



# Ignorance and concession with superlative modifiers: a cross-linguistic perspective

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**Abstract** Superlative modifiers (SMs) are known to demonstrate an ambiguity between an epistemic reading (EPI) conveying speaker ignorance and a concessive reading (CON) conveying speaker concession. Such EPI-CON ambiguity has often been taken, implicitly or explicitly, to be a lexical coincidence. While there may be some justification for such a position when a single language is considered, we argue for an intrinsic connection between the two readings based on cross-linguistic considerations. This paper focuses on English *at least* and Mandarin *zhi-shao* as representative of superlative modifiers across a wide range of languages to propose a unified account of the two readings. The proposal builds on Biezma (2013) in relying on the role of focus and scalarity in developing a unified semantics for the two readings, but differs in capitalizing on the fact that cross-linguistically superlative modifiers use the same morphological formants as quantity superlatives. It also follows Biezma (2013) in taking pragmatic factors as crucial in deriving the variation between EPI and CON readings. Elaborating on her account, it offers a more nuanced picture of the ways in which EPI is sensitive to the question of informativity while CON relates to issues of evaluativity. The paper shows how the proposed semantics and pragmatics account for several well-known properties of superlative modifiers. It ends by noting several open issues in the literature on this topic that the current proposal sheds new light on.

**Keywords** Superlative modifiers · Scalar particles · Focus · Alternative semantics

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## 1 The two readings of superlative modifiers

In this section I introduce the core phenomenon that is the focus of this paper, the epistemic and concessive readings of the superlative modifier *at least* and its correspondents in other languages. I briefly discuss three earlier proposals and argue that from a cross-linguistic perspective a unified account of the phenomenon is desirable.

### 1.1 Epistemic and concessive readings across languages

Nakanishi and Rullmann (2009) (henceforth N&R) observe that English sentences containing *at least* can have two readings: an epistemic reading (EPI) and a concessive reading (CON), the first conveying ignorance on the part of the speaker about the actual state of affairs and the second conveying an evaluation by the speaker about a settled fact. Assuming a context in which there are three medals, the highest being gold, followed by silver and then bronze, the examples in (1) can be used to illustrate the distinction. EPI conveys that the speaker is sure that Mary won either a silver medal or a gold medal, but cannot say anything more definitive. CON conveys that Mary's winning a silver, while less desirable than winning a gold medal, is better than her winning a bronze.<sup>1</sup>

- (1) a. **EPI:** Mary won **at least** a [silver]<sub>F</sub> medal (for all I know, she may have won a gold medal).  
 b. **CON:** (Mary didn't win a gold medal, but) **at least** she won a [silver]<sub>F</sub> medal.

As we can see, the use of *at least* signals that the prejacent is interpreted against other possible propositions and that these propositions are ranked in order of preference. N&R note that CON has a "settle-for-less" flavor in the sense that it falls short of the intended goal of winning the gold. EPI is neutral in the sense that there need not be any prior expectations about what Mary would win, or whether she would win anything. The ambiguity is also in evidence in statements with numerals, where the logic of numbers determines the ranking between the relevant propositions. Assuming that we are talking about a game of dice where the minimum someone can get is 1 and the maximum is 6, (2a) is consistent with Mary getting three, four, five or six; Instead, (2b) takes it as settled that she did not get

<sup>1</sup> Kay (1992: 311) distinguishes three uses of *at least* in English: a scalar use, an evaluative use and a rhetorical use. The three uses are illustrated in (i).

- (i) a. Mary received calls from **at least** three soldiers.   Scalar  
 b. **At least**, this one is cooked.   Evaluative  
 c. I see her every day, **at least** when I'm in town.   Rhetorical

Kay's scalar use corresponds to N&R's EPI and his evaluative use corresponds to N&R's CON. This paper is concerned with these two uses of *at least* and its cross-linguistic counterparts. I believe that the analysis to be proposed in this paper can be extended to the rhetorical use but I do not make specific claims about it here.

four or more, which would have been better, but indicates that she could have got one or two, which would have been worse:

- (2) Mary got **at least** three. **EPI**  
**At least** Mary got three. **CON**

More generally, while the source of the ranking between the preadjacent and the alternatives may vary, the contribution of *at least* remains fixed. It leads to two distinct readings, EPI and CON.

The question that arises in cases like this is whether the two readings represent a case of accidental homonymy or whether there is a deeper connection between them. Cross-linguistic considerations are often used in such cases to arbitrate between the two possibilities. It turns out that the availability of EPI and CON holds across many languages, making a strong case for treating EPI and CON as having a common core of meaning.

Grosz (2011) has noted the existence of the EPI-CON ambiguity of superlative modifiers in Greek (*tulachiston*), Hebrew (*le-faxot*), Czech (*aspoň*) and Spanish (*al menos*). Based on an informal survey, I have found the same ambiguity replicated in Brazilian Portuguese (*pelo menos*), Dutch (*tenminste*), French (*au moins*), Hindi (*kam se kam*), Italian (*almeno*), Indonesian (*paling-sedikit*), Japanese (*sukunakutomo*), Korean (*cek-eto*), Magahi (*kam se kam*), Mandarin (*zui-shao*), Russian (*po krajnej mere*), Turkish (*en azından*), and Vietnamese (*ít-nhất*). The relevant lexical items in all these languages lead to EPI and CON readings. The following are some representative examples.<sup>2</sup>

- (3) Liubei **zhi-shao** mai-le [san]<sub>F</sub>-ke pingguo. Mandarin  
 Liubei at least buy-ASP three-CL apple  
 ‘Liubei at least bought three apples.’
- (4) Jaun [tin]<sub>F</sub>-go Seo **kam se kam** kharidlai. Magahi  
 John three-CL apple less than less bought  
 ‘John at least bought three apples.’
- (5) John **en az-in-dan** [üç]<sub>F</sub> elma al-dı. Turkish  
 John SUP little-3SGPOSS-ABLATIVE(from) three Apple buy-PAST  
 ‘John at least bought three apples.’
- (6) John **ít-nhất** đã-mua [ba]<sub>F</sub>-trái-táo. Vietnamese  
 John SUP-little ASP-buy three-CL-apple  
 ‘John at least bought three apples.’

