built up a semantic analogy between disjunction and nonspecific de re indefinites. The proper response to (DPr), I think, is to take a further page from the literature on indefinites. According to [36]'s seminal treatment, indefinite DPs like ‘a friend of mine’, at LF, are *free* variables that can be bound by various “unselective” quantificational operators.[40] It is widely known that *negation* is non-Boolean in dynamic systems like Heim's: it quantifies universally over assignment functions. As a consequence, under Heimian negation free indefinites uniformly revert to their *classical* existentially-quantified force.[41]

(36) Rick doesn't own a donkey.
$\neg$ [donkey x_1] [Rick own x_1]
Interpreted: $\forall x \neg(donkey(x) \wedge Own(Rick, x))$;
equivalent to $\neg\exists x(donkey(x) \wedge Own(Rick, x))$

This is exactly the pattern we see in (DPr): when an existential—this time, disjunction—is embedded under negation, it reverts to its classical behavior. But from Heim's independently motivated point of view, this is not because the existential is *itself* classical. It is because negation is *not* classical. The upshot is that we can take Heimian negation “off the shelf”, so to speak, and get an account of the validity of (DPr). In the toy system explored here, quantification over the y parameter will do:

$$h, y, x \vDash \neg\phi \text{ iff there is no } y' \text{ s.t. } h, y', x \vDash \phi$$

(see Appendix, Fact 5). The move to treat disjunctions and free variables alike with respect to binding patterns enjoys autonomous support. It was originally

[39] The (DPr) schema is written, in Starr's paper, for general $\Diamond$ modals, rather than ‘can’, though (35) is an obvious application; see Starr *op. cit.*, pg. 3. Starr and Barker themselves, of course, do not take (DPr) to motivate a return to a classical semantics. For another treatment of Free Choice sensitive to (DPr), see [30].

[40] The summary of Heim's semantics in this paragraph and the next is, of course, crudely compressed.

[41] See esp. Heim Ch. 2, §2. *Inter alia*, [82, §3.2.3 ff.] provides additional discussion of the motivations and ramifications of this change in negation.

proposed by [65], who motivated it on grounds completely independent of the interaction of disjunction and negation. In particular, Rooth & Partee noted that disjunctive NPs license donkey-like anaphora, and also that, like indefinite NPs, disjunctions have more scope-taking possibilities than their "universal" cousin (viz., conjunction).

As Rooth & Partee wrote at the time, "considerable work remains [to be done] to turn [the free-variable analysis of disjunction] into an explicit set of rules" (*op cit.*, pg. 9). I too have not offered such a treatment here, as it goes beyond the scope of the present project. However, I submit that it is a natural, forceful response to the objection to (FC$^+$) from (DPr).

Appendix

In this appendix, I work with a univocal lexical entry for 'OR', and hence omit the subscript 'w'. To simplify proof by induction, I will assume that the parse of a disjunction $\ulcorner a_1 \text{ OR } a_2 \text{ OR } a_3 \text{ OR } \ldots \text{ OR } a_n \urcorner$ has the LF $((\ldots(a_1 \text{ OR } a_2) \text{ OR } a_3) \text{ OR } \ldots \text{ OR } a_n)$. Hence any n-ary disjunction is at most disjunctive in its left argument.

Syntax Let $\mathtt{At}$ be a set of propositional letters $a_1, a_2 \ldots$. We define three languages, $\mathcal{L}_{bl}$ (the Boolean fragment of $\mathcal{L}$), $\mathcal{L}_{nonm}$ (the nonmodal fragment of $\mathcal{L}$), and $\mathcal{L}$.

$$\mathcal{L}_{bl}\phi ::= a_i \mid \neg\phi \mid (\phi \wedge \phi)$$
$$\mathcal{L}_{nonm}\phi ::= a_i \mid \neg\phi \mid (\phi \wedge \phi) \mid (\phi \text{ OR } \phi)$$
$$\mathcal{L}\phi ::= a_i \mid \neg\phi \mid (\phi \wedge \phi) \mid (\phi \text{ OR } \phi) \mid \blacklozenge\phi \mid \boxdot\, \phi \mid C\phi$$

Semantics A **model** M is a triple $\langle W, R, \mathcal{I}\rangle$ where W a nonempty set of possible worlds, R is a reflexive binary relation on W, and $\mathcal{I}$ is a function from the elements of $\mathtt{At}$ to $\mathcal{P}(W)$ ("the interpretation function").

