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THE LOGIC AND MEANING OF PLURALS. PART I

ABSTRACT. Contemporary accounts of logic and language cannot give proper treatments of plural constructions of natural languages. They assume that plural constructions are redundant devices used to abbreviate singular constructions. This paper and its sequel, "The logic and meaning of plurals, II", aim to develop an account of logic and language that acknowledges limitations of singular constructions and recognizes plural constructions as their peers. To do so, the papers present natural accounts of the logic and meaning of plural constructions that result from the view that plural constructions are, by and large, devices for talking about many things (as such). The account of logic presented in the papers surpasses contemporary Fregean accounts in its scope. This extension of the scope of logic results from extending the range of languages that logic can directly relate to. Underlying the view of language that makes room for this is a perspective on reality that locates in the world what plural constructions can relate to. The papers suggest that reflections on plural constructions point to a broader framework for understanding logic, language, and reality that can replace the contemporary Fregean framework as this has replaced its Aristotelian ancestor.

KEY WORDS: aggregate, irreducibility of plurals, logic, natural language, plural, regimentation of plurals, second-order logic, semantics, set, singular, the many, the one

1. INTRODUCTION

Study of logic dates back to Aristotle. His theory of logic, the logic of syllogism,[1] was not surpassed until the late 19th century, when modern logicians developed powerful theories of logic that eclipsed it. The key figure among them was Gottlob Frege, who "invented logic in the modern form" as Benson Mates (1972, p. 227) puts it. Frege devised a system of logic that has much wider scope than its predecessors, and it has become the archetype of contemporary systems. The breakthrough in logic is due to Frege's analysis of language that led to the development of rich regimented languages, which his system of logic can relate to. The superiority of the system to its predecessors results from the richness of the languages developed by Frege. Aristotle's system and its descendants can relate only to categorical sentences, such as "Every circle is a figure", but the categorical sentences amount to just a small fragment of Frege's languages. But the languages have conspicuous limitations in comparison with natural languages. This leads to limitations of Frege's system that its contemporary descendants cannot overcome.

The Fregean systems, Frege's system and its descendants, cannot deal with the logic of the *plural constructions* (in short, *plurals*) of natural languages, such as the following:

[P1] *Venus and Serena* are tennis players, and *they* won a U.S. Open doubles title.
[P2] *Venus and Serena* are *the females who won a Wimbledon doubles title in 2000*.
[P3] **There are some** tennis players who won a U.S. Open doubles title.
[P4] *The females who won a Wimbledon doubles title in 2000* won a U.S. Open doubles title.

We cannot apply the systems to natural language sentences without paraphrasing them into Frege's languages (or their minor variants), which the systems relate to. But the languages have no adequate paraphrases of many plural constructions. They are *singular languages*, languages with no counterparts of natural language plurals,[2] because they result from refining *singular constructions* (in short, singulars) of natural languages, such as the following:

[S1] *Serena* is a tennis player, and *she* won a U.S. Open singles title.
[S2] *Serena* is *the female who won a Wimbledon singles title in 2002*.
[S3] **There is** a tennis player who won a U.S. Open singles title.
[S4] *The female who won a Wimbledon singles title in 2002* won a U.S. Open singles title.

But plurals are as prevalent in natural languages as singulars, and they are as much subject to conspicuous logical relations. We can see that [P1] logically implies [P3] just as we can see that [S1] logically implies [S3]. Similarly, we can see that [P1] and [P2] logically imply [P4] as we can see that [S1] and [S2] logically imply [S4]. We can also see that [P3] does not logically imply [P1] just as [S3] does not logically imply [S1]. And we can see logical relations that crisscross singulars and plurals. We can see that "Venus and Serena are tennis players" logically implies "Venus is a tennis player", but that "Venus and Serena won a Wimbledon doubles title" does not logically imply "Venus won a Wimbledon doubles title." Similarly, we can see that "There are some tennis players who are female" logically implies "There is a tennis player who is female", but that [P3] does not logically imply "There is a tennis player who won a U.S. Open doubles title."

To explain the logic of plurals,[3] we need a system of logic, a post-Fregean system, that has wider scope than the Fregean systems. To formulate such a system, it is necessary to analyze plurals as Frege has analyzed their singular cousins to formulate his system of logic.

We can analyze the singular constructions [S1]–[S4] as consisting of expressions of three kinds:

[a] *Singular Terms*: the proper name "Serena", the pronoun "she", and the definite description "the female who won a Wimbledon singles title in 2002";
[b] *Predicate Phrases of the Singular Form*: "is a tennis player", "won a U.S. Open singles title" (or "won"), and "is";
[c] *Singular Quantifier Phrases*: "there is" (or "there is a").

By refining these expressions, we can obtain the key expressions of *elementary languages*, the basic Fregean languages that form the cores of the other Fregean languages:

[a*] *Singular Terms* (e.g., the singular variable "x");
[b*] *Singular Predicates* (e.g., "wins" or, in symbols, "W");
[c*] *Singular Quantifiers* (e.g., "$\exists$").[4]

So we can paraphrase [S1]–[S4] into elementary languages, and use *elementary logic*, the core of the Fregean systems that relates directly to elementary languages, to explain their logical relations.

Similarly, we can analyze the plural constructions [P1]–[P4] as consisting of plural cousins of [a]–[c]:

[A] *Plural Terms*: the conjunctive term "Venus and Serena", the pronoun "they", and the definite description "the females who won a Wimbledon doubles title in 2000";
[B] *Predicate Phrases of the Plural Form*: "are tennis players", "won a U.S. Open doubles title" (or "won"), and "are";
[C] *Plural Quantifier Phrases*: "there are" (or "there are some").

There are no counterparts of these expressions in elementary languages or other Fregean languages (e.g., the higher-order extensions of elementary languages[5]). But we can augment the languages with the expressions that result from refining [A]–[C] as we refine their singular cousins to obtain [a*]–[c*]:

[A*] *Plural Terms* (e.g., the plural variable "*xs*");
[B*] *Plural Predicates* (e.g., "**win**" or, in symbols, "$\mathbf{W}_{\mathrm{i}}$");
[C*] *Plural Quantifiers* (e.g., "$\boldsymbol{\Sigma}$").[6]

Adding these expressions to elementary languages yields the regimented languages that I call (*first-order*) *plural languages*. Plural languages contain natural paraphrases of basic plural constructions (e.g., [P1]–[P4]), as elementary languages contain natural paraphrases of their singular cousins (e.g., [S1]–[S4]). So we can explain the logic of the plural constructions using a system of logic that characterizes the logical relations among plural language sentences. To do so, I formulate a system of logic that I call (*first-order*) *plural logic*.[7] Plural logic extends elementary logic to plural languages.[8]

The account of the logic of plurals sketched above results from my conception of the function of plurals. I think that plurals are potent devices that complement their singular cousins. They are developed first and foremost as devices for talking about *many things*[9] (as such[10]). Their singular cousins, by contrast, are devices for talking about *one thing* (at a time). Using purely singular constructions, we can make statements that pertain specifically to some one thing (e.g., Venus), and make generalizations thereof. We can say what the thing in question is, what it is like, what it does, and so on, and we can say what everything is like, whether there is something of a certain kind, and so forth. When we say "Venus won, but Serena didn't" or "Venus beat Serena", we talk about many things (viz. two humans), but only in the sense that we attribute something to *each* one of them or say something that relates one of them to another. Plurals (e.g., the pronoun "they") work quite differently. They can relate to many things *without setting each one of them apart from the others*. We can use them to talk about two humans, for example, without separating them. We can say that *they* are two tennis players, or that *they* won a U.S. Open doubles title. So, I think, plurals enable us to say more than what we can say using only singulars.

Contemporary accounts of plurals deny this. They rest on a view shared by Aristotle and Frege: we can say all there is to say using only singulars. This view results from the conception of plurals as redundant devices that serve only the purpose of abbreviating singular constructions. On this traditional conception, plurals must have singular equivalents, singular constructions that they abbreviate and, thus, are logically equivalent to. Those who are committed to the conception see no need to expand the Fregean languages. They hold that the singular character of the languages does not diminish their expressive power, and conclude that the only thing to do to explain the logic and meaning of plurals is to formulate a suitable scheme for paraphrasing them into the languages. So contemporary accounts of plurals rest on various schemes for paraphrasing plurals into singular languages.[11] I think that this longstanding and prevalent approach is ruled out by the logic of plurals. Robust logical relations that pertain to plurals, I hold, show that many plural constructions have no singular equivalents.

What I aim to do in this paper and its sequel, Yi (LMP II), is to develop an account of logic and language that acknowledges limitations of singulars and recognizes plurals as their peers. In the next section, I motivate my new approach to plurals by arguing that the traditional approach cannot be carried through. To do so, I clarify robust logical relations that hold among plurals and argue that plurals are not reducible to singulars. In, Section 3, the last section of this paper, I take the first step in presenting the non-reductive account of plurals that results from the conception of plurals as devices for talking about the many. I regiment basic plural constructions of natural languages as Frege regimented their singular cousins, and present their refinements, which can complement the elementary language refinements of the singular cousins. In the sequel to this paper, I continue to present the non-reductive account of plurals. In its first section, Section 4, I present languages, (first-order) plural languages, that result from adding the refinements of plural constructions to elementary languages. In Section 5, I give the semantics of plurals and characterize the truth and falsity of plural language sentences. In Section 6 and Section 7, I account for the logic of plurals by characterizing the logical relations among plural language sentences. In Section 8, I conclude by highlighting the main ideas running through the previous sections.

2. Singular Languages and the Irreducibility of Plurals

Contemporary accounts of plurals take the reductionist approach. They try to explain the logic and meaning of plurals by paraphrasing them into the standard regimented languages, contemporary descendants of Frege's languages. This approach rests on the traditional view that plural constructions of natural languages are mere abbreviation devices. In this section, I examine this view to challenge the viability of the reductionist approach. By considering robust logical relations that pertain to plurals, I argue that some plural constructions have no singular equivalents. In the next sections, I pursue an anti-reductionist approach driven by the view that plurals are potent devices on a par with their singular cousins.

2.1. *Elementary Languages and Their Higher-Order Extensions*

To examine whether plurals are reducible to singulars, it is necessary to have an overview of the range of singular constructions. To do so, it is useful to consider expressions available in the standard regimented languages because they result from refining singular fragments of natural languages.

We can distinguish the standard regimented languages into two kinds: elementary languages, and the higher-order languages built on them. Elementary languages can be taken to contain five kinds of primitive expressions:

[a] *Singular Constants*
"*Academica*" (or, in symbols, "a"), "Bob" (or "b"), "Bill" (or "b_i"), "Carol" (or "c"), "Cicero" (or "c_i"), "Ezra" (or "e"), Frege (or "f"), "Genie" (or "g"), "John" (or "j"), "London" (or "l"), "*Principia mathematica*" (or "p"), "Russell" (or "r"), "Thomas" (or "t"), "Whitehead" (or "w"), etc.
[b] *Singular Variables*
"x", "y", "z", etc.
[c] *Predicates*

[i] *One-place predicates*
"is-a-child" (or "C"), "is-a-critic" (or "C_r"), "is-healthy" (or "H"), "is-a-philosopher" (or "P"), "is-a-piano" (or "P_i"), etc.
[ii] *Two-place predicates*
"is-identical-with" (or "="), "admires" (or "A"), "is-an-author-of" (or "A_u"), "lifts" (or "L"), "is-a-member-of" (or "ε"), "is-offspring-of" (or "O"), "is-a-part-of" (or "$\leq$"), "resides-in" (or "R"), "works-with" (or "W"), etc.
[iii] *Three-place predicates*
"gives . . . to" (or "G"), etc.
Etc.

[d] *Truth-functional Sentential Connectives*
The conjunction symbol "and" (or "$\wedge$"), the negation symbol "It-is-not-the-case-that" (or "$\sim$"), etc.
[e] *Standard Quantifiers*
The singular existential quantifier "There-is-something . . . such-that" (or "$\exists$")
The singular universal quantifier "Anything . . . is-such-that" (or "$\forall$")

We can get different elementary languages by choosing different groups of singular constants or predicates. Sentences of an elementary language are constructed from its primitive expressions in the familiar way.[12] An n-place predicate combines with n occurrences of singular terms (viz. singular constants or variables) to yield an atomic sentence: "x is-a-child" (or "C(x)"), "John admires y" (or "A(j, y)"), etc. A standard quantifier (e.g., "There-is-something . . . such-that") combines with a variable (e.g., "x") and a sentence (e.g., "C(x)") to yield a complex sentence: the existential generalization "*There-is-something x such-that x* is-a-child" (or

"$\exists x\mathrm{C}(x)$"), etc. Finally, a sentential connective combines with a suitable number of occurrences of sentences to form a complex sentence: the negation "*It-is-not-the-case-that* x is-a-child" (or "$\sim\mathrm{C}(x)$"), the conjunction "[x is-a-child *and* John admires y]" (or "[$\mathrm{C}(x) \wedge \mathrm{A}(\mathrm{j}, y)$]"), etc.[13]

We can get *higher-order languages* built on elementary languages by adding higher-order predicates, variables, or quantifiers. Higher-order predicates are predicates with argument places that *can be filled with* (in short, *admit*) other predicates; higher-order variables are variables that can replace predicates; and higher-order quantifiers are quantifiers that can bind higher-order variables. Higher-order expressions contrast with their cousins in elementary languages. Predicates of elementary languages have no argument place that admits other predicates (their argument places admit only terms); their variables cannot replace predicates (they can replace only terms); and their quantifiers cannot bind higher-order variables (they can bind only variables that can replace terms). Such predicates, variables, or quantifiers are called *first-order expressions*. Accordingly, regimented languages that, like elementary languages, contain only first-order expressions are called *first-order languages*. So the higher-order expressions at the lowest level are *second-order* expressions. Second-order predicates have at least one argument place that admits first-order predicates, but no argument place that admits higher-order predicates; second-order variables (e.g., the monadic "X") can replace first-order predicates; and second-order quantifiers (e.g., the existential "$\exists$") can bind second-order variables.

I call regimented languages that contain second-order expressions of any kind (but no expressions at a higher-level) *second-order languages*. This might seem to deviate from the usual definition of second-order language. The usual definition appears to exclude languages that contain *second-order predicates*.[14] The main reason for this, I think, is that the definition is formulated with regard only to languages without higher-order predicates (i.e., 'predicate constants' as distinguished from higher-order variables). To examine whether plurals are reducible to singulars, however, it is necessary to consider languages that contain second-order predicates as well as second-order quantifiers and variables.