<sup>2</sup> Abbreviations: ACC for accusative case; ASP for aspect marker; CL for classifier; MOD for modification marker; NOM for nominative case; PAST for past tense; SFP for sentence-final particle; SUP for superlative morpheme; TOP for topic marker; 3SG for third person singular.

Interestingly, not only does the ambiguity repeat itself across genetically-unrelated languages, it also repeats itself with synonymous items within languages. In addition to *zhi-shao* in (3), we find two other Mandarin expressions (*zui-shao* and *qima*) that also lead to EPI and CON readings, as shown in (7).

- (7) Liubei **zui-shao/ qima** mai-le [san]<sub>F</sub>-ke pingguo. **EPI, CON**  
 Liubei at least/ at minimum buy-ASP three-CL apple  
 ‘Liubei at least bought three apples.’

An anonymous reviewer questions the availability of CON readings for Mandarin *zui-shao* in examples like (7). The sentences in (8), due to Beibei Xu (p.c.), help put this doubt to rest. These sentences are embedded in contexts that do not support EPI readings and yet they are fully acceptable, clearly under the concessive reading. I take this to be compelling evidence of the possibility of concessive readings with all three items.<sup>3</sup>

- (8) a. Jishi/Jiusuan ni bu-yong zhengde  
 wo de tongyi,  
 Even.if/ Even.though you not-need ask.for  
 I POSS permission  
**Zui-shao/ zhi-shao/ qima,** ni de zhihui  
 wo yi-sheng ba!  
 At least you have.to inform  
 I one-sound SFP  
 ‘Even though you don’t need to ask for my permission, at least, you have to inform me about what happened.’
- b. **Zui-shao/zhi-shao/ qima** yao gei ta  
 yi-dian yanse kankan!  
 At least should give him  
 a.little color look  
 ‘At least we should teach him a lesson!’
- c. Ni **zui-shao/zhi-shao/qima,** yao zhengde  
 wo de tongyi!  
 You at least should ask.for  
 I POSS permission  
 ‘You should at least ask for my permission!’

<sup>3</sup> Although both *zui-shao* and *zhi-shao* can convey the concessive reading, the latter is much more frequently used with respect to the concessive reading. This may be the source of an anonymous reviewer’s comment that *zui-shao* lacks the CON reading. The reviewer also notes that *zh-ishao* cannot be used as a quantity superlative. I address this in Sect. 3.3 when we look at further issues related to the morphology-meaning mapping.

To return to the cross-linguistic evidence for a unified account, there is evidence from multiple lexical variants in English as well. *At minimum* also conveys speaker ignorance and speaker concession, as shown in (9):<sup>4</sup>

- (9) a. Every electoral district has **at minimum** three seats in **EPI**  
the chamber.  
b. **At minimum**, you should warn your neighbors the building **CON**  
was on fire.

This further reinforces the point that the EPI-CON ambiguity cannot be accidental and must arise from the core meaning of a superlative modifier.

## 1.2 More cross-linguistic evidence

The EPI and CON readings of the superlative modifier *at least* share three common properties: focus sensitivity, scalarity and end-of-scale effects. These properties have been noted for English, mostly in relation to EPI. They have not been systematically discussed in relation to CON, neither have they been discussed in connection with superlative modifiers in other languages. Here I compare English and Mandarin to argue for their cross-linguistic validity but my claim goes beyond these two languages. They are expected to hold in the other languages mentioned in Sect. 1.1. as having lexical items conveying both readings.

Starting with the first common property, focus-marking makes explicit the piece of information that the speaker's ignorance or concession is about. It has been well-established in the literature that EPI conventionally associates with focus (see e.g., Krifka, 1999). For example, (10a) and (10b) are truth-conditionally distinct: (10a) conveys the speaker's ignorance about *who John invited*, while (10b) conveys the speaker's ignorance about *what John did*.<sup>5</sup>

- (10) a. John **at least** invited [Bill]<sub>F</sub>. EPI  
b. John **at least** [invited Bill]<sub>F</sub>. EPI

This truth-conditional difference in turn is related to question-answer congruence. Specifically, (10a) is a felicitous answer to the question in (11a) but not to (11b); in contrast, (10b) is a felicitous answer to the question in (11b) but not to (11a).

- (11) a. Who did John invite?  
b. What did John do?

<sup>4</sup> I thank an anonymous reviewer for bringing (9a) to my attention. The sentence in (9b) was found on a Google search.

<sup>5</sup> In English, a focus associate typically bears prosodic prominence, with the so-called *A*-accent (see e.g., Jackendoff, 1972; Büring, 2003; Beaver and Clark, 2008: chapter 2). Thus, (10a) and (10b) are not only truth-conditionally different, they are also prosodically distinct: it is the noun *Bill* in (10a) that bears the pitch accent, while it is the verb phrase *invited Bill* as a whole that bears the pitch accent in (10b).

It is worth highlighting that CON is similarly focus-sensitive, as shown in (12), where the two sentences are truth-conditionally different: (12a) conveys the speaker's concession about *who John invited*, while (12b) conveys the speaker's concession about *what John did*. As noted in Sect. 1, we can identify this reading by the fact that the status of a higher alternative, in this case *inviting Adam* or *cooking dinner*, is settled in the discourse:

- (12) a. John **at least** invited [Bill]<sub>F</sub>. CON  
 b. John **at least** [invited Bill]<sub>F</sub>. CON

Again, this truth-conditional difference (varying with the position of focus) is related to question-answer congruence, as shown in (13).

- (13) a. Who did John invite? Did John invite Adam?  
 i. No, but **at least** he invited [Bill]<sub>F</sub>.  
 ii. #No, but **at least** he [invited Bill]<sub>F</sub>.  
 b. What did John do? Did he cook the dinner?  
 i. #No, but **at least** he invited [Bill]<sub>F</sub>.  
 ii. No, but **at least** he [invited Bill]<sub>F</sub>.