We define **the standard intension of** ϕ, $V(\phi)$, on $\mathcal{L}_{nonm}$ as follows:

$$V(a) = \mathcal{I}(a)$$
$$V(\neg\phi) = W \setminus V(\phi)$$
$$V(\phi \wedge \psi) = V(\phi) \cap V(\psi)$$
$$V(\phi \text{ OR } \psi) = V(\phi) \cup V(\psi)$$

A **point of evaluation in** M is a triple $\langle h, y, x\rangle$ such that h is a *serial, reflexive subset* of W ($\forall w \in h, wRw$), and a pair of worlds $y, x \in h$.

Truth at a Point of Evaluation For any model M and point of evaluation $\langle h, y, x\rangle$ in M, propositional letter a, wffs ϕ, ψ:

$$
\begin{array}{rll}
h, y, x \vDash a & \text{iff} & x \in V(a) \\
h, y, x \vDash \neg\phi & \text{iff} & \text{there is no } y' \text{ such that } h, y', x \nvDash \phi \\
h, y, x \vDash (\phi \wedge \psi) & \text{iff} & h, y, x \vDash \phi \text{ and } h, y, x \vDash \psi \\
h, y, x \vDash \blacklozenge\phi & \text{iff} & \exists w \in h : h, y, w \vDash \phi \\
h, y, x \vDash \boxdot\phi & \text{iff} & \forall x' \in h\text{: if } xRx',\ \text{then } h, y, x' \vDash \phi \\
h, y, x \vDash C\phi & \text{iff} & \exists w \in h \text{ s.t.: } \forall v \text{s.t. } wRv : h, y, v \vDash \phi.
\end{array}
$$

...these entries are the entries of the toy language in Table 2 (§2), with a free y parameter added.

Given a pair of sentences ϕ_1 and ϕ_2 in $\mathcal{L}$, the w-relative **answer set** of ϕ_1 and ϕ_2 is

$$
Ans_w(\phi_1, \phi_2) = \begin{cases} \{\phi_1\} & \text{if } h, w, w \vDash \phi_1 \text{ and } h, w, w \nvDash \phi_2 \\ \{\phi_2\} & \text{if } h, w, w \vDash \phi_2 \text{ and } h, w, w \nvDash \phi_1 \\ \{\phi_1, \phi_2\} & \text{otherwise.} \end{cases}
$$

Now disjunction can be added:

$$
h, y, x \vDash (\phi \text{ OR } \psi) \quad \text{iff} \quad \exists\beta : \beta \in Ans_y(\phi, \psi) \text{ and } h, y, x \vDash \beta
$$

Two interdefinitions of C hold, given the reflexivity of R: (i) $C\phi := \blacklozenge\boxdot\phi$ [39, pg. 606]; (ii) $C\phi := \blacklozenge(\phi \wedge \boxdot\phi)$.

Consequence

There are four notions of consequence available in our system, corresponding to some choice of *local* or *global*, and *diagonal* or *two-dimensional*.

	global	local
diagonal	$\vDash_1$	$\vDash_2$
two-dimensional	$\vDash_3$	$\vDash_4$

We are interested primarily in the preservation of **diagonal acceptance**, which corresponds to $\vDash_1$: ϕ is accepted at h iff $\forall w \in h$: $h, w, w \vDash \phi$. For short, we use $h \triangleright \phi := \forall w \in h$: $(h, w, w \vDash \phi)$.