It is important to distinguish such languages from the usual second-order languages, which contain no second-order predicates. Second-order variables are not comparable to second-order predicates, but to first-order predicates.[15] They can take terms to form a predication (i.e., atomic sentence), and the resulting sentences are *first-order* predications. For example, the monadic second-order variable "X" can take the singular term "j" to form the predication "$X(\mathrm{j})$", as the first-order predicate "C" can take the same term to form the predication "$\mathrm{C}(\mathrm{j})$". Second-order predications require second-order predicates (or their peers, *third-order* variables). So the usual second-order languages have no second-order predications. This leads to an important disparity between their logic and the logic of the richer second-order languages that contain second-order predicates: the principle of substitutivity of extensionally equivalent predicates holds, by logic, for the former languages, but not for the latter.[16] To distinguish the two kinds of second-order languages, I call the former *meager second-order languages*, and the latter *plenary second-order languages*.

We can see that the standard regimented languages are singular languages. Expressions of elementary languages amount to components of purely singular constructions of natural languages: singular proper names, predicates of singular form (used in purely singular constructions[17]), singular pronouns (used anaphorically), and quantifiers that singular pronouns can take as antecedents. Higher-order languages built on elementary languages do not add counterparts of natural language plurals. The higher-order expressions in the languages are no more refinements of plurals than the predicate phrase "is Russell or Whitehead" or its analogues in regimented languages are refinements of the plural term "Russell and Whitehead".[18]

2.2. *Plurals and Elementary Languages*

Expressions of elementary languages result from refining singular constructions of natural languages. So it is straightforward to paraphrase into the languages basic[19] singular constructions of natural languages, such as the following:

[1] John is a child who is healthy.
[2] A child is healthy (i.e., there is a child who is healthy).

These can be paraphrased into elementary languages that contain the predicates "is-a-child" and "is-healthy" (i.e., elementary language counterparts of the English predicate phrases "is a child" and "is healthy") as follows:

[1a] John is-a-child and John is-healthy (in symbols, $[\mathrm{C(j)} \wedge \mathrm{H(j)}]$).
[2a] There-is-something x such-that [x is-a-child and x is-healthy] (in symbols, $\exists x[\mathrm{C}(x) \wedge \mathrm{H}(x)]$).

So elementary logic can yield adequate treatment of logical relations among basic singular constructions of natural languages. [1a] logically implies [2a], on elementary logic, and the logical relation between them can be transferred to the English sentences paraphrased by them.

The situation is quite different with plurals. Consider some basic plural constructions:

[3] John and Carol are children who lift Bob.
[4] Some children lift Bob.
[5] John and Carol lift Bob.
[6] There are some things that lift Bob.

(In the sentences discussed in this paper and its sequel, "Bob" is used as the name of a specific piano that is so huge that it is impossible for any one person to lift it.) We can see that [3] and [5] logically imply [4] and [6], respectively, just as we can see that [1] logically implies [2]. Still, we cannot use elementary logic to explain the logical relations among those plural constructions, because they cannot be paraphrased into elementary languages. Elementary languages have no counterparts of their components, such as the plural term "John and Carol", the plural predicate "*to* lift",[20] or the plural quantifier "some".

It has not been clearly and widely recognized that this impoverishes elementary languages. A sizable range of plurals have conspicuous logical relations to singulars, and this has led to the prevalence of implicit use of partial schemes for paraphrasing plurals away. But the schemes have unmistakable limitations. They cannot cope with such basic plural constructions as [3]–[6], because they are not logically equivalent to their singular cousins.[21]

Consider the following plural constructions:

[7] John and Carol are children.
[8] Some children are healthy.

We can intuitively recognize that they are logically equivalent to their singular cousins:

[9] John is a child and Carol is a child.
[2] A child is healthy.

And these can be paraphrased into elementary languages as follows:

[9a] John is-a-child *and* Carol is-a-child (in symbols, $[\mathrm{C}(\mathrm{j}) \wedge \mathrm{C}(\mathrm{c})]$).
[2a] There-is-something x such-that [x is-a-child and x is-healthy] (in symbols, $\exists x[\mathrm{C}(x) \wedge \mathrm{H}(x)]$).

So it is usual to take these to paraphrase [7] and [8] as well. But we cannot apply the scheme implicit in the usual practice to [3]–[6]. Applying the scheme to [5] would yield "$[\mathrm{L}(\mathrm{j}, \mathrm{b}) \wedge \mathrm{L}(\mathrm{c}, \mathrm{b})]$", which paraphrases the following into elementary languages:

[10] John lifts Bob and Carol lifts Bob.

But [5] and [10] cannot be paraphrased by the same sentence. They are not logically equivalent. Suppose that John and Carol cooperate to lift Bob (which neither of them can lift alone). In that case, [5] would be true but [10] false. Similarly, one cannot use the scheme that renders [8] to [2a] to paraphrase [6], because [6] is not logically equivalent to its singular cousin:

[6a] There is something that lifts Bob.

Some might object that [5] is logically equivalent to [10] (and [6] to [6a]) because "John lifts Bob", for example, is interchangeable with "John lifts Bob *by himself or together with others*." The verb "*to* lift" may sometimes be used in this way. But it can also be used differently, as in "John cannot lift a piano upstairs, but he can do that with others" or "John cannot lift a piano upstairs, but he and Carol can do that" (these sentences can be used to make true statements). It can be used in such a way that "*John and Carol* lifted Bob upstairs", for example, is true if the two children collaborated to lift Bob upstairs (so that neither can be said to have lifted it alone) while the achievement does not make "*John* lifted Bob upstairs" true. So let it be understood that the verb is used in this way in [3]–[6] and other sentences discussed below.[22] Some might still assimilate [5] to [10] by failing to distinguish [5] from the following:

[10a] *Both* John *and* Carol lift Bob.

[10a] is logically equivalent to [10], but this does not mean that [5] is so as well. For "John *and* Carol" and "*both* John *and* Carol" are not always interchangeable. Compare "John and Carol are two children" and "Both John and Carol are two children." The latter cannot be true, but the former might be (it can be used to make a true statement if John is not Carol). Similarly, [5], as usually understood, is not interchangeable with [10a]. [5] may sometimes be used as a shorthand for [10a], but this does not mean that it cannot be used in the way described above.

Now, some might propose other schemes for paraphrasing plurals into elementary languages. One might hold that we can paraphrase [5] into elementary languages using a three-place predicate, a predicate that can yield a sentence by combining with three singular terms (e.g., "John", "Carol", and "Bob").[23] This view cannot accommodate the obvious connections that [5] has to such sentences as the following:

John, Carol, and Bill *lift* Bob.
John, Carol, Bill, and Hillary *lift* Bob.
John, Carol, Bill, Hillary, and Chelsea *lift* Bob.

It should be clear that all these sentences (including [5]) have the same predicate (i.e., the italicized). This conflicts with the proposed scheme of paraphrase. The proponents of the scheme must hold that the sentences contain three-, four-, five-, and six-place predicates, respectively. They might reply that the sentences indeed fails to have the same predicate or, more plausibly, that correct paraphrases need not preserve *syntactic* relations among the sentences paraphrased by them. But the scheme that uses a sequence of homonymous predicates to paraphrase the English sentences mentioned above cannot preserve important *logical* relations that pertain to them. Note, for example, that they all logically imply [6], which is not a logical truth. One cannot do justice to this without recognizing a common predicate in the sentences.[24]

Let me now examine the standard view among advocates of elementary languages. On this view, the recalcitrant plurals can be paraphrased using singular constructions that pertain to composite objects of some kind: sets or classes; aggregates, fusions, or mereological sums; or the so-called totalities, pluralities, or plural objects.[25] Many of those who hold the view think that the plural term "John and Carol" in [5], for example, is tantamount to the singular term "the set {John, Carol}", which refers to the set that consists of John and Carol,[26] and paraphrase [5] as follows:

[5a] The set {John, Carol} lifts Bob (in symbols, $\exists x[\forall y(y\ \varepsilon\ x \leftrightarrow y = \mathrm{j} \vee y = \mathrm{c}) \wedge \mathrm{L}(x, \mathrm{b})]$).

This logically implies the following:

[11] There is something of which both John and Carol are members (in symbols, $\exists x[\mathrm{j}\ \varepsilon\ x \wedge \mathrm{c}\ \varepsilon\ x]$).

So the standard view yields the conclusion that [5] logically implies [11]. This, I think, is clearly counterintuitive.

We can confirm the correctness of the intuition by clarifying its grounds. Compare [5] with the following:

[12] John and Carol *are two offspring of* Ezra.

We can get this sentence from [5] by replacing "carry" and "Bob" with "are two offspring of" and "Ezra", respectively. So if [5] logically implies [11], [12] must also do so. But we can see that [12] does not logically imply [11]; "John is offspring of Ezra, Carol is offspring of Ezra, and John is not Carol" logically implies [12], but not [11].

Some who hold the standard view might argue that plural quantifications yield a stronger case for the view than plural predications. But one cannot hold that the plural quantification [6], for example, logically implies the existence of a composite object while granting that the plural predication [5] does not. For [5] logically implies [6].

It might be useful to elaborate how this point applies to the plural quantification, called the *Geach–Kaplan sentence*, that has been the focus of much of recent discussion on plurals:

[13] There are some critics who admire only one another.

David Kaplan has proved that [13] cannot be paraphrased into elementary languages,[27] assuming that it can be paraphrased into meager second-order languages as follows:

[14] $\exists X[\exists y X(y) \wedge \forall y(X(y) \rightarrow [\mathrm{C_r}(y) \wedge \forall z(\mathrm{A}(y, z) \rightarrow y \neq z \wedge X(z))])]$.[28]

In response to this result, Quine holds that [13] can be paraphrased by the elementary language analogue of [14] that involves first-order quantification over sets (or the like) of critics:

[14a] $\exists x[\exists y\, y \,\varepsilon\, x \wedge \forall y(y \,\varepsilon\, x \rightarrow [\mathrm{C_r}(y) \wedge \forall z(\mathrm{A}(y, z) \rightarrow y \neq z \wedge z \,\varepsilon\, x)])]$.

This is a straightforward paraphrase of a singular construction that logically implies the existence of a set (or something else of its sort):

[14b] There is something (e.g., a set) every member of which is a critic and admires only its other members.

So Quine concludes that those who assert [13] commit themselves to the existence of sets because what Kaplan's proof shows, he holds, is that [13] cannot be paraphrased without quantifying over sets (or the like).[29] But it is wrong to paraphrase [13] by [14a], because [13] and [14b] are not logically equivalent. To see this, consider the following sentences:

[15] Ezra and Thomas are critics who admire only each other.
[16] Ezra is a critic, Thomas is a critic, Ezra is not identical with Thomas, Ezra admires only Thomas, and Thomas admires only Ezra.

[16] logically implies [15], and [15] does [13]. But [14b] is not logically implied by [16]; [14a] is not logically implied by the straightforward paraphrase of [16] into elementary languages:

[16a] $[\mathrm{C_r(e)} \wedge \mathrm{C_r(t)}] \wedge \mathrm{e} \neq \mathrm{t} \wedge [\forall z(\mathrm{A(e,} z) \rightarrow z = \mathrm{t}) \wedge \forall z(\mathrm{A(t,} z) \rightarrow z = \mathrm{e})]$.

Defenders of the standard view might hold that the above objection applies only to schemes that invoke abstract objects (e.g., sets) to paraphrase plurals, and propose alternative schemes that invoke only concrete objects (e.g., aggregates).[30] But the objection does not assume that the singular sentence taken to paraphrase [13] invokes abstract objects. It applies to any view on which [13] implies the existence of a composite object (abstract or not) that cannot be considered a critic. Consider schemes that, invoking aggregates, render [13] to the following:[31]

[14c] $\exists x \forall y(y \leq x \rightarrow [\mathrm{C_r}(y) \wedge \forall z(\mathrm{A}(y, z) \rightarrow y \neq z \wedge z \leq x)])$
[14d] $\exists x \forall y(y \prec x \rightarrow [\mathrm{C_r}(y) \wedge \forall z(\mathrm{A}(y, z) \rightarrow y \neq z \wedge z \prec x)])$

where "$\leq$" amounts to the predicate "is a part of" that indicates[32] the relation between parts (e.g., Ezra) and wholes (e.g., the aggregate of Ezra and Thomas), and "$\prec$" to "is an *atomic* part of".[33] These cannot be taken to paraphrase [13], either, because [16a] does not logically imply them.

Some might reply that one can use truths about composites (e.g., the following) to derive [14a] or [14d] from [16a]:

[17] $\forall x \forall y \exists z\ z = \{x, y\}$ (viz. $\forall x \forall y \exists z \forall u[u\ \varepsilon\ z \leftrightarrow u = x \vee u = y]$).
[18] $\forall x \forall y \exists z\ z = [x \oplus y]$ (viz. $\forall x \forall y \exists z \forall u[x \leq u \wedge y \leq u \rightarrow u \leq z]$).

This is beside the point. I assume that [17] is true,[34] but this does not help one to defend paraphrasing [13] by [14a]. The defense requires that [17] be a *logical* truth, whereas set theory (on the standard conception) is not a theory of logic. Similarly, invoking mereology, of which [18] is a theorem,[35] does not yield a defense of paraphrasing [13] by [14d]. It is one thing to accept mereology, quite another to hold that its theorems are logical truths.[36]

What if defenders of the standard view insist that some of the so-called proper theorems of set theory, mereology, or the like are logical truths? They might hold that elementary logic is too weak even for elementary languages: [16a] *logically* implies the existence of a composite object, because [17], for example, is a *logical* truth.[37] But beefing up elementary logic cannot make up for the poverty of elementary languages. The standard view conflicts directly with basic logical relations that pertain to plurals.

To see this, consider, for example, "Russell and Whitehead" (or "the authors of *PM*"[38]). Suppose that there is an object, a composite, that the plural term refers to, and let "Genie" be a singular term that refers to that object (one may take "Genie" to abbreviate "the set {Russell, Whitehead}", "the aggregate of Russell and Whitehead", or the like). Then the following sentences, [19] and [20], must have the same truth value, because [19] results from replacing "Genie" in [20] with "Russell and Whitehead":

[19] Genie is one of *Russell and Whitehead*.
[20] Genie is one of *Genie*.