The responses in (13a-i)/(13b-i) convey the speaker's concession about *who John invited*, while (13a-ii)/(13b-ii) convey the speaker's concession about *what John did*. As in the case of epistemic *at least*, the truth-conditional difference between the (i)-examples and the (ii)-examples in (13) impacts the question-answer congruence. In (13a), the main discourse question (super-question) concerns *who John invited*, while that in (13b) it concerns *what John did*. Imagine a scenario where Adam, Bill and Chris are the three relevant individuals in the discourse, and there is a priority ranking on the invitation: *Adam > Bill > Chris*. In this scenario, (13a-i) is a felicitous answer to the question in (13a), but (13a-ii) is not. Similarly, if there is a priority ranking on the to-do list for John: *cook the dinner > invite Bill > clean the house*, (13b-ii), but not (13b-i), is a felicitous answer to the question in (13b).<sup>6</sup>

We can demonstrate the same focus-sensitivity in Mandarin as well, and with respect to all three variants of *at least*. The Mandarin example (14a) conveys speaker ignorance or concession about *who Liubei invited*, while (14b) does so about *what Liubei did*:

<sup>6</sup> An anonymous reviewer points out that the stress may fall on the particle *at least* itself and wonders whether the proposed analysis can capture the pattern of stress in sentences with *at least*. Unfortunately, the current study, like some of the previous studies of focus adverbs, does not delve into the issue of stress pattern concerning the interface of phonology-semantics, and thus I do not have much to say here. However, it is certainly an important line of research concerning whether the two readings can be further teased apart based on the stress pattern of *at-least* sentences. We may then have not only a syntactic cue (the syntactic position of *at least*; see Sect. 3.1) but also a prosodic cue (the stress pattern) to further distinguish the two readings. I thank the reviewer for raising the question of stress pattern.

- (14) a. Liubei **zui-shao/ zhi-shao/ qima** yaoqing-le [Bill]<sub>F</sub>. **EPI, CON**  
 Liubei at least/ at least/ at minimum invite-ASP Bill  
 ‘Liubei at least invited Bill.’
- b. Liubei **zui-shao/ zhi-shao/ qima** [yaoqing-le Bill]<sub>F</sub>. **EPI, CON**  
 Liubei at least/ at least/ at minimum invite-ASP Bill

The second common property of EPI and CON is that both meanings are compatible with various scales. The fact that EPI has this property is well-established (see e.g., Mendia, 2016a). The key observation here is that CON is also compatible with various scales, and this holds in both English and Mandarin. The following are some examples that illustrate this property:

- (15) a. **Numeral Scales** (a contextual ranking:  $4 > 3 > 2$ )  
 John **at least** wrote [three]<sub>F</sub> novels. **EPI, CON**
- b. **Plurality Scales** (a contextual ranking:  $adam \oplus bil \oplus chris > adam \oplus bill > adam$ )  
 John **at least** hired [Adam and Bill]<sub>F</sub>. **EPI, CON**
- c. **Lexical Scales** (a contextual ranking:  $gold\ medal > silver\ medal > bronze\ medal$ )  
 John **at least** got a [silver]<sub>F</sub> medal. **EPI, CON**
- d. **Pragmatic Scale s** (a contextual ranking:  $cherries > apples > bananas$ )  
 John **at least** bought [apples]<sub>F</sub>. **EPI, CON**
- (16) a. **Numeral Scales**  
 Liubei **zhi-shao** xie-le [san]<sub>F</sub>-ben-xiaoshuo. **EPI, CON**  
 Liubei at least write-ASP three-CL-novel  
 ‘Liubei at least wrote three novels.’
- b. **Plurality Scales**  
 Liubei **zhi-shao** guyong-le [Adam he Bill]<sub>F</sub>. **EPI, CON**  
 Liubei at least hire-ASP Adam and Bill  
 ‘Liubei at least hired Adam and Bill.’
- c. **Lexical Scales**  
 Liubei **zhi-shao** na-le [yin]<sub>F</sub>-pai. **EPI, CON**  
 Liubei at least take-ASP silver-medal  
 ‘Liubei at least got a silver medal.’
- d. **Pragmatic Scales**  
 Liubei **zhi-shao** mai-le [pingguo]<sub>F</sub>. **EPI, CON**  
 Liubei at least buy-ASP apple  
 ‘Liubei at least bought apples.’

Finally, both EPI and CON display End of Scale Effects: *at least* cannot be used to modify propositions at the highest or the lowest end of the scale, which I call the Top of the Scale Effect (TSE) and Bottom of the Scale Effect (BSE) and refer to

them jointly as End of Scale Effects.<sup>7</sup> For the scale with the three medals, for example, (16a) and (16b) are both unacceptable, regardless of the reading involved, in a context where it is known that Mary has won a medal and the inquiry is about what kind of medal Mary has won.

- (17) a. #Mary **at least** won a [gold]<sub>F</sub> medal. **Top-of-the-scale Effect (TSE)**  
 b. #Mary **at least** won a [bronze]<sub>F</sub> medal. **Bottom-of-the-scale Effect (BSE)**

Similarly, consider the scenario in (18). We see TSE for EPI and CON in (19) and BSE for the two readings in (20):

- (18) Scenario: Adam, Bill and Chris are playing dice. In each round, whoever gets a bigger number wins; scores are not cumulated. A dice has six numbers on it: Six is the upper bound and one the lower bound. Chris threw the dice but Adam missed the result. During his turn, Adam asks about the result.