Lemma 1 (Nondisjunctive Stability) *For any $\phi \in \mathcal{L}_{bl}$, any $h \subseteq W$, and $x, y, y' \in h$: $h, y, x \vDash \phi$ iff $h, y', x \vDash \phi$.*

Proof A trivial induction on the complexity of $\phi \in \mathcal{L}_{bl}$. □

Theorem 1 (Diagonal Classicality) *For any* $h \subseteq W, w \in h,$ *and* $\phi \in \mathcal{L}_{nonm}$*:* $h, w, w \vDash \phi$ *iff* $w \in V(\phi)$*.*

Proof By induction. The atomic, negation, and conjunction cases are trivial.

Disjunction. We need to show: $h, w, w \vDash (\phi \text{ OR } \psi)$ iff $w \in (V(\phi) \cup V(\psi))$. Assume for the Inductive Hypothesis that (i) $h, w, w \vDash \phi$ iff $w \in V(\phi)$, and (ii) $h, w, w \vDash \psi$ iff $w \in V(\psi)$.

($\Rightarrow$) If $h, w, w \vDash (\phi \text{ OR } \psi)$, then $w \in (V(\phi) \cup V(\psi))$.

If $h, w, w \vDash (\phi \text{ OR } \psi)$, then $\exists\beta$: $\beta \in Ans_w(\phi, \psi)$ and $h, w, w \vDash \beta$. For any such h, w, and β: $\beta \in \{\phi, \psi\}$. Hence if $h, w, w \vDash \beta$, then $h, w, w \vDash \phi$ or $h, w, w \vDash \psi$. Hence (by Inductive Hypothesis) $w \in V(\phi)$ or $w \in V(\psi)$. Hence $w \in (V(\phi) \cup V(\psi))$.

($\Leftarrow$) If $w \in (V(\phi) \cup V(\psi))$, then $h, w, w \vDash (\phi \text{ OR } \psi)$.

If $w \in (V(\phi) \cup V(\psi))$, then $w \in V(\phi)$ or $w \in V(\psi)$.

Case 1. $w \in V(\phi)$. Then by IH, $h, w, w \vDash \phi$. By the definition of the *Alt* function, it follows that $\phi \in Ans_w(\phi, \psi)$. Hence $\exists\beta(=\phi) \in Ans_w(\phi, \psi)$ such that $h, w, w \vDash \beta$. Hence $h, w, w \vDash (\phi \text{ OR } \psi)$. Case 2 is similar, but with ψ/ϕ.

□

Lemma 2 (Classical Theoremhood) *For any* $\phi \in \mathcal{L}_{nonm}, \vDash_1 \phi$ *iff* ϕ *is a theorem of classical propositional logic.*

Application: The Dartboard [49]

We identify worlds with ordered pairs $\langle\tau(n), m\rangle$ consisting of a position tried for (n), and a position hit (m).[42] $\langle\tau(n), m\rangle$ is globally possible—possible with respect to the modal base—if $|n - m| \leq \Delta$, where Δ is the agent's margin of error. For the local accessibility relation R on worlds, we assume $\langle\tau(n), m\rangle R \langle\tau(n'), m'\rangle$ iff

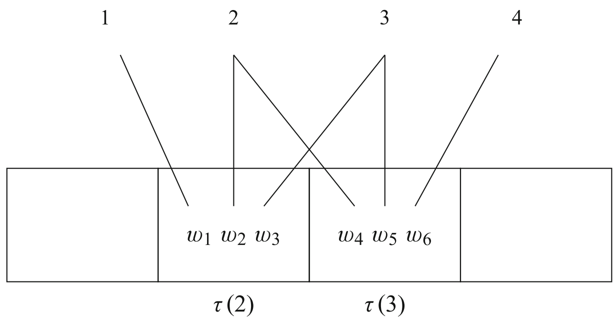
- $n = n'$ (the agent is omniscient w.r.t. her tryings); and
- $\langle\tau(n), m\rangle$ and $\langle\tau(n'), m'\rangle$ are both globally possible.

We can show that:

Fact 1 (FC$^-$) $h \rhd (E \text{ OR } O)$ but $h \not\rhd C(E \text{ OR } O)$.

