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A FIRST ORDER THEORY OF FUNCTIONAL PARTHOOD

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ABSTRACT. This paper contains a formal theory of functional parthood. Since the relation of functional parthood is defined here by means of the notion of design, the theory of functional parthood turns out to be a theory of design. The formal theory of design I defend here is a result of introducing a number of constraints that are to express the rational aspects of designing practice. The ontological background for the theory is provided by a conception of states of affairs. The theory is accompanied with a formal model. I prove that the theory is sound and complete with respect to this model.

KEY WORDS: artifact, functional parthood, mereology

Mereology is one of the most popular logical tools in contemporary philosophy. When a philosopher says this is part of that, he or she usually has in mind the notion whose logical properties are defined by the classical theory invented by Stanislaw Lesniewski ([16] and [17]). However, it is also well known that application of this system is not straightforward as witnessed by a number of objections to the mereological axioms. One of the first objections was advanced by N. Rescher in [23]; one of the latest objections may be found in [26]. Usually these objections boil down to 'real-world counterexamples' to one of the mereological theses. For instance, those who doubt whether the relation of parthood is transitive point out to such claims as:

- The handle is part of the door. The door is part of the house. Still, the handle is not part of the house.
- The platoon is part of the company. The company is part of the battalion. Still, the platoon is not part of the battalion.
- Hydrogen is part of water. Water is part of the cooling system. Still, hydrogen is not part of the cooling system.

The examples are taken from, respectively, [4], [13], and [23].

One of the strategies for dismissing such counterexamples suggests that besides the general notion of parthood, there are other, more specific, notions, which might be non-transitive. Thus, besides the notion of part *simpliciter*, there are various notions of φ-part. It seems that this strategy is recommended in [2], p. 33–34. Another strategy is the equivocation strategy, according to which the non-transitivity phenomena are illusory. Each alleged counterexample is claimed to hinge upon ambiguity because it involves at least two notions of parthood. Every kind of parthood relation is transitive, but if we claim that *x* is part of *y* (in one sense of the word 'part') and that *y* is part of *z* (in another sense), then it is no wonder that we cannot claim that *x* is not part of *z* (in any sense) ([33]). The third strategy suggests that we should limit the domain of application of Lesniewski's system to regions of space/time ([22]). One of these more specific notions is the notion of functional parthood. This paper is an attempt at establishing the logical properties of the relation of functional parthood.

Section 1 contains a philosophical framework in which to define the notion of functional part. Gathering various preformal intuitions concerning artefacts and their functions, I gradually construct a formal theory of the notion at stake. In Section 2 I present a semantics for this theory. The following two sections are devoted to the proofs that the theory is sound and complete with respect to this semantics. Section 5 compares my proposal with other theories of functional parthood. The last Section outlines the perspectives for further work.

1. From a Philosophy of Artefacts to a Formal Theory of Functional Parthood

Unless otherwise stated, I will speak only about the so-called artefact-types and not about artefact-tokens. Roughly speaking, artefact-types are construed here as objects intentionally constructed from artefact tokens when the individual or accidental properties of the latter are neglected. An artefact token is an artefact in the ordinary sense of the word. It is located in space and time. It is artefact tokens that we use as vehicles, medicines, garments, etc. An artefact type collects the features common to a group of artefact tokens. Artefact types are not located in space or time, therefore you cannot write philosophical papers with them; nonetheless, artefact types are useful conceptual devices. From the formal point of view, an artefact type may be represented as an equivalence class of the relation of being the same artefact as. Thus, your Lenovo T43 laptop is the same artefact as my T43 while being different physical objects.

I start with a general definition of functional parthood. *x* is a *functional part of y* iff there is a function *z* such that *x* performs *z* in *y*. This definition is considered here as the minimal lexical characterisation of the notion in question.

The problem with the definition (and with any definition which resorts to the notion of function) is that the notion of function seems to be inconsistent. The competent users of the languages in which the term ‘function’ occurs express inconsistent beliefs about the essential features of artefact functions. In an influential paper, Johann De Kleer claims that a function of an artefact is what the artefact is for, in contradistinction to a behavior of the artefact, which is what the device does (see [14], p. 205). On the other hand, Mark Rosenman and John Gero claim that a function of an artefact is what the artefact does and its behavior is how it does ([24], p. 167–168). Similarly, in [25] it is maintained that artefact functions are independent from the context of their applications, but in [15] it is maintained that artefact functions are dependent on the context. Finally, [1] has it that artefact functions are more abstract than artefact behaviors and [34] expresses the contrary claim. These divergences are not of minor importance as they concern the constitutive, so to speak, aspects of the notion at stake. We cannot downplay them saying that they are just terminological controversies because the notions of function and behaviour are deeply embedded in engineering practice. As a result, we do not know what artefact functions are and in some cases we cannot judge whether this and this is a function of an artefact. It is symptomatic that all these claims come from the domain of engineering design, where one can expect a greater unanimity of thought. Of course, debates in philosophy are much more fundamental and disagreement is far more prevalent.[1]

Nonetheless, even if one cannot specify what a function of an artefact is, one may be in a position to specify whether the artefact performs some function. Consequently, avoiding the controversial issues involved in the debate on the nature of functions, I will attempt to provide a definition of functional parthood in which no reference to a particular function is made, but which at the same time refers to the fact that one entity fulfills some function in another entity.

Namely, I will pursue the solution to the effect that artefact functions have something to do with artefact designs. In engineering design any process of designing is claimed to consist of four phases: planning and clarification of design task, conceptual design, embodiment design, and detailed design ([21], p. 64–70). The functions of the designed artefact play a crucial role during the conceptual design phase.

> After completing the task clarification phase, the conceptual design phase determines the principle solution. This is achieved by abstracting the essential problems, establishing the function structures, searching for suitable working principles and then combining them into a working structure. Conceptual design results in the specification principle. ([21], p. 67)

The specification principle is further developed in the subsequent phases into the specification of production. The latter specification determines:

> [...] the arrangements, forms, dimensions and surface properties of all the individual parts are finally laid down, the materials specified, production possibilities assessed, costs estimated and all the drawings and other production documents produced [...]. ([21], p. 69)

It is obvious that some features or properties specified by the design of an artefact, e.g., lengths, diameters, etc., do not concern the functions of the artefact. Still, it is equally obvious that for any rational design process, any object mentioned in this design, e.g., a bolt, piston, etc., plays some function in the artefact; otherwise, the object would be redundant and as such would not occur in a rational design. In other words, even if a design mentions more than just artefact functions, each object mentioned in the design performs some function in the artefact which is represented by this design.

This argument requires four comments. First, the argument is based on the distinction between entities that are specified (or qualified) by a design, such as bolts, pistons, and capacitors, and specifications (or qualifications), such as diameters, temperatures, etc. For example, when an engineer requires that the diameter of a bolt be 10 mm, then the bolt is the entity which is thereby specified and the diameter is the corresponding specification. I assume that only the former entities, which later will be called objects, are, so to speak, eligible candidates for function bearers. Thus, it is the bolt, and not its diameter, of which we may say that it has a function. The notion of object at stake is roughly equivalent to the notion of bearer of properties. Nonetheless, having certain qualities as their properties, objects participate in processes, are related by relations, etc.

Secondly, one should distinguish between the broad and the narrow notion of function. A function in the broad sense is anything which is desired or intended by some agent ([3], p. 172–177). If an engineer speaks about functions or functional requirements, he usually has in mind this broad meaning ([24], p. 166–167). A function in the narrow sense is a role which something plays in some structure as opposed to a purpose of the structure construed as a whole. Peter McLaughlin explains this distinction as follows:

> In the case of the functions of whole artifacts the determination of their functions or purposes is completely external. It lies in the actual intentions of the designer, manufacturer, user, etc., however socially determined these intentions may in fact be. [...] On the other hand, the

> functions of parts of an artificial system are in a sense internal and somewhat more objective insofar as these functions are always relative to their contribution to the capacities of the system of which they are part, and this contribution is part of the causal structure of the material world. It is the implied reference to the containing system [...] that distinguishes such (relative) functions from purposes. We can plausibly distinguish between a knife that has a purpose [..] and a gear that has a function within a machine [...]. ([19], p. 52)

The argument presupposes the narrow notion of function.

Still, and this is the third comment, the notion of function at stake is broad enough to include the aesthetic and ergonomic functions. Thus, even if a decorative trim around your car does not physically contributes to the overall function of being a means of transportation, still it performs some function, which justifies the designer's decision to fix the trim to the chassis.

Fourthly, it is worth to emphasise that the argument requires a relatively modest assumption concerning the rationality of designing practice. I do not claim that any detail of an artefact token performs some function because the theory espoused here is consistent with the claim that some details of the artefact token are not specified by its design.[2] Moreover, I do not deny that some functions are redundant, in which case we may eliminate them from the respective designs. Similarly, I do not deny that functions may be inconsistent, in which case performing one of them inhibits performing the other. I just claim that if an engineer mentions some object in his design, this means that he considers this object as performing some function in the artefact he designs and that this opinion of his is, so to speak, ontologically reliable for the notions at stake in the sense that the object actually performs some function.

I argued in another paper ([9]) that in philosophical ontology engineering designs (e.g., drawings, bills of materials, schemas, flow charts, etc.) may be represented by complex states of affairs.[3] However, as far as the formal aspect of the theory is concerned, this assumption is not crucial. One may identify artefact designs with other kinds of entities, e.g., with material objects of some sort, provided that this identification makes room for the definitions of the relations homomorphic to $<$ and Occ (see below).

In [9] I also provided a formal framework in which to express the relevant aspects of states of affairs and objects occurring therein. As a matter of fact, this framework amounts to a minimal conception of states of affairs and objects, which contains just these claims which I need for a formal description of artefact designs. The claims in question are the most trivial assumptions about states of affairs and objects.

Let ‘Occ(x, y)’ abbreviate the expression ‘(an object) x *occurs in* (a state of affairs) y.’ The meaning of this expression is explained by means of the following examples and axioms.

- The object named ‘John’ occurs in the state of affairs that John is tall and no other object occurs therein.
- The object named ‘John’ occurs in the state of affairs that John runs quickly and no other object occurs therein.
- The objects named ‘John’ and ‘Mary’ occur in the state of affairs that John hates Mary and no other object occurs therein.
- The objects named ‘John,’ ‘Mary,’ ‘Paul,’ and ‘Peter’ occur in the state of affairs that Mary and John sit between Peter and Paul and no other object occurs therein.

Thus, x occurs in y if x has some property and x’s having this property constitutes, so to speak, y. Similarly,

- If x participates in some process and x’s participating in this process constitutes y, then x occurs in y,
- If $x_1, x_2, \ldots, x_n$ are related by some relation and their being so related constitutes y, then $x_1, x_2, \ldots, x_n$ occur in y.

We may now define the notion of object (1.1) and the notion of state of affairs (1.2):

$$\text{Obj}(x) \equiv \exists y \ \ \text{Occ}(x, y) \tag{1.1}$$

$$\text{Soa}(x) \equiv \exists y \ \ \text{Occ}(y, x) \tag{1.2}$$

Let ‘$x < y$’ abbreviate the expression ‘(a state of affairs) x is *part of* (a state of affairs) y.’ The meaning of this expression is explained by means of the following examples and axioms.

- The state of affairs that John is tall is part of the state of affairs that John is tall and smart.
- The state of affairs that John is an accountant is part of the state of affairs that John is a handsome accountant.
- The state of affairs that John runs is part of the state of affairs that John runs quickly.

I assume that the relation $<$ is a strict partial order (on any set of states of affairs).

$$\text{Soa}(x) \rightarrow x \nless x. \tag{1.3}$$

$$x < y \rightarrow \ y \nless x. \tag{1.4}$$

$$x < y \wedge y < z \rightarrow x < z. \tag{1.5}$$

$$x < y \rightarrow \ \text{Soa}(x) \wedge \ \text{Soa}(y). \tag{1.6}$$

Axiom (1.6) restricts both place-holders of '$<$' to state of affairs.

I also define the relation of improper parthood on the set of states of affairs.

$$x \leq y \equiv x < y \ \vee (x = y \wedge \mathrm{Soa}(x)). \tag{1.7}$$

The relation $<$ is to mirror the structure of states of affairs. Consequently, it is supposed to mirror complexity of designs as well.

On any plausible account of states of affairs and objects, the former are different from the latter.

$$\mathrm{Obj}(x) \rightarrow \neg \mathrm{Soa}(x). \tag{1.8}$$

Since a state of affairs involves somehow the objects that occur therein, if one state of affairs is part of another, then any object occurring in the former occurs in the latter:

$$x \leq y \rightarrow \forall z(\mathrm{Occ}(z, x) \rightarrow \mathrm{Occ}(z, y)). \tag{1.9}$$

Given the notions of state of affairs and object as defined by definitions (1.1), (1.2), and axioms (1.3), (1.4), (1.5), (1.6), (1.8), and (1.9), I construct a formal theory of artefact designs. I assume, without any further argument, that artefacts are objects which are produced by human beings or, more broadly, by intentional agents. Any production process is at least partially determined by the design(s) of the artefact(s) that is/are produced. Nonetheless, the adequate representation of an artefact should contain more information than just its design. For instance, the way in which the artefact is to be used (or deployed) is an essential feature of this artefact (see e.g., [3] or [8]). However, for the sake of simplicity I ignore here these additional dimensions of artefacts. Namely, I assume that any object that is produced according to some design is an artefact. Let 'design(x, y)' means that x is a *design of* an artefact y.

$$\mathrm{design}(x, y) \rightarrow \mathrm{Soa}(x) \wedge \mathrm{Obj}(y). \tag{1.10}$$

$$\mathrm{Art}(x) \equiv \exists y \ \mathrm{design}(y, x). \tag{1.11}$$

How many designs does an artefact have? If you construe designs as material objects of some sort, e.g., as inscriptions, drawings, pictures, etc., there is no definite answer to this question because you may duplicate at will and subsequently destroy such material representations. Since we construe designs as states of affairs, the answer should be straightforward: Any artefact is bound to have at most one design. However, this solution presupposes a narrow concept of design which involves only specifications of production. As we saw, the notion of design is ambiguous since it may refer to any specifications produced during design process. If we construe the notion of design broadly and count as a design (of x) any representation (of x) used by engineers in their engineering activities, then we should admit that (at least) some artefacts are represented by more than one design. An artefact may be part of another artefact and a design of the latter may not specify all details of the former. A diode is part of a power supply. The design of the diode that is part of the design of the power supply specifies only two parts of the diode: the anode and the cathode. Still, a more detailed design of the diode, for instance the design you may find in a handbook on general electronics, mentions also a semi-conductor junction between the anode and cathode. Consequently, the diode has at least two designs. My claim that an artefact may have more than one design is related to the fact that engineers do not represent all details of the artefacts they use as parts of the artefacts they design. If an engineer incorporates an artefact x as part of an artefact y, then he may, i.e., it is not irrational for him to, ignore some details of x. As a result, x acquires a new design representation. It is important to notice that this new representation is used as a means of identification of x. Subsequently, the representation cannot be too unspecific on pain of malmanufactoring, and subsequently, malfunctioning of y. Finally, the practice of creating different representations of artefacts is not a contingent or praxeologically reproachable fact. Human beings are bound to represent only some details of the products of their rational activities because their representational capabilities are strictly limited. These limits makes it praxeologically reasonable for them to constrain the complexity of these products.

Artefact designs may be ordered with respect to their specificity. I will identify the relation of being less specific with the relation $<\restriction\{x : \exists y\ \mathrm{design}(x, y)\}$. i.e., the relation $<$ restricted to the set of artefact designs. It is obvious that every artefact has the most specific design (1.12), which is the design according to which the artefact is manufactured. If we agree that some artefacts have also less specific design representations, we should acknowledge the existence of the least specific representation (1.13). In order to support this claim let me observe that any artefact design consists of finitely many elements, i.e., of finitely many states of affairs, as a product of an intentional agent (or a finite group of intentional agents) with strictly limited representational capabilities. This entails that any artefact has a (possibly non-unique) $<$-minimal design. Since all these minimal designs represent one artefact, they have something in common, i.e., the set of states of affairs which are parts of all minimal designs is not empty. This common core is represented here by the notion of least specific design.

Definitions (1.12) and (1.13), and axiom (1.14) entail that for any artefact x, the most and the least specific design of x are unique. I will denote them by, respectively, 'Design(x)' and 'design$_0$(x).' The former will be called the *full design* of x; the latter will be called the *minimal design* of x.

$$\mathrm{Design}(x,y) \equiv \mathrm{design}(x,y) \wedge \forall z[\mathrm{design}(z,y) \rightarrow z \leq x]. \quad (1.12)$$

$$\mathrm{design}_0(x,y) \equiv \mathrm{design}(x,y) \wedge \forall z[\mathrm{design}(z,y) \rightarrow x \leq z]. \quad (1.13)$$

Axiom (1.14) states that every artefact has the most and the least specific design.

$$\mathrm{Art}(x) \rightarrow \exists y \exists z[\mathrm{design}_0(y,x) \wedge \mathrm{Design}(z,x)]. \quad (1.14)$$

Now I will introduce a few constraints which are to exclude the most obvious cases of irrational designs. First, although it can be argued that all designs are underdetermined, i.e., no design adequately represents all details of the respective artefact tokens, there are some limits to this indeterminacy. For instance, any full design should contain at least the minimal designs of the artefacts which occur in this design.

$$\mathrm{Occ}(x, \mathrm{Design}(y)) \wedge \mathrm{Art}(x) \rightarrow \mathrm{design}_0(x) \leq \mathrm{Design}(y). \quad (1.15)$$

Secondly, the above axioms do not guarantee that artefact designs are not circular. There are at least two kinds of circularity at stake. The first one is more straightforward. Both artefacts and non-artefacts may occur in artefact designs, but on pain of infinite regress I assume that no artefact occurs in its own design.

$$\mathrm{Design}(x,y) \rightarrow \neg \mathrm{Occ}(y,x). \quad (1.16)$$

This axiom does not proscribe the design supported by the sentence 'The hammer x consists of the haft y and ...', but it does proscribe the design supported by the sentence 'The hammer x consists of the hammer x and ...'. Equation (1.16) establishes the special meaning of the expression 'x is a design of an artefact y'. Namely, if x is a design of y, then x does not contain the states of affairs in which y occurs but the states of affairs in which the objects composing y occur.