- (19) Adam: What number did Chris get?  
 a. Bill: #I don't know for sure but Chris at least got [six]. **EPI**  
 b. Bill: #Although he could have done worse, Chris at least got [six]. **CON**
- (20) Adam: What number did Chris get?  
 a. Bill: #I don't know for sure but Chris at least got [one]. **EPI**  
 b. Bill: #Although he didn't get any of the high numbers, Chris at least got [one]. **CON**

Crucially, BSE and TSE are both attested in Mandarin as well. Under the same scenario as (18), neither of Bill's utterances in (21a) is felicitous as an answer to Adam's question, regardless of whether EPI or CON is intended. Like English *at least*, we see BSE in (21a) with *zhi-shao* and TSE with *zhi-shao* in (21b).

- (21) a. #Chris **zhi-shao** shai-le [yi]<sub>F</sub>. **BSE**  
 Chris at least one  
 'Chris at least got one.'  
 b. #Chris **zhi-shao** shai-le [liu]<sub>F</sub>. **TSE**  
 Chris at least dice-ASP six  
 'Chris at least got six.'

These facts from English and Mandarin together indicate that the two end-of-scale effects are systematic and intrinsically connected to SMSs, irrespective of the particular reading involved; they are not language-specific and cannot be ascribed to lexical idiosyncrasies.

<sup>7</sup> To my knowledge, the fact that *at least* demonstrates TSE and BSE on both readings has not been documented before, and exactly how and why TSE and BSE arise with *at least* has thus not been fully explored in previous studies.



To sum up, we have seen in this section three common properties of the two meanings: both EPI and CON are focus-sensitive, compatible with variety of scales, and demonstrate end-of-scale effects. Although I have given examples only from English and Mandarin, to the best of my knowledge, these facts also hold in other languages with superlative modifiers that have both EPI and CON meanings.

### 1.3 Previous accounts of the EPI-CON ambiguity

In this section I review three current accounts of the phenomenon, N&R, Cohen and Krifka (2014), and Biezma (2013). Of the three, the first two can be characterized as taking an ambiguity approach to the two readings and the last as offering a unified account. While there are several other accounts in the literature that bear on EPI and/or CON readings of superlative modifiers, my brief discussion of these three studies will serve to contextualize the proposal that I will develop in Sect. 2. For a fuller discussion and overview, I refer the reader to Sect. 4 of Coppock and Brochhagen (2013) as well as to Mihoc (2019).

#### 1.3.1 Ambiguity accounts

N&R suggests a non-uniform account of English *at least*: one lexical entry for EPI and another for CON. In the case of EPI, N&R adopts Geurts and Nouwen (2007)'s view that the speaker ignorance conveyed by *at least* stems from a covert epistemic modal:<sup>8</sup>

(22) Epistemic *at least* (N&R 2009: slide 16)

a. Truth conditions

$$\exists q \in C [q \geq p \wedge q(w) = 1]$$

“There is a proposition  $q$  which ranks higher than or as high as the prejacent  $p$ , and which is true”

b. Conventional implicature

$$\exists w' [\text{Epist}(w, w') \wedge \exists q \in C [q > p \wedge q(w') = 1]]$$

“It is epistemically possible that some proposition  $q$  that ranks higher than  $p$  is true”

For CON, N&R refer to propositions that are ranked higher as well as lower than the prejacent, with an appeal to conventional implicatures of CON.

(23) Concessive *at least* (N&R 2009: slides 18)

a. Truth conditions

$$p(w) = 1$$

“The prejacent proposition  $p$  is true”

b. Conventional implicatures

<sup>8</sup> Although both studies assume a covert epistemic modal in the semantics of English *at least*, N&R crucially differs from Geurts and Nouwen (2007). The former takes ignorance to be due to a conventional implicature, while the latter takes it to be part of the truth-conditions. We will return to this point in Sect. 2.3.1.

- i.  $\forall r, r' \in C [r' > r \leftrightarrow r' \text{ is preferred to } r]$   
“The scalar ranking reflects a preference ranking”
- ii.  $\exists q \in C [q > p]$   
“There is a proposition q that ranks higher than p”
- iii.  $\exists q \in C [q < p]$   
“There is a proposition q that ranks lower than p”

Another study, explicitly arguing that epistemic *at least* and concessive *at least* are two independent lexical items, is due to Cohen and Krifka (2014). Cohen and Krifka propose that epistemic *at least* is a meta-speech act operator. Informally put, the meta-speech act of a GRANT of a proposition is a denial to assert the negation of the proposition. Thus, by saying *Adam petted at least three rabbits* the speaker conveys that n is the minimal number such that the speaker GRANTS the proposition that Adam petted n rabbits (here n = 3). More specifically, by saying *Adam petted at least three rabbits*, the speaker denies that they can assert that Adam petted zero, one or two rabbits; the hearer then infers that the speaker GRANTS that Adam petted that number of rabbits (i.e., three, four, five... rabbits).

On this view, to interpret epistemic *at least*, one must compute an implicature but because scalar implicatures tend to disappear in downward-entailing environments, it makes a prediction. Cohen and Krifka point out that epistemic *at least* is infelicitous under negation: e.g., *Adam didn't eat at least three cookies*. However, English *at least* can be felicitous in other downward-entailing environments such as the antecedent of conditionals or the restrictor of universals. Crucially, Cohen and Krifka argue that this is a different sense of *at least*, a so-called evaluative sense (concessive *at least* in N&R's term) and that this evaluative sense comes with its own constraints, as shown by the contrast below.

- (24) a. Everyone who donates **at least** 10 BGN will get a thank you postcard.  
b. #Everyone who donates **at least** 10 BGN is a fool.

The important point here is that the evaluative interpretation is sensitive to the polarity of the property in the conversation: (24a) is felicitous because donating 10 BGN is considered a good thing, having a positive polarity; (24b) is degraded because it suggests that donating 10 BGN is a bad thing, having a negative polarity.

Cohen and Krifka's (2014: 81) prediction is that as long as English *at least* carries an evaluative flavor, it should be acceptable in any downward-entailing environment. However, as they note, in negative declaratives, the evaluative interpretation is unavailable, regardless of whether the polarity of the property is understood as positive or negative.

- (25) a. ??The hotel isn't **at least** centrally located.  
b. ??The hotel isn't **at least** far away.