But they do not have the same truth value. [20] is true (for Genie is Genie); [19] is false, because it is logically equivalent to the following sentence (which is false because Genie is neither Russell nor Whitehead):

[21] Genie is Russell or Genie is Whitehead.

So there can be no object that the plural term "Russell and Whitehead" refers to.[39]

Defenders of the standard view might object that [20] is ungrammatical (or false) because the predicate "is one of" must be followed by a plural term to form a grammatical sentence (or to be true). But it is straightforward to reformulate the argument to circumvent this objection. The following two sentences result from each other by replacing the italicized terms:

[19a] Genie is one of Frege and *Russell and Whitehead*.[40]
[20a] Genie is one of Frege and *Genie*.

But [20a] is true whereas [19a] is false. [20a] is logically equivalent to "Genie is Frege or Genie is Genie"; and [19a] to the following:

[21a] Genie is Frege or Genie is one of Russell and Whitehead.

This is logically equivalent to the following singular construction that we can clearly see is false (for Genie is neither Frege, nor Russell, nor Whitehead):

[21b] Genie is Frege or Genie is Russell or Whitehead.

Notice that the above argument appeals to logical relations pertaining to plurals that we can intuitively recognize, such as the logical equivalence between [19] and [21], or that between [19a] and [21a]. The logical relations can be captured by the following two schemas:[41]

[22] Something is one of [S_1 and S_2] if and only if it is S_1 or it is S_2.
[23] Something is one of [S and T] if and only if it is S or it is one of T.

We can see that instances of these schemas that result from replacing "S", "S_1", and "S_2" with singular terms, and "T" with a plural term are logical truths. This, we have seen, foils the standard view on plurals.[42]

2.3. *Plurals and Higher-Order Languages*

Most of those who recognize the poverty of elementary languages in coping with plurals resort to their higher-order extensions. They reject the standard view on plurals held by advocates of elementary languages, but retain the view of plurals as redundancies. To do so, they propose an analysis of plurals that can be seen to lie under the standard view: plural terms amount to predicates. On their view, "Russell and Whitehead" (or "the authors of *PM*") is a device for abbreviating the complex predicate "is Russell or Whitehead" (or "is an author of *PM*") that indicates a property, such as *being Russell or Whitehead* (or *being an author of PM*). The standard view results from this view by replacing the property with a composite object meant to encapsulate it, and the predicate with a singular term that involves it. Advocates of higher-order languages might retain the original view of plurals as predicates by rejecting this move, which collapses those languages to elementary languages by assimilating properties to objects. Michael Dummett, who argues that "a plural subject of predication . . . cannot stand for any . . . composite object", holds that "a plural noun phrase . . . under a correct analysis . . . is seen to figure predicatively" (1991, p. 93).[43] And Barry Schein holds bluntly that "[a] plural term . . . is a predicate" (1993, p. 4),[44] a view that he attributes to George Boolos, whose works on plural quantification have done much to loosen the grip of the standard view.[45] On their view, plural constructions (e.g., a plural predication, which involves a plural term as a subject of predication) can be analyzed as higher-order constructions (e.g., a second-order predication, which involves a predicate as a subject of predication). If so, plurals can be paraphrased into higher-order languages. So they might think that one can retain the view of plurals as redundant devices because the predicates corresponding to plural terms (e.g., "is Russell or Whitehead" or "is an author of *PM*"), it might seem, can be taken to combine only with singular terms.[46]

I think that the view of plurals as predicates is better than the standard view. Proponents of the view can meet the objections raised above against the standard view. This, however, does not mean that their view can be squared with the logic of plurals. There remain basic logical relations among plurals that one cannot accommodate by analyzing them as predicates, as we shall see.

Consider a simple plural construction, such as the following:

[19] Genie *is one of* Russell and Whitehead.

On the view of plurals as predicates, one can regard the predicate phrase "is one of" in [19] as a particle that merely signals predication[47] and paraphrase the sentence using a complex predicate amounting to the plural term "Russell and Whitehead". Using the complex predicate "is Russell or Whitehead" or, more precisely, its refinement "⟨is Russell or Whitehead⟩" in regimented languages, one might paraphrase [19] as follows:

[19b] ⟨is Russell or Whitehead⟩(g).[48]

And one might paraphrase some plural quantifications using second-order quantifiers and variables. One might paraphrase [13], for example, as follows:

[14] $\exists X[\exists y X(y) \wedge \forall y(X(y) \rightarrow [\mathrm{C_r}(y) \wedge \forall z(\mathrm{A}(y, z) \rightarrow y \neq z \wedge X(z))])]$.

Given such quantifiers and variables, moreover, we can do without complex predicates. [19b], for example, is logically equivalent to the following:

[19c] $\exists X[\forall x(X(x) \leftrightarrow [x = \mathrm{r} \vee x = \mathrm{w}]) \wedge X(\mathrm{g})]$.

So one might take this sentence to paraphrase [19] into meager second-order languages. But the languages do not have paraphrases of many basic plural constructions, such as the following:[49]

[5] John and Carol *lift* Bob.
[6] There are some things that *lift* Bob.

To apply higher-order schemes of paraphrase to these sentences, it is necessary to use *second-order predicates*, predicates with argument places that admit first-order predicates, such as "is-a-child" or "⟨is John or Carol⟩".[50] Using "*lift*" as the second-order analogue of the predicate "*to* lift", one might paraphrase [5] and [6] into plenary second-order languages as follows:[51]

[5b] $\textit{LIFT}(\langle\text{is John or Carol}\rangle, \mathrm{b})$.
[6b] $\exists X\ \textit{LIFT}(X, \mathrm{b})$.

We can now see how proponents of the higher-order scheme presented above can meet my objections to the standard view. To do so, it is necessary to draw a sharp distinction between second-order sentences (e.g., [5b] or [14]) and their elementary language analogues (e.g., [5a] or [14a]), and between predicable entities (e.g., properties) and non-predicable entities (in short, objects).[52] Those who do so can hold that [5b], unlike [5a], does not logically imply the existence of any *object* (simple or composite) except John, Carol, and Bob, although it implies the existence of a *property*, e.g., the property of *being John or Carol*, a predicable entity indicated by the predicate "is John or Carol" (or "⟨is John or Carol⟩").[53] Similarly, they can hold that [6b] or [14] states the existence of a property while denying that it logically implies the existence of an *object* encapsulating the property.

This defense of the scheme brings to the fore a serious problem, because different properties can be instantiated by the same objects. To see this, consider:

[24] Russell and Whitehead cooperate.
[25] Russell and Whitehead are the authors of *PM*.
[26] The authors of *PM* cooperate.

Clearly, [24] and [25] logically imply [26]. Consider, however, the second-order sentences that result from applying the scheme to [24]–[26]:

[24a] $\textit{COOPERATE}(\langle\text{is Russell or Whitehead}\rangle)$.[54]
[25a] $\forall x[\langle\text{is Russell or Whitehead}\rangle(x) \leftrightarrow \langle\text{is-an-author-of } \textit{PM}\rangle(x)]$.
[26a] $\textit{COOPERATE}(\langle\text{is-an-author-of } \textit{PM}\rangle)$.

[24a] and [25a] fail to logically imply [26a]. The two complex predicates in [25a] are coextensive (i.e., [25a] is true), but they do not indicate the same property: *being Russell or Whitehead* is not the same property as *being an author of PM*. So one of the properties may instantiate a second-order property (e.g., what "*COOPERATE*" indicates) that the other does not.

Some might attempt to avoid this problem by proposing a variant of the straightforward higher-order scheme discussed above. One can preserve the logical relations among [24]–[26] using a scheme that renders [24] and [26] not to second-order predications but to second-order existential generalizations, such as the following:[55]

[24b] $\exists X[\forall x(X(x) \leftrightarrow \langle\text{is Russell or Whitehead}\rangle(x)) \wedge \textit{COOPERATE}(X)]$.
[26b] $\exists X[\forall x(X(x) \leftrightarrow \langle\text{is-an-author-of } PM\rangle(x)) \wedge \textit{COOPERATE}(X)]$.

But the scheme fails to preserve logical relations among the following sentences:

[24] Russell and Whitehead cooperate.
[27] Russell and Whitehead are philosophers who write *PM*.
[28] There are some philosophers who write *PM* and cooperate

[24] and [27] logically imply [28]. But the scheme renders [24] and [27] to sentences that do not logically imply the second-order analogue of [28]. The scheme renders [27] to the following:[56]

[27a] $\exists X[\forall x(X(x) \leftrightarrow \langle\text{is Russell or Whitehead}\rangle(x)) \wedge [\forall x(X(x) \to \mathrm{P}(x)) \wedge \textit{WRITE}(X, \mathrm{p})]]$.

And [28] can be paraphrased as follows:

[28a] $\exists X[\forall x(X(x) \to \mathrm{P}(x)) \wedge [\textit{WRITE}(X, \mathrm{p}) \wedge \textit{COOPERATE}(X)]]$.

[28a] is not logically implied by [24b] and [27a]. The truth of these sentences can be witnessed by two different, if coextensive, properties.

Notice that one cannot meet the above objections to higher-order schemes by adopting the extensional conception of property that identifies coextensive properties.[57] One cannot hold, e.g., that [24a] and [25a] logically imply [26a][58] unless the 'identity' between the first-order properties in question is a logical consequence of their coextensiveness. But it is one thing to hold the extensional conception, quite another to hold, more implausibly, that the truth of the conception rests on logic alone.[59] Similarly, one cannot meet the objections by deriving [26a] from [24a] and [25a] under the assumption that the property indicated by "*COOPERATE*" is one that Russell calls *extensional* (that is, a second-order property instantiated by any first-order property coextensive with one that instantiates it).[60] This does not help unless the assumption holds by logic, but one cannot show this while taking the predicate "*COOPERATE*" to be an unanalyzed primitive.[61]

This completes of my discussion of the attempts to understand plurals by paraphrasing them away. The attempts stem from the longstanding *bias against plurals*. The bias leads one to view plurals as redundant devices, and the view comes with the promise that one can understand the syntax, semantics, or logic of plurals by reducing them to singulars – whether to those amenable to elementary languages or to those allowed only in their higher-order extensions. The promise, as we have seen, cannot be kept. The bias thrives only on the lack of an adequate account of plurals that defies its grip. In the next section, I take the first step in presenting such an account.

3. REGIMENTING PLURALS

In this section, I regiment basic plural constructions of natural languages as Frege regimented their singular cousins. This prepares for the account of plurals that I present in the sequel to this paper, Yi (LMP II). The account is based on the view that plurals are substantial devices with their own semantic functions. To give such an account, it is essential to regiment plurals as expressions belonging to categories on a par with those of their singular cousins.

The basic plural constructions of natural languages that I shall examine include the following:

[4] Some children lift Bob.
[5] John and Carol lift Bob.
[7] John and Carol are children.
[13] There are some critics who admire only one another.
[15] Ezra and Tomas are critics who admire only one another.
[19] Genie is one of Russell and Whitehead.
[24] Russell and Whitehead cooperate.
[25] Russell and Whitehead are the authors of *PM*.

We can analyze them as involving the following expressions:

[a] plural terms: the plural pronoun "they", "Russell and Whitehead", "the authors of *PM*", etc.
[b] predicates or predicate phrases that can combine with plural terms: "lift", "is one of", "cooperate", "are", "are children", etc.
[c] quantifier phrases that plural pronouns can take as antecedents: "Some children", "There are some critics who . . .", etc.

Adding refinements of these expressions to elementary languages yields regimented languages that I call *plural languages*. Plural languages have natural paraphrases of basic plural constructions of natural languages as elementary languages have natural paraphrases of their singular cousins.

3.1. *Plural Terms*

The standard regimented languages, elementary languages and their higher-order extensions, have no counterparts of plural terms of natural languages. They have two kinds of terms: singular constants and singular variables. Singular constants are refinements of singular proper names (e.g., "Russell"), and singular variables refinements of the singular pronouns (e.g., "he", "she", or "it") as used anaphorically (as in "There is a dog and a cat over there, and he is chasing her"). In addition to these terms, plural languages contain refinements of plural terms of natural languages: [i] plural variables, which result from refining the plural pronoun "they" as used anaphorically (as in "There are some animals over there, and they chase each other"), and [ii] counterparts of such plural terms as "Russell and Whitehead". I discuss plural terms of the second kind in this subsection, and return to discuss plural variables in Section 3.3, where I discuss them together with quantifiers that bind them.

Consider the following sentence:

[5] John *and* Carol lift Bob.

It has the plural term "John and Carol", which results from combining two singular terms (i.e., "John" and "Carol") with the connective "and". This use of the word "and" contrasts with its use in the following:

[9] John is a child *and* Carol is a child.

In this sentence, it is used to combine two sentences to yield a complex sentence. Now, the conjunction sign available in elementary languages amounts only to the English connective "and" as used in [9]. The languages have no counterpart of the connective as used in [5]. To deal with [5] and the like, we can add a connective that amounts to the word "and" as used in [5]: "**and**" (in symbols, "**@**"). In contrast to *sentential connectives* (e.g., "$\wedge$"), which operate on sentences to form more complex sentences, the connective "**and**" operates on terms to form a complex term. It combines two terms (e.g., "John" and "Carol") to yield a plural term (e.g., "[John **and** Carol]" (in symbols, "[j**@**c]"). Call such connectives, those that combine with one or more terms to yield more complex terms, *term connectives*.[62]

The lack of the term connective "**and**" in elementary languages is somewhat compensated by a close interplay between the two uses of "and" in English. Consider another sentence:

[7] John *and* Carol are children.

In this sentence, too, the word "and" is used as a term connective. But it is standard practice to paraphrase the sentence into elementary languages using the sentential connective "$\wedge$". [7] is usually paraphrased by the straightforward paraphrase of the conjunction [9]:

[9a] John is-a-child *and* Carol is-a-child (in symbols, $[C(j) \wedge C(c)]$).

The justification for this practice lies in the conspicuous logical equivalence between the two English sentences, [7] and [9], as noted in Section 2. But there are limits to the interplay between the two uses of "and". [5], for example, is not logically equivalent to its singular cousin, "John lifts Bob and Carol lifts Bob" (sentence [10]). So the term connective use of "and" cannot be reduced to its use as a sentential connective.

3.2. *Plural Predicates*

We cannot paraphrase [5] into the standard regimented languages. Part of the problem is that the languages have no counterpart of the plural term "John and Carol". But the problem runs deeper. Augmenting them with only refinements of plural terms (e.g., "[John **and** Carol]") does not work, because they have no counterpart of the predicate "*to* lift" that occurs in [5]. This predicate can combine with plural terms. It can take the plural term "John and Carol" in its first argument place (while taking "Bob" in its second) to yield [5].[63] Call such predicates, predicates with at least one argument place that admits plural terms, *plural predicates*, and sentences that, like [5], result from combining plural predicates with one or more plural terms *plural predications*. We can then see that the main problem confronting the attempt to paraphrase [5] into the standard regimented languages is that [5] is a plural predication formed by a plural predicate whereas the languages have no plural predicates. Plural languages extend elementary languages by including plural predicates, such as the predicate "**lift**" (in symbols, "**L**") that results from refining the English predicate "*to* lift". Using this predicate, we can paraphrase [5] as follows:

[5c] [John **and** Carol] **lift** Bob (in symbols, **L**([j**@**c], b)).

This sentence is a plural predication of plural languages that results from combining the plural predicate "**lift**" with two terms, "[John **and** Carol]" and "Bob".

A plural predicate that requires special attention is the English predicate "*to* be one of". This, like the identity predicate, is a logical predicate, and is involved in various complex predicates. We can take "John and Carol are children" (sentence [7]), for example, to be a plural predication formed by a complex predicate defined in terms of the logical predicate, and paraphrase the sentence by a plural predication of plural languages. We can then explain its logical relations to other sentences (e.g., its singular cousin [9]) as due to the logic of the plural constructions that it involves. The usual practice of paraphrasing [7] by the elementary language counterpart of [9], by contrast, cannot lead to an explanation of the logical relations, because the practice *presupposes* the logical equivalence between [7] and [9].

3.2.1. *Singular Predicates versus Plural Predicates*

Predicates of elementary languages cannot combine with plural terms, because they result from refining singular forms of natural language predicates (as used in purely singular constructions). The predicate "is-a-child" results from refining the predicate phrase "is a child", the singular form of an English predicate. So the plural language predicate does not fit the plural term "[John **and** Carol]", just as the English predicate phrase does not fit the plural term "John and Carol". Similarly, the two-place predicate "lifts" (or "L") of elementary languages cannot take "[John **and** Carol]" in its first argument place, because it results from refining the verb "lift" as used in purely singular constructions, such as [29] and [6a] (i.e., "John lifts Bob" and "There is something that lifts Bob"). To obtain suitable paraphrases of plural predications, we need predicates that can combine with plural terms, such as the predicate "**lift**" (in symbols, "**L**"). This predicate results from refining the English predicate "*to* lift" that takes its plural form in [5]. Such predicates, which are allowed only in plural languages, are what I call *plural predicates*, whereas the predicates available in elementary languages are what I call *singular predicates*. To give a precise account of the difference between singular and plural predicates, it is useful to examine their counterparts in natural languages.

Consider the following sentences:

John and Carol *lift* Bob. (sentence [5])
John, Carol, and Bill *lift* Bob.
John, Carol, Bill, and Hillary *lift* Bob.
John, Carol, Bill, Hillary, and Chelsea *lift* Bob.
They *lift* Bob.[64]

They contain the same predicate, as noted above. So the common predicate must be a two-place predicate.[65] Its second argument place is filled with the same singular term "Bob" in the sentences, and its first argument place with different plural terms: "John and Carol", "John, Carol, and Bill", etc. So the common predicate has an argument place that admits (i.e., can be filled with) a plural term. Call such an argument place a *plural argument place*. Now, can plural argument places admit singular terms as well? The answer is yes. The first argument place of the predicate mentioned above admits a singular term as well. Let me explain.

Compare the predicates of the sentences mentioned above with those of related singular constructions, such as the following:

[29] John *lifts* Bob.

I think that this sentence has literally the same predicate that [5] has: "*to* lift". On my view, the common predicate takes two different forms (viz. the (present) singular and plural forms) in [29] and [5] because of the difference in grammatical number of the terms filling its first argument place. Some might disagree. They might hold that what I take to be two forms of the same predicate (i.e., "lifts" and "lift") are distinct predicates. I think that this is a mistaken view, and I return shortly to argue against it. But it is useful to consider the nature of the three expressions taken to be predicates by those who hold one or the other of the two views: "*to* lift", "lifts", and "lift". The first argument place of the putative predicate "lifts" is not plural. It admits *only* a singular term. "John and Carol *lifts* Bob", for example, is not grammatically correct. Call such an argument place *singular*. By contrast, the first argument places of the other putative predicates (i.e., "lift" and "*to* lift") admit the plural term "John and Carol" to yield grammatically correct constructions (e.g., [5]). Call such argument places *plural*. Can plural argument places admit singular terms as well? The first argument place of the putative predicate "lift" (i.e., the plural form of "*to* lift") does not admit any singular term. "John *lift* Bob", for example, is not grammatically correct. Call such an argument place *exclusively plural*. By contrast, the first argument place of the predicate "*to* lift" admits a singular term (e.g., "John") as well. It takes the singular form "lifts" to yield [29] while taking the plural form "lift" to yield [5]. Call such an argument place, a plural argument that admits a singular term as well, *neutral*.

Now, we can classify predicates according to what kind of argument places they have. Say that a predicate is *singular*, if *all* of its argument places are singular; and *plural*, if *some* of its argument places are plural.[66]

The above discussion applies, *mutatis mutandis*, to plural languages as well. Plural languages, like English, contain both singular and plural terms, and they draw a syntactic distinction between the two kinds of terms. So we can carry the classification of English predicates over to plural language predicates. Consider the predicates "**lift**" (or "**L**") and "lifts" (or "L"). The former is introduced as the refinement of the predicate "*to* lift" that, on my view, occurs in both [5] and [29], whereas the latter, like other elementary language predicates, is introduced as the refinement of the singular form of the English predicate. So we can identify the former as a plural predicate, and the latter as a singular predicate.[67]

The first argument place of the plural language predicate "**lift**", like that of "*to* lift", is not exclusively plural, but neutral.[68] In designing plural languages, I ignore predicates with exclusively plural argument places, although it is straightforward to add such predicates to the languages and to characterize the logic of the extended languages that result from doing so. One can defend the design on grounds of simplicity of exposition, but there is more than that.

I think that most, if not all, of the predicates of, e.g., English are neutral predicates. The view that [5] and [29] have two different predicates results from identifying predicates with the various forms that they take in particular sentences, but it is wrong to individuate predicates (or, in general, words) in this way. Those who do so cannot take "I *am* John" or "You *are* John" to contain the predicate of identity that occurs in "He *is* John." Some might concede that the same predicate can take different forms, but still insist that the so-called singular and plural forms of a predicate are special cases that must be considered different predicates on their own right. Consider, however, the passive forms of [5] and [29]:

Bob *is lifted by* John.
Bob *is lifted by* John and Carol.

With no difference even in the form of the verb, it is highly implausible to hold that these sentences have different predicates and, thus, that the phrase "is lifted by" is used equivocally in the sentences. On this view, the sentence "Bob is lifted" must be ambiguous because the phrase "is lifted" may derive from two homonymous predicates. Moreover, it would be wrong to abbreviate "Either Bob is lifted by Russell or it is lifted by the children" as "Bob is lifted by either Russell or the children" just as it is wrong to abbreviate "Either John is blue [in mood] or the diamond is blue [in color]" (where "blue" is used equivocally) as "Either John or the diamond is blue." So the passive forms of [5] and [29] must be seen to contain a common predicate, "*to* be lifted by", whose second argument place (placed after "by") is neutral. If so, [5] and [29] must also contain a common predicate – the neutral predicate "*to* lift", which is the converse of "*to* be lifted by".[69]

3.2.2. *Neutral Expansions*

Compare:

[7] John and Carol are children.
[5] John and Carol lift Bob.

The two sentences have the same subject: the plural term "John and Carol". The term combines with the predicate phrase "are children" to yield [7], and enters the first argument place of the predicate "*to* lift" (while "Bob" entering its second argument place) to yield [5]. So the connective "and" is used as a term connective in both [7] and [5], not as a sentential connective as in the following:

[9] John is a child *and* Carol is a child.
[10] John lifts Bob *and* Carol lifts Bob.

There is no denying that [7] involves a plural term. Clearly, [7] and [9] are two different sentences. Still, one might think that [7] is a mere abbreviation of [9]. This does not mean that the term connective "and" is merely a device for abbreviating sentential conjunctions (e.g., [9]), as we have seen, because [5] is not logically equivalent to [10]. But some might hold that the use of the plural term in [7] diverges from its use in [5] because "and" is used differently in them. On their view, the term connective is used in [7] as a device for abbreviating sentential conjunctions though it is not so used in [5]. One might hold this because it seems to explain the close, if limited, interplay between the sentential and term connective uses of "and": [7] and [9] are logically equivalent, one might hold, because [7] is an abbreviation of [9] that results from using "and" as an abbreviation device; [5] and [10] are not, because the connective is not so used in [5]. But it is wrong to think that the connective "and" is used equivocally in [7] and [5]. To see this, consider the following:

[3] *John and Carol* are children who lift Bob.
[3a] *John and Carol* are children and *John and Carol* lift Bob.

Clearly, [3] is a legitimate contraction of [3a]. But this must be denied by those who hold the plural term "John and Carol" is used equivocally in [3a] because "and" is used differently in its two conjuncts (i.e., [7] and [5]). They must conclude that it is wrong to contract [3a] to [3] just as it is wrong to contract "John is blue [in mood] and the diamond is blue [in color]" to "John and the diamond are blue."

The logical equivalence between [7] and [9], we have seen, is not due to a special use (or sub-use) of the term connective in [7]. If so, what explains their logical equivalence? To explain it, we need to attend to the predicate of [7].

We can formulate the predicate that takes two different forms (i.e., "are children" and "is a child") in [7] and [9] as "*to* be child$^{\varnothing}_{\text{ren}}$". And to highlight the contrast between [7] and [5], take [5] and [10] to contain the predicate phrases "lift Bob" and "lifts Bob" that can be considered different forms of the same complex predicate "*to* lift Bob". This helps us to see that [5] and [10] have the same syntactic relation to each other that [7] and [9] have. Now, despite this syntactic parallelism, [7] is logically equivalent to [9] whereas [5] is not logically equivalent to [10]. We can describe this as a disparity between the two predicates that form [7] and [5]: "*to* be child$^{\varnothing}_{\mathrm{ren}}$" and "*to* lift Bob". The former predicate *distributes over* the term connective "and", although the latter does not.[70] We can then see that the distributivity of the former is due to the special character that it has on account of its construction. The neutral predicate can be considered a complex predicate that involves the underlying singular predicate "is a child". It is designed so that *it is true of some things* (taken together) *if and only if the singular predicate is true of each one of them* (so "Some things are children if and only if any one of them is a child" is a logical truth).[71] So the former predicate is true of, e.g., John and Carol (taken together) if and only if the latter is true of John *and also of* Carol, and this yields the distributivity of the former. Call the neutral predicates designed as described above the *neutral expansions* of the underlying predicates. Then we can see that all neutral expansions are distributive.[72]

This explanation, to be sure, does not apply to all (one-place) neutral predicates because not all of them are neutral expansions of underlying predicates. The predicate "*to* lift Bob", for example, cannot be considered the neutral expansion of its singular cousin "lifts Bob": the former can be true of John and Carol (as such) without the latter being true of either of them. So neutral predicates are not in general distributive.

We can generalize the notions of neutral expansion and distributivity. [5], on my analysis, is formed by the two-place plural predicate "*to* lift", and this predicate contrasts with the predicate "*to* be offspring of" that occurs in the following:

[30] John and Carol *are offspring of* Ezra.
[31] John *is offspring of* Ezra and Carol *is offspring of* Ezra.

These sentences, unlike [5] and [10], are logically equivalent. We can explain this using the generalized notions of distributivity and plural expansion. Using the generalized notions, we can see that the predicate "*to* be offspring of" *distributes* (over the term connective) *on* its first argument place (for [30], for example, is logically equivalent to its singular cousin [31]), and that it is the *neutral expansion* of the singular predicate "is offspring of" *on* its first argument place. And we can show that the predicate distributes on its first argument place because it is the neutral expansion of an underlying singular predicate on its first argument place. By contrast, "*to* lift" cannot be considered a neutral expansion of another predicate on its first argument place, and it does not distribute on its first argument place.

On the above analysis, the predicate of [7], for example, is a complex predicate that involves an underlying singular predicate, whose elementary language counterpart is the predicate "is-a-child" (or "C"). We can introduce plural language counterparts of such complex predicates as the neutral expansions of underlying predicates, such as the elementary language predicate "is-a-child" (or "C"). To do so, write the neutral expansion of a one-place predicate by adding the superscript "N" to the predicate as in "is-a-child^{N}" (or "C^{N}");[73] and the *i-th neutral expansion* of a predicate (i.e., its neutral expansion on the i-th argument place) by adding the superscript "$N(i)$" to the predicate as in "is-offspring-of$^{N(1)}$" (or "O$^{N(1)}$").[74] Then we can paraphrase [7] and [30] as follows:

[7a] [John **and** Carol] is-a-child^{N} (in symbols, $\mathrm{C}^{N}([\mathrm{j}\,@\,\mathrm{c}])$).
[30a] [John **and** Carol] is-offspring-of$^{N(1)}$ Ezra (in symbols, $\mathrm{O}^{N(1)}([\mathrm{j}\,@\,\mathrm{c}], \mathrm{e})$).

We can use the same predicates to paraphrase their singular cousins, [9] and [31], as follows:

[9b] John is-a-child^{N} and Carol is-a-child^{N} (in symbols, $[\mathrm{C}^{N}(\mathrm{j}) \wedge \mathrm{C}^{N}(\mathrm{j})]$).
[31a] John is-offspring-of$^{N(1)}$ Ezra and Carol is-offspring-of$^{N(1)}$ Ezra (in symbols, $[\mathrm{O}^{N(1)}(\mathrm{j}, \mathrm{e}) \wedge \mathrm{O}^{N(1)}(\mathrm{c}, \mathrm{e})]$).

Then we can show that [7] and [30a] are logically equivalent to [9b] and [31a], respectively, using the *principle of distributivity of neutral expansions*, which can be formulated as follows:

$$\begin{aligned}&\pi^{N(i)}(\tau_1, \ldots, \tau_{i-1}, [\tau\,@\,\mu], \tau_{i+1}, \ldots, \tau_n)\\ &\quad\leftrightarrow [\pi^{N(i)}(\tau_1, \ldots, \tau_{i-1}, \tau, \tau_{i+1}, \ldots, \tau_n)\\ &\qquad\wedge \pi^{N(i)}(\tau_1, \ldots, \tau_{i-1}, \mu, \tau_{i+1}, \ldots, \tau_n)]\end{aligned}$$

where π is an n-place predicate, μ a term suitable for the i-th argument place of π, and $\tau_1, \ldots, \tau_{i-1}, \tau, \tau_{i+1}, \ldots, \tau_n$ terms suitable for its 1st, 2nd, $\ldots$, n-th argument places of π, respectively.[75] Clearly, the principle does not yield the logical equivalence between paraphrases of [5] and [10]:

[5c] [John **and** Carol] **lift** Bob (in symbols, $\mathbf{L}([\mathrm{j}\,@\,\mathrm{c}], \mathrm{b})$).
[10a] John **lift** Bob and Carol **lift** Bob (in symbols, $[\mathbf{L}(\mathrm{j}, \mathrm{b}) \wedge \mathbf{L}(\mathrm{c}, \mathrm{b})]$).

Moreover, we can define neutral expansions using a logical predicate "*to* be one of" (or its refinement), and show that the definition, which codifies the above account of neutral expansion, yields the principle of distributivity of neutral expansions. This result fulfills the explanation, outlined above, of the logical equivalence between, for example, [7] and [9].[76]

Before considering the logical predicate involved in neutral expansions, let me note how the predicate "*to* lift" relates to its singular cousin "lifts". The neutral predicate "*to* lift" must be considered a basic predicate, we have seen, because it cannot be taken to derive from an underlying singular predicate. But its singular cousin can be taken to result from, to put it figuratively, projecting it to the horizon of singular terms. Write the predicate that I call the *i-th singular reduct* of a base predicate by adding the superscript "$S(i)$" to the base predicate (or, simply, "S" if the predicate is one-place) as in "*to* lift$^{S(1)}$" (or "$\mathbf{L}^{S(1)}$"). The i-th singular reduct of a base predicate works exactly like the base predicate except that its i-th argument place is singular. So "$\forall x \forall y[\mathbf{L}^{S(1)}(x, y) \leftrightarrow \mathbf{L}(x, y)]$" is a logical truth. Now, we can identify the singular cousin of the predicate "*to* lift" as its singular reduct. And we can identify the elementary language predicate "L" with the singular reduct "$\mathbf{L}^{S(1)}$" of the plural language counterpart "**L**" of "*to* lift", because "L" is introduced into elementary languages as the refinement of the singular form of the same English predicate. So we may use the singular reduct "$\mathbf{L}^{S(1)}$" (instead of "L") to paraphrase singular constructions involving "*to* lift" (e.g., [29] or [10]). Given the neutral predicate "**L**", however, we do not need its singular reduct to paraphrase such singular constructions. We can use the neutral predicate to paraphrase [29] as follows:

[29a] John **lift** Bob (in symbols, **L**(j, b)).

And we can define singular reducts using the singular predicate for identity. "$\mathbf{L}^{S(1)}$", for example, can be defined as follows:[77]

$$\mathbf{L}^{S(1)}(x, y) \equiv_{\text{df}} \exists z[z = x \wedge \mathbf{L}(x, y)]$$

The definition yields the logical truth "$\forall x \forall y[\mathbf{L}^{S(1)}(x, y) \leftrightarrow \mathbf{L}(x, y)]$".[78]

3.2.3. *The Logical Predicate "is one of"*

Elementary languages have one primitive predicate marked as a logical expression: "is-identical-with" (in symbols, "="), the singular predicate that indicates the identity relation. Plural languages have another logical predicate: "**is-one-of**" (in symbols, "**H**"),[79] a two-place plural predicate whose second argument place is neutral while its first argument place is singular. It is the refinement of the English predicate "*to* be one of" (or its singular form "is one of").[80] So we can use it to paraphrase [19], [19a], and [20], for example, into plural languages as follows:

[19d] Genie **is-one-of** [Russell **and** Whitehead] (in symbols, g**H**[r**@**w]).

[19e] Genie **is-one-of** [Frege **and** [Russell **and** Whitehead]] (in symbols, g**H**[f**@**[r**@**w]]).

[20b] Genie **is-one-of** Genie (in symbols, g**H**g).[81]

We can also paraphrase "Some philosophers admire Frege, but Cicero *is* not *one of* them" using the predicate, but this requires quantifiers and variables discussed in the next subsection.

Like the identity predicate, the predicate "**is-one-of**" (or its natural language counterpart) is implicit in many constructions. Consider, for example, sentences involving "each other", such as "Ezra and Thomas are critics who admire only each other" (sentence [15]). Using the logical predicate, we can paraphrase [15] as follows:

[15a] $\forall y[y\mathbf{H}[\mathrm{e}@\mathrm{t}] \rightarrow (\mathrm{C_r}(y) \wedge \forall z[\mathrm{A}(y, z) \rightarrow y \neq z \wedge z\mathbf{H}[\mathrm{e}@\mathrm{t}]])]$.[82]

And we can use the predicate to paraphrase sentences involving neutral expansions, such as "John and Carol *are children*" (sentence [7]). We can define neutral expansions, as noted above, in terms of their base predicates as follows:[83]

$$\pi^{N(i)}(\tau_1, \ldots, \tau_{i-1}, \tau_i, \tau_{i+1}, \ldots, \tau_n) \equiv_{\mathrm{df}} \forall \upsilon[\upsilon\mathbf{H}\tau_i \rightarrow \pi(\tau_1, \ldots, \tau_{i-1}, \upsilon, \tau_{i+1}, \ldots, \tau_n)]$$

where π is an n-place predicate, $\tau_1, \ldots, \tau_n$ terms suitable for its 1st, ..., n-th argument places of π, and υ a singular variable that occurs in none of those terms. On the definition, [7a], the natural paraphrase of [7], abbreviates the straightforward paraphrase of "*Any one of* John and Carol *is a child*" into plural languages:

[7b] $\forall x[x\mathbf{H}[\mathrm{j}@\mathrm{c}] \rightarrow \mathrm{C}(x)]$.

Now, we can show that this is logically equivalent to the usual paraphrase of [7] into elementary languages: "$[\mathrm{C(j)} \wedge \mathrm{C(c)}]$" (sentence [9a]). The equivalence results from logical truths that pertain to the predicate "**H**", such as "$\forall x(x\mathbf{H}[\mathrm{j}@\mathrm{c}] \leftrightarrow [x = \mathrm{j} \vee x = \mathrm{c}])$."[84] Similarly, we can show that [7b] is logically equivalent to [9b], "$[\mathrm{C}^N(\mathrm{j}) \wedge \mathrm{C}^N(\mathrm{c})]$", because we can derive the distributivity of neutral expansions from the logic of the predicate.

One of the neutral expansions, "**is-one-of**$^{N(1)}$" (in short, "$\sqsubseteq$"), is of special significance. It is a logical predicate, because it can be defined using only logical resources (its base is "**is-one-of**"). It amounts to the English predicate "*to* be among", "*to* be some of", or the converse of "*to* include" in the following sentences:

Ezra and Thomas *are among* my favorite poets.
My favorite poets *include* Ezra and Thomas.
John and Carol *are some of* Carol and Russell, and John.

So the third sentence can be paraphrased by the plural language sentence "$[\mathrm{j}@\mathrm{c}] \sqsubseteq [[\mathrm{c}@\mathrm{r}]@\mathrm{j}]$", which we can show is a logical truth.[85]

We can define another logical predicate, "**be-the-same-as**" (in symbols, "$\approx$"), as follows:

$$\tau \approx \mu \equiv_{\text{df}} \forall \upsilon[\upsilon \mathbf{H} \tau \leftrightarrow \upsilon \mathbf{H} \mu]$$

where τ and μ are any terms, and υ a singular variable that does not occur in τ or μ ("$\approx$" is a two-place predicate both of whose argument places are neutral). The predicate is the plural (viz. neutral) cousin of the identity predicate "$=$", which is the refinement of the singular form of the predicate "*to* be" that is used in the following:

Cicero *is* Tully.
He *is* John.
Chelsea's mother *is* Hillary.
John *is* not Carol.
There is a philosopher who wrote *Academica*, and he *is* Cicero.

So we can use "**be-the-same-as**" to paraphrase sentences that contain the plural form of the predicate "*to* be", such as the following:

Cicero and Russell *are* Russell and Tully.
They *are* Cicero and Russell.
Chelsea's parents *are* Bill and Hillary.
John and Bob *are* not Bob and Carol.
There are some critics who admire only one another, and they *are* Ezra and Thomas.[86]

The first of these, for example, can be paraphrased as follows:[87]

[Cicero **and** Russell] **be-the-same-as** [Russell **and** Tully] (in symbols, $[c_i \textbf{@} r] \approx [r \textbf{@} t]$).

Why must the predicate "**is-one-of**" (or its natural language counterparts) count as a logical expression? It must do so for the same reason that the predicate "$=$" does: there are logical relations that rest on the special character of the predicate (or its natural language counterparts). For example, "Cicero *is one of* Cicero and Caesar" is a logical truth; "Cicero *is one of* Tully and Caesar" and "Cicero is Tully or Caesar" (or "Cicero *is one of* the Roman philosophers" and "Cicero is a Roman philosopher") are logically equivalent; and so are "John is a child and Carol is a child" (i.e., [9]) and "Any *one of* John and Carol is a child." Replacing the predicate in these sentences with another predicate belonging to the same syntactic category can disturb the logical relations.

We can confirm the logical status of the predicate "**is-one-of**" by considering the logic of predicates that we can use it to define, such as "**be-the-same-as**". Consider the following sentences:

Russell and Cicero *are* Russell and Cicero.
If Russell and Cicero *are* Tully and Russell, then Tully and Russell *are* Russell and Cicero.
If Russell and Cicero wrote *PM* and they *are* Tully and Russell, then Tully and Russell wrote *PM*.
If some philosophers wrote *PM* and they *are* Tully and Russell, then Tully and Russell wrote *PM*.

They are plural cousins of the following logical truths:

Cicero *is* Cicero.
If Cicero *is* Tully, then Tully *is* Cicero.
If Cicero wrote *Academica* and he *is* Tully, then Tully wrote *Academica*.
If a philosopher wrote *Academica* and he *is* Cicero, then Cicero wrote *Academica*.

We can see that those plural constructions are also logical truths. We cannot explain this without acknowledging the special logical status of the predicate "*to* be" (or its plural form "are"), as we cannot explain the logic of their singular cousins without acknowledging the special logical status of the singular form "is" of the predicate. So the predicate "*to* be" must be a logical expression. Then the predicate "**is-one-of**" (or its English counterpart) must also be a logical expression because it is essential to analyzing the plural cousin of the identity predicate.

3.3. *Plural Quantifiers and Variables*

Compare the following sentences:

[2] *A* child is healthy (i.e., *there is a* child who is healthy).
[4] *Some* children lift Bob.

Elementary languages have variables and quantifiers that one can use to paraphrase the italicized "a" as used in [2]. Using the variable "x" and the existential quantifier "$\exists$", one can paraphrase [2] as follows:

[2a] There-is-something x such-that [x is-a-child and x is-healthy] (in symbols, $\exists x[\mathrm{C}(x) \wedge \mathrm{H}(x)]$).

The variable amounts to the singular pronoun "it" of English as used anaphorically, and the quantifier to the singular quantifier phrase "there is something such that". But elementary languages (or their higher-order extensions) have no expressions that one can use to paraphrase the plural quantifier "some" in [4]. Plural languages extend elementary languages by including the plural cousins of elementary language variables and quantifiers:

Plural variables: "*xs*", "*ys*", etc.[88]
Plural quantifiers: "**There-are-some-things** . . . **such-that**" (in symbols, "$\boldsymbol{\Sigma}$"), and "**Any-things** . . . **are-such-that**" (in symbols, "$\boldsymbol{\Pi}$").

The plural variables are refinements of plural pronouns of English (e.g., "they"), and the plural quantifiers of plural quantifier phrases of English (e.g., "there are some things such that"). Using the variable "*xs*" and quantifier "$\boldsymbol{\Sigma}$", we can paraphrase [4] as follows:

[4a] **There-are-some-things** *xs* **such-that** [*xs* is-a-child^N and *xs* **lift** Bob] (in symbols, $\boldsymbol{\Sigma} xs[\mathrm{C}^N(xs) \wedge \mathbf{L}(xs, \mathrm{b})]$).

We can justify paraphrasing [4] by [4a] in the same way that we justify paraphrasing [2] by [2a].

3.3.1. *Plural Quantifiers versus Singular Quantifiers*

The quantifier "some" used in [4] is the plural cousin of the quantifier "a" used in [2]. Both are existential quantifiers and contrast with the universal quantifier "every" in "Every child is healthy." But "some" in [4] combines with nouns of plural form (e.g., "children") to yield quantifier phrases (e.g., "some children") that combine with predicates of the plural form (e.g., "lift"), whereas "a" in [2] combines with nouns of the singular form to yield quantifier phrases combining with predicates of the singular form. So we can say that "some" is a *plural quantifier* whereas "a" is a *singular quantifier*. How about quantifiers of elementary languages? We cannot apply to them the above account of the distinction between singular and plural quantifiers, because predicates of elementary languages do not take singular or plural forms. Still, we can identify them as singular quantifiers. They result from refining singular quantifier phrases of natural languages and match with variables that result from refining singular pronouns of natural languages.

To see this, consider pronouns used anaphorically for cross-reference, such as the italicized pronouns in "There is a baby in the room and *it* is healthy" or "There is a cat over there, there is also a dog, and *he* is chasing *her*." It is useful to introduce a device that clarifies which phrases are the antecedents of the pronouns. One such device, which is not adopted in English, is to tag the pronouns to the antecedent phrases as in "There is <u>a cat</u> [*she*] and there is <u>a dog</u> [*he*] over there and *he* is chasing *her*", where "she" and "he" in parenthesis are tagged to "a cat" and "a dog", respectively, to indicate that the pronouns (in their later occurrences) take the phrases as their antecedents. Variables of elementary languages are refinements of pronouns used in this way: they are to the quantifiers that bind them what anaphoric pronouns are to the quantifier phrases that they take as antecedents.[89] The elementary language quantifier "There-is-something . . . such-that" has an argument place (viz. the first one marked by ". . .") that admits a variable to yield, for example, "*There-is-something* x *such-that* [x is-a-child and x is-healthy]." The argument place is the slot for tagging a variable to the quantifier to clarify that it is the antecedent of the variable in its 'later' occurrences (viz. its free occurrences in the sentence that fill the second argument place of the quantifier).

Now, variables of elementary languages are refinements of singular pronouns of natural languages: "he", "she", "it", "this", "that", etc. Elementary languages have no counterparts of plural pronouns, such as "they", "these", and "those". So we can identify elementary language variables as singular variables. Accordingly, we can identify quantifiers of elementary languages as singular quantifiers because they can be the antecedents of only singular variables. The criteria applied to identify them as singular variables and quantifiers can be stated as follows: a variable is *singular* (or *plural*), if it is the refinement of a singular (or plural) pronoun used anaphorically; a quantifier is *singular* (or *plural*), if it can be the antecedent of only singular (or plural) variables.[90] We can see that these criteria are straightforward extensions of those applied to the English pronouns or quantifier phrases. The singular quantifier phrase "a child" in [2] can be the antecedent of only singular pronouns, whereas the plural quantifier phrase "some children" can be the antecedent of only plural pronouns.

Now, we can augment elementary languages with the plural cousins of the singular variables and quantifiers thereof. The anaphoric use is not limited to singular pronouns in natural languages. The languages have plural pronouns and quantifier phrases, and the plural pronouns can also be used anaphorically to take plural quantifier phrases as antecedents. In the sentence "There are <u>some children</u> in this room and *they* are lifting Bob", for example, the plural pronoun "they" is used to take the plural quantifier phrase "some children" as its antecedent. So we can introduce the plural cousins of elementary language variables and quantifiers as refinements of the plural pronouns and quantifier phrases. Plural languages include such expressions: *plural variables*, such as "*xs*" or "*ys*"; and *plural quantifiers*, such as "$\boldsymbol{\Sigma}$". Plural variables result from refining plural pronouns as we refine singular pronouns to obtain the variables of elementary languages, and plural quantifiers from refining plural quantifier phrases of natural languages as we refine their singular cousins to obtain the quantifiers of elementary languages.

So plural languages have straightforward paraphrases of plural quantifications of natural languages. [4], for example, can be paraphrased by

[4a]. And we can justify paraphrasing [4] by [4a] in the same way that we justify paraphrasing the singular quantification [2] by [2a].

To see this, consider the following sequence of sentences that runs from [2] to [2a]:

[a] A child is healthy. (sentence [2])
[b] There is a child who is healthy.
[c] There is a child and he is healthy.
[d] There is something that is a child and it is healthy.
[e] There is something such that it is a child and such that it is healthy.
[f] There is something x such that [x is a child and x is healthy].
[g] *There-is-something* x *such-that* [x is a child and x is healthy].
[h] *There-is-something* x *such-that* [x is-a-child $\wedge$ x is-healthy]. (sentence [2a])

Any two consecutive sentences in the sequence can be seen to abbreviate or elaborate each other – either in English as it is or using the new, quasi-English devices (e.g., the variable "x" in [f]) introduced in the later sentence as refinements of their siblings in the earlier sentence (e.g., the pronoun "it" in [e]). So we can justify paraphrasing [2] by [2a] by clarifying the new devices while invoking connections among pure English expressions. Similarly, we can justify paraphrasing [4] by [4a] by considering a parallel sequence:

[A] Some children lift Bob. (sentence [4])
[B] There are some children who lift Bob.
[C] There are some children and they lift Bob.
[D] There are some things that are children and they lift Bob.
[E] There are some things such that they are children and such that they lift Bob.
[F] There are some things *xs* such that [*xs* are children and *x*s lift Bob].
[G] **There-are-some-things** *xs* **such-that** [*xs* are children and *x*s lift Bob].
[H] **There-are-some-things** *xs* **such-that** [*xs* is-a-child^N $\wedge$ *xs* **lift** Bob]. (sentence [4a])

It is straightforward to justify the steps in this sequence except those from [E] to [G]. [F] results from [E] by replacing the pronoun "they" with the variable "*xs*" (while tagging the variable to the quantifier phrase "some things" to specify its antecedent), and [G] from [F] by replacing the English phrase "There are some things . . . such that" with the plural language quantifier "**There-are-some-things** . . . **such-that**". We can justify these two steps by invoking the fact that the new devices are refinements of the English expressions that they replace.

3.3.2. *Singularizable Plural Quantifications*

Some plural quantifications are logically equivalent to their singular cousins. We can say that such plural quantifications are *singularizable*. It is usual to paraphrase them into elementary languages by paraphrasing their singular cousins. This usual practice yields the result that those plural quantifications are logically equivalent to their singular cousins. But the practice does not yield an explanation of this logical relation, because one cannot justify the practice without presupposing the logical relation. To explain it, it is necessary to analyze the logic of the basic expressions involved in the plural quantifications. By regimenting plurals, we are well prepared to undertake this task. Plural languages have natural paraphrases of plural quantifications, and we can justify the paraphrases without assuming their logic. So we can explain the singularizability of some of the plural quantifications by showing that their plural language paraphrases are logically equivalent to the natural paraphrases of their singular cousins.

Consider a singularizable plural quantification:

[8] *Some* children are healthy.

It is usual to paraphrase it as follows:

[2a] There-is-something x such-that [x is-a-child and x is-healthy] (in symbols, $\exists x[\mathrm{C}(x) \wedge \mathrm{H}(x)]$).

But the straightforward paraphrase of the singular cousin of [8] is this:

[2] *A* child is healthy.

We can justify paraphrasing [2] by [2a] by appealing to the gradual and smooth transition from [2] to [2a], as we have seen, because the quantifier and variable in [2a] relate directly to the singular quantifier "a" in [2]. But we cannot give a similar justification for paraphrasing [8] by [2a], because the quantifier and variable do not relate to the plural quantifier "some" in [8]. To justify this paraphrase, it is necessary to invoke the logic of [8]. There are plural quantifications that we cannot paraphrase by paraphrasing their singular cousins, because they fail to be logically equivalent to their singular cousins. We cannot paraphrase [4] by paraphrasing its singular cousin, "A child carries Bob", because they are not logically equivalent. But we have a clear intuition that [8] is logically equivalent to its singular cousin, [2].[91] We can see this just as we can see the logical equivalence between the plural predication [7] and its singular cousin (i.e., "John and Carol are children" and "John is a child and Carol is a child"). So one may appeal to the intuition about [8] to justify its usual paraphrase: the elementary language paraphrase of [2] can be taken to paraphrase [8] as well, because [8] is logically equivalent to [2]. But we cannot explain the logic of [8] by accommodating it into elementary languages in this way. To explain its logic, it is necessary to account for its logical relation to [2], but the justification of its paraphrase into elementary languages must *presuppose* the logical relation.

By attending to the plural constructions involved in singularizable plural quantifications, we can explain their logical equivalence with their singular cousins. The disparity in logical behavior between [8] and the usual plural quantification [4], for example, must be due to the difference between the predicates that they do not share with each other: "*to* be healthy" and "*to* lift Bob". The former can be seen to be the neutral expansion of an underlying singular predicate, the English counterpart of the elementary language predicate "is-healthy". So we can paraphrase [8] using the neutral expansion of the elementary language predicate as follows:

[8a] **There-are-some-things** *xs* **such-that** [*xs* is-a-child^N and *xs* is-healthy^N] (in symbols, $\mathbf{\Sigma} xs[\mathrm{C}^N(xs) \wedge \mathrm{H}^N(xs)]$).

[8a] is the natural paraphrase of [8] that we can get by taking steps parallel to those from [2] to [2a]. And it is straightforward to justify the steps without presupposing the logic of [8]. The quantifier and variable in [8a] are refinements of the plural quantifier "some" in [8], and the predicates in [8a] of those in [8]. So we can explain the logical equivalence between [2] and [8] by showing that their natural paraphrases, [8a] and [2a], are logically equivalent. We can show this, as we shall see, by analyzing the logic of the basic expressions in them.[92] So plural logic yields the desired explanation of the logic of singularizable plural quantifications, and justifies their usual paraphrases into elementary languages.

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NOTES

[1] The word "logic" is equivocal. It is used sometimes to indicate certain features of arguments or relations among sentences, and sometimes a discipline that studies those features or relations. I see no elegant way to use it in just one of the two ways.

[2] Not all natural languages have singular or plural constructions. Many of them (e.g., English or German) distinguish singular and plural forms of verbs and draw syntactic distinctions between singulars and plurals, but some of them (e.g., Korean) have no plurals (or singulars) because they draw no such distinctions. (This, however, does not mean that the latter languages, like Frege's languages, have no counterparts of plurals.)

[3] By "the logic of plurals", I mean *the logical relations that pertain to plurals*. These include the logical properties of plurals and the logical relations that crisscross singulars and plurals as well as the logical relations among plurals.

[4] Here I omit sentential connectives. See Section 2.1, where I discuss elementary languages and their higher-order extensions.

[5] Higher-order extensions of elementary languages result from adding to elementary languages higher-order expressions: variables that can replace predicates, quantifiers that can bind such variables, or predicates that can combine with predicates. They are also singular languages. See the last paragraph of Section 2.1.

[6] [A*]–[C*] are the plural cousins of [a*]–[c*], just as [A]–[C] are the plural cousins of [a]–[c]. See Section 3 for discussions of plural terms, plural predicates, and plural quantifiers.

[7] What I call *elementary languages* and *elementary logic* are often called *first-order languages* and *first-order logic*, respectively. I avoid this terminology, because it suggests contrasts only with higher-order languages and logic. There is a clear sense in which the plural languages presented in the sequel to this paper (Yi (LMP II)), like elementary languages, are first-order languages. They have no higher-order variables, predicates, or quantifiers. They are, so to speak, horizontal extensions of elementary languages, whereas the usual higher-order languages are their vertical extensions. We can add higher-order expressions to those plural languages and obtain their vertical extensions, *higher-order plural languages*, that include the usual higher-order languages as their singular fragments. We can then formulate systems of logic, *higher-order plural logics*, that relate to higher-order plural languages. It is straightforward to combine first-order plural logic with the usual higher-order logics to obtain higher-order plural logics. But I leave it for another occasion to present higher-order plural languages and logic.

[8] See Yi (LMP II, Sections 6–7). Elementary logic can be considered the singular fragment of plural logic, because plural logic is its *conservative* extension (that is, plural logic agrees with elementary logic as far as elementary language sentences are concerned).

[9] By "many", I mean: *two or more*. I add "first and foremost" because plurals do not relate exclusively to two or more things. Some plural terms (e.g., "Cicero and Tully"), on my view, refers to some one thing (e.g., Cicero).

[10] I add "as such" (usually in parenthesis) as a disambiguation device that clarifies that in the above sentence, for example, it is *not* meant that they are developed as devices for talking about *each one of* many things. To elaborate on the content of the phrase, I use the phrase "without setting them apart from each other" or "without separating them" (see below).

[11] See Lønning (1996) for a survey of contemporary accounts of plurals. The accounts differ from each other by adopting different regimented languages, by assuming different accounts of the languages, or by proposing different schemes of paraphrase.

[12] On one usual terminology, "x is-a-child" (or "C(x)")" is a formula but not a sentence (i.e., closed formula); on my terminology, it is a sentence but not a closed one. I avoid the word "formula" because it is often loaded with the view that formulas (open or closed), unlike natural language sentences, are composed of *meaningless* 'symbols'. What I call *elementary languages* (or, in general, *regimented languages*) are not the 'formal languages' that contain formulas in this sense (which I do not call *sentences* or *symbols*), but languages in the robust sense that contain meaningful expressions: primitive expressions and complex expressions (e.g., sentences) built from them in a definite number of rigid ways of construction.

[13] I ignore parentheses in the discussion of syntax. We can regard them as parts of sentential connectives by identifying, e.g., the conjunction symbol as "[$\ldots \wedge \ldots$]".

[14] Church says, "The functional calculus of second order ... has, in addition to the notations of the functional calculus of first order, quantifiers with propositional or functional variables" (1956, p. 294); Kleene, "Another form of predicate calculus treats the ions [i.e., predicates] as variables which may also be quantified" and is "called *second-order* predicate calculus" (1967, p. 85); and Boolos and Jeffrey, "A second-order formula ... is a formula that contains at least one occurrence of a function, sentence, or predicate variable" (1989, p. 198).

[15] The variables can enter the argument places of second-order predicates and replace first-order predicates.

[16] In this respect, the latter languages are comparable to the usual third-order languages, which contain second-order predications.

[17] I add the qualification in parenthesis, because plural constructions can have predicates of the singular form. Consider, e.g., "Bob is carried by John's friends" or "John has collected all the U.S. stamps." In refining predicates of singular forms to obtain elementary language predicates, it is also required that they cannot admit plural terms in any of its argument places.

[18] And the second-order variables of such languages need to replace only singular first-order predicates, which result from refining singular forms of natural language predicates.

[19] I add "basic" because it is not straightforward to paraphrase into elementary languages singular constructions whose counterparts are not available in them, such as adverbs (e.g., "happily") or collective nouns (e.g., "committee") that can combine with predicates of singular form (as well as those of plural form). My view is that adverbs are higher-order expressions and that most collective nouns are hybrids between singulars and plurals.

[20] I put the italicized "*to*" in the quotation names of predicates (simple or complex) of English (as in, e.g., "*to* lift") to distinguish them from their plural forms (e.g., "lift").

[21] Moreover, one cannot *explain* the logic of even the docile plurals by just paraphrasing them using those schemes, because the schemes *presuppose* their logical equivalence to their singular cousins. See the discussion of singularizable plural quantifications (e.g., [8]) in Section 3.3.2.

[22] I think that this is the way it is usually used.

[23] On this view, [5] is akin to "Bob *is between* John *and* Carol", which is often taken to contain a 3-place predicate; or "John and Carol are different", which one may paraphrase using a 2-place predicate (for it is logically equivalent to "John *is different from* Carol").

[24] The common predicate must be a two-place predicate whose first argument place admits plural terms (e.g. "John and Carol"). I call such predicates, which are not available in elementary languages, *plural predicate*s (see Section 3.2). They must be clearly distinguished from predicates known as *multigrade* (or *variably polyadic*) *predicates* (that are not plural), which can take different numbers of singular terms in various occurrences (but no plural term). [6] must be seen to result from a plural predicate combining with the plural quantifier phrase "There are some things that", not from a multigrade predicate taking several singular quantifiers. For the germination and change of the notion of multigradicity, see Leonard and Goodman (1940), Kenny (1963, p. 156f), Davidson (1967), Morton (1975), Grandy (1976, p. 398f), and Taylor and Hazen (1992). Some of them confuse plural predicates with multigrade predicates (or perhaps presuppose reduction of the former to the latter). Morton (1975, p. 311), for example, uses bracketed variables (e.g., "[x]"), which he says "roughly correspond to English pluralization", and quantifiers binding them (e.g., the universal "([x])") in addition to multigrade predicates to paraphrase "The Mortons live together"; to combine with the plural variable "[x]", however, the predicate corresponding to "live together" must have one special argument place, not a varying number of argument places that can admit only the usual, non-bracketed, singular variables (e.g., "x"). On account of this confusion, I think, van Inwagen (1990, Chapter 2) misuses the term "variably polyadic" for plural predicates (and "multigrade" for plural properties or relations, properties or relations indicated by plural predicates). See Yi (1998) and (1999a, Section 2) for more on problems with attempts to paraphrase plurals using multigrade predicates.

[25] Some of those composite objects (e.g., sets or classes) are usually considered abstract, and others (e.g., aggregates, fusions, or mereological sums) concrete. Some who hold the standard view (e.g., Link, 1998) raise objections to taking plurals to pertain to abstract objects, but argue that the objections do not apply to the standard view itself. The objection to the view that I raise below does not assume that the composite object that "John and Carol" is taken to refer to is abstract; they rest only on the assumption that it is considered an *object* that is *neither John nor Carol*.

[26] The sortal "set" is not essential in this term. The term (or, more precisely, "{John, Carol}") can be considered an abbreviation of the singular definite description "the thing of which something is a member if and only if it is John or Carol", which does not contain "set". (And the definite description, though not available in elementary languages, can be paraphrased away in the usual way; see the sentence inside the parentheses in [5a].)

[27] His proof is reproduced in Boolos (1984, p. 432f, note 7). The gist of the proof is that the negation of [14] can be seen to have the same structure as the second-order induction principle, which helps to yield a complete axiomatization of arithmetic. In Yi (LMP II, Section 6), I give an alternative proof, which does not rest on the assumption that [13] can be paraphrased by a second-order sentence.

[28] [14] can be seen to be the second-order analogue of a plural construction that one can get from [13] by analyzing the predicate phrase "admire only one another":

> There are some things that are critics and any one of which admires nothing but another one of them.

[29] See Quine (1972, p. 239) and (1973, p. 111). Boolos (1984) and (1985a) and Lewis (1991) criticize Quine's view, and Resnik (1988) and Hazen (1993) defend it against their

criticisms. I follow Boolos and Lewis in rejecting the view, but I do not think that they have made convincing cases against Quine. Boolos holds that Quine's paraphrase of [13] has the "weird outcome" that "assertively uttering the Geach–Kaplan sentence commits one to the existence of classes" (1985a, p. 331), and thus rejects "the thought that ... to assess the commitment of a theory, we must first put it into [elementary languages] as well as we can" (*ibid.*, p. 332); but defenders of Quine respond by upholding this thought (while assuming that elementary languages must be rich enough to accommodate plurals) and arguing that the intuition against its outcome must be overridden. Lewis argues against the "singularist dogma" (1991, p. 68) that "[p]lurals ... are the means whereby ordinary language talks about classes" (*ibid.*, p. 65), and concludes that "plural quantification is irreducibly plural" (*ibid.*, p. 68). And Boolos urges the readers to "[a]bandon ... the idea that use of plural forms must *always* be understood to commit one to the existence of sets (or 'classes,' 'collections,' or 'totalities') of those things to which the corresponding singular forms apply" (1984, p. 442; my italics). Clearly, however, the Quinean view must be distinguished from this much less plausible view, on which to assert even "Some critics live in New York" or "Ezra and Thomas are critics" is to assert (or imply) the existence of sets (or the like) of critics; notice that Resnik, for example, distinguishes "genuine" plural quantifications from apparent ones that "can be plausibly put into" elementary languages without invoking (additional) sets (1988, p. 80). To be fair to Boolos and Lewis, they aim at more than just refuting the "dogma". So they consider plurals that they take to be of the same kind as [13], such as "There are some sets of which every set that is not a member of itself is one" (Boolos, 1984, p. 442) or "There are the non-self-membered classes" (Lewis, 1991, p. 65). But they fail to explain why *these* plurals, unlike those humdrum ones, must be taken to be on a par with [13]. (They are logically equivalent to singular constructions that do not invoke additional composites, such as "There is a set that is not a member of itself." See *Ths. 11 & 13* in Yi (LMP II, Section 7).)

[30] See, e.g., Link (1995, p. 208) and (1998, p. 314f). There is a long tradition of relating plurals to aggregates (or the like) that dates at least back to Frege, who holds that "Bunsen and Kirchoff" in "Bunsen and Kirchoff laid the foundation of spectral analysis" refers to "a whole or system ... consisting of parts" (and distinguishes this from the class consisting of the parts) in his letter to Russell (Frege, 1902, p. 140). See also Russell (1903, Chapter V). There are different conceptions of sums, aggregates, or the like. For the standard account thereof, see Leonard and Goodman (1940) or Tarski (1929, p. 24f). Burge (1977) gives an alternative account by distinguishing the relation of *being a member component of* from that of *being a part of*. Link distinguishes the "individual part relation" and "(individual) sums or plural objects" (which pertain to plurals) from the "material part" relation and "collections in the portions of matter sense" (which pertain to mass terms), respectively (1983, p. 305ff). See also Kamp and Reyle (1993, Chapter 4), who follow Link on the semantics of plurals.

[31] Link (1998, p. 339) proposes to paraphrase [13] by [14d] (with "$\prec$" defined using the predicate "$\leq_i$" for his *individual* part-whole relation). See also Kamp and Reyle (1993, Chapter 4). [14d] avoids a problem with [14c]: [14c], which implies "$\exists x \forall y[y \leq x \rightarrow \mathrm{C_r}(y)]$", is not made true by the existence of an aggregate of, e.g., three critics who admire only one another (the aggregate of two of the three may not be a critic). Note in passing that [14c] and [14d] do not imply the existence of a composite object (e.g., a non-atomic aggregate) because they hold even if there is one critic who is an atomic object that admires nothing. But the schemes in question render closely related plural constructions (e.g., "There are *at least two* critics who admire only one another" or "Some critics *admire someone* but only one another") to sentences that invoke composites.

[32] I use “indicate” for the relation between any linguistic expression and its metaphysical counterpart, and reserve “refer” for the more specific relation pertaining to non-predicable expressions (e.g., proper names). See Yi (LMP II, Section 5).

[33] We can define this predicate using “is a part of”: x *is an atomic part of* y if, and only if, [i] x is a part of y, and [ii] x is *atomic*, i.e., x has no proper parts (z *is a proper part of* x, if z is a part of x but is not identical with x).

[34] [17] is an axiom, the Pair Set Axiom, in standard formulations of set theory.

[35] [18] follows from an instance of the schema that asserts the existence of the aggregate of the things that satisfy a non-empty predicate; and instances of the schema, which Link calls “the Axiom of Definable Completeness” (1998, p. 157), are axioms or theorems of mereology. Note, however, that one cannot derive [14d] from [16a] without making further assumptions that are not its theorems, assumptions to the effect that both Ezra and Thomas are atomic. (The assumptions follow from “$\forall x(x \prec [\mathrm{e} \oplus \mathrm{t}] \leftrightarrow x = \mathrm{e} \vee x = \mathrm{t})$”. This is Link’s paraphrase of “Something is one of Ezra and Thomas if and only if it is Ezra or Thomas.” This English sentence is a logical truth that involves plurals [see [23] below and *Th. 6* in Yi (LMP II, Section 7)], but Link cannot take it to be a logical truth because his paraphrase of it is not even a theorem of mereology.)

[36] I doubt that mereology is true, but my case against the standard view does not rest on my skepticism about mereology.

[37] I doubt that many advocates of the standard view would flatly reject elementary logic to rescue elementary languages, but some of them might. Kamp and Reyle (1993) seem to suggest this approach. They hold that the pronoun “they” in the sentence “They had a lousy time” that is followed by the antecedent sentence “John took Mary to Acapulco” refers to a so-called “set” (i.e., sum) “whose existence is entailed by the antecedent . . . by means of logical deduction” (*ibid*., p. 307). See also Link (1998). Link calls his system, a variant of atomistic mereology, “The Logic of Plurality” (*ibid*., p. 159), although he separates axioms that belong to its “logical basis” (*ibid*., p. 139), a free-logic variant of the standard system of elementary logic, from its “proper” axioms (*ibid*., p. 141).

[38] I use “*PM*” henceforth as an abbreviation of “*Principia mathematica*”.

[39] Note that the argument applies only to non-degenerate plural terms, such as “Russell and Whitehead”, which is not degenerate because Russell is not Whitehead. I think that the degenerate plural term “Cicero and Tully”, for example, refers to an object: Cicero (i.e., Tully).

[40] “Frege and *Russell and Whitehead*” is the term resulting from flanking “and” with “Frege” and “Russell and Whitehead”, not the one resulting from combining “and” (as a three-place connective) with the three singular terms “Frege”, “Russell”, and “Whitehead”. So it has the plural term “Russell and Whitehead” as a component, and results from substituting this term for “Genie” in “Frege and *Genie*”. Those with qualms about “Frege and *Russell and Whitehead*” may consider “Frege and the authors of *PM*” instead. (“Genie is one of the authors of *PM*” is logically equivalent to “Genie is an author of *PM*”.)

[41] See *Th. 6* in Yi (LMP II, Section 7).

[42] The argument as presented above seems to rest on the additional assumption that the composite object in question is neither Russell nor Whitehead nor Frege, but it is not an assumption that one can challenge to defend the standard view. Moreover, we can improve the argument to eliminate the assumption (see *Th. 3* in Yi (LMP II, Section 7)). See Yi (preprint) for further discussion and refinements of the argument. See also Yi (1999b, pp. 146–9).

[43] Dummett attributes this view to Frege (*ibid.*). As Oliver (1994) points out, it is not a view that Frege had consistently, if ever, held (see note 30). Frege (1884) held, however, that numerical sentences involving plurals (e.g., "The Kaiser's carriage is drawn by *four* horses") can be analyzed as attributing numbers (e.g., four) to the 'concepts' indicated by predicates, although I think that he eventually replaces this analysis with one that analyzes them as attributing 'numbers' (i.e., properties involving numbers, which he takes to be objects) to the *extensions* of 'concepts'. He analyzed the above-mentioned sentence as attributing four to the 'concept' indicated by the predicate "is a horse that draws the Kaiser's carriage" (1884, Section 47). Dummett adds that "no one has subsequently found an improvement" on this analysis of the sentence (1991, p. 93). But the analysis has an obvious problem: it is acceptable only when one reads the sentence as meaning that there are four horses *each of* which draws the Kaiser's carriage, whereas there is another (and I think usual) reading, on which it means that there are four horses that together draw the Kaiser's carriage. Fregeans would need to resort to a second-order analogue of the predicate phrase "is drawn by" as used in the sentence on the second reading (see the discussion, below, of "*to* lift" in [5]), but this subjects them to the objections presented below to the schemes resorting to plenary second-order languages.

[44] Schein rejects the standard view. He calls it "the *objectual* view of plurals", and formulates it as the view that "there are plural objects", where he uses "plural object" to mean *object that a* (*non-degenerate*) *plural term refers to* (1993, p. 4).

[45] Similarly, Lewis says "Boolos identifies plural quantification with ... second-order quantification" (1991, p. 70), and Quine that Boolos "shows that [second-order logic] can be interpreted as a mere regimentation of plurals" (1991, p. 224). See Boolos (1984, 1985a, 1985b) for his discussion of plurals and second-order logic. I do not think that he, in any of these works, explicitly propounds the views that Schein and Lewis attribute to him or one that Quine suggests. His primary aim in the works, as in his earlier discussion of second-order logic in Boolos (1975), is to defend the logical status of second-order logic against Quine's charge that it is "Set theory in sheep's clothing" (Quine, 1970, p. 66). To do so, he proposes a deviant interpretation of monadic second-order 'languages' and clarifies limitations of elementary languages. He proposes to interpret second-order *formulas* by reading them using natural language plurals, and argues for the limitations of elementary languages in coping with plurals (see note 12 for my use of "formula" and "language"). But he gives no theory of the logic or semantics of the plurals used to interpret second-order formulas, and his discussion of the limitations of elementary languages vis-à-vis plurals proceeds by rendering them to second-order formulas. This can lead one to attach undue significance to the second-order analogues of plurals in considering their syntax, semantics, or logic and to assume the undefended views mentioned by Schein or Lewis. Notice that Boolos himself says that "The rocks rained down" involves "a new sort of predication" (1985b, p. 343), by which he means that the sentence involves second-order predication (viz. one that involves the second-order predicate amounting to "rained down"), whereas the Geach–Kaplan sentence, [13], involves only first-order predication because the second-order variable "X" in its second-order analogue [14] occurs only in the predicate position (compare [14] with [5b], the second-order analogue of [5]). One cannot hold this view without identifying the syntax of plurals with that of their higher-order analogues. Both plural constructions, on my view, involve the same kind of predication (viz., *plural predication*), combining of a plural term with *plural predicates*: "rain down" and "admire only one another" (see the discussion of plural predicates in Section 3.2). The difference lies in that "admire only one another" is a complex predicate that one can analyze using "admires" and "is one of" (see note 28), but the analysis of [13] that results from analyzing

the predicate does not eliminate plural predication because it requires another plural predicate (viz. "is one of"), whose second argument place is filled with the plural term "them" to yield the analysis.

[46] I add "it might seem" because one cannot find a predicate (of the desired kind) that corresponds to, e.g., "those who write *PM*" or "the (two) children who lift Bob upstairs".

[47] On this view, "is one of" is akin to the verb "is" (or "*to* be") used in "John *is* happy", "John *is* a human", etc. (On my view, by contrast, "is one of" is a robust predicate on a par with the identity predicate. See Section 3.2.3 below.) Note that Russell takes plural definite descriptions (e.g., "the humans") as referring to classes (e.g., the class of humans) and uses the membership predicate "ε" to indicate predication. This yields the view that "John *is one of* the humans", for example, is merely an alternative form of "John *is* a human" (and so is "John *is a member of* the class of humans"). See, e.g., Whitehead and Russell (1910–3, Vol. I, p. 25), who mention Peano (1889, p. 25) as relating membership to predication, and Russell (1919, p. 181).

[48] The usual elementary languages or their higher-order extensions have no complex predicates, but we can augment them with refinements of such predicates. The "λ"-operator, for example, applies to sentences (e.g., the open sentence "$[x = \mathrm{r} \vee x = \mathrm{w}]$") to yield complex predicates (e.g., "$\lambda x[x = \mathrm{r} \vee x = \mathrm{w}]$"). (Note that in languages with such predicates, the predication "$\lambda x[x = \mathrm{r} \vee x = \mathrm{w}](\mathrm{g})$" can be distinguished from the disjunction "$[\mathrm{g} = \mathrm{r} \vee \mathrm{g} = \mathrm{w}]$". The former contains a complex predicate, the latter only the simple predicate "$=$".) We may take, e.g., "$\langle$is Russell or Whitehead$\rangle$" to be a casual form of "$\lambda x[x = \mathrm{r} \vee x = \mathrm{w}]$".

[49] So, as Lønning (1996, p. 1050) notes, Boolos cannot paraphrase [5] or [6] into the second-order languages that he uses to render such plurals as [13]. See the discussion of Boolos's example "The rocks rained down" in note 45. By "second-order language", Boolos seems to mean *meager* second-order languages (see note 14).

[50] Schein (1993) takes plural terms to be predicates and presents a second-order scheme that does without second-order predicates. The scheme renders, e.g., "they lift Bob" to a singular quantification over events: "$\exists e[e$ is-a-lifting-event $\wedge \forall x(\mathrm{A}(e, x) \leftrightarrow T(x)) \wedge \forall x(\mathrm{O}(e, x) \leftrightarrow x = \mathrm{b})]$", where "$T$" is a second-order variable amounting to the plural pronoun "they" and "$\mathrm{A}(e, x)$" and "$\mathrm{O}(e, x)$" abbreviate "x is-an-agent-of e" and "x is-an-object-of e", respectively. The scheme leads to a contradiction (a version of the so-called Russell's paradox), assuming that there are at least two objects, because it requires that given any objects (e.g., Russell and Whitehead), which are as many as themselves, there be one single object (viz. an event witnessing that they are as many as themselves) that 'represents' them (as taken together). For the proof of this, see Yi (1999a, p. 186, note 34).

[51] So "LIFT" is a second-order predicate whose first argument place admits first-order predicates. Note that [5b] can be taken to abbreviate "$\exists X[\forall x(X(x) \leftrightarrow [x = \mathrm{j} \vee x = \mathrm{c}]) \wedge \mathit{LIFT}(X, \mathrm{b})]$".

[52] I think that it is right to distinguish second-order sentences from their elementary language analogues by distinguishing predicable entities from objects. The distinction that I draw between objects and predicable entities (e.g., properties) is tantamount to Frege's distinction between objects and 'concepts'. See Yi (1999b, p. 168) and Frege (1892).

[53] And they deny that there is an object (composite or not) that the plural term "Russell and Whitehead" in, e.g., [19] refers to; the predicate amounting to the term indicates a property.

[54] "*COOPERATE*" is a 1-place second-order predicate tantamount to "*to* cooperate".

[55] This is the key idea behind Russell's "No-Class Theory", on which the talk of sets (or "classes") can be reduced to the talk of properties (or "propositional functions"). See, e.g., Whitehead and Russell (1910–3, Vol. I, p. 75ff), and Russell (1919, p. 187f).

[56] "*WRITE*" is the second-order analogue of "*to* write"; its first argument place admits first-order predicates.

[57] The conception is implausible given such properties as the above-mentioned.

[58] Or that [24a] and [27a] logically imply [28a].

[59] Similarly, it is one thing to say that the principle of extensionality of sets is true, quite another to say that it is a logical or analytical truth. (My view is that it is not an analytic truth.)

[60] Russell says, "We will call a statement involving a function . . . an 'extensional' function of the function . . . if its truth-value is unchanged by the substitution of any formally equivalent [i.e., coextensive] function; and when a function of a function is not extensional, we will call it 'intensional' " (1919, p. 186). He thinks that "I believe that all men are mortal", for example, is a second-order predication that attributes an 'intensional function', related to the verb "believe", to the 'functions' indicated by the predicates "is human" and "is a mortal" (*ibid*.). This view must be distinguished from the view that the verb generates an 'intensional' (viz. *opaque*) context, where substitutivity of co-referential expressions fails. Russell, unlike those who hold the latter view, thinks that the sentence mentioned above can be analyzed as a second-order predication; the 'intensionality' of "believe" arises, on his view, only because "two formally equivalent functions may not be identical" (*ibid*.). I think that the sentence cannot be analyzed as a second-order predication because it involves opacity (see Yi, 1999a, p. 186f), but I also think that Russell's reason for holding that higher-order functions cannot be assumed to be 'extensional' is correct. Note in passing that Russell holds that "the functions of functions with which mathematics is specially concerned are all extensional" (1908, p. 172) because, on his view, they are higher-order functions of special kinds – functions (or quantifiers) whose definitions yield their 'extensionality'. See also Whitehead and Russell (1910–3, Vol. I, p. 21f).

[61] So it is wrong to paraphrase, e.g., [24] as follows:

$$\exists X(\forall x[X(x) \leftrightarrow \langle \text{is Russell or Whitehead}\rangle(x)] \wedge \forall Y[\forall x(X(x) \leftrightarrow Y(x)) \\ \rightarrow \textit{COOPERATE}(Y)]).$$

This paraphrase rests on the assumption that "*COOPERATE*" indicates an extensional property.

[62] Some languages (e.g., Korean) have two different words that amount to the sentential and term connective uses of the English word "and".

[63] It is this analysis of the composition of [5] that gives the ultimate justification for the view that "and" is used in [5] as a term connective that yields "John and Carol" as an integral component of the sentence.

[64] This sentence occurs in natural paraphrases of [4] or [6] (e.g., "There are some children and *they lift Bob*", or "There are some things and *they lift Bob*").

[65] See note 24.

[66] We may also say that a predicate is *neutral* (or *exclusively plural*), if some of its argument places are neutral (or exclusively plural). Then some predicates are both neutral and exclusively plural.

[67] One could regard them as neutral predicates that happen to take only singular forms when used in elementary languages. But I think that they are designed as singular predicates by those who have developed the languages (and regarded as such by those who have

learned the languages from them). Their thought, I think, is not that elementary languages happen to have no plural terms although they have predicates capable of combining with them, but that there cannot possibly be any such predicates. This, I think, explains why they think that apparent plural terms of natural languages *must* be paraphrased away.

68 So we can use the same predicate to paraphrase sentences that contain the singular form of "*to* lift". [29], for example, can be paraphrased by "John **lift** Bob (in symbols, **L**(j, b))." (Note that the neutral predicates of plural languages, unlike their English counterparts, do not take different forms to combine with singular or plural terms.) See the last paragraph of Section 3.2.2.

69 Some might hold that at least some predicates of English (e.g., "*to* cooperate", or "*to* play together") are exclusively plural. Even granting this, not much is lost by using neutral predicates to paraphrase them into regimented languages. And, as noted above, it is straightforward to add exclusively plural predicates to my plural languages.

70 I use "distributive" differently from the way it is sometimes used in semantics. The predicate "*to* be red", for example, is distributive (over the term connective), as I use the term (for "John and Carol are red" and "John is red and Carol is red" are logically equivalent), but not "distributive" in the sense that "John is red" and "Any part of John is red" are equivalent. See also Link (1983, p. 309), who uses the term in yet another way. My use of the term is analogous to the mathematical use: multiplication is said to distribute over addition, because, e.g., $2 \times (3 + 5) = 2 \times 3 + 2 \times 5$.

71 In this paper and its sequel, I use "some things" or "any things" interchangeably with "some *one or more* things" or "any *one or more* things" (not with "some two or more things" or "any two or more things"). I think that this is how those quantifier phrases are actually used in English (see the discussion of *Ax. 6* in Yi (LMP II, Section 7)). Some might disagree, but even those who do so should have no difficulty in understanding what I mean by them.

72 The converse does not hold. There are distributive predicates that are not *neutral expansions*, such as "*to* be finitely many human beings" or "*to* be one or many humans".

73 Link, I think, has essentially the same idea in introducing the operator "*" (1983, p. 306) or "the distributivity operator 'D' " (1991, p. 52) that applies to 1-place predicates. But he does so in the context of a traditional approach to plurals (see note 30): the predicates "*C" and "^{D}C" are available in elementary languages, but have wider extensions than "C" ("*C" is true of any aggregate or "individual sum" composed of some things that "C" is true of, and "^{D}C" of any aggregate composed of some atomic things that "C" is true of). So Link cannot explain that [7] and [9] are *logically* equivalent. His paraphrase of [7] that involves "*C" (or "^{D}C") cannot be shown to be equivalent to [9a] in his mereological system that he calls "The Logic of Plurality" (see note 35). See also Kamp and Reyle (1993, p. 327), and Lønning (1996, p. 1031f).

74 We can then take "N" in, e.g., "is-a-child^{N}" to abbreviate "$N(1)$" in, e.g., "is-a-child$^{N(1)}$".

75 It is to be understood that i and n are natural numbers such that $1 \leq i \leq n$. I use Greek letters (with or without subscripts) as metavariables. See the 4th paragraph of Section 4 of Yi (LMP II).

76 See the discussion of neutral expansion in the second paragraph of Section 3.2.3, and the discussion of *Ths. 4–5* in Yi (LMP II, Section 7). Using *Th. 4*, we can also show that "[C(j) ∧ C(c)]", the usual paraphrase of [9], is logically equivalent to [7a], and, similarly, that the usual paraphrase of [31] is logically equivalent to [30a].

77 This does not yield a proper definition of "$\mathbf{L}^{S(1)}$" unless the second argument place of "**L**" is singular, but it is straightforward to modify it to obtain proper definitions of singular reducts. See *Def. 3* in Yi (LMP II, Section 4).

78 For a formulation of the logic of singular reducts, see Yi (1995, Chapter 2).

79 "**H**" is the uppercase eta in boldface. The predicate must be clearly distinguished from the singular predicate "is-a-member-of" (in symbols, "ε") that indicates the membership relation that a member of a set has to the set. This is a non-logical predicate available in elementary languages.

80 I think that "John and Carol *is one of* the children in London" and "John and Carol *are one of* the children", unlike "John and Carol *are among* (or *some of*) the children in London", are not grammatically correct constructions. So I distinguish "**is-one-of**" from "**be-among**" (or "**be-some-of**"), whose first argument place, too, is neutral. The latter can be defined as the neutral expansion of the former (see below).

81 One might object that the English predicate "*to* be one of" accepts only a plural term into its second argument place. If so, "**is-one-of**" can be introduced as the plural language counterpart of the predicate "is-one-of" that one can define using the English predicate as follows:

S is-one-of $T \equiv_{\mathrm{df}}$ anything is such that S is one of [T and it],

where "S" can be replaced with any singular term, and "T" any term (singular or plural).

82 We can use it to paraphrase [13], "There are some critics who admire only one another", as well, but this requires plural quantifiers and variables.

83 So in plural languages proper, which have no complex predicates, sentences containing neutral expansions are considered abbreviations of their definitions. See *Def. 3* in Yi (LMP II, Section 4).

84 See [22] and [23] in Section 2.2, and *Th. 6* in Yi (LMP II, Section 7).

85 See *Ths. 5–6* in Yi (LMP II, Section 7).

86 The usual distinction between two uses of "is" applies to its plural form "are" as well. The singular form "is" is used as the identity predicate in "Cicero *is* Tully", but merely as the so-called copula that indicates predication (or, on my view, as a part of a primitive predicate that is not significant on its own) in "Cicero *is* dead." Similarly, its plural form "are" is used in the sentences mentioned above differently from the way it is used in "John and Carol *are* alive." Just as the singular predicate "$=$" amounts only to the first use of "is", so does the neutral predicate "$\boldsymbol{\approx}$" amount only to the first use of "are" (or, more precisely, the predicate "*to* be").

87 The sentence must be distinguished from "Cicero and Russell are Russell and Tully*, respectively*." The former is true, but the latter false.

88 As refinements of plural pronouns, plural variables are primitive (viz. simple) expressions. But I include the italicized "*s*" in them to emphasize their kinship to, as well as distinctness from, their elementary language cousins. I have taken this notation over from Gödel (1947, p. 475, note 11), who used it presumably for a different purpose, in Yi (1995). See also van Inwagen (1990, p. 25), who introduces plural variables using the same notation. Kamp and Reyle (1993, Section 5) might seem to introduce similar variables, ones that correspond to plural pronouns used anaphorically (*ibid*., p. 339ff), but their singular and plural variables are not primitive expressions but only devices for abbreviating sentences that contain the primitive, 'neutral' variables and the sortal predicate "atomic" (*ibid*., p. 332). So their languages turn out to have only one kind of quantifiers and variables, and not to distinguish singular and plural argument places. The languages are

elementary languages (not proper extensions thereof) that include the singular predicates "atomic" and "is-a-part-of".

89 Here I agree with Kamp and Reyle (1993) and van Inwagen (1990, p. 25), and reject the regrettably influential view proposed by Kaplan (1989, p. 485). Kaplan assimilates variables to pronouns used demonstratively, and suggests that they are paradigmatic examples of the so-called *directly referential terms*. I think that variables are not even referential terms. They result from refining pronouns used anaphorically, but these must be distinguished clearly from pronouns used demonstratively (which I think are referential terms).

90 One might imagine quantifiers of another kind, *neutral* quantifiers, whose first argument places admit both singular and plural variables. The idea is that neutral quantifiers may take singular or plural *forms* depending on whether their first argument places are filled with singular or plural variables. I do not include such quantifiers in plural languages, but this decision can be considered a matter of convenience in exposition. (It is another question whether quantifiers of elementary languages can be considered neutral quantifiers that *happen* to take only singular forms in elementary language sentences. One could perhaps understand them as such, but I doubt that the architects or users of the languages have done so.)

91 It is wrong to conclude from this that [8] involves no 'genuine' plural constructions. In [8], as in [4], the quantifier "some" combines with a noun and predicate of the plural form.

92 See the discussion of *Th. 2* in Yi (LMP II, Section 7).

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