The second type of circularity is more complex. Assume that a design of an artefact x is less specific than a design of an artefact y and a design of y is less specific than a design of x. If we conceded that each artefact has exactly one design, such a case would be impossible by the asymmetry of $<$. However, since I argued that some artefacts may have more than one design, it is possible that one design of x is part of a design of y and another design of y is part of another design of x. In this case it seems that x is a proper part of y (because a design of x is a proper part of a design of y) and y is a proper part of x (because a design of x is a proper part of a design of y), which conclusion is absurd. In order to exclude such cases I introduce the following axiom:

$$\mathrm{design}_0(x) < \mathrm{Design}(y) \wedge \mathrm{design}_0(y) < \mathrm{Design}(x) \rightarrow x = y. \tag{1.17}$$

Remember that x and y are artefact types.

My conception of minimal design has two more consequences. The first one has to do with the claim that minimal designs are conceptual means of artefact identification. That is to say that artefacts (i.e., artefact types) with the same minimal designs are identical. Needless to say, if two artefacts share the same non-minimal design, then they are identical as well because by definition any design of an artefact which is not minimal describes the artefact in more detail than the respective minimal design. The conjunction of these two claims is axiom (1.18). The other consequence (i.e., axiom (1.19)) is related to the first one: any artefact preserves the structure determined by its minimal design, i.e., the minimal design is part of any state of affairs in which the artefact occurs. If the minimal design of x was not part of some state of affairs in which x occurs, then x could not be identified within this state of affairs.

$$\mathrm{design}(x, y_1) \wedge \mathrm{design}(x, y_2) \rightarrow y_1 = y_2. \tag{1.18}$$

$$\mathrm{Occ}(x, y) \wedge \mathrm{design}_0(z, x) \rightarrow z \leq y. \tag{1.19}$$

Assuming that any object which occurs in a design of an artefact is not redundant within this design, we may say that x performs some function in y if there is a state of affairs in which x occurs and which is part of the full design of y.

$$\mathrm{Func}(x, y) \equiv \exists z[\mathrm{Occ}(x, z) \wedge z \leq \mathrm{Design}(y)]. \tag{1.20}$$

The formal theory of functional parthood (FTFP) which I present in this paper is expressed in a first order language (with identity) with three primitive binary predicates: ‘Occ’, ‘<’, and ‘design.’ I presuppose that the standard proof-theoretical definition of consequence operation is defined for this language. The expression ‘$X \vdash \varphi$’ means that a formula φ is a *logical consequence* of a set X of formulas. FTFP is based on definitions (1.1), (1.2), (1.7), (1.11), (1.12), (1.13), and (1.20), and axioms (1.4), (1.5), (1.6), (1.8), (1.9), (1.10), (1.14), (1.15), (1.16), (1.17), (1.18), and (1.19). If a well formed formula φ of FTFP (for short, a formula) is a logical consequence of the union of a set X and the set of these axioms and definitions, then I will denote this fact by ‘$X \vdash_{\mathrm{FTFP}} \varphi$’. When $\emptyset \vdash_{\mathrm{FTFP}} \varphi$, I will say that φ is a *thesis of FTFP* (also written: $\vdash_{\text{FTFP}} \varphi$). A set X of formulas is *consistent* iff there is no formula φ such that both $X \vdash_{\text{FTFP}} \varphi$ and $X \vdash_{\text{FTFP}} \neg\varphi$.

The solution to the effect that a theory of functional parthood is in fact a theory of design has three advantages over other approaches. First, the notion of design is far better understood and far less controversial than the notion of function. Being aware of the aforementioned problems with an adequate definition of the latter notion, one may appreciate an approach in which the notion of function is not taken for granted.

Secondly, in FTFP the logical properties of functional parthood are not simply assumed as axioms but may be derived as theses. This fact is of some importance for those who esteem the epistemological value of the debates on the mereological principles. FTFP allows us to discuss the controversies over the relation of functional parthood in a broad framework which makes room for discovering the sources of disagreement. For instance, it can be shown that the relation defined by Equation (1.20) is irreflexive and asymmetric (Equations (1.21) and (1.22)) but in general not transitive (Equation (1.23)). Still, we can show why or when the relation is transitive (Equations (1.24) and (1.25)).

$$\vdash_{\text{FTFP}} \neg\text{Func}(x,x). \tag{1.21}$$

Proof. Assume that for some x_0, $\text{Func}(x_0, x_0)$. Definition (1.20) entails that for some z_0, $\text{Occ}(x_0, z_0)$ and $z_0 \leq \text{Design}(x_0)$. Consequently, $\text{Occ}(x_0, \text{Design}(x_0))$ (by Equation (1.9)). Axiom (1.16) entails now that $\neg\text{Design}(\text{Design}(x_0), x_0)$, which is inconsistent with the definitions of FTFP. □

$$\vdash_{\text{FTFP}} \text{Func}(x,y) \rightarrow \neg\text{Func}(y,x). \tag{1.22}$$

Proof. Suppose that for some x_0 and y_0, $\text{Func}(x_0, y_0)$ and $\text{Func}\,(y_0, x_0)$. The former entails (*) and the latter entails (**).

(*) $\exists z[\text{Occ}(x_0, z) \wedge z \leq \text{Design}(y_0)]$,
(**) $\exists v[\text{Occ}(y_0, v) \wedge v \leq \text{Design}(x_0)]$.
It follows from (*) that $\text{Occ}(x_0, \text{Design}(y_0))$. It follows from (**) that $\text{Art}(x_0)$. (*) and axiom (1.15) entail now (***):

(***) $\text{design}_0(x_0) \leq \text{Design}(y_0)$.
Similarly, (****) follows from (**).
(****) $\text{design}_0(y_0) \leq \text{Design}(x_0)$.
(***), (****), and axioms (1.17) and (1.18) entail that $x_0 = y_0$ what contradicts both assumptions of the proof (1.21). □

$$\nvdash_{\text{FTFP}} \text{Func}(x,y) \wedge \text{Func}(y,z) \rightarrow \text{Func}(x,z). \tag{1.23}$$

Proof. Consider the following model of FTFP (see Section 2 below). The model consists of ten elements: $1, 2, 3, 4, 5, A, B, C, D,$ and E.

1. $\mathrm{Occ}(1, A), \mathrm{Occ}(1, C)$,
2. $\mathrm{Occ}(2, B)$, $\mathrm{Occ}(2, C), \mathrm{Occ}(2, D), \mathrm{Occ}(2, E)$,
3. $\mathrm{Occ}(3, C)$,
4. $\mathrm{Occ}(4, D), \mathrm{Occ}(4, E), \mathrm{Occ}(5, E)$,
5. $A < C$,
6. $B < C, B < D, B < E$,
7. $D < E$,
8. $\mathrm{design}(A, 3)$,
9. $\mathrm{design}(B, 4), \mathrm{design}(C, 4)$,
10. $\mathrm{design}(D, 5)$.

It is easy to verify that all axioms of FTFP are satisfied in this model. Moreover, for some $x, y,$ and, z if the formulas '$\mathrm{Func}(x, y)$' and '$\mathrm{Func}(y, z)$' are satisfied, then the formula '$\mathrm{Func}(x, z)$' is not. □

$$\vdash_{\mathrm{FTFP}} \forall x\ \mathrm{design}_0(x) = \mathrm{Design}(x) \rightarrow$$
$$\rightarrow [\mathrm{Func}(x, y) \wedge \mathrm{Func}(y, z) \rightarrow \mathrm{Func}(x, z)]. \tag{1.24}$$

Proof. Assume that for all $x, \mathrm{design}_0(x) = \mathrm{Design}(x)$. Now let $\mathrm{Func}(x, y)$ and $\mathrm{Func}(y, z)$. The former entails (*) and the latter entails (**).

(*) $\exists v_1[\mathrm{Occ}(x, v_1) \wedge v_1 \leq \mathrm{Design}(y)]$,
(**) $\exists v_2[\mathrm{Occ}(y, v_2) \wedge v_2 \leq \mathrm{Design}(z)]$.

As in the proof of Equation (1.22), (**) entails that $\mathrm{design}_0(y) \leq \mathrm{Design}(z)$. Since $\mathrm{design}_0(y) = \mathrm{Design}(y)$, (*) gives us that $v_1 \leq \mathrm{Design}(z)$. Consequently, we get (***):

(***) $\exists v_1[\mathrm{Occ}(x, v_1) \wedge v_1 \leq \mathrm{Design}(z)]$,

This obviously completes the proof (1.20). □

$$\vdash_{\mathrm{FTFP}} \forall x, y\,[\mathrm{Func}(x, y) \rightarrow \mathrm{Design}(x) \leq \mathrm{Design}(y)]$$
$$\rightarrow [\mathrm{Func}(x, y) \wedge \mathrm{Func}(y, z) \rightarrow \mathrm{Func}(x, z)]. \tag{1.25}$$

Proof. Assume that for all x and y, $[\mathrm{Func}(x, y) \rightarrow \mathrm{Design}(x) \leq \mathrm{Design}(y)]$. Now let $\mathrm{Func}(x, y)$ and $\mathrm{Func}(y, z)$. The former entails (*) and the latter entails (**).

(*) $\exists v_1[\mathrm{Occ}(x, v_1) \wedge v_1 \leq \mathrm{Design}(y)]$,
(**) $\exists v_2[\mathrm{Occ}(y, v_2) \wedge v_2 \leq \mathrm{Design}(z)]$.

Moreover, both assumptions entail that

(***) $\mathrm{Design}(y) \leq \mathrm{Design}(z)$.

(*) and (***) gives us that $v_1 \leq \text{Design}(z)$. Consequently, as in the previous proof we get (****):

$$(****)\ \exists v_1[\text{Occ}(x, v_1) \wedge v_1 \leq \text{Design}(z)]. \qquad \square$$

Equation (1.24) reveals one of the sufficient conditions for the transitivity of functional parthood: if each artefact in a set X has exactly one design, then the relation of functional parthood is transitive in X. Equation (1.25) reveals another condition: if for any artefact from a set X, its full design contains the full designs of all its functional parts, then the relation of functional parthood is transitive in X. In a similar way, one can discuss in FTFP other principles of the standard mereology, e.g., the principle of extensionality.

The third advantage of FTFP over other approaches consists in the fact that FTFP does not take for granted such crucial logical properties of functional parthood as transitivity or extensionality, but relates them to the actual engineering designs. Thus, whether this relation is transitive or extensional depends eventually on the way in which artefacts are designed.

2. Semantics

Let O and Aux be two sets.[4] Let $S \subseteq \wp(O \cup Aux)$. I assume in this paper the system of set theory with the axiom of foundation which entails Equation (2.1).

$$X \cap \wp(X) = \emptyset. \tag{2.1}$$

Consequently, $O \cap S = \emptyset$.

In the set $(O \cup Aux) \cup S$ I define two relations.

$$< x, y > \in occ \equiv x \in y \wedge x \in O \wedge y \in S. \tag{2.2}$$

$$< x, y > \ \in < \ \equiv x \subset y \wedge x, y \in S. \tag{2.3}$$

By '$\leq$' I will denote the union of $<$ and the set $\{< x, x >: x \in S\}$. Moreover, let *designof* $\subseteq S \times O$.

A *structure* is a pair $< \mathfrak{U}, \mathfrak{f} >$, where

1. $\mathfrak{U} = (O \cup Aux) \cup S$ is a universe of the structure,
2. $\mathfrak{f}$ is a function defined on the set {'Occ,' '<,' 'design'} of FTFP predicates, which function satisfies the following conditions:

- $\mathfrak{f}$("Occ")=*occ*,
- $\mathfrak{f}$("<")=<,
- $\mathfrak{f}$("design")=*designof*.

A structure $<\mathfrak{U}, \mathfrak{f}>$ is an *FTFP-structure* if it satisfies conditions 2.11, 2.12, 2.13, 2.14, 2.15, 2.16, 2.17, and 2.18. In order to introduce these conditions in a concise way, I need five auxiliary definitions.

$$design := \{x : \exists y < x, y > \in designof\}. \tag{2.4}$$

$$art := \{x : \exists y < y, x > \in designof\}. \tag{2.5}$$

$$\begin{aligned} x \in \mathfrak{Design} \equiv \exists y \in x\ \exists z\, [< y, z > \in designof \wedge \\ \wedge \forall v\, (v \in x \equiv < v, z > \in designof)]. \end{aligned} \tag{2.6}$$

$$Design := \{x \in design : \exists y \in \mathfrak{Design}\ x = \bigcup y\}. \tag{2.7}$$

$$design_0 := \{x \in design : \exists y \in \mathfrak{Design}\ x = \bigcap y\}. \tag{2.8}$$

Here are the aforementioned conditions.[5]

$$O \subseteq \bigcup S. \tag{2.11}$$

$$x \in S \to x \cap O \neq \emptyset. \tag{2.12}$$

Any element of $\mathfrak{Design}$ has (in $\mathfrak{Design}$) the least and the *greatest element* (*with respect to* $\subseteq$). (2.13)

The relation *designof* is a function. (2.14)

$$[designof \cap (design_0 \times O)] \mid [occ \cap (art \times Design)] \subseteq \leq . \tag{2.15}$$

$$[designof \cap (design_0 \times O)] \mid occ \subseteq \leq . \tag{2.16}$$

$$designof \cap occ^{-1} = \emptyset. \tag{2.17}$$

$$\begin{aligned} & < x_1, x >, < x_2, x >, < y_1, y >, < y_2, y > \in designof \to \\ & \to [x_1, y_1 \in design_0 \wedge x_2, y_2 \in Design \to \\ & \to (x_1 < x_2 \wedge y_1 < y_2 \to x = y)]. \end{aligned} \tag{2.18}$$

An *assignment* in a structure $<\mathfrak{U}, \mathfrak{f}>$ is a function $\mathfrak{g}$ such that $\mathfrak{g}$ maps the set of individual variables of FTFP into the set $\mathfrak{U}$. An *interpretation* of FTFP is a pair $<< \mathfrak{U}, \mathfrak{f} >, \mathfrak{g} >$, where $< \mathfrak{U}, \mathfrak{f} >$ is an FTFP-structure and $\mathfrak{g}$ is an assignment in this structure. Following [7] I will identify an interpretation $\mathfrak{I} = << \mathfrak{U}, \mathfrak{f} >, \mathfrak{g} >$ of FTFP with an assignement $\mathfrak{g}$, i.e., $\mathfrak{I}(\alpha) = \mathfrak{g}(\alpha)$. In the usual way I define the interpretation $\mathfrak{I}\frac{\alpha}{x}$ which maps

x to α ($x \in \mathfrak{U}$) and agrees with an interpretation $\mathfrak{I}$ on all variables distinct from α.

The expression '$\mathfrak{I} \models_{\mathrm{FTFP}} \varphi$' means as usual that an interpretation $\mathfrak{I}$ is a *model of* a formula φ (or that $\mathfrak{I}$ *satisfies* φ). The clauses of the definition of satisfaction relation for the classical connectives are standard. Besides them, we have in FTFP three specific primitive predicates:

$$\mathfrak{I} \models_{\mathrm{FTFP}} \mathrm{Occ}(\alpha, \beta) \equiv < \mathfrak{I}(\alpha), \mathfrak{I}(\beta) > \in occ,$$
$$\mathfrak{I} \models_{\mathrm{FTFP}} \alpha < \beta \equiv < \mathfrak{I}(\alpha), \mathfrak{I}(\beta) > \in <,$$
$$\mathfrak{I} \models_{\mathrm{FTFP}} \mathrm{design}(\alpha, \beta) \equiv < \mathfrak{I}(\alpha), \mathfrak{I}(\beta) > \in designof. \quad (2.19)$$

The clauses for other specific predicates and function symbols of FTFP follow from Equation (2.19) and the respective definitions (i.e., Equations (1.1), (1.2), (1.7), (1.11), (1.12), (1.13), and (1.20)).

If there is an interpretation which is a model of all formulas from a set X, I will say that X is *satisfiable*. A formula φ is *valid in FTFP* (written: $\models_{\mathrm{FTFP}} \varphi$) iff for any interpretation $\mathfrak{I}$ of FTFP, $\mathfrak{I} \models_{\mathrm{FTFP}} \varphi$.

3. Soundness

THEOREM 1. If $\vdash_{\mathrm{FTFP}} \varphi$, then $\models_{\mathrm{FTFP}} \varphi$.

Proof. I will prove theorem 1 only for the case when φ is a specific axiom of FTFP. The remaining part of the proof is similar to the classical case.

1. φ = 1.3, φ = 1.4, and φ = 1.5.
In this case theorem 1 boils down to $\neg x \subset x$, $x \subset y \rightarrow \neg y \subset x$, and $x \subset y \wedge y \subset z \rightarrow x \subset z$ (Equation (2.3) and (2.19)).
2. φ = 1.16.
Let $\mathfrak{I} \models_{\mathrm{FTFP}} x < y$. Definition (2.3) entails that $\mathfrak{I}(x)$, $\mathfrak{I}(y) \in S$. Condition 2.12 guarantees that there are some $z_1, z_2 \in O$ such that $z_1 \in x$ and $z_2 \in y$. Given definition (2.2) it follows that $\mathfrak{I} \models_{\mathrm{FTFP}} \exists z\, \mathrm{Occ}(z, x)$ and $\mathfrak{I} \models_{\mathrm{FTFP}} \exists z\, \mathrm{Occ}(z, y)$. Given definition (1.2), the former is equivalent to $\mathfrak{I} \models_{\mathrm{FTFP}} \mathrm{Soa}(x)$ and the latter is equivalent to $\models_{\mathrm{FTFP}} \mathrm{Soa}(y)$.
3. φ = 1.8.
Assume that $\mathfrak{I} \models_{\mathrm{FTFP}} \mathrm{Obj}(x)$, i.e., that for some y, $\mathfrak{I}(x) \in y$, $\mathfrak{I}(x) \in O$, and $y \in S$. Due to Equation (2.1), $\mathfrak{I}(x) \notin S$. As a result, there is no such z that $z \in \mathfrak{I}(x)$, $z \in O$, and $\mathfrak{I}(x) \in S$. Consequently, $\neg \exists z\, \mathrm{Occ}(z, x)$, i.e., $\neg \mathrm{Soa}(x)$.
4. φ = 1.9.

Suppose that $\mathfrak{I} \models_{\text{FTFP}} x \leq y$ and that $\mathfrak{I} \models_{\text{FTFP}} \text{Occ}(z, x)$. If $\mathfrak{I}(x) = \mathfrak{I}(y)$, $\mathfrak{I} \models_{\text{FTFP}} \text{Occ}(z, y)$. If $\mathfrak{I}(x) < \mathfrak{I}(y)$, this means that $\mathfrak{I}(x) \subset \mathfrak{I}(y)$. Given that $\mathfrak{I}(z) \in \mathfrak{I}(x)$, we get that $\mathfrak{I} \models_{\text{FTFP}} \text{Occ}(z, y)$.

5. φ = 1.10.

Let $\mathfrak{I} \models_{\text{FTFP}} \text{design}(x, y)$. Given the definition of relation *designof* the assumption entails that $\mathfrak{I}(x) \in S$ and $\mathfrak{I}(y) \in O$. As in the previous proof, the former entails that $\mathfrak{I} \models_{\text{FTFP}} \text{Soa}(x)$. The latter together with condition 2.11 entails that $\mathfrak{I} \models_{\text{FTFP}} \exists z\ \text{Occ}(y, z)$, i.e., $\mathfrak{I} \models_{\text{FTFP}} \text{Obj}(y)$.

6. φ = 1.14.

Assume that for some $y \in \mathfrak{U}, <y, \mathfrak{I}(x)> \in \mathit{designof}$. Subsequently, the set $\{z : <z, \mathfrak{I}(x)>\} \in \mathit{designof}$, i.e., for some $z \in \mathfrak{Design}, y \in z$. It follows from condition 2.13 that z contains two elements z_1 and z_2 such that $z_1 = \bigcap z$ and $z_2 = \bigcup z$. Definition (2.6) entails that $<z_1, \mathfrak{I}(x)> \in \mathit{designof}$ and $<z_2, \mathfrak{I}(x)> \in \mathit{designof}$. Let $< v, \mathfrak{I}(x) > \in \mathit{designof}$. Then $v \in z$ because of Equation (2.6). Consequently, $z_1 \subseteq v \subseteq z_2$. This leads us to $\mathfrak{I} \models_{\text{FTFP}} \text{design}_0(z_1, x)$ and $\mathfrak{I} \models_{\text{FTFP}} \text{Design}(z_2, x)$ as desired.

7. φ = 1.15.

First observe that $\mathfrak{I} \models_{\text{FTFP}} \text{Art}(x) \equiv \mathfrak{I}(x) \in \mathit{art}$. Let $\mathfrak{I} \models_{\text{FTFP}} \text{Occ}(x, \text{Design}(y))$. Obviously, $\mathfrak{I}(x) \in O$. This entails that for some $z \in \mathit{Design}, <\mathfrak{I}(x), z> \in \mathit{occ}$. Moreover, let $\mathfrak{I} \models_{\text{FTFP}} \text{Art}(x)$, i.e., that for some $v \in \mathfrak{U}, < v, \mathfrak{I}(x) > \in \mathit{designof}$. Condition 2.13 entail that there exists w such that $< w, \mathfrak{I}(x) > \in \mathit{designof}$ and $w \in \mathit{design}_0$. Consequently, $< w, \mathfrak{I}(x) > \in \mathit{designof} \cap (\mathit{design}_0 \times O)$. On the other hand, $< \mathfrak{I}(x), z > \in \mathit{occ}$ and $< \mathfrak{I}(x), z > \in \mathit{art} \times \mathit{Design}$, i.e., $< \mathfrak{I}(x), z > \in \mathit{occ} \cap (\mathit{art} \times \mathit{Design})$. This means that $<w, z> \in \mathit{designof} \cap (\mathit{design}_0 \times O) \mid \mathit{occ} \cap (\mathit{art} \times \mathit{Design})$. Condition 2.16 gives that $v \subseteq z$, which means that $\mathfrak{I} \models_{\text{FTFP}} \text{design}_0(x) \leq \text{Design}(y)$.

8. φ = 1.16.

Assume that $\mathfrak{I} \models_{\text{FTFP}} \text{Design}(x, y)$. This entails that $<\mathfrak{I}(x), \mathfrak{I}(y)> \in \mathit{designof}$. Then it follows from condition 2.17 that $<\mathfrak{I}(y), \mathfrak{I}(x)> \notin \mathit{occ}$, i.e., $\mathfrak{I}$ is not a model of Occ(y, x).

9. φ = 1.17.

Suppose that

(*) $\mathfrak{I} \models_{\text{FTFP}} \text{design}_0(x) < \text{Design}(y)$,
(**) $\mathfrak{I} \models_{\text{FTFP}} \text{design}_0(y) < \text{Design}(x)$.

(*) entails that there is such z_1 that $z_1 \in \mathit{design}_0$ and $< z_1, \mathfrak{I}(x) > \in \mathit{designof}$. Moreover, there is such v_1 that $v_1 \in \mathit{Design}$, $< v_1, \mathfrak{I}(y) > \in \mathit{designof}$ and $z_1 < v_1$. (**) entails that there is such z_2 that $z_2 \in \mathit{design}_0$ and $< z_2, \mathfrak{I}(y) > \in \mathit{designof}$. Moreover, there is such v_2 that $v_2 \in \mathit{Design}$, $< v_2, \mathfrak{I}(x) > \in \mathit{designof}$ and $z_2 < v_2$. It follows directly from Equation (2.18) that $\mathfrak{I}(x) = \mathfrak{I}(y)$.

10. $\varphi = 1.18$.
Assume that $\mathfrak{I} \models_{\text{FTFP}} \text{design}(x, y_1)$ and $\mathfrak{I} \models_{\text{FTFP}} \text{design}(x, y_2)$. This entails that $< \mathfrak{I}(x), \mathfrak{I}(y_1) > \in \textit{designof}$ and $< \mathfrak{I}(x), \mathfrak{I}(y_2) > \in \textit{designof}$. The required result, i.e., that $y_1 = y_2$, follows from condition 2.14.

11. $\varphi = 1.19$.
Let $\mathfrak{I} \models_{\text{FTFP}} \text{design}_0(z, x)$ and $\mathfrak{I} \models_{\text{FTFP}} \text{Occ}(x, y)$. The former assumption entails that $< \mathfrak{I}(z), \mathfrak{I}(x) > \in \textit{designof}$, $\mathfrak{I}(z) \in \textit{design}_0$, and $\mathfrak{I}(x) \in O$. The latter assumption entails that $< \mathfrak{I}(x), \mathfrak{I}(y) > \in \textit{occ}$. These consequences entail together that $< \mathfrak{I}(z), \mathfrak{I}(y) > \in [\textit{designof} \cap (\textit{design}_0 \times O)] \mid \textit{occ}$. Thus, Equation (2.17) gives the desired result, namely that $\mathfrak{I}(z) \subseteq \mathfrak{I}(y)$, i.e., $\mathfrak{I} \models_{\text{FTFP}} z \leq y$. □

4. Completeness

THEOREM 2. If $\models_{\text{FTFP}} \varphi$, then $\vdash_{\text{FTFP}} \varphi$.

Proof. In order to prove theorem 2 I will show that every consistent set of formulas is satisfiable. In order to show this I follow Henkin's proof of the completeness of the first-order logic. Consequently, the proof of 2 consists of two stages. First, it is to be proved that every negation complete consistent set of formulas that contains witnesses is satisfiable. Then one shows how to extend a consistent set of formulas into a negation complete and consistent set that contains witnesses ([7], p. 75–82).

Let X be a set of formulas. In the standard way I will define the term structure determined by X. I start with a definition of the following relation on the set of FTFP terms:

$$\alpha \sim \beta \equiv X \vdash_{\text{FTFP}} \alpha = \beta. \tag{4.1}$$

It is easy to show that the relation $\sim$ is an equivalence relation. By "$\overline{\alpha}$" I will denote the $\sim$-equivalence class of α. I put $T^X := \{\overline{\alpha} : \alpha$ is a term of FTFP$\}$.

$$O^X := \{\overline{\alpha} \in T^X : \text{for some } \beta, X \vdash_{\text{FTFP}} \text{Occ}(\alpha, \beta)\}. \tag{4.2}$$

$$\textit{Aux}^X := \{\overline{\alpha} \in T^X \setminus O^X : \text{ for no term } \beta, X \vdash_{\text{FTFP}} \text{Occ}(\beta, \alpha)\}. \tag{4.3}$$

$$\overline{\overline{\alpha}} := \{\overline{\beta} \in T^X : X \vdash_{\text{FTFP}} \text{Occ}(\beta, \alpha)\} \cup \{\overline{\beta} \in T^X : X \vdash_{\text{FTFP}} \beta \leq \alpha\}. \quad (4.4)$$

$$S^X := \{\overline{\overline{\alpha}} : \text{ for some } \beta,\ X \vdash_{\text{FTFP}} \text{Occ}(\beta, \alpha)\}. \quad (4.5)$$

The universe of the term structure is the set $\mathfrak{U}^X$.

$$\mathfrak{U}^X := (O^X \cup Aux^X) \cup S^X. \quad (4.6)$$

In $\mathfrak{U}^X$ I define three relations:

$$occ^X(\overline{\alpha}, \overline{\overline{\beta}}) \equiv X \vdash_{\text{FTFP}} \text{Occ}(\alpha, \beta). \quad (4.7)$$

$$<^X(\overline{\overline{\alpha}}, \overline{\overline{\beta}}) \equiv X \vdash_{\text{FTFP}} \alpha < \beta. \quad (4.8)$$

$$designof^X(\overline{\overline{\alpha}}, \overline{\beta}) \equiv X \vdash_{\text{FTFP}} \text{design}(\alpha, \beta). \quad (4.9)$$

The remaining sets and relations, i.e., $\leq^X$, art^X, $\mathfrak{design}^X$, $design^X$, $design_0^X$, and $Design^X$, might be defined along the lines of definitions (2.5), (2.6), (2.7), and (2.8).

The function $\mathfrak{f}^X$ maps the primitive predicates of FTFP into the set of these relations, i.e.,

- $\mathfrak{f}^X$(“Occ”)=occ^X,
- $\mathfrak{f}^X$(“<”)=$<^X$,
- $\mathfrak{f}^X$ (“design”)=$designof^X$.

LEMMA 1. *A pair* $< \mathfrak{U}^X, \mathfrak{f}^X >$ *is a structure.*

Proof. The proof of this lemma boils down to a proof that the relations defined by Equations (4.7) and (4.8) satisfy, respectively, definitions (2.2) and (2.3), and that relation $designof^X$ is a subset of $S^X \times O^X$.

1. $occ^X = occ$.
First, I show that $occ^X \subseteq occ$. Let $X \vdash_{\text{FTFP}} \text{Occ}(\alpha, \beta)$. Given definitions (4.7), (4.2), (4.4), and (4.5), this entails that $\overline{\alpha} \in O^X$, $\overline{\overline{\beta}} \in S^X$, and $\overline{\alpha} \in \overline{\overline{\beta}}$. This means that $< \overline{\alpha}, \overline{\overline{\beta}} > \in occ$. Assume now that for $\overline{\alpha}, \overline{\overline{\beta}}$, the following assumptions hold.
(*) $\overline{\alpha} \in O^X$,
(**) $\overline{\overline{\beta}} \in S^X$,
(***) $\overline{\alpha} \in \overline{\overline{\beta}}$.
(*) is equivalent to the fact that for some $\beta, X \vdash_{\text{FTFP}} \text{Occ}(\alpha, \beta)$, which means that $X \vdash_{\text{FTFP}} \text{Obj}(\alpha)$. Axioms (1.8) and (1.6) entail that

$X \vdash_{\text{FTFP}} \forall\beta\ \beta \nleq \alpha$. Consequently, definition (4.4) and (***) gives us the required result, namely that $X \vdash_{\text{FTFP}} \mathrm{Occ}(\alpha, \beta)$.

2. $<^X = <$.

First I will show that if $X \vdash_{\text{FTFP}} \alpha < \beta$, then $\overline{\overline{\alpha}} \subset \overline{\overline{\beta}}$. Assume otherwise. Since it holds that $X \vdash_{\text{FTFP}} \alpha < \beta$, therefore (*) also holds because of axiom (1.9).

(*) $X \vdash_{\text{FTFP}} \forall\gamma(\mathrm{Occ}(\gamma, \alpha) \to (\mathrm{Occ}(\gamma, \beta))$.

As relation $<$ is irreflexive and asymmetric, $\beta \neq \alpha$ and $\beta \nless \alpha$, i.e., $\overline{\overline{\alpha}} \neq \overline{\overline{\beta}}$ because $\overline{\beta} \notin \overline{\overline{\alpha}}$ but $\overline{\beta} \in \overline{\overline{\beta}}$. Consequently, it suffices to show that any $\overline{\gamma} \in \overline{\overline{\alpha}}$ belongs to $\overline{\overline{\beta}}$. Suppose that $\gamma \in \overline{\overline{\alpha}}$. This means that either (**) or (***) is the case.

(**) $X \vdash_{\text{FTFP}} \mathrm{Occ}\,(\gamma, \alpha)$,

(***) $X \vdash_{\text{FTFP}} \gamma \leq \alpha$.

That $\overline{\gamma} \in \overline{\overline{\beta}}$ is entailed both by (**) (due to (*)) and (***) (due to axiom (1.5)).

Suppose now that

(*) $\overline{\overline{\alpha}} \subset \overline{\overline{\beta}}, \overline{\overline{\alpha}} \in S^X, \overline{\overline{\beta}} \in S^X$.

Due to the reflexivity of $\leq$ it is always the case that $\overline{\alpha} \in \overline{\overline{\alpha}}$. Thus, $\overline{\alpha} \in \overline{\overline{\beta}}$. The assumption that $\overline{\overline{\alpha}} \in S^X$ entails that $X \vdash_{\text{FTFP}} \mathrm{Soa}(\alpha)$. It follows from axiom (1.8) that $X \vdash_{\text{FTFP}} \forall\gamma\ \neg\mathrm{Occ}(\alpha, \gamma)$. Since $\overline{\alpha} \in \overline{\overline{\beta}}$, $X \vdash_{\text{FTFP}} \alpha \leq \beta$. As $\alpha \neq \beta, X \vdash_{\text{FTFP}} \alpha < \beta$.

3. $designof^X \subseteq S^X \times O^X$.

Given axiom (1.10) and definition (4.9), the proof is straightforward. □

LEMMA 2. *A structure* $<\mathfrak{U}^X, \mathfrak{f}^X>$ *is an FTFP-structure.*

Proof. In order to prove this lemma I will show that (given definitions (2.5), (2.6), (2.7), and (2.8)) conditions 2.11, 2.12, 2.13, 2.14, 2.15, 2.16, 2.17, and 2.18 are satisfied for $\mathfrak{U}^X$ and $\mathfrak{f}^X$.

1. Condition 2.11

If $\bar{x} \in O^X$, then for some $y, X \vdash_{\text{FTFP}} \mathrm{Occ}(x, y)$. Consequently, there exists such $\overline{\overline{y}} \in S^X$ that $\bar{x} \in \overline{\overline{y}}$.

2. Condition 2.12

Suppose that $\overline{\overline{x}} \in S^X$. This entails that for some $y, X \vdash_{\text{FTFP}} \mathrm{Occ}(y, x)$. This means, on the one hand, that $\overline{y} \in \overline{\overline{x}}$; on the other hand, we get that $\overline{y} \in O^X$.

3. Condition 2.13

Suppose that $x \in \mathfrak{design}^X$. Definition (2.6) entails that for some $\overline{\overline{y}} \in x$ and $\bar{z}$,

(*) $<\overline{\overline{y}}, \bar{z}> \in designof^X$,

and

(**) $\forall\overline{\overline{v}}(\overline{\overline{v}} \in x \equiv <\overline{\overline{v}}, \bar{z}> \in designof^X)$.

Then definition (4.9) yields that $X \vdash_{\text{FTFP}} \text{design}(y, z)$. Now definition (1.11) and axiom (1.14) entail that $X \vdash_{\text{FTFP}} \exists v \exists t\, [\text{design}_0(v, z) \wedge \text{Design}(t, z)]$. It follows that

(***) for some v_0, $X \vdash_{\text{FTFP}} \text{design}(v_0, z) \wedge \forall w(\text{design}(w, z) \rightarrow v_0 \leq w)$,
(****) for some $t_0, X \vdash_{\text{FTFP}} \text{design}(t_0, z) \wedge \forall w(\text{design}(w, z) \rightarrow w \leq t_0)$.
Given definitions (4.9), (4.8), and lemma 1 we get from (***) that $< \overline{\overline{v_0}}, \overline{z} > \in \mathit{designof}^X$ and $\forall \overline{\overline{w}} (< \overline{\overline{w}}, \overline{z_0} > \in \mathit{designof}^X \rightarrow \overline{\overline{v_0}} \subseteq \overline{\overline{w}})$. (**) entails that $\overline{\overline{v_0}} \in x$ and $\forall \overline{\overline{w}} (\overline{\overline{w}} \in x \rightarrow \overline{\overline{v_0}} \subseteq \overline{\overline{w}})$ This means that $\overline{\overline{v_0}}$ is the $\subseteq$-least element of x. In a similar way, we can show that $\overline{\overline{t_0}}$ is the $\subseteq$-greatest element of x.

4. Condition 2.14
Assume $<\overline{\overline{x_0}}, \overline{v_1}> \in \mathit{designof}^X$ and $\overline{\overline{x_0}}, \overline{v_2} \in \mathit{designof}^X$. Definition (4.9) entails that $X \vdash_{\text{FTFP}} \text{design}(x_0, v_1)$ and $X \vdash_{\text{FTFP}} \text{design}(x_0, v_2)$, what yields the required result in face of Equation (1.18).

5. Condition 2.15
Suppose that $<\overline{\overline{x}}, \overline{y}> \in [\mathit{designof}^X \cap (\mathit{design}_0^X \times O^X)] \mid [\mathit{occ}^X \cap (\mathit{art}^X \times \mathit{Design}^X)]$. This means that for some $\overline{x_0}$ it holds that

(*) $< \overline{\overline{x}}, \overline{x_0} > \in \mathit{designof}^X, \overline{\overline{x}} \in \mathit{design}_0^X, \overline{x_0} \in O^X$,
(**) $\overline{x_0}, \overline{\overline{y}} > \in \mathit{occ}^X, \overline{x_0} \in \mathit{art}^X, \overline{\overline{y}} \in \mathit{Design}^X$.
From (*), Equation (4.9), and axiom (1.18) it follows that $X \vdash_{\text{FTFP}} \text{design}_0(x, x_0)$. Now (**), Equations (4.7), (4.9), and (1.18) gives us that $X \vdash_{\text{FTFP}} \text{Occ}(x_0, y)$, $X \vdash_{\text{FTFP}} \text{Art}(x_0)$, and $X \vdash_{\text{FTFP}} \text{Design}(y, x_0)$. Axiom (1.15) entails that $X \vdash_{\text{FTFP}} x \leq y$. Given Equation (4.8) this means that $< \overline{\overline{x}}, \overline{\overline{y}} > \in \leq^X$.

6. Condition 2.16
Suppose that $<\overline{\overline{x}}, \overline{y}> \in [\mathit{designof}^X \cap (\mathit{design}_0^X \times O^X)] \mid \mathit{occ}^X$. This means that for some $\overline{x_0}$ it holds that

(*) $< \overline{\overline{x}}, \overline{x_0} > \in \mathit{designof}^X, \overline{\overline{x}} \in \mathit{design}_0^X, \overline{x_0} \in O^X$,
(**) $< \overline{x_0}, \overline{\overline{y}} > \in \mathit{occ}^X$.
From (*), Equations (4.9), and (1.18) it follows that $X \vdash_{\text{FTFP}} \text{design}_0(x, x_0)$. Now (**), Equations (4.7), and (1.18) gives us that $X \vdash_{\text{FTFP}} \text{Occ}(x_0, y)$. Axiom (1.19) entails that $X \vdash_{\text{FTFP}} x \leq y$, i.e., that $< \overline{\overline{x}}, \overline{\overline{y}} > \in \leq^X$.

7. Condition 2.17
Assume otherwise. Let $< \overline{\overline{x_0}}, \overline{y_0} >$ be a member of the intersection of $\mathit{designof}^X$ and $(\mathit{occ}^X)^{-1}$. It follows from Equations (4.9) and (4.7) that $X \vdash_{\text{FTFP}} \text{design}(x_0, y_0)$ and $X \vdash_{\text{FTFP}} \text{Occ}(y_0, x_0)$. The former claim, axiom (1.14), definitions (1.11) and (1.12) entail that for some $z_0, X \vdash_{\text{FTFP}} \text{Design}(z_0, y_0)$ and $X \vdash_{\text{FTFP}} x_0 \leq z_0$. Consequently, $X \vdash_{\text{FTFP}} \text{Occ}(y_0, z_0)$ (by axiom (1.9)). However, axiom (1.16) yields that $X \vdash_{\text{FTFP}} \neg\text{Occ}(y_0, z_0)$.

8. Condition 2.18
Let

(*) $<\overline{\overline{x_1}},\overline{x}>$, $<\overline{\overline{x_2}},\overline{x}>\in$ *designof*^{X}, $\overline{\overline{x_1}} \in$ *design*$_0^X$, $\overline{\overline{x_2}} \in$ *Design*^{X},
(**) $<\overline{\overline{y_1}},\overline{y}>$, $<\overline{\overline{y_2}},\overline{y}>\in$ *designof*^{X}, $\overline{\overline{y_1}} \in$ *design*$_0^X$, $\overline{\overline{y_2}} \in$ *Design*^{X},
(***) $\overline{\overline{x_1}} <^X \overline{\overline{x_2}}, \overline{\overline{y_1}} <^X \overline{\overline{y_2}}$.

Given definition (4.9) (and axiom (1.18)) (*) entails (#) and (**) entails (##).
(#) $X \vdash_{\text{FTFP}} \text{design}_0(x_1, x), X \vdash_{\text{FTFP}} \text{Design}(x_2, x)$,
(##) $X \vdash_{\text{FTFP}} \text{design}_0(y_1, y), X \vdash_{\text{FTFP}} \text{Design}(y_2, y)$. Moreover Equation (4.8) (***) yields (###).
(###) $X \vdash_{\text{FTFP}} x_1 < x_2 \wedge y_1 < y_2$. (#), (##), and (###) together with axiom (1.17) entail that $X \vdash_{\text{FTFP}} x = y$, as required. □

Since the pair $< \mathfrak{U}^X, \mathfrak{f}^X >$ has been shown to be an FTFP-structure, I define in this structure the interpretation $\mathfrak{I}^X$ of FTFP along the lines of definition (2.19). In order to prove that every negation complete consistent set of formulas that contains witnesses is satisfiable I follow again the proof from [7]. This proof has two steps. First it is shown that condition (*) holds for atomic formulas, and then (*) is shown to hold for all formulas if X is negation-complete and contains witnesses.
(*) $\mathfrak{I}^X \models_{\text{FTFP}} \varphi \equiv X \vdash_{\text{FTFP}} \varphi$.

LEMMA 3 *If φ is an atomic formula of FTFP, then (*) holds.*

Proof. In FTFP there are three types of atomic (FTFP-specific) formulas:

1. Case $\varphi = \text{Occ}(\alpha, \beta)$
$\mathfrak{I}^X \models_{\text{FTFP}} \text{Occ}(\alpha, \beta) \equiv$ (because of Equation (2.19) and lemma 1) $< \overline{\alpha}, \overline{\overline{\beta}} >\in$ *occ*^{X} $\equiv$ (because of Equation (4.7)) X $\vdash_{\text{FTFP}} \text{Occ}(\alpha, \beta)$.
2. Case $\varphi = \alpha < \beta$
$\mathfrak{I}^X \models_{\text{FTFP}} \alpha < \beta \equiv$ (because of Equation (2.19) and lemma 1) $\equiv< \overline{\overline{\alpha}}, \overline{\overline{\beta}} >\in <^X$ $\equiv$ (because of Equation (4.8)) $X \vdash_{\text{FTFP}} \alpha < \beta$. □
3. Case $\varphi = \text{design}(\alpha, \beta)$
$\mathfrak{I}^X \models_{\text{FTFP}} \text{design}(\alpha, \beta) \equiv$ (because of Equation (2.19) and lemma 1) $< \overline{\overline{\alpha}}, \overline{\beta} >\in$ *designof*^{X} $\equiv$ (because of Equation (4.9)) $X \vdash_{\text{FTFP}}$ design (α, β).

The following three lemmas, which complete the proof of theorem 2, may be proven along the lines of the standard proof ([7]).

LEMMA 4. *If X is a consistent set of FTFP formulas that is also negation complete and contains witnesses, then condition (*) holds for all formulas of FTFP.*

LEMMA 5. *If X is a consistent set of FTFP formulas and the number of variables in the formulas from X is finite, then there is a consistent set of FTFP formulas which is a superset of X and which contains witnesses.*

LEMMA 6. *If X is a consistent set of FTFP formulas then there is a consistent set of FTFP formulas which is a superset of X and which is negation complete.* □

5. Comparisons

Artefacts are somewhat outside the mainstream of philosophical thinking ([6], p. 7–8). Consequently, there are very few conceptions which contain a definition of functional parthood or which at least characterise this notion in a more loose way. The author of this paper is aware of three accounts that may be compared with FTFP in this respect: the formal theory of artefacts constructed by Athanassios Tzouvaras ([28] and [29]), the theory of functional dependence proposed by Laure Vieu and Michel Aurnague ([31]), and the definition of functional parthood formulated by Ingvar Johansson ([13]). Other theories of functions, e.g., [6], [20], or [10], are not expressive enough to provide any informative formal characterisation of functional parthood.

5.1. *Formal Theory of Artefacts*

As a matter of fact, Tzouvaras presents two theories: a theory of (artefact) parts and a theory of significant (artefact) parts. From the formal point of view, both theories differ from FTFP in that they are expressed in a second-order language of set theory. The theory of artefact parts has two primitive constants: the binary predicate "$\mathfrak{F}$" and the binary function symbol "□". "$\mathfrak{F}(x,y)$" means: an artefact x fits (i.e., may be assembled with) an artefact y. The symbol "$x \Box y$" denotes the assembly of x and y (when x fits y). Axiom (5.1) describes the relation between these notions:

$$\mathfrak{F}(x,y) \equiv \exists z\, z = x \Box y. \tag{5.1}$$

The relation of parthood may be defined by means of these terms as the transitive closure of the relation of direct parthood. The latter relation is defined by Equation (5.2).

$$x <_0 y \equiv \exists z\, y = x \square z \tag{5.2}$$

The theory of significant (artefact) parts adds to the symbols of Tzouvaras' theory of parts one new predicate "S." The expression "$\mathrm{S}(x, y)$" means: x is a significant part of y.

It is not clear whether we should compare our relation $<$ with the transitive closure of $<_0$ or with S. In fact it is not even clear that the formal theory of artefacts is a theory of functional parthood. In any case, Tzouvaras' proposal does not explicitly refer to the notion of design. Thus, it is unclear what relation it bears to the actual design specifications. Apparently, the notion of design is somehow involved in the notion expressed by "$\mathfrak{F}$" (or the notion expressed by "$\square$") but the exact nature of this relation remains cryptic. For example, one may wonder why x fits y and does not fit, say, z. Moreover, some theses of the formal theory of artefacts diverge from engineering practise. For instance, one thesis has it that each artefact has exactly two direct parts, which claim is inconsistent with the actual part lists. One may also raise the objection that definition (5.2) is too broad. If x may be assembled with y, then it may not be the case that x actually is assembled with y, i.e., that there is a further artefact z such that x and y are parts of z. If x may be assembled with y, then there may exist a further artefact z such that x and y are parts of z, i.e., they may be parts of z.

From the philosophical point of view, it is important to notice that according to Tzouvaras both of his theories concern not the artefacts themselves but the states of artefacts. Needless to say, this assumption significantly hinders any attempt to test this theory against the real-world data, which are rarely rendered in terms of states of objects. Suppose that one wishes to verify the thesis to the effect that any artefact has exactly two direct parts. How can she proceed? The most obvious way is closed for her because she cannot check the actual lists of parts produced by engineers as these lists refer to such things as bolts, resistors, and pistons, i.e., to objects and not to states of objects.

5.2. *Theory of Functional Dependence*

Vieu and Aurnague ([31]) propounds a theory of the relation between components and integral wholes.[6] I assume that this relation is a counterpart of my $<$. Vieu and Aurnague define four kinds of the component–integral whole relations, but for our purposes it suffices if we consider just two of them. Both definitions of the component–integral whole relation are rendered in terms of the functional dependence. The theory of functional dependence is a first order modal theory with five primitive constants.[7] "$\mathrm{CF}(x,y,t)$" means here that (an entity) x is classified at time t as (a lexical type) y. "$\mathrm{F}(x,y,t)$" means that (an entity) x functions at time x as (a lexical type) y. "$t \subseteq t'$" means that a time region t is part of a time region t'. "$\mathrm{Obj}(x)$" means that x is a material object. Finally, "$\mathrm{PP}(x,y,t)$" means that (an entity) x is a mereological part of (an entity) y at time t. As we see, the crucial notion of this theory is the notion of lexical type. Although Vieu and Aurnague does not define it explicitly, the way in which they use it suggests that it corresponds to the philosophical notion of kind provided that we drop any (substantial) ontological constraints associated with the latter notion. More perspicuously speaking, a lexical type constituted by a linguistic description denotes a set of entities that satisfy this description.

First, we define the notion of generic functional dependence. Symbol "$\mathrm{GFD}(x,y)$" is to be read as: (a lexical type) x is generically functionally dependent on (a lexical type) y.

$$\begin{aligned}\mathrm{GFD}(x,y) \equiv \Box\forall z,t[&\mathrm{CF}(z,x,t) \wedge \mathrm{F}(z,x,t) \rightarrow \\ &\exists v(v \neq z \wedge \mathrm{CF}(v,y,t) \wedge \mathrm{F}(v,y,t))]\wedge \\ &\wedge \Diamond\exists z,t\,\mathrm{CF}(z,x,t) \wedge \neg\Box\forall t\exists z \mathrm{CF}(z,y,t).\end{aligned} \tag{5.3}$$

Then we define the notion of individual functional dependence.

$$\begin{aligned}\mathrm{IFD}(x,y,z,v,t) \equiv\ &\mathrm{GFD}(y,v) \wedge \mathrm{CF}(x,y,t) \wedge \mathrm{CF}(z,v,t)\wedge \\ &\forall t'[t' \subseteq t \wedge \mathrm{F}(x,y,t') \rightarrow \mathrm{F}(z,v,t')].\end{aligned} \tag{5.4}$$

The expression "$\mathrm{IFD}(x,y,z,v,t)$" means that (an entity) x as (a lexical type) y functionally depends on (an entity) z as (a lexical type) v at time t.

Having at our disposal the notion of individual functional dependence, we can define two relations of being a functional part:

$$\mathrm{FP}_1(x,y,z,v,t) \equiv \mathrm{Obj}(x) \wedge \mathrm{Obj}(y) \wedge \mathrm{PP}(x,y,t) \wedge \mathrm{IFD}(x,y,z,v,t). \tag{5.5}$$

$$\mathrm{FP}_2(x,y,z,v,t) \equiv \mathrm{Obj}(x) \wedge \mathrm{Obj}(y) \wedge \mathrm{PP}(x,y,t) \wedge \mathrm{IFD}(z,v,x,y,t). \tag{5.6}$$

From the formal point of view, the theory of functional dependence is more expensive than FTFP because the former, but not the latter, requires a modal logic as its prerequisite. As it stands, the former theory refers to any kind of functions, including biological functions. If, for the sake of comparison, we constrained it to the artefactual world, we could say that it refers both to artefact tokens and types. The theory defines the notion of functional parthood by means of the notion "function as." FTFP defines the notion of functional parthood by means of the notion "function in." Moreover, in contradistinction to FTFP, the term expressing the former notion is assumed a primitive constant. As in the case of the formal theory of artefacts, there is no direct reference to the notions used in engineering design. Consequently, it is not clear how we can assess the adequacy of the proposal in question. It is also not clear which definition we should employ when we describe artefact functions: (5.5) or (5.6). The last problem is not a contingent matter, but is a consequence of the theoretical decision to define the notion of functional parthood by means of the notion of functional dependence. In one sense a part of a whole is dependent on the whole; in the other sense the whole is dependent on its parts. Subsequently, if we say that x is a functional part of y provided that x is functionally dependent on y, we get two types of functional parthood, which consequence seems to be unsupported by ordinary parlance. Even if the locution "x is a functional part of y" is ambiguous, the ambiguity at stake has nothing to do with the fact that in one sense x functionally depends on y and in the other sense y functionally depends on x.

5.3. *Johansson's Definition of Functional Parthood*

The part of [13] which is relevant for our purposes contains the following informal definition of functional parthood. x is a *functional part of* y iff (a) y is a functional unity or integral object of some kind, (b) for some z, x makes something to happen to z, which is relevant for the fact that y is a functional unity or integral object of some kind, and (c) x is a spatial part of y.

The most acute problem with this definition is that [13] lacks any specific characterisation of the notions which appear in the definiendum. For instance, we do not know what it means that y is a functional unity. This may be a potentially serious drawback because one may doubt whether the definition is not circular. If you define the notion of functional unity by means of the notion of functional parthood, you cannot define the latter in terms of the former. Similarly, we do not know what it means that x makes something to happen to z, which is relevant for the fact that y is a functional unity or integral object of some kind. If x secretes a fluid which is harmful for the integrity of y, is x a functional part of y?

Presumably, all the notions involved in Johansson's definition are related somehow to the notion of design or some other notion used in engineering design, but it is not clear what kind of relationship is at stake here. If "being relevant for" has something to do with the notion of design, then the definition bears a closer resemblance to Equation (1.20) than it may appear at first sight. Moreover, Johansson's definition is similar to Equation (1.20) in that both definitions attempt to avoid any direct reference to functions while defining the relation of functional parthood. Finally, let me observe that Johansson does not make it clear whether his conception refers to artefact tokens or artefact types.

6. Further Work

Basically, there are three directions in which FTFP may be developed. First, one may require a stronger version of the theory of states of affairs. For instance, we may require that the relation $<$ be extensional. Another obvious extension consists in adding some more constraints on the relation of design which would express other rational aspects of designing processes. But the most important development would be to define the notion of artefact function itself, i.e., instead of saying that x performs in y some function, one might wish to say that x performs in y a function z.

FTFP is a philosophical theory of artefact functions which ought to be tested against the real-world problems. One method of evaluation is to compare its tenets with the actual engineering designs and the conceptual structures determined thereby. It is a well-known fact that engineering practice involves multiple parthood structures. The latter are usually called bills of materials. There are at least three kinds of bills of materials: engineering bills of materials, manufacturing bills of materials, and logistical bills of materials. An engineering bill of materials represents the abstract physical architecture of a given artefact. A manufacturing bill of materials represents the manufacturing scheme for constructing the artefact. Finally, a logistical bill of materials represent those entities that are required to maintain the artefact in a state of readiness ([27], p. 268–271). It goes without saying that it is the engineering bills of materials with which to compare FTFP. P. Simons and Ch. Dement claim that a bill of materials is typically acyclic graph that mirrors the mereological structure of an artefact (p. 266–267). However, [32] maintains that bills of materials are unstructured part lists. In the former case, the comparison would be more straightforward; the latter case would involve more complex investigation.

Notes

[1] [30] is an up-to-date survey paper which reviews a number of philosophical theories of artefact functions.

[2] Subsequently, I do not commit to a principle which is called by D. Dennett a default assumption of reverse engineering. According to Dennett, a reverse engineer must start

his analysis with the assumption that any detail of an artefact whose design he is to reconstruct is there for some reason, i.e., it performs some function in the artefact [5], p. 212.

[3] Strictly speaking, I employed in [9] the distinction between real and intentional states of affairs, which distinction goes back to the work of the Polish philosopher Roman Ingarden ([11] and [12]). Given Ingarden's distinction, artifact designs may be represented by intentional states of affairs.

[4] The semantics for FTFP follows the formal framework from [7].

[5] Symbol "|" denotes the relation of relative product of binary relations:

$$<x,y> \in R \mid S \equiv \exists z[<x,z> \in R \wedge <y,z> \in S] \tag{2.9}$$

Symbol "$^{-1}$" denotes the converse of a relation:

$$<x,y> \in R^{-1} \equiv <y,x> \in R \tag{2.10}$$

[6] The relation "(a component) x is a part of (an integral whole)" was one of the parthood relations distinguished in [33] as the sources of the ambiguity problem.

[7] The theory presupposes the S5 logic. Strictly speaking, the theory requires more primitives because it is an extension of the theory of roles from [18], where the number of primitive constants is much higher. Still, only five primitive terms occur in the definitions we need.

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