While the behavior of EPI and CON in downward-entailing environments is interesting, not only for what it may tell us about the semantics and pragmatics of superlative modifiers, but also for the light it can shed on the nature of polarity items, to explore it here would take us too far afield. I refer interested readers to Mihoc (2019: chapter 5) for a detailed discussion of issues related to Cohen and Krika’s proposal.

1.3.2 A unified account

To the best of my knowledge, Biezma (2013) is the first study arguing for a unified account of *at least*. In contrast to N&R, Biezma argues that the two meanings of *at least* are pragmatic variants of a single lexical item, for which she proposes the lexical entry shown below (see Biezma, 2013: (22)).

- (26) a. Let  $\alpha$  be a proposition, and  $[\alpha]_{A,i}$  the set of alternatives of  $\alpha$  ordered according to  $\leq_i$ , where  $\leq_i$  is a contextually salient order of alternatives and  $\forall \gamma \in [\alpha]_{A,i}, \gamma \in \text{QUD}$ :  
 $\llbracket \text{at least } \alpha \rrbracket = \lambda w. \exists \beta, \gamma \in [\alpha]_{A,<i}, \text{ s.t. } \gamma <_i \alpha <_i \beta \ \& \ [\alpha(w) \vee \beta(w)] \ \& \ \forall \mu \in [\alpha]_{A,i}, \mu <_i \alpha \ [\neg \mu(w) \vee \alpha \text{ entails } \mu]$
- b. If  $\alpha$  in  $\llbracket \text{at least } \alpha \rrbracket$  is not a proposition,  $\alpha$  is of type  $\langle a, \langle s, t \rangle \rangle$ , where  $a$  is any type  
 $\llbracket \text{at least } \alpha \rrbracket = \lambda X_{\langle a \rangle} \lambda w. \exists \beta, \gamma \in [\alpha]_{A,<i}, \text{ s.t. } \gamma <_i \alpha <_i \beta \ \& \ [\alpha(X)(w) \vee \beta(X)(w)] \ \& \ \forall \mu \in [\alpha]_{A,i}, \mu <_i \alpha \ [\neg \mu(X)(w) \vee \alpha(X) \text{ entails } \mu(X)]$

Under this account, the two readings differ only in whether in a given discourse, the relevant higher alternatives are left open (EPI) or known to be false (CON). Biezma captures the intrinsic connection between the two meanings by encoding a disjunction into the semantics of *at least*:  $\alpha(w) \vee \beta(w)$  in (26a).

Biezma’s analysis is insightful in at least four respects. One, the EPI-CON ambiguity can be systematically derived from one single semantic entry. Two, both meanings are predicted to be focus-sensitive because the unified entry operates on a set of focus alternatives sensitive to different Questions-under-discussion (QUDs), in the sense of Roberts (1996/2012). Three, both meanings are predicted to be compatible with different scales because the relevant ordering is contextually-valued by an assignment function in the unified entry. Four, Biezma suggests that the “settle-for-less” flavor can be derived from two pragmatic ingredients: **(a)** when the set of focus alternatives is evaluated and ranked against the speaker’s goals and interlocutors’ interests in the discourse, and **(b)** when the relevant higher alternatives are known to be false. An additional bonus is that Biezma’s entry also captures the fact that the prejacent is entailed under CON (cf. 23a): given the disjunctive statement made by *at least* (i.e.,  $p(w) \vee q(w)$ ), when the relevant higher alternative is false (i.e.,  $q(w)$  does not hold), the fact that the prejacent  $p$  is entailed is derived (i.e.,  $p(w)$  is true). Above all, an account such as this meshes well with the cross-linguistic perspective that I am advocating in this paper.

## 1.4 Taking stock

There are, of course, other studies that bear on superlative modifiers but the ones surveyed here give us enough background to anchor the proposal I forward in this article. I have argued that the pervasiveness of the phenomenon across diverse languages, and distinct lexical items within languages, favor a unified approach and the one presented in Biezma (2013) serves as a particularly successful example of such an account. There are, however, at least two points that Biezma's account does not provide satisfactory explanations for.

The status of the two end-of-scale effects, TSE and BSE, for example, calls for further clarification. Consider a situation in which it is known that Adam won a medal and the relevant discourse question is *What kind of medal did Adam win?* In this scenario, neither response in (27) is felicitous:

- (27) A contextually-given ranking: *a gold medal* > *a silver medal*  
       > *a bronze medal*
- a. #Adam **at least** won a [gold]<sub>F</sub> medal.   **Top-of-the-scale Effect (TSE)**  
 b. #Adam **at least** won a [bronze]<sub>F</sub> medal.   **Bottom-of-the-scale Effect (BSE)**

Although both sentences in (27) are infelicitous, speakers report an intuitive difference in the nature of the infelicity in the two cases. (27a) with TSE sounds *more unacceptable* than (27b) with BSE. The proposal we develop in Sect. 2 will try to provide an explanation for this intuition.

Secondly, Biezma's account does not pay attention to the cross-linguistically pervasive fact that degree morphology is involved in superlative modifiers. SMs, as a class of focus adverbs, present an intriguing morpho-semantic puzzle: Cross-linguistically, these focus adverbs typically involve quantity adjectives (Q-adjectives) and degree morphemes in their morphological makeup. Taking English for example, the same component *least* is shared in SMs (*Adam won at least three medals*) and quantity superlatives (*Adam drank the least amount of water*). Similarly, we find that the Mandarin expression *zui-shao* demonstrates the EPI-CON ambiguity, but morphologically consists of the superlative morpheme *zui* and Q-adjective *shao* 'little'. I hope to shed more light on this in Sect. 3. Our proposal for the semantics of SMs will take superlative morphology seriously.

In sum, the proposal I develop in Sect. 2 addresses these two gaps in a way that brings together different people's work on EPI and CON in the earlier literature, while capturing the two readings in terms of a unified semantics of *at least* coupled with different pragmatic conditions. After the proposal is fleshed out in Sect. 2, I will turn to some further implications in Sect. 3.

The rest of this paper proceeds as follows. Sections 2.1–2.2 offer a unified semantics of *at least*, using morphology as a guide to meaning and capturing the three common properties of EPI and CON identified above: focus-sensitivity, compatibility with a variety of scales and the two end-of-scale effects. Section 2.3 shows that the two readings deal with different issues: EPI addresses the issue of informativity and CON the issue of evaluativity. Section 3 returns to some open

issues in discussions of *at least* in the literature, assessing them from the perspective of the proposal made in Sect. 2.

## 2 A unified semantics for *at least*

In this section I propose a unified semantics for the superlative modifier *at least* and show how the proposal captures the three core properties of SMs shown in Sect. 1 to hold cross-linguistically: focus sensitivity, compatibility with a variety of scales and end-of-scale effects. In fleshing out the account, I try to connect the details of my proposal with earlier proposals in the literature, making it clear where I draw on the insights of other researchers and where I depart from them. After presenting core aspects of the proposal in Sects. 2.1 and 2.2, I turn in Sect. 2.3 to those aspects of meaning that depends on the pragmatic profile of SMs.

### 2.1 A Superlative-based semantics for *at least*

The following is a proposal for English *at least* and its Mandarin counterparts such as *zhi-shao*, and by extension for other items with the same profile. We focus on the case where the prejacent is a proposition.<sup>9,10</sup>

(28) **A propositional version of *at least***

$$\llbracket \textit{at least} \rrbracket^{w,c} = \lambda \alpha_{\langle s,t \rangle} . \exists \gamma [\gamma \in \text{SUP}(C, \alpha) \wedge \gamma_w]$$

where  $\text{SUP}(C, \alpha) = \lambda \beta: \beta \in C. \beta \neq \alpha \rightarrow \mu_c(\beta) > \mu_c(\alpha)$

Let us unpack this. We are defining the superlative modifier *at least* as a scalar focus item that is associated with a contextual restriction *C*, denoting a set of propositional alternatives (as in Rooth, 1992). The second ingredient of the analysis is a measure function,  $\mu_c$  (of type  $\langle \eta, d \rangle$ : in principle.  $\eta$  could be any type), mapping the focus alternatives to their corresponding positions along a contextually-valued scale. The third piece of the analysis is the ordering between alternatives, represented in terms of a (strict) comparison relation between the prejacent  $\alpha$  and its alternatives along the contextually-given scale:  $\mu_c(\beta) > \mu_c(\alpha)$ . Finally, we also introduce a superlative component,  $\text{SUP}(C, \alpha)$  in the semantic representation of *at least*, which imposes an additional restriction on the propositional space: The relevant alternatives in *C* that are not identical to the prejacent are included only if they are ranked above the prejacent (i.e.,  $\lambda \beta: \beta \in C. \beta \neq \alpha \rightarrow \mu_c(\beta) > \mu_c(\alpha)$ ).<sup>11</sup> This excludes any potential

<sup>9</sup> For cases where the prejacent is not a proposition, I suggest the following entry, which can be obtained by the Geach Rule (Jacobson 1999).

(i)  $\llbracket \textit{at least} \rrbracket^{w,c} = \lambda \alpha_{\langle a, \langle s,t \rangle} \lambda \beta_{\langle a \rangle} \exists \gamma [\gamma \in \text{SUP}(C, \alpha(\beta)) \wedge \gamma_w]$

See Coppock and Brochhagen (2013: (21)) for a similar proposal for mediating between propositional and non-propositional *only* and *at least*.

<sup>10</sup> I am very grateful to an anonymous reviewer for pointing out some problems with the semantics of *at least* in an earlier version of the paper.

<sup>11</sup> The meaning of SUP introduced here as a domain restrictor of *at least* is essentially the same as the meaning of superlatives discussed in Heim (1999), except that instead of enforcing universal

lower alternatives in  $C$ , restricting  $C$  to a set where the prejacent marks the lower bound. Thus, the domain  $C$ , further restricted by the superlative component, now denotes a set consisting of the prejacent and its higher-ranked focus alternatives. Taken together, (28) asserts that there is one proposition  $\gamma$  in this restricted domain that is true.

The semantic representation in (28) draws on and preserves many insights of Biezma (2013)'s analysis. Given that an existential claim over a set amounts to a disjunctive statement over the elements in the set, it yields a disjunctive statement without hard-wiring a disjunction into the semantics (cf. Krifka, 1999; Büring, 2008). It leaves open whether the relevant higher alternatives are true in a given discourse, which as we will see Sect. 2.3, crucially makes room for pragmatics to play a role in delivering the ambiguity between EPI and CON. The key novelty in the proposed semantics for *at least* is that the prejacent marks the lower bound in the restricted set of focus alternatives (i.e.,  $\text{SUP}(C, \alpha)$ ).

Note that the first two common properties of EPI and CON, focus-sensitivity and compatibility with a variety of scales, are built into the proposed lexical entry in (28). Focus marking evokes the set of alternatives against which the contribution of *at least* is to be evaluated. Focus-sensitivity follows from this. The compatibility with various scales follows because the unified entry requires the set of focus alternatives to be ordered along a scale. There are, however, no restrictions on what those scales may be. Thus, *at least* is predicted to combine with a variety of scales, familiar from analyses of other scalar terms.

Let us consider a concrete example. In (29), speaker A raises an issue about the academic ranking of John and speaker B replies with an assertion using epistemic *at least*. The relevant LF and semantic computation of speaker B's utterance are given in (30).<sup>12</sup>

(29) A: What type of professor is John?

B: John is at least an [associate]<sub>F</sub> professor.

- (30) a. LF:  $[_{IP} \text{ at least}(C) [_{IP} [_{IP} \text{ John is an [associate]}_F \text{ professor}] \sim C]]$ <sup>13</sup>  
 b.  $\alpha \sim C$  is defined iff  $[[\alpha]]^o \in C \wedge \exists \alpha' [\alpha' \neq \alpha \wedge [[\alpha']]^o \in C] \wedge C \subseteq [[\alpha]]^f$   
 c.  $C = \{\lambda w. \text{John is a full professor in } w, \lambda w. \text{John is an associate professor in } w, \lambda w. \text{John is an assistant professor in } w\}$   
 d.  $\text{SUP}(C, \alpha) = \{\lambda w. \text{John is a full professor in } w, \lambda w. \text{John is an associate professor in } w\}$

Footnote 11 continued

quantification over the set of elements  $\beta$  compared with  $\alpha$ , we use lambda-abstraction to create a set serving as the domain for *at least*.

<sup>12</sup> The traditional analysis of English *even* invokes the scale of likelihood. However, this traditional view has been recently challenged by Greenberg (2016, 2018). Specifically, Greenberg (2016, 2018) argues that *even* invokes a contextually-given scale and then gives a gradability-based analysis of *even*. In this spirit, this paper shares a similar line of research in applying formal tools from previous studies on gradability to the phenomena of scalarity shown by SMs.

<sup>13</sup> I use the logical form *at least* (C) to indicate the fact that *at least* contains the domain argument C.

$$e. \llbracket(30a)\rrbracket^{w,c} = 1 \text{ iff } \exists \gamma[\gamma \in \text{SUP}(C, \alpha) \wedge \gamma_w]$$

The superlative component SUP takes the domain  $C$  and the prejacent  $\alpha$ , and returns a set consisting of only the prejacent  $\lambda w. \textit{John is an associate professor in } w$  and its higher alternative  $\lambda w. \textit{John is a full professor in } w$ . Speaker B's utterance with *at least* asserts that there is one element in the resulting set that is true. That is, one of the two propositions in (30d) has to be true.<sup>14</sup>

With the essentials of the semantics in place, we are now ready to tackle the third signature property of superlative modifiers, namely the end-of-scale effects.

## 2.2 Deriving end of scale effects

To see how the proposal in Sect. 2.1 derives the end-of-scale effects, let us start with a context where Chris has rolled the dice and the relevant options are that he got between 1 and 6, the following sentences are judged to be infelicitous:

- (31) #Chris at least got one.
- (32) #Chris at least got six.

While both are infelicitous, (31), as an instance of a BSE violation, is reported by speakers to be *less infelicitous* than (32), an instance of a TSE violation. Before we can make theoretical sense of the reported intuition, let us note that the deviance of the TSE violating sentence and the BSE violating sentence arises in different ways under the proposal outlined in Sect. 2.1. In the first case, SUP has no effect on  $C$ ; in the second case, SUP restricts the set  $C$  to a singleton.

- (33) a. #Chris **at least** got [one]<sub>F</sub>.  
 b.  $C = \{\text{Chris got one, Chris got two, Chris got three, Chris got four, Chris got five, Chris got six}\}$   
 c.  $\text{SUP}(C, \alpha) = \{\text{Chris got one, Chris got two, Chris got three, Chris got four, Chris got five, Chris got six}\}$
- (34) a. #Chris **at least** got [six]<sub>F</sub>.  
 b.  $C = \{\text{Chris got one, Chris got two, Chris got three, Chris got four, Chris got five, Chris got six}\}$   
 c.  $\text{SUP}(C, \alpha) = \{\text{Chris got six}\}$

<sup>14</sup> As pointed out by two anonymous reviewers, the proposed analysis shares some features with the proposal in Coppock and Brochhagen (2013). However, the empirical issues covered are distinct. Coppock and Brochhagen focus on the EPI reading and use Inquisitive Semantics to account for it. My focus here is on the possibility of EPI and CON readings cross-linguistically. It is not clear to me that their account extends straightforwardly to CON readings. They also do not discuss the end-of-scale effects that I discuss in Sect. 2.2. The perspective I take on them would have to be incorporated into C&B's account, for both EPI and CON, in order to account for them. A more pointed comparison between the current proposal and theirs, unfortunately, lies outside the scope of this paper.

Let us assume a fairly standard view of discourse, stemming from Stalnaker (1978, 1998, 2002) and much subsequent work. On this view, an assertion is felicitous if it reduces the context set. The idea is that discourse (un)informativity depends on whether asserting a proposition  $p$  updates the context set non-trivially by removing those worlds where  $p$  is false from the original context set  $C$ .<sup>15</sup> In the context under consideration, where it is known that Chris has rolled the dice, a simple assertion about the number obtained would be informative in this sense. The use of the focus sensitive expression *at least* imposes further restrictions. (31), in effect, says that Chris got some number between 1 and 6, which is uninformative in the same way that saying *Chris got one or more* is uninformative: it does not eliminate any worlds from the context set. The same is not true of (32). Here the initial context set is reduced by eliminating the worlds in which Chris got between 1 and 5. The problem here lies in the fact that once  $SUP(C)$  comes into play, the result is vacuity in the role of focus.

For completeness, let us illustrate with a felicitous example where both informativity and non-vacuity are respected:

- (35) a. Chris **at least** got [four]<sub>F</sub>.  
 b.  $C = \{\text{Chris got one, Chris got two, Chris got three, Chris got four, Chris got five, Chris got six}\}$   
 c.  $SUP(C, \alpha) = \{\text{Chris got four, Chris got five, Chris got six}\}$

Note that an assertion of (35a) is informative because  $SUP(C)$  eliminates from the original context set those worlds in which Chris got one, two or three. The set that remains under consideration after the restriction imposed by  $SUP(C)$  is a plurality and this allows for a meaningful connection between the focus domain of *at least* and the existential quantification over this domain. As we will see shortly, this allows for EPI as well as CON readings, depending on the context. A few points are worth noting before we move on to that task.

As far as I know, the following TSE effects with quantifiers have not previously been noted in the literature on *at least* but analogous effects have been noted for English *only* (Al Khatib 2013: 17):

(36) Adam **at least** saw #[every student]<sub>F</sub>/ [some students].

(37) Adam **only** saw #[every student]<sub>F</sub>/ [some students].

Al Khatib explains the contrast in (37) by pointing out that *only* cannot be vacuous. The examples (37) and (38) illustrate his point.<sup>16</sup>

<sup>15</sup> There are other, possibly more appropriate, ways of characterizing the contribution of an assertion as a discourse move that leads to an update of the context set when the discourse move is accepted by the discourse interlocutors (e.g., Farkas and Bruce 2010, among others).

<sup>16</sup> The interaction between focus particles and quantifiers of individuals is more complicated than it is reported here. For example, English *only* does not seem to be compatible with the quantifier *most* or *many*, either: #*John only saw most/ many students*. Chen (2008) observes that the use of the Mandarin particle *dou* is felicitous only when the assertion meets or exceeds the speaker's expectation about the core predication. Crucially, she suggests that the content about speaker's expectation is a presupposition



- (38) a. #Of Mary and Sue, Adam **only** saw [Mary and Sue]<sub>F</sub>.
- b. Of Mary and Sue, Adam **only** saw [Mary]<sub>F</sub>.
- c. Of Mary and Sue, Adam **only** saw [Sue]<sub>F</sub>.

The paradigm can be replicated with *at least* under its two readings.

- (39) a. #Of Mary and Sue, Adam **at least** saw [Mary and Sue]<sub>F</sub>.
- b. Of Mary and Sue, Adam **at least** saw [Mary]<sub>F</sub>.                   √EPI, √CON
- c. Of Mary and Sue, Adam **at least** saw [Sue]<sub>F</sub>.                   √EPI, √CON

There are, of course, many differences between *only* and *at least*, and I do not mean to conflate the two cases. At the same time, there are also some clear parallels. The comparison with *only* shows that the contribution of SUP(C) in the semantics of *at least* allows for a similar explanation for TSE in terms of the ban on the vacuous use of focus adverbs in natural language.

Next, an utterance with BSE can become *felicitous* if it can be understood as the speaker joking or being sarcastic. That is, the speaker is being *intentionally uncooperative* and *flouting* the maxim of quantity (Grice, 1989). In short, the proposed semantic entry predicts TSE and BSE to be different in nature. In the case of *at least*, TSE results from semantic vacuity while BSE from discourse uninformativity. This is evidenced by the fact that BSE, but not TSE, can be *pragmatically repaired* by certain conversational strategies.

We should also note in this connection another way in which BSE can be repaired: This can be illustrated by considering an example like the following, which has a CON reading:

- (40) At least, Mary got a bronze medal.

Here focus on *bronze* can be seen as evoking a distinct alternative set *C*: <Mary won no medal, Mary won a bronze medal, Mary won a silver medal, Mary won a gold medal> or perhaps <Mary won no medals, some medals, many medals, all medals>. By switching scales, *getting a bronze medal* is no longer the bottommost element in the set *C* of alternatives based on this scale.<sup>17</sup> Of course, not every context allows for such repair. Our dice rolling example in (31) cannot be repaired in this way because it is not possible to switch to a scale with *zero* in it once the dice has been rolled.

To sum up, we started with the observation that speakers report a difference in the nature of the infelicity resulting from TSE and BSE violations. We argued that

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Footnote 16 continued

encoded by *dou*; moreover, it is considered as the opposite case of what is presupposed by English *only*: *only three students came* [the speaker’s expectation: more than three students would come]. Thus, the use of English *only* triggers an expectation about “*more*”, while the use of Mandarin *dou* triggers an expectation about “*fewer*”. The key point is that the presupposition of a given focus particle may also play a role in the interaction with quantifiers, besides the asserted content.

<sup>17</sup> Note that appealing to a two-membered scale may suggest that either the BSE or TSE will be violated no matter which member sets the value of the prejacent: <no medal, some medal>.

the way the focus sensitivity of *at least* is calculated imputes the infelicity in the two cases to distinct sources. Our next task is to show how the use of *at least* that does not incur either of these violations can have two different readings, EPI and CON, depending upon the discourse.

### 2.3 The pragmatics of *at least*

In this section I discuss how a single lexical entry for the superlative modifier *at least* ends up having two distinct readings, EPI and CON. Although EPI is typically associated with ignorance inferences and CON with lack of ignorance, these correlations are not so clear-cut. This section tries to shed light on the relationship between a statement with a superlative modifier and the question under discussion that it addresses. In Sect. 2.3.1, we show that the EPI reading arises when the statement can be seen as a direct answer to an information seeking question. A necessary condition for EPI, then, is that the answer to the question not already be in the common ground, a requirement that is compatible with contexts in which the speaker may or may not have full knowledge of the facts. In Sect. 2.3.2, we show that the CON reading arises when the statement can be seen as a comment about the relative position of the prejacent in the set of alternatives. A necessary condition for CON is that the speaker has full knowledge of the facts, a condition that is compatible with contexts in which those facts are part of common ground knowledge as well as contexts in which they are not. Although I draw on the current literature on this topic in illustrating the sources of EPI and CON readings of *at least*, I believe the generalization as presented here is more nuanced than earlier characterizations of this phenomenon.

#### 2.3.1 EPI with and without ignorance

Let me start by taking a sentence like (41) and considering it in the three distinct contexts in (42):

(41) John has won at least a silver medal.

(42) a. Competition context 1:

The winners of the competition are on stage. The results have not been formally announced but have been disclosed to the winners. John is looking visibly happy. The speaker knows that John has previously won the bronze medal several times. He has tried again, hoping to do better. The speaker says (41).

b. Competition context 2:

The results of the competition have been announced. The speaker knows the result. She calls John's mom Mary to tell her the good news but tries to keep the suspense up and have some fun. She says (41).

## c. Competition context 3:

The results of the competition have been announced. The speaker and hearer are both watching the results being posted. The speaker says (41).

The question under discussion in all three contexts is *what medal did John win?* Context 1 is a canonical EPI context where the speaker is ