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Modalising Plurals

Simon Thomas Hewitt

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Abstract There has been very little discussion of the appropriate principles to govern a modal logic of plurals. What debate there has been has accepted a principle I call (NecInc); informally if this is one of those then, necessarily: this is one of those. On this basis Williamson has criticised the Boolosian plural interpretation of monadic second-order logic. I argue against (NecInc), noting that it isn't a theorem of any logic resulting from adding modal axioms to the plural logic PFO+, and showing that the most obvious formal argument in its favour is question begging. I go on to discuss the behaviour of natural language plurals, motivating a case against (NecInc) by developing a case that natural language plural terms are not de jure rigid designators. The paper concludes by developing a model theory for modal PFO+ which does not validate (NecInc). An Appendix discusses (NecInc) in relation to counterpart theory.

> Of course, it would be a mistake to think that the rules for "multiple pointing" follow automatically from the rules for pointing proper.
>
> Max Black—The Elusiveness of Sets

In some influential articles during the 1980s George Boolos proposed an interpretation of monadic second-order logic in terms of plural quantification [4, 5]. One objection to this proposal, pressed by Williamson [22, 456-7], focuses on the modal behaviour of plural variables, arguing that the proposed

S. T. Hewitt (✉)
Department of Philosophy, Birkbeck College, University of London, London, WC1E 7HX, UK
e-mail: lhewi01@students.bbk.ac.uk

interpretation yields the wrong results in respect of the modal status of atomic predications. In the present paper I will present this objection and argue against it. In the course of developing the argument, I will have cause to consider the under-investigated question of how a logic for plurals should be extended to incorporate modal operators.

Keywords Second-order logic · Plural quantification · Modal logic

1 A Problem with Predication

To understand the modal objection to Boolos' interpretation of monadic second-order logic (**MSOL**), it is helpful to consider Rayo's formalisation of the key clauses from Boolos' translation scheme [16, 26]:[1]

$$Tr'(X_j x_i) = x_i \prec xx_j$$

$$Tr'(\exists X_j.\phi) = \exists xx_j.Tr'(\phi) \vee Tr'(\phi*)$$

where $\phi*$ is the result of substituting $x_i \neq x_i$ everywhere for $X_j x_i$ in ϕ.

Here 'xx' is a plural variable, taking as its values some things together in plurality, and '$\prec$' denotes plural inclusion. We read '$x \prec xx$' in inelegant logicians' English as 'x is among xx'. Throughout this paper I will follow Rayo in taking the background (non-modal) plural logic[2] to be **PFO**$^+$, although for reasons which will become clear in due course I will follow McKay [14] in

[1]It should be empahasised that Rayo's plural notation is replacing here Boolos' use of natural language plurals. A prevalent misreading of Boolos understands him as arguing that second-order quantifiers just *are* plural quantifiers. or that predicates just *are* plural terms. Boolos does not demur from the Fregean position that a thought/ sentence is not simply a list of constituents. Roughly, the first of the translation clauses presented here conveys the proposal that predicate letters are incomplete symbols, the concatenation of which with an appropriate number of singular terms communicates plural inclusion. Thank you to an anonymous referee for raising questions about this.

[2]For details see [13] and [16]. More general issues around the development of a formal logic of plurals are discussed in [14]. Two minor differences from Rayo's **PFO+** will be operative in the present paper. First, as will be clear, I offer a simplified rendering of the extensionality axiom. Second, I do not allow that plural predicates are mixed, in the sense of having potentially both singular and plural argument places. This seems to me to be an unnecessary complication in the development of a formal logic of plurals. Given the admission of single-membered pluralities, we can simply allow that plural terms denoting such pluralities can stand proxy for the relevant singular terms. Whilst this is may not faithful to the precise logical form of natural language plural usage (although this requires more examination), it is precisely the kind of simplification that is usual when regimenting discourse in a formal language.

writing '$xx \approx yy$' to mean 'xx are the same as yy', carefully distinguishing the *same things* relation from singular identity, denoted '='. Like McKay also, I will understand '$\approx$' as defined rather than primitive, although unlike him I will take the salient definition to be in terms of '$\prec$':[3]

$$(xx \approx yy) \leftrightarrow_{df} \forall z\, z \prec xx \leftrightarrow z \prec yy \tag{$\approx$}$$

There are good, although in my view not defeating, objections which can be made to the second of the Boolosian translation clauses. Our focus here, however, is on the first. This invites us to understand predication in terms of plural inclusion. But—the objection now announces itself—this gives us incorrect results when we consider the (alethic) modal behaviour of predications. Plurals conform to the modal principle:

$$x \prec xx \rightarrow \Box x \prec xx \tag{NecInc}$$

The corresponding principle for predication—$Xx \rightarrow \Box Xx$—is clearly false. So predication must involve something quite distinct from plural inclusion, not just at the superficial level of natural language grammar, but in terms of fundamental logical strucutre. Boolos' proposed translation scheme must be incorrect. Or so it is argued.

Here are two bad ways to respond to the modal complaint against Boolos. The first is to claim the objection has no force since Boolos is offering us a translation of **MSOL**, and the language of **MSOL** does not contain modal operators. Thus any objection to his translation on the grounds that it involves an unacceptable account of the interaction of plurals with modal operators is beside the point. This response has some value as a salutary reminder to philosophers that any serious study of formal logic should involve a rigorous specification of the language of the system under consideration That having been said, it remains the case that the proposed defence misses a vital point. One supposed virtue of the Boolos interpretation of **MSOL** is that it safeguards the system's status as part of logic, the thought being that plural inclusion and quantification do not bring with them ontological commitments injurious to logical status and do not require cognitive resources beyond those which

[3]The alternative, faithful to McKay, is to define sameness in terms of the relation written '$xx \sqsubseteq yy$'; the relation of some things being amongst some other things. McKay treats what I call 'inclusion'—the relation $\prec$—as a special case of what he calls inclusion—the relation $\sqsubseteq$—in which the plural term on the LHS denotes a plurality of one. I can define '$\sqsubseteq$' in terms of my primitive '$\prec$': $xx \sqsubseteq yy \leftrightarrow_{df} \forall z\, z \prec xx \rightarrow z \prec yy$. I use the term 'inclusion' for $\prec$ (rather than $\sqsubseteq$) since I wish to avoid the set theoretic connotations which come with the more frequently employed 'membership'.

might be thought logical.[4] Whilst there is little agreement on what is at issue in debates about logicality, a broad consensus could be assembled around the maxim that logic is a *topic neutral* enquiry. Logic can be used to talk about anything. But there are many topics where modal considerations loom large in our everyday language and thought. An obvious example here is the study of ordinary concreta: the occupants of my office are philosophers, but we might have been lion-tamers. Both my flasks are full of coffee at t, the present moment; alas, they might have been empty. It would be curious indeed if a supposedly topic neutral non-modal logic could not be extended in a straightforward fashion in order to facilitate formalisation of these sorts of claim.