Suppose the agent will try to hit either 2 or 3 in the figure below, and that $\Delta = 1$, and so the dart will fall in the range [1,4]. Our modal base is $\{w_1 \ldots w_6\}$, where $w_1 = \langle\tau(2), 1\rangle$, $w_2 = \langle\tau(2), 2\rangle$, $w_4 = \langle\tau(3), 2\rangle$, and so on. $\mathcal{I}(E) = \{w_2, w_4, w_6\}$. $\mathcal{I}(O) = h \setminus \mathcal{I}(E)$.

[42] C.f. [81, pg. 985].



Proof By **Classicality**, $h \triangleright (E \text{ OR } O)$ iff $h \triangleright (E \vee O)$. That this latter claim is true is clear by inspection of the model.

Now, we evaluate the claim that $h \triangleright C(E \text{ OR } O)$. Using the second paraphrase ($C(\phi) := \blacklozenge(\phi \wedge \boxdot\phi)$), $h \triangleright C(E \text{ OR } O)$ iff $\forall w \in h$: $\exists v \in h$ s.t.: (i) $h, w, v \vDash (E \text{ OR } O)$ and (ii) $h, w, v \vDash \boxdot(E \text{ OR } O)$. We instantiate w with w_2. Hence:

$$\begin{aligned}
\exists v \in h \text{ s.t.: } (i)\ h, w_2, v \vDash (E \text{ OR } O) \text{ and } (ii)\ h, w_2, v \vDash \boxdot(E \text{ OR } O) \text{ iff}\\
\exists v \in h \text{ s.t.: } (i)\ \exists \beta \in Ans_{w_2}(E, O) : h, w_2, v \vDash \beta \text{ and } (ii)\ h, w_2, v \vDash \boxdot(E \text{ OR } O) \text{ iff}\\
\exists v \in h \text{ s.t.: } (i)\ \exists \beta \in \{E\} : h, w_2, v \vDash \beta \text{ and } (ii)\ h, w_2, v \vDash \boxdot(E \text{ OR } O) \text{ iff}\\
\exists v \in h \text{ s.t.: } (i)\ h, w_2, v \vDash E \text{ and } (ii)\ h, w_2, v \vDash \boxdot(E \text{ OR } O) \text{ iff}\\
\exists v \in h \text{ s.t.: } (i)\ h, w_2, v \vDash E \text{ and } (ii)\ \forall w' \text{ s.t. } vRw' : h, w_2, w' \vDash E \text{ iff}\\
\exists v \in h \text{ s.t.: } (i)((v = w_2) \vee (v = w_4) \vee (v = w_6)) \text{ and } (ii)\ \forall w' \text{ s.t. } vRw' : w' \in \mathcal{I}(E)\\
\text{...but there is no such } v\text{: each } v \in E \text{ is s.t. } \exists w' : vRw' \text{ and } w' \notin \mathcal{I}(E).
\end{aligned}$$

Hence $h \not\triangleright C(E \text{ OR } O)$. □

For the next Fact, it is in the interest of generality not to presume a dartboard model. (A dartboard-specific version of the proof, in terms of margins of error, appears below (Fact 3).)

Fact 2 (FC$^+$ for historically possible and mutually exclusive disjuncts) $C(p \text{ OR } q)$, $\blacklozenge(p \wedge \neg q)$, $\blacklozenge(q \wedge \neg p) \vDash C(p) \wedge C(q)$.

Proof $h \triangleright C(p \text{ OR } q)$ iff $\forall w \in h$: $\exists v \in h$ s.t.: (i) $h, w, v \vDash (p \text{ OR } q)$ and (ii) $h, w, v \vDash \boxdot(p \text{ OR } q)$. By the premise $\blacklozenge(p \wedge \neg q)$, $\exists w' \in h$ such that $h, w', w' \vDash (p \wedge \neg q)$ (call this world "w_p"). By the premise $\blacklozenge(q \wedge \neg p)$, $\exists w'' \in h$ such that $h, w'', w'' \vDash (q \wedge \neg p)$ (call this world "w_q").

Case 1. First, we instantiate w with w_p. As above, it follows that $\exists v \in h$ (call it w_{p*}) s.t. (i) $h, w_p, w_{p*} \vDash (p \text{ OR } q)$ and (ii) $h, w_p, w_{p*} \vDash \boxdot(p \text{ OR } q)$.

For the first conjunct: $h, w_p, w_{p*} \vDash (p \text{ OR } q)$ iff $\exists \beta \in Ans_{w_p}(p, q)$ s.t. $h, w_p, w_{p*} \vDash \beta$. Because $Ans_{w_p}(p, q)$ is the singleton $\{p\}$, it follows that $h, w_p, w_{p*} \vDash p$.

For the second conjunct: $h, w_p, w_{p*} \vDash \boxdot(p \text{ OR } q)$ iff $\forall v$ s.t. $w_{p*}Rv$, $\exists\beta \in Ans_{w_p}(p, q)$ s.t. $h, w_p, v \vDash \beta$. Again, because $Ans_{w_p}(p, q)$ is the singleton $\{p\}$, it follows that $\forall v$ s.t. $w_{p*}Rv$: $h, w_p, v \vDash p$. Hence $h, w_p, w_{p*} \vDash \boxdot p$.

Hence for any $w \in h$: $\exists v$ (viz., w_{p*}) s.t. (i) $h, w, v \vDash p$ and (ii) $h, w, v \vDash \boxdot p$. It follows that $h \rhd \blacklozenge(p \wedge \boxdot p)$, and hence that $h \rhd C(p)$. ✓

Case 2. Second, we instantiate w with w_q. A symmetric argument to the argument in Case 1 with q/p will show that $h \rhd C(q)$. ✓ □

Fact 3 (I$^-$) $C(\phi) \nvDash C(\phi \text{ OR } \psi)$.

For this example, we consider $\phi = K$ (composite) and $\psi = P$ (prime) as in the main text, restricting for convenience to n between 46 and 54 (Fig. 3).

If the agent's margin of error is 2 or less, she can reliably guarantee K in this range by aiming for e.g. 50. However, she cannot reliably guarantee P. But by a similar proof to the proof of Fact 3 above, $C(K \text{ OR } P) \vDash C(K) \wedge C(P)$. Since $h \not\rhd C(P)$, $h \not\rhd C(K \text{ OR } P)$.

General (n-ary) Disjunction

Suppose $\phi, \psi \in \mathcal{L}_{nonm}$. Then for any disjunctive wff $(\phi \text{ OR } \psi)$, $\psi \in \mathcal{L}_{bl}$. We want to show, where $\otimes$ is exclusive-or:

Fact 4 (Procedure for disjunction) $h, y, x \vDash (\phi \text{ OR } \psi)$ iff either
(i) $h, y, y \vDash (\phi \otimes \psi)$ and $h, y, x \vDash \chi$, where χ is the $\alpha \in \{\phi, \psi\}$ s.t. $h, y, y \vDash \alpha$, or
(ii) $h, y, y \nvDash (\phi \otimes \psi)$ and $h, y, x \vDash (\phi \vee \psi)$.

Proof This follows from inspection of the clause for "OR". (i) covers the first two cases of the $Ans_y(\phi, \psi)$ function, while (ii) covers the third case. □

For the next theorem, we use the following

Notation. For $\phi \in \mathcal{L}_{bl}$ and $x, y \in W$: $x \sim_\phi y$ iff $h, y, y \vDash \phi$ and $h, x, x \vDash \phi$. NB that by Nondisjunctive Stability, above, this is equivalent to: $x \sim_\phi y$ iff $h, y, y \vDash \phi$ and $h, y, x \vDash \phi$.

Theorem 2 (Characterization of 2D Disjunction.) *If, for $\phi_1 \ldots \phi_n \in \mathcal{L}_{bl}$: $h, y, x \vDash (\phi_1 \text{ OR } \ldots \text{ OR } \phi_n)$ and $\exists!\phi_i \in \{\phi_1 \ldots \phi_n\}$ s.t. $h, y, y \vDash \phi_i$, then $y \sim_{\phi_i} x$.*

Proof by induction on the length of n.

Proof Atomic case (viz., two disjuncts in $\mathcal{L}_{bl}$.)

We show that for $\phi, \psi \in \mathcal{L}_{bl}$, if $h, y, x \vDash \phi \text{ OR } \psi$ and $\exists!\alpha \in \{\phi, \psi\}$ s.t. $h, y, y \vDash \alpha$, then $y \sim_\alpha x$.

Assume $h, y, x \vDash (\phi \text{ OR } \psi)$ for $\phi, \psi \in \mathcal{L}_{bl}$ and $\exists!\phi_i \in \{\phi, \psi\}$ s.t. $h, y, y \vDash \phi_i$. Then either (i) $h, y, y \vDash \phi$ and $h, y, y \nvDash \psi$, or (ii) $h, y, y \vDash \psi$ and $h, y, y \nvDash \phi$. We show that in either case, $y \sim_{\phi_i} x$.

Case (i). In this case, $Ans_y(\phi, \psi) = \{\phi\}$. Hence $h, y, x \vDash (\phi \text{ OR } \psi)$ iff $h, y, x \vDash \phi$; hence $h, y, x \vDash \phi$. Hence $\phi_i = \phi$. Since $h, y, y \vDash \phi$ and $h, y, x \vDash \phi$, it follows that $y \sim_{\phi_i} x$.

Case (ii) is symmetric, with ψ instead of ϕ. In this case, $\phi_i = \psi$ and $y \sim_{\phi_i} x$.

Inductive Step (number of disjuncts > 2.)

Assume that if, for $\phi_1 \ldots \phi_{(n-1)} \in \mathcal{L}_{bl}$, $h, y, x \vDash (\phi_1 \text{ OR } \ldots \text{ OR } \phi_{(n-1)})$ and $\exists! \phi_i \in \{\phi_1 \ldots \phi_{(n-1)}\}$ s.t. $h, y, y \vDash \phi_i$, then $y \sim_{\phi_i} x$ (viz., that $h, y, x \vDash \phi_i$).

Show: if, for $\phi_1 \ldots \phi_n \in \mathcal{L}_{bl}$: $h, y, x \vDash ((\phi_1 \text{ OR } \ldots \text{ OR } \phi_{(n-1)}) \text{ OR } \phi_n)$, and $\exists! \phi_i \in \{\phi_1 \ldots \phi_n\}$ s.t. $h, y, y \vDash \phi_i$, then $y \sim_{\phi_i} x$. □

Proof If, by " OR ", $h, y, x \vDash ((\phi_1 \text{ OR } \ldots \text{ OR } \phi_{(n-1)}) \text{ OR } \phi_n)$ and $\exists! \phi_i \in \{\phi_1 \ldots \phi_n\}$ s.t. $h, y, y \vDash \phi_i$, then either

(i) $\exists! \phi_i \in \{\phi_1 \ldots \phi_{(n-1)}\}$ s.t. $h, y, y \vDash \phi_i$ and $h, y, y \nvDash \phi_n$; or
(ii) $\exists! \phi_i \in \{\phi_n\}$ s.t. $h, y, y \vDash \phi_i$ and $h, y, y \nvDash (\phi_1 \text{ OR } \ldots \text{ OR } \phi_{(n-1)})$.

Case (i). Then by IH, $y \sim_{\phi_i} x$ for $i < n$ and hence $\exists! \phi_i \in \{\phi_1 \ldots \phi_n\}$ s.t. $y \sim_{\phi_i} x$.

Case (ii). Then $Ans_y(\phi \text{ OR } \ldots \text{ OR } \phi_{(n-1)}, \phi_n) = \{\phi_n\}$. Hence $h, y, x \vDash ((\phi_1 \text{ OR } \ldots \text{ OR } \phi_{(n-1)}) \text{ OR } \phi_n)$ iff $h, y, x \vDash \phi_n$. Hence $x \sim_{\phi_n} y$; hence there is some unique $\phi_i \in \{\phi_i\}$ s.t. $x \sim_{\phi_i} y$. □

Theorem 3 (OR -elim$^+$) *Let* $\boxplus$ *be a normal modal operator. We show that, for* $\phi_1 \ldots \phi_n \in \mathcal{L}_{bl}$*: if (Premise 1)* $h, y, y \vDash \boxplus(\phi_1 \text{ OR } \ldots \text{ OR } \phi_n)$ *and (Premise 2)* $h, y, y \vDash ((\bigwedge_{j \neq i} \neg \phi_j) \wedge (\phi_i))$*, then (C)* $h, y, y \vDash \boxplus \phi_i$.

Proof $h, y, y \vDash \boxplus(\phi_1 \text{ OR } \ldots \text{ OR } \phi_n)$ iff $\forall x$ s.t. yRx, $h, y, x \vDash (\phi_1 \text{ OR } \ldots \text{ OR } \phi_n)$. By Theorem 2 and (Premise 2), $h, y, x \vDash (\phi_1 \text{ OR } \ldots \text{ OR } \phi_n)$ entails $h, y, x \vDash \phi_i$. Hence by (Premise 1), $\forall x$ s.t. xRy: $h, y, x \vDash \phi_i$. Hence $h, y, y \vDash \boxplus \phi_i$. □

(DPr) Under Quantificational Negation

Fact 5 ((DPr) for (FC$^+$)) Here, we show that with the alternative entry for negation proposed in §5:

($\neg_2$) $h, y, x \vDash \neg \phi$ iff there is no y' s.t. $h, y', x \vDash \phi$

a form of (DPr) follows. We focus on the simple case $\neg_2 C(p \text{ OR } q) \vDash \neg_2 C(p) \wedge \neg_2 C(q)$, adding (as in the proof of Fact 2) the assumption that $\blacklozenge(p \wedge \neg q)$ and $\blacklozenge(q \wedge \neg p)$.

Proof $\neg_2 C(p \text{ OR } q), \blacklozenge(p \wedge \neg q), \blacklozenge(q \wedge \neg p) \vDash \neg_2 C(p) \wedge \neg_2 C(q)$

$$\begin{aligned} h \rhd \neg_2 C(p \text{ OR } q) \quad & \text{iff} \;\; \forall w \in h : h, w, w \vDash \neg_2 C(p \text{ OR } q) \\ & \text{iff} \;\; \forall w \in h : \text{ there's no } y' \in h : h, y', w \vDash C(p \text{ OR } q) \\ & \text{iff} \;\; \forall w \in h : \text{ there's no } y' \in h : \exists w' \in h : h, y', w' \vDash \boxdot(p \text{ OR } q) \\ & \text{iff} \;\; \text{there's no } y', w' \in h : h, y', w' \vDash \boxdot(p \text{ OR } q) \\ & \text{iff} \;\; \forall y' : \text{ there's no } w' \in h : h, y', w' \vDash \boxdot(p \text{ OR } q) \end{aligned}$$

By the second premise, we know that $\exists w \in h$ such that $h, w, w \vDash p$ and $h, w, w \nvDash q$ (call this world w_p). Instantiating w_p for y' above, we can conclude that:

$$\begin{aligned} & \text{there's no } w' \in h : h, w_p, w' \vDash \boxdot(p \text{ OR } q) \\ & \text{iff there's no } w' \in h : h, w_p, w' \vDash \boxdot p \end{aligned}$$

Because the truth-conditions of p are not actuality-parameter sensitive, this is equivalent to:

$$\begin{aligned} & \forall y' \in h : \text{ there's no } w' \in h : h, y', w' \vDash \boxdot p \\ & \text{iff there's no } y', w' \in h : h, y', w' \vDash \boxdot p \end{aligned}$$

Hence $h \rhd \neg_2 C(p)$. A similar argument, leveraging the third premise, shows that $h \rhd \neg_2 C(q)$. □

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