Topic neutrality gives us one reason for rejecting a second bad response. This proceeds by noting that Boolos' motives for his plural interpretation were confined to the foundations of mathematics—he wished to allow for a non paradox-incurring set theory with only finitely-many axioms, for example. Modal considerations, the response continues, are simply irrelevant in mathematics; the truths of mathematics are necessary truths, and mathematical falsehoods are necessary falsehoods. Again, the criterion of topic neutrality blocks this response. **MSOL** might be especially useful for talking about mathematical entities, but that in no way means that it shouldn't be available for talking about other things as well. Indeed, if it is truly logic it *must* be so available; if it can't be used to talk about cabbages and kings, it isn't available *as a logic* to talk about sets and real numbers. The response even faces trouble even with respect to its claim that mathematics is modally uninteresting. The modal structuralism of [8] follows Putnam in claiming that ordinary mathematical statements are implicitly modal.

A better response to the anti-Boolosian's complaint dinstinguishes between a maximal and a minimal understanding of the Boolos interpretation of **MSOL**. On a minimal understanding, Boolos' translation should be understood as an interpretation only in a limited mathematical sense. Plural logic may be used to interpret **MSOL** only in the same sense that set theory may be used to interpret arithmetic. All that follows from the translation is a result about the relative expressive power of two formal systems; nothing is disclosed about the meaning of second-order formulae, or about the natural language and thought they regiment, or about logicality or ontological commitment. The maximal interpretation draws bolder philosophical conclusions from Boolos' work. It maintains that the Boolosian interpretation is in some way canonical, that it is informative with respect to what second-order logic *really* involves, and presumably also with respect to the philosophy of predication. Williamson's criticisms of the Boolosian interpretation assume a maximal understanding, as does our discussion to this point. If we settle on a minimal

[4]The primary source of the felt need to defend the logicality of second-order logic is, of course, Quine [15], although see also [19]. For scepticism about the invocation of the Boolos interpretation as a defence of logicality see Resnik's [17] and Linnebo's [12].

understanding, there is no worry about the necessitation of predication, since we never expected the Boolos translation scheme to tell us anything deep about the nature of predication. Now, whilst the minimal understanding does not empty the Boolosian interpretation of all philosophical significance,[5] it does deprive it of importance for debates about the nature and logicality of second-order systems and the semantics of predication, so it would be nice to be able to defend a maximal understanding of the Boolosian interpretation against Williamson.

Unsurprisingly, neither bad response is any good. And the better response robs the Boolos project of a substantial amount of its philosophical significance. We must do better if we are to defend the Boolosian interpretation of **MSOL**, and doing better will have to involve attacking head on the claim that the interpretation yields $Xx \to \Box Xx$. This necessitation of atomic predication is clearly unacceptable. Our only option then is to undermine the claim about the modal behaviour of plural variables which, in combination with Boolos' translation scheme, entails the unwelcome result about predication. Consider the *necessary inclusion thesis*, formalised by our (NecInc): $x \prec xx \to \Box x \prec xx$. What reasons are there to believe it? (NecInc) is certainly not a theorem of any system which would result from extending a standard plural logic by the addition of modal operators and any of the usual axiom systems for normal modal logics.[6] What we do have as a theorem is:

$$\Box\big(\forall xx \forall yy\, xx \approx yy \leftrightarrow \forall x(x \prec xx \leftrightarrow x \prec yy)\big) \qquad \text{(NecExt)}$$

Which is the necessitation of ($\approx$), and a **K**-thoerem. Now, (NecInc) is not derivable from (NecExt) Nor is it even derivable from the significantly stronger extensionality principle:

$$\Box\big(\forall xx \forall yy\, xx \approx yy \leftrightarrow \forall x \Box(x \prec xx \leftrightarrow x \prec yy)\big) \qquad \text{(StrongNec)}$$

That (StrongNec)$\nvdash$(NecInc) might seem surprising. It is most readily verified via. the construction of a countermodel in a suitable model theory. We will develop such a theory later. For present purposes, though, and in

[5]Here is an interesting implication of the Boolosian translation scheme, preserved even on a minimal understanding: consider second-order ZF (closely related to the class theory MK). If the first-order variables are taken to range with absolute generality over all sets then standard interpretations of the range of the second-order variables in terms of sets of objects in the domain encounter problems: we seem to incur commitment to a universal set. The problem disappears if we interpret the second-order variables plurally. But the Boolos interpretation shows that there is a logic of plurals in which **MSOL** can be interpreted. So why not just axiomatise set theory using a plural logic?

[6]Why does this matter? it will ultimately turn out that the issue of whether we should accept (NecInc) turns on distinctively *philosophical*, rather than formal, considerations, confirming the Kripkean thought that there are no mathematical solutions to be had to philsophical questions. However, it is important to exclude any likely avenues of formal confirmation at the outset, since its following logically from antecendently accepted plural principles would be decisive support for (NecInc). Thank you to an anonymous referee for comments on this question.

sketch: take $\mathcal{F} = \langle \mathcal{S}, \mathcal{R} \rangle$, where $\mathcal{S} = \{0, 1\}$ and $\mathcal{R} = \{\langle 0, 1 \rangle \langle 1, 0 \rangle\}$. Now let $\mathcal{M} = \langle \mathcal{S}, \mathcal{R}, \mathcal{D} \rangle$,[7] where $\mathcal{D} = \{i, e\}$. There is a valuation v which assigns both i and e to each xx and yy with respect to 0, but which assigns only i to each variable with respect to 1. In virtue of this valuation, (NecInc) is not true in $\mathcal{M}$. Nonetheless, the valuation respects (StrongNec).

All is not quite lost for (StrongNec). Williamson [23], working within a constant domain framework, has shown recently that (NecInc) *can* be derived from (StrongNec) *plus* the plausible additional principle,

$$\forall xx \exists yy \left(xx \approx yy \wedge \forall x \left(\Diamond x \prec yy \rightarrow \Box x \prec yy \right) \right)$$

In the light of this result, we are forced to ask what motivation there might be for assenting to (StrongNec). The principle is not derivable within a basic plural modal logic of the sort under present consideration, and it is unclear why someone not already prepared to accept (NecInc) would be any more prepared to accpt (StrongNec). Thus a proof of (NecInc) on the assumption of (StrongNec) has little dialectical force.

Something should be said at this point on the question of contingent existence. One immediately obvious ground for rejecting (NecInc) would be that one believed that some *xx*s exist contingently. Were this the case, (NecInc) will be false, even if one believes that the plural variable '*xx*' is a rigid designator. This avenue for rejecting (NecInc) does not get to the philosophical heart of the matter, though, and the issue can be neutralised readily through Rumfitt's formulation of (NecInc)[8] [18, 113]:

$$x \prec xx \rightarrow \Box(E^P xx \rightarrow x \prec xx) \tag{NeutrNec}$$

Where 'E^P' is a plural existence predicate. The ready availibility of (NeutrNec) allows the question at issue to be framed in terms acceptable to all. For ease of presentation, my discussion here will be in terms of (NecInc), but nothing of present philosophical importance turns on this choice. As we have seen, (NecInc) is not a theorem of the bare system which results from modalising a plural logic. The introduction of (NecInc) to a system—either as an axiom itself, or through the addition of something like (StrongNec) as an axiom so as to ensure (NecInc)'s theoremhood—is a move which requires philosophical motivation in order to be principled. Our attention will turn shortly to considering what such motivation might consist in. First, we should minute the fact that we are handing the proponent of (NecInc) a dialectical advantage by excluding from consideration certain current accounts of the semantics for modal claims.

[7] Models, as we will see later, usually also contain an interpretation function for the non-logical vocabulary. This is superfluous for present purposes.

[8] I have altered Rumfitt's notation to bring it into line with that used here.

1.1 Two-Dimensional Semantics and Counterpart Theory

Any theory of modality on which, say, ‘Hesperus is Phosphorus’ can express a contingent truth is going to alter radically the lie of the land with respect to (NecInc). Suppose necessarily y is among xx, and that ‘$x = y$’ is true, but only contingently so. Then whilst it is true that x is among xx, it is not true that: necessarily, x is among xx. Two significant projects allow that there might be contingently identity statements involving exclusively proper names. The *counterpart theory* of David Lewis [10] denies that any entity can exist at more than one possible world[9] Given this world-boundness of individuals, in order to assess identity claims within the scope of modal operators, we need to consider the counterparts of entities at distinct worlds. At this point complications arise for the modal status of identit statements. An entity may have numerically distinct counterparts at one and the same world. Now consider the Kripkean claim:

$$x = y \to \Box x = y$$

On a counterpart theoretic reckoning, the RHS of this is interpreted as claiming that, at any given world, any counterpart of x is identical to any counterpart of y. This can be false, since there may be worlds where the individual which is both x and y has more than one counterpart.[10]

Two-dimensional semantics, associated in particular with David Chalmers [6], has it that individual tokens of ‘Hesperus is Phosphorus’ possess two distinct propositions. One, the *primary intension*, is associated with the means by which language-users determine the referents of ‘Hesperus’ and ‘Phosphorus’. Suppose that we come to identify some heavenly body as Hesperus because it is the body which appears in such-and-such a place in the evening, and that we come to identify some heavenly body as Phosphorus necause it is the the body which appears in such-and-such a place in the morning. In actual fact, one and the same body is picked out in both cases. It could have been otherwise, though. Now, for the two-dimensionalist, the primary intension of ‘Hesperus is Phosphorus’ is a proposition[11] which is only contingently true—namely that the object which the utterer designates with ‘Hesperus’ is identical with the object she designates with ‘Phosphorus’. The secondary intension of ‘Hesperus is Phosphorus’ is, by contrast, the necessarily true proposition that Venus is self-identical. Thus whether or not identity statements express

[9] Worlds, for Lewis’ genuine modal realism, are of course concrete existents [11]. On this view, the world-boundness of individuals seems to follow from Leibniz’s Law. The path to this conclusion is not strictly compulsorary for the modal realist. For a critical discussion of Lewis’ rejection of the view that ordinary individuals are transworld worms with modal parts, see Section 7 of Weatherson’s [21].

[10] Note that ‘$x = x \to \Box x = x$’ will come out true for counterpart theory. Any counterpart of x is *self*-identical.

[11] Standardly understood in the literature as a function from worlds to truth-values.

necessary truths depends on whether the primary or the secondary intension is being considered. Even so, it is something of moot point whether there is any sense in which the two-dimensionalist countenances contingent *identity*; Chalmers himself writes of the primary intension as corresponding to an epistemic modality, and the secondary to a metaphysical modality—we might well think that only the latter is relevant to considerations of identity.

A theory for which singular identity is contingent is likely to be a ready source of counter-examples to (NecInc). In order to allow the proponent of (NecInc) the best available case, then, both counterpart theory and two-dimensional semantics will be bracketed for the reaminder of the body of this paper.[12] Nonetheless, we will sketch out in an Appendix a plural extension of counterpart theoretic semantics for quantified modal logic which fails to validate (NecInc).

2 Arguments for the Necessary Inclusion Thesis

It might appear as though there is a straightforward argument to be had for (NecInc). This begins by noting that, at least if we permit ourselves the logical resource of infinitary disjunction, an instance of (non-modal) plural inclusion will be equivalent to that of some disjunction:

$$x \prec yy \leftrightarrow \bigvee_{\alpha} (x = y_{\alpha}) \tag{DIS}$$

Where 'α' ranges over some ordinals.[13] In order for (DIS) to hold for all the pluralities the Boolosian is interested in—crucially, for the sets (all of them)—the infinitary resources required will be substantial. The logic with the least expressive power suitable for the task is a plural version of $L_{\infty\omega_1}$, admitting arbitrarily large infinite disjunctions and conjunctions, with the additional feature that an arbitrarily large infinite number of free variables are available. Whilst $L_{\infty\omega_1}$ in its non-plural form has been the object of logical study, and whilst extending it to admit plurals will be technically straightforward, there may be philosophical questions about the legitimacy of infinitary resources. We will not engage with these here. This is dialectically permissible for the same reason as the exclusion of counterpart theory and two-dimensional semantics: in not querying the availibility of infinitary logic for

[12] This dialectical generosity aside, I do in fact favour a Barcan-Kripke view of singular identity. Part of the burden of the present paper is to moot the consideration that singular identity/ reference might be significantly different from plural sameness/ reference, such that reaching a view about the singular cases need not commit one to a view about the plural cases (*need not*—note that, say, the counterpart theorist does seem to have her room for manovure restricted here).

[13] Care is needed here. Considering (DIS), assign the empty set to 'x' and the ordinals to 'yy'. If we take 'α' to range over a set, we incur the Burali-Forti paradox. My own view is that it is best to understand the range of 'α' itself *plurally*. So, in the case under consideration, 'α' ranges over the ordinals (all of them). More standard would be the invocation of proper classes. Note also that a principle of global choice is implicit in (DIS).

present philsoophical purposes, an advantage is being handed to my ultimate opponent, the proponent of (NecInc).

The next move is to claim that (DIS) is definitional of inclusion; to be one of *these* simply is to be either *this*, or *this*, and so on. This is used to licence the necessitation of (DIS):

$$\Box\left(x \prec yy \leftrightarrow \bigvee_{\alpha}(x = y_{\alpha})\right) \tag{NDIS}$$

Apart from (NDIS), the other premiss required is the more familiar assertion that identity is necessary:[14]

$$x = y \rightarrow \Box x = y \tag{NIDN}$$

Given these premises, (NecInc) follows in **T**. We substitute the plural free variable '*yy*' for '*xx*' in (NecInc) for notational clarity:

Proof

$$x \prec yy \qquad \text{(Ass. for} \rightarrow I) \tag{1}$$

$$\bigvee_{\alpha}(x = y_{\alpha}) \qquad (1, \text{DIS}, \leftrightarrow E, \rightarrow E) \tag{2}$$

$$\bigvee_{\alpha}\Box(x = y_{\alpha}) \qquad (2, \vee E_{inf}, \text{NIDN}, \vee I_{inf}) \tag{3}$$

$$\Box\bigvee_{\alpha}(x = y_{\alpha}) \qquad (3, \vee E_{inf}, \mathbf{T}) \tag{4}$$

$$\Box x \prec yy \leftrightarrow \Box\bigvee_{\alpha}(x = y_{\alpha}) \qquad (\text{NDIS}, \mathbf{T}) \tag{5}$$

$$\Box x \prec yy \qquad (4, 5, \leftrightarrow E, \rightarrow E) \tag{6}$$

$$x \prec yy \rightarrow \Box x \prec yy \qquad (1\text{–}6, \rightarrow I) \tag{7}$$

□

The vulnerable point in this chain of argumentation is (NDIS). It is unclear why we should believe the claim it makes, unless we already believe (NecInc). But (NecInc) is precisely what we are seeking to establish through the invocation of (NDIS). The justification, mooted above, that (DIS) is *definitional* of inclusion and that its necessitation ought to be admitted, is question-begging. In itself (DIS) is platitudinous; there is nothing which is amongst some things without being a particular thing amongst those things. Conversely, if something is either y_1, or y_2, through to y_i, then there are some things which it is amongst. It does not follow that (DIS) is definitional of inclusion. For that to be the

[14]The canonical case for this is Barcan's [2].

case would require that no thing could be amongst *these* without being *this*, or *this*, and so on. Yet this is what the supporter of (NecInc) needs to show. Perhaps some philosophical motivation for necessitation might be hoped for through the drawing of a connection between (singular) identity and *plural identity*. The thought here will be that acceptance of (NecInc) opens the door to understanding plural identity as nothing over and above repeated instances of identity between objects. This position might draw some succour both from the belief that (StrongNec) may be viewed as a statement of modal plural identity conditions and from the fact that (StrongNec) follows from (NecInc) in a **B**-logic equipped with a plural existence predicate.[15]

Superficially attractive though the stance which draws an intimate connection between singular and plural identity might be, it is mistaken. There is no such thing as plural identity, so in particular there is no such thing as plural identity which can be intimately connected to singular identity (or *identity* as we should call it). Integral to Boolos' motivation in developing the logic of plurals was their ontological neutrality. Our talk of the set of the three wise men is committed to the existence of a fourth entity, the set, over and above the magi. Our plural talk about them does not incur a similar commitment; or so I wish to claim. Like almost everything in the philosophical dispute over plurals, this has not gone uncontested.[16] I concur with (what I take to be) the majority position in taking Boolos' *Cheerios consideration* to be decisive here:

> Entities are not to be multiplied beyond necessity. one might doubt, for example, that there is such a thing as the set of Cheerios in the... bowl on the table. There are, of course, quite a lot of Cheerios in the bowl, well over two hundred of them. But is there, in addition to the Cheerios, also a set of them all? And what about the $>10^{60}$ subsets of that set (And don't forget the sets of sets of Cheerios in the bowl.) It is haywire to think that when you have some Cheerios you are eating a *set*—what you're doing is: eating THE CHEERIOS. Maybe there are some reasons for thinking there is such a set... but it doesn't follow from the fact that there are some Cheerios in the bowl that, as some who theorize about the semantics of plurals would have it, there is also a set of them all. [5, 72].

What Boolos says here about sets applies doubly to reified pluralities, if these are supposed to be something other than sets. There are no such things as pluralities, however convenient the grammatically singular term 'plurality' might be for talking about some things together. It follows that plural variables are not eligible to flank an identity predicate, and so that '$\approx$' is not an identity predicate. As we have already remarked '$\approx$' denotes a sameness relation. It is the modal behaviour of that relation into which we are enquiring.

[15]Rumfitt derives $(E^P xx \wedge \neg x \prec xx) \rightarrow \Box\neg x \prec xx$ [18, 115]. From this plus (NecInc), already assumed for the derivation, (StrongNec) follows immediately.
[16]See [17].

Now, the supporter of (NecInc) who sought to draw on a close relationship between $\approx$ and $=$ in support of her position might protest that, whilst the considerations mooted in the previous paragraph disagree with the letter of her position, they concur with its spirit. It is precisely because there are no such things as pluralities, she can be imagined as saying, that we can only understand '$\prec$' in terms of '$=$'. To say this, she will conclude, is to *define* '$\prec$' in terms of identity plus disjunction, and to to licence the necessitation of (DIS). Stalemate.

If progress is to be made, a new strategy is required. That adopted here will be one of examining whether all natural language plural terms are rigid. I understand the claim that a plural term '*tt*' is rigid to be the following: in all contexts of evaluation[17] in which '*tt*' denotes anything at all, (a.) anything that at any other context is amongst the things denoted by '*tt*' is amongst the things denoted by '*tt*' at that context, and (b.) anything that is not among the things denoted by '*tt*' at any context in which '*tt*' denotes anything is not among the things denoted by '*tt*' at that context. A distinction needs to be made now: (NecInc) is a claim about the world, not about language: (NecInc) says that if this is one of these, then necessarily this is one of these. It does *not* say of a plural variable that it denotes rigidly.[18] This vital difference having been marked, it is nonetheless apparent that the latter metalinguistic claim is intimately related to the former claim about the world in the following way: if necessarily this is one of these, then—regardless of whether it is embedded within the scope of modal operators—any plural variable which denotes these must denote some things which include this. This implication of (NecInc) for language is a necessary, but not sufficient, condition for the *rigidity* of plural variables. It follows that if arguments can be had to the effect that plural variables ought not to be construed as rigid, then these same arguments might undermine (NecInc).[19] Presently I will survey some examples from natural language of rigid plural terms, which might be thought to count in favour of the rigidity of plural variables. I will then go on to draw attention to non-rigid plural terms in natural language, and will cite these in support of not viewing plural variables as rigid designators. Before that, we should pause and ask why evidence from natural language is useful for making progress in this area.

[17] By a 'context of evaluation' here I mean much the same as some authors mean by a 'world'. I avoid this usage so as not to prejudge important metaphysical questions.

[18] Thanks to Øystein Linnebo for insisting on clarity here.

[19] *Might* because it is just conceivable that someone might admit (NecInc) along with the **K** co-consistent principle $\exists yy \exists x\, \neg x \prec yy \wedge \Diamond x \prec yy$, or something similar. Since the conjunction of this formula with (NecInc) is only satisfied by models for theories whose logic lacks **B**, and given the widespread acceptance of **B** in the analysis of alethic modality, we shall ignore this subtlety from this point onwards and talk simply of 'rigidity'.

2.1 How Should We Assess Considerations from Natural Language?

We want to know whether '*xx*' is a *de jure* rigid designator[20] (DJRD). I am proposing examining natural language as a way of making progress in this debate. There is an objection which has some force against this strategy. The plural variable '*xx*' is a item from the lexicon of a formal, aritificial, language. The motivation for our present investigation is that we wish to use plural variables to interpret another formal, aritificial language—that of **MSOL**. But given that our concerns are exclusively with formal languages, and the correct formal semantics for these, why do we need to give any consideration to natural language? Can we not just stipulate that the plural variables of our modal plural logic are not DJRDs, supply a model theory which respects this stipulation, and use the resulting system to interpret modal **MSOL**?

We can, of course, devise a system with variables 'xx_1', 'xx_2' etc. which do not designate rigidly, and proceed to use this system to interpret second-order logic. However, a major part of the philosophical appeal of Boolos' interpretation of **MSOL** is that it is widely (although not universally [12]) believed to have secured the *logicality* of the second-order system through interpreting it using, self-evidently logical, plural locutions. What, if anything, logicality consists in is a vexed question, but a frequent thought is that logic enjoys a cognitively basic status,[21] and that the occurence of a candidate logical form in natural language has evidential value with respect to its being appropriately basic. Suppose now that we are considering a defence of the plural interpretation of **MSOL** which denies that plural variables are DJRDs, and so permits us to deny (NecInc). A conceivable worry is that, whilst non-rigid plural variables are readily entertainable model theoretically, they lack the cognitively basic status required for logicality. A response to this worry would be to point to the occurence of non-rigid plural terms in natural language. On the other hand, a way of weightening the worry would be to argue that there are no such terms.

What is meant by 'plural terms' in this context? Our ultimate interest is in plural variables in a formal language. The nearest natural language correlates to these, in the case of free variables, are plural demonstratives: 'these', 'those', 'ces', and so on, as my previous usage indicates. The function of demonstratives (and here the analogy with variables is apparent) is to stand in for terms of other sorts: compare 'The Western Isles are remote' with 'These are remote' and 'Those are remote'. Similarly, bound plural variables correspond to plural pronouns, which also have a place holding function: compare 'Les Beatles

[20] A *de jure* rigid designator is a linguistic item which designates rigidly in virtue of the semantic category to which it belongs. For example, a major thesis of *Naming and Necessity* is that singular proper names are *de jure* rigid designators. Some NPs are *de facto* rigid designators, in spite of not being *de jure* rigid designators. Consider 'the even prime'; this cannot denote anything other than the natural number 2, but yet the members of its semantic category (singular definite descriptions) are not in general rigid designators.

[21] See e.g. Linnebo's [12, 75].

sont formidables' with 'Ils sont formidables'. But, as the use of the English 'these' in the above discussion of the status of (NecInc) attests, the modal status of plural demonstratives and pronouns is no immediately clearer than that of their formal correlates.[22] There is an important contrast here with singular demonstratives and pronouns, the status of which as rigid designators is uncontroversial. This lack of clarity need not represent an *impasse*, though. Given that we have noted the place-holding function of demonstratives and pronouns, we can make progress by addressing the modal status of admissible substituends for these categories, and by making an inference back from our conclusions about these to the modal status of plural demonstratives and pronouns themselves. Our attention turns, then, to those plural terms which are not themselves demonstratives or pronouns. With Rumfitt [18, 86], I will include amongst these compound names, such as 'Amélie et Bernard', collective names, such as 'Radiohead', and plural indexicals, such as 'we'. I exclude plural definite descriptions from consideration,[23] although as in the singular case ('the Holy Roman Empire') there are collective names which commence with the definite article. Here is Rumfitt on 'the Channel Islands':

> This expression does not mean 'the islands in the English Channel'. The Isle of Wight is an island in the English Channel, but it is not one of the Channel Islands. Rather, the term refers to the islands in a certain archipelago off the western coast of Normandy. [18, 120]

We shall return to 'the Channel Islands' shortly.

2.2 Natural Language Plural Rigidity

It is immediately clear that one subcategory of plural term is *de jure* rigid, namely compound names. Nobody other than Alice or Bob could have been one of the things referred to by 'Alice and Bob', and each of Alice and Bob could not but have been one of the things referred to by 'Alice and Bob'. Once we move beyond compound names, matters are far less transparent. Rumfitt argues in favour of the rigidity of collective names, and against some considerations which might be thought to support the claim that these are non-rigid. I will now outline the considerations which are Rumfitt's target, and will then present, and cast doubt on, Rumfitt's counter-argument.

Here is one reason one might suppose collective names not to be DJRDs. As we have already seen, 'the Channel Islands' is a collective name. Now here are two sentences which seem to express truths: 'Herm might not have been one of the Channel Islands' and 'There might have been another one of the Channel Islands'. The first, on the face of it, is true because it is possible that

[22] Although, we will see below a *prima facie* example of a non-rigid plural demonstrative phrase, which becomes compelling once certain philosophical arguments against non-rigid plural terms are answered.

[23] I take it that these are not terms. Even if readers demur on this point, they will presumably concede that, special cases aside ('the even numbers'), plural definite descriptions are not rigid.

Herm not be in its actual location but isolated in the Channel. If this were the case, Herm would exist but would not be one of the Channel Islands. The truth of the second appears immediate: is it not possible that there be an island in the archipelego additional to the ones there actually are. Where '*aa*' is a collective name for the Channel Islands, we then have:

$$h \prec aa \wedge \Diamond\neg(h \prec aa) \tag{HERM}$$

and,

$$\neg b \prec aa \wedge \Diamond b \prec aa \tag{EXTRA}$$

(HERM) entails the negation of the universal closure of (NecInc). The truth of each of the formulae requires that '*aa*' function non-rigidly.

Rumfitt argues that the appearance of truth is deceptive in the case of both sentences. In the first case, he argues that its having approximately its actual spatial location is an essential property of a geographical entity, so that no island which is isolated in the Channel could be Herm. If there were such an Island, but no island in Herm's actual position, then Herm would not exist (and so the Channel Islands would not exist.) Intuitions regarding essential properties are frequently difficult to adjudicate, but I cannot agree with Rumfitt on this: suppose that by some miracle the mass of rock referred to by 'Herm' were transported instantaneously fifty miles West, along with all its inhabitants. Would they not very naturally, and correctly, describe the situation as one in which Herm had changed location? But then approximate spatial location is not an essential property of Herm's afer all. In any case, one can readily come up with similar sentences whose truth doesn't turn on the pecularities of the metaphysics of geographical objects. Here is a true sentence: 'Charlie Watts might not have been one of the Rolling Stones'.[24]

Rumfitt's approach to 'There might have been another one of the Channel Islands' presents more of a challenge. He suggests that the intuition of its truth rests on treating 'the Channel Islands' as being descriptive in a fashion incompatible with the claim that it is a genuine plural term. The argument here is that in order for 'There might have been another one of the Channel Islands' to be true, 'the Channel Islands' has to function as a disguised definite

[24] Possible objection: words for rock groups are not plural, but singular terms, referring to special objects—namely groups (in a non-mathematical sense). One reply might call evidence from British English to its aid: we say 'the Rolling Stones *are* performing next Tuesday' (although, note the US English usage: 'Pulp *is* playing Boston next Friday'). In my view, a better reply is metaphysical: if rock groups are entities, they are very curious entities. Why on earth should I believe that, over and above George, Paul, George and Ringo, there is a fifth entity, referred to by 'The Beatles'? One reason perhaps would be that my best theory of the world needs to make mention of the Beatles, perhaps because the band is causally efficacious. If I am wedded to the Quinean dogma that I may only regiment my theory canonically into a first-order language with exclusively singular vocabulary, then it will turn out that I am committed to the Beatles-entity on the Quinean criterion of ontological commitment. But the example only serves to show that we should jettison the dogma: admit a canonical notation with plural vocabulary [16], and evict the Beatles-entity from our desert landscape.

description, along the lines of ‘the islands which are an archipelego comprising of Jersey, Gurnsey…’. So, we are faced on Rumfitt’s reckoning with a choice; either the sentence in question is not true, or else ‘the Channel Islands’ is not a plural term. Either way we do not have a counter-example to the thesis that plural terms are not *de jure* rigid.

There are (at least) two ways in which one might respond to Rumfitt here. The first consists in, what one might call, *ostrich nonrigidism* about NPs such as ‘the Channel Islands’.[25] This position takes as its departure point the belief that our philosophical theories about language owe a duty to the intuitions of competent language users. It takes on board the strong intuition that it is obviously true that there might have been another one of the Channel Islands, but thinks that there are good reasons to deny that ‘the Channel Islands’ is a disguised definite description of the type described by Rumfitt. After all, can’t one be a perfectly competent user of the phrase without being in a position to know a priori, say, that the Channel Islands are an archipelego in the English Channel? Yet if it is part of the *meaning* of ‘the Channel Islands’ that its denotation (if any) is an archipelego in the English Channel, surely one ought to be able to know a priori that the Channel Islands are an archipelego in the English Channel simply by examining one’s usage. The ostrich nonrigidist stops here. ‘The Channel Islands’ is a non-rigid term. There might have been Channel Islands which are not amongst the actual Channel Islands. More than this, the ostrich nonrigidist will not say. She offers us no theory of how a non-rigid plural term refers. Ostrich nonrigidism is a fall-back position. I note it in order to emphasise that the philosopher who is convinced that there a plural terms which are not rigid need not admit defeat if she cannot offer an account of how such a term might succeed in referring. This having been said, it is not difficult to feel as though there is something unsatisfactory about ostrich nonrigidism. What is the other option?

What I will call the *conditional reference theory* (CRT) about collective names holds that these names refer directly, such that their contribution to the truth-apt content of sentences in which they occur is simply the objects to which they refer.[26] In particular, collective names have no descriptive content. Thus far, CRT says about collective names what the direct reference theorist says about singular proper names. The direct reference theorist, however, holds that proper names denote rigidly, whereas CRT allows for non-rigidity in the plural case. Motivated by intuitions about the kind of case we have discussed, the CRT theorist argues as follows: consider an arbitrary singular proper name ‘a’. Counterfactually, by virtue of rigidity, ‘a’ refers to a. We determine which thing in that context a is by means of a’s identification

[25] My inspiration for this coinage is Armstrong’s term ‘ostrich nominalism’ [1].

[26] Immediately, then, the problem of empty collective names rears its head. My position is that there are no such names. Every genuine collective name, in a non-modal context, has a referrent. This finds expression in the model theory for plural modal logic expounded below. Defence of this position will have to take place elsewhere.

conditions. These are not part of the content of '*a*'. They are, rather, criteria for (as it were) locating *a* counterfactually. So far, so much orthodox direct reference theory. Now: consider an arbitrary plural name *aa*. In a counterfactual context '*aa*' refers to *aa*. Disanalogously with the singular case, however, this does not imply rigidity since different things might be *aa* counterfactually. How do we determine which things conterfactually are *aa*? There are *aa-conditions* which enable us to determine which things are *aa*. No more than in the somewhat analogous singular case, are these conditions part of the semantic content of '*aa*'. Some amongst the community of language users may be explicitly aware of the *aa*-conditions, indeed they may have introduced the conditions by stipulation. Yet neither is the reference of collective names mediated by its associated conditions, nor is it up to language-users, rather than the world, which things are (actually, or counterfactually) *aa*.[27] CRT is a direct and semantically externalist theory of the reference of collective names.

CRT requires far more investigation and development than is possible here; we have here a programme for future research, not a completed theory of plural reference. Nonetheless, I think that CRT has the potential to be developed into a credible theory which respects intuitions about the behaviour of collective names. Crucially, in the current context, it provides a provisional theoretical basis for disagreement with Rumfitt. Armed with this theory, let us assess where things stand with (NecInc).

3 Against the Necessary Inclusion Thesis

A single counter-example will serve to falsify (NecInc). If we have:

$$\exists xx\, x \prec xx \wedge \Diamond\neg(x \prec xx) \tag{8}$$

Then we have the negation of the universal closure of (NecInc). Equation 8 is entailed by:

$$x \prec aa \wedge \Diamond\neg(x \prec aa) \tag{9}$$

If the argument about (HERM) above is correct, and given the legitimacy of reading across conclusions from natural language to an account of the correct rules for a modal plural logic, then Eq. 9 is secure. Further evidence against (NecInc) may be supplied by amassing examples of plural terms which are not rigid, since the claim that all plural terms are DJRDs compels acceptance of (NecInc). CRT places the intuition that there are such terms (for instance, 'The Rolling Stones') on a sure footing. The cumulative case against (NecInc) constructed on the basis suffices in my estimation to justify rejection of the

[27] Consider the plural name of some mountains 'the Munros'. This is not a description. After all, I can know that Mary has climbed every one of the Munros without knowing that Mary has climbed every Scottish mountain which exceeds 3000 ft. in height. 'The Munros' was introduced by stipulation. It is up to the world, however, not to language users that the Munros exceed 3000 ft in height. Yet, by CRT, it is not possible that there be a Munro with a height of 2000 ft.

principle and gives us philosophical permission to develop a semantics for the modal logic of plurals in which variables are not *de jure* rigid. As it happens, however, I think there are further considerations which should encourage us in this development. Two deserve attention before we proceed:

1. Non-rigid plural demonstratives. These are of particular interests for present purposes, given the close similarity between natural language demonstratives and variables in formal languages. An example, owing to Dorothy Edgington, is given by Rumfitt:

   > In remonstrating at your indiscretion in relaying to a crowd some gossip about Smith, I might say, 'You shouldn't have said that. If Smith hadn't been delayed, he would have been one of those people. [18, 120-1]

   Rumfitt's response, as in the second 'Channel Islands' case is to argue that the sentence in question is either false, or doesn't involve the demonstrative functioning rigidly. His reasoning here is that if Smith could have been one of those people, there must be an answer to the question which of those people he could have been. If we imagine otherwise, this is because 'we surreptitiously imagine a fresh use of the demonstrative, made in circumstances in which Smith has joined the throng'. Note that Rumfitt is relying here on the natural language equivalent of (NDIS), a principle we have seen reason to doubt. Once we abandon the belief that someone couldn't have been one of those people without being *a*, or *b*, or *c* etc., the natural reading of 'if Smith hadn't been delayed, he would have been one of those people' as involving a non-rigid plural demonstrative commends itself.
2. The nihilist recourse to plurals. The *mereological nihilist* believes that there are only simples, and that no composite objects exist. This belief is likely[28] to issue in the belief that there are no such things as coffee cups or computers, but that there might be quarks or leptons—or whichever particles (if any[29]) the physicists end up telling us are fundamental. Call this position *microphysical nihilism*. An immediate, and unfortunate, seeming consequence of microphysical nihilism is that we end up with an error theory about swathes of quotidien natural language. As I type this I am quite prepared to affirm 'There is a coffee cup on my desk, near my computer'—and I am sure you would be prepared to assert a sentence with the same propositional content if you had perceptual access to the present contents of my room. Yet, there are—on the hypothesis of microphysical nihilism—no computers, or desks, or coffee cups. One strategy for avoiding such an heroic error theory, supported by van Inwagen [20], involves

[28]But not inevitably: the nihilist could consistently believe that coffee cups and computers are themselves simples. She then faces the unenviable task of accounting for our strong intuition that, if such things exist at all, they have parts.

[29]The possibility of *gunk* rears its head here.

> understanding prima facie singular NPs as in fact plural noun-phrases.[30] Now suppose if (unlike van Inwagen) I am a thoroughgoing nihilist about not only inanimate objects, but also about organisms. Then, unless I affirm a very particular sort of substance dualism about mind,[31] the plural response to the error theory is going to move me to claim that 'Quine' is a plural term, referring to some simples arranged Quine-wise. But 'Quine', thus understood, cannot be rigid. Quine might easily have lacked a given molecule, and so its constituent simples. Now, the metaphysical theories here are deeply controversial, but it seems peculiar to suggest that they can be ruled out a priori simply because of an account of plurals.[32]

The development of a semantics for modal plural logic in which plural variables are not *de jure* rigid designators is well-motivated. The motivating considerations are not without more general philosophical implications: for example, our argument thus far involves the rejection of the claim, associated with Gareth Evans, that it is characteristic of referential parts of language that they designate rigidly. The questions arise: what, if anything, is essentially a feature of all and only referring parts of language, if rigid designation isn't? Is reference better understood as a family resemblance concept, rather than a tightly delimited concept under which proper names and NPs with close semantic similarities, and nothing else, fall? These issues will have to be addressed in another setting. In terms of our immediate concerns, it remains to cash out our investigations in terms of a semantics for modal plural logic.

4 A Non-Rigid Variables Semantics for a Modal Plural Logic

It is straightforward to supply a model theoretic semantics for a plural language in which plural variables are not rigid. We present here a semantics for the language of **PFO+**, including singular and plural constants and supplemented with modal operators. For ease of exposition, we work with a constant domain semantics; no particular issues related to plurals arise in modifying what follows for the varying domain case. For details of varying domain semantics see any competent text on first-order modal logic, for example [7].

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[30] For a formal fleshing-out of this view see Hossack's [9].

[31] Namely a version in which I am an immaterial simple mind which is somehow associated with material simples, which are not constituents of me.

[32] Thank you to Sam Lebens for drawing my attention to the use of plurals by mereological nihilists. Another interesting consideration arising from mereology, and owing to conversations with Gabriel Uzquiano, is as follows: given the non-admission of gunk, there is an exact formal similarity between mereology and the principles governing pluralities: both are incomplete Boolean algebras, lacking a null element. It is at the very least interesting, then, that mereology and plural logic are conventionally treated very differently when it comes to the addition of modal principles. Whilst philosophers routinely acquiesce to (NecInc), few would accept the corresponding mereological claim: $xPy \rightarrow \Box xPy$.

One clarificatory point should be made before we commence the exposition of the model theory. For the sake of ease, we will be assigning (non-empty) *sets* to plural constants and variables. This model theoretic convenience should not be understood as carrying with it the claim that plurals are really a means of making disguised singular reference to sets. Instead, the formal semantics should be understood as modelling circumstances in which plurals denote the elements of the sets in question. It would be an interesting exercise to attempt the formulation of a formal semantics that avoided this kind of recourse to sets, but this is beyond our present scope.[33]

Define a frame as usual: $\mathcal{F} = \langle S, R \rangle$, where S is a non-empty set and R a relation on the elements of S. A model $\mathcal{M}$ on $\mathcal{F}$ is defined as $\mathcal{M} = \langle S, R, D, I \rangle$. Here D is a non-empty set and I an *interpretation* of the modalised plural language. I makes assigments to the non-logical vocabulary as follows:

- To each *singular constant* some $d \in D$.
- To each *n-adic predicate*, for each $s \in S$, some $d \subseteq D^n$.
- To each *plural constant*, a non-empty set of ordered pairs P such that $\forall \langle x, y \rangle \in P\, x \in S \wedge y \subseteq D \wedge \forall z\, (\langle x, z \rangle \in P \rightarrow z = y)$.
- To each *plural predicate*, for each $s \in S$ some $p \subseteq \wp(D)$.

Note that plural constants are not, in general, rigid. Next we define a *valuation* in $\mathcal{M}$. A valuation v assigns to each free singular variable 'x' some $v(x) \in D$ and to each free plural variable 'xx' some $v(xx)$, where $v(xx)$ is a nonempty set of ordered pairs, the first co-ordinate of each of which is an element of S, the second co-ordinate a non-empty subset of D. It is a constraint on valuations that, for an aribtrary plural variable 'xx' and for any $s \in S$, $v(xx)$ has no more than one element with s as its first co-ordinate.

We can now define truth in a model. We write $\mathcal{M}, s \Vdash_v \phi$ for '$\mathcal{M}$ satisfies ϕ on v with respect to s', where $s \in S$. In what follows we write 't_n' for a *singular term*. Singular constants and singular variables are singular terms. We define $v(t)$ as $I(t)$ when 't' is a singular constant:

(1) For an n-adic predicate 'F', $\mathcal{M}, s \Vdash_v Ft_1 \ldots t_n$ iff $\langle v(t_1 \ldots v(t_n)) \rangle \in I(F, s)$.
(2) For a plural constant 'aa', $\mathcal{M}, s \Vdash_v t \prec aa$ iff $\exists \langle m, n \rangle \in I(aa)\, (m = s \wedge t \in n)$.
(3) For a plural variable 'xx', $\mathcal{M}, s \Vdash_v t \prec xx$ iff $\exists \langle m, n \rangle \in v(xx)\, (m = s \wedge t \in n)$.
(4) For a plural predicate 'FF' and a plural constant 'aa', $\mathcal{M}, s \Vdash_v FFaa$ iff $\exists \langle m, n \rangle \in I(aa)\, (m = s \wedge n \in I(FF, s)$.
(5) For a plural predicate 'FF' and a plural variable 'xx', $\mathcal{M}, s \Vdash_v FFaa$ iff $\exists \langle m, n \rangle \in v(xx)\, (m = s \wedge n \in I(FF, s)$.

[33]See Hewitt, S. (An extended logic of plurals, forthcoming).

We specify recursive rules for molecular wffs in the usual fashion:

(1) $\mathcal{M}, s \Vdash_v \neg\phi$ iff $\mathcal{M}, s \nVdash_v \phi$.
(2) $\mathcal{M}, s \Vdash_v (\phi \wedge \psi)$ iff $\mathcal{M}, s \Vdash_v \phi$ and $\mathcal{M}, s \Vdash_v \psi$.
(3) $\mathcal{M}, s \Vdash_v \Box\phi$ iff $\forall u \in S$ if sRu then $\mathcal{M}, u \Vdash_v \phi$.
(4) $\mathcal{M}, s \Vdash_v \forall x\, \phi$ iff for every valuation w, which differs from v at most with respect to the assignment to 'x', $\mathcal{M}, s \Vdash_w \phi$.
(5) $\mathcal{M}, s \Vdash_v \forall xx\, \phi$ iff for every valuation w, which differs from v at most with respect to the assignment to 'xx', $\mathcal{M}, s \Vdash_w \phi$.

Other connectives and operators are understood as abbreviations. We say that some formula ϕ is *true* with respect to[34] s in $\mathcal{M}$ iff for every valuation v, $\mathcal{M}, s \Vdash_v \phi$.

We define truth in a model, which we write $\mathcal{M} \vDash \phi$:

$$\mathcal{M} \vDash \phi \text{ iff } \phi \text{ is true with respect to every } s \in S \text{ in } \mathcal{M}$$

Validity is defined relative to a frame or class of frames—$\vDash \phi$ iff ϕ is true in every model based on the frame(s). If confusion is likely to arise, the turnstile can be subscripted to indicate which frames validity is understood relative to. An understanding of consequence arises natural from our definition of validity:

> $\Gamma \vDash \phi$ iff for every $s \in S$ in every model $\mathcal{M}$ based on the relevant frames, just in case if every element of Γ is true w.r.t. s then ϕ is also true w.r.t. s.

It is easy to show the **K**-invalidity of (NecInc):

Proof Let $\mathcal{F} = \langle\{0, 1\}, R\rangle$. Now consider a model $\mathcal{M}$. Let $R = \{\langle 0, 1\rangle, \langle 1, 0\rangle\}$ and let $D = \{\pi, e\}$. Consider a valuation v including $v(x) = \pi$ and $v(xx) = \{\langle 0, \{\pi\}\rangle, \langle 1, \{e\}\rangle\}$. With respect to 0, '$x \prec xx$' is true, but '$\Box x \prec xx$' is false, since $0R1$ and $\mathcal{M}, 1 \nVdash_v x \prec xx$. Hence (NecInc) is false with respect to 0. Thus (NecInc) is **K**-invalid. □

It remains to prove that this semantics validates the distinctive axioms of **PFO+**. This is straightforward, and we omit the details. The axioms are:

Comprehension: $\exists x\, \phi \rightarrow \exists xx \forall x\, (x \prec xx \leftrightarrow \phi)$
Nonemptiness: $\forall xx \exists y\; y \prec xx$
Extensionality: $\forall xx \forall yy\; xx \approx yy \leftrightarrow \forall x (x \prec xx \leftrightarrow x \prec yy)$

[34] Or 'at s'. I demur from this terminology in the text in order to disassociate myself from the dubious metaphysics which frequently arises from thinking of the elements of S as possible worlds. After all, there are no possible worlds, so in particular there are no possible worlds available as elements of sets. Or at least, so we are prone to say before considerable exposure to a certain type of metaphysics. For the algebraic purposes of model theory anything whatsoever—my coffee cup, Cheryl Cole or the Taaj Mahal—can be elements of S.

5 Conclusion

So then, we have provided philosophical motivation for the suggestion that plural variables are not *de jure* rigid designators, and have constructed a model theory for a plural modal logic which implements formally this suggestion. (NecInc) is not valid in the resulting logic. If we use this logic to interpret **MSOL**, the complaint that Boolos' reading of **MSOL** involves commitment to '$Xx \to \Box Xx$' is defeated. That is by no means all that needs to be done in order to shore up the Boolosian reading, but it is not without significance.

Nor is it without significance elsewhere. In a recent paper, Timothy Williamson invokes (NecInc) in the course of developing an argument for *necessitism*, the claim that everything exists necessarily [23]. In outline, Williamson argues that once plurals are admitted to a modal language, the necessitist can construct a mapping to extract the 'cash value' from the *contingentist*'s modal utterances, but the contingentist cannot do the same with the necessitist's. Williamson's argument, the sophistication of which it is impossible to do justice to here, is to the effect that this counts against necessitism. Crucial for present purposes is the fact that Williamson relies on (NecInc) in the course of his argument.

But detailed response to necessitism was not my purpose here. There are some papers which discuss necessitism. This is not one of those (although it might have been).

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Appendix: (NecInc) and Counterpart Theory

We develop a counterpart theoretic model theory for the language of **PFO+**, enhanced with modal operators and without constants or nonlogical constants.

Define a C-model to be an ordered triple $\mathcal{M} = \langle W, C, @\rangle$, such that W is a set of pairwise disjoint nonempty sets, C is a binary relation on $\bigcup W$, and $@ \in W$. In order to conform with Lewis' description of the counterpart relation in [10], we make the following further stipulations about C:

C1: $\forall x \in \bigcup W \exists y \in C\ y = \langle x, x\rangle$
C2: $\forall w \in W\ (\exists x \in w \exists y \in C \exists q \in w\ [y = \langle x, q\rangle] \to x = q)$

Now we define a *valuation* in $\mathcal{M}$. A valuation v assigns to each free singular variable 'x' a set of pairs, $v(x)$, the first co-ordinate of each of which is an element of W, and the second co-ordinate of each of which is an element of the first, such that the assignment to each singular variable contains precisely one pair for each element of W. We further require that all second co-ordinates of elements of $v(x)$ stand in the C-relation to the second co-ordinate of the element whose first co-ordinate is[35] @. We write '$v(x)_w$' for the unique element of $v(x$ whose first co-ordinate is w. To each free plural variable v assigns a set of pairs $v(xx) = \langle w \in W, s \subset w\rangle$ such that $\forall x \in s \exists y \in @ \ \langle y, x\rangle \in C$. We require that every $v(xx)$ contain an element whose first co-ordinate is @, and that no element of W occur more than once as that first co-ordinate of any $v(xx)$. Note how concessive to instincts supportive of (NecInc) is our assignment to plural variables by comparison to the model theory developed in the main body of the article. We wish to show that the invalidity of (NecInc) follows from counterpart theory alone.[36]

Now we define truth in a C-model. We write $\mathcal{M}, w \Vdash_v \phi$ for '$\mathcal{M}$ satisfies ϕ on v with respect to w', where $w \in W$:

1. $\mathcal{M}, w \Vdash_v x = y$ iff $v(x)_w = v(y)_w$.
2. $\mathcal{M}, w \Vdash_v x \prec xx$ iff $\exists m \subset w\ [\langle w, m\rangle \in v(xx) \wedge v(x)_w \in m]$).

Moleculars are dealt with recursively:

1. $\mathcal{M}, w \Vdash_v \neg\phi$ iff $\mathcal{M}, w \nVdash_v \phi$.
2. $\mathcal{M}, w \Vdash_v (\phi \wedge \psi)$ iff $\mathcal{M}, w \Vdash_v \phi$ and $\mathcal{M}, w \Vdash_v \psi$.
3. $\mathcal{M}, w \Vdash_v \Box\phi$ iff $\forall u \in W\ \mathcal{M} \Vdash_v \phi$.
4. $\mathcal{M}, w \Vdash_v \forall x\, \phi$ iff for every valuation u, which differs from v at most with respect to the assignment to 'x', $\mathcal{M}, w \Vdash_v \phi$.
5. $\mathcal{M}, w \Vdash_v \forall xx\, \phi$ iff for every valuation u which differs from v at most with respect to the assignment to 'xx', $\mathcal{M}, w \Vdash_v \phi$.

We say that ϕ is *true* with respect to w in $\mathcal{M}$ iff for every valuation v, $\mathcal{M}, w \Vdash_v \phi$.

The definition of truth in a C-model follows:

$$\mathcal{M} \vDash \phi \text{ iff } \phi \text{ is true with respect to every } w \in W \text{ in } \mathcal{M}$$

C-validity is defined as truth in every C-model. and the definition of consequence is the obvious one.

(NecInc) is C-invalid.

Proof Consider a model $\mathcal{M}$ such that $W = \{\{0\}, \{1, 2\}\}$ and $@ = \{0\}$. Now let the elements of C be all and only the following pairs: $\langle 0, 0\rangle, \langle 1, 1\rangle, \langle 2, 2\rangle, \langle 0, 1\rangle, \langle 0, 2\rangle$. Now consider a valuation v such that $v(x) = \{\langle @, 0\rangle, \langle\{1, 2\}, 1\rangle\}$ and $v(xx) = \{\langle @, \{0\}\rangle, \langle\{1, 2\}, \{2\}\rangle\}$. We have $\mathcal{M}, w \nvDash$ (NecInc). $\square$

[35]This modifies Lewis, keeping the closest counterpart theoretic analogue of the constant domain assumption from the main body of the article.

[36]The ommission of an accessibility relation, and the concomitant assumption of universal accessibility implicit in the truth clauses is similarly in the spirit of counterpart theory. It would, however, be easy to modify this feature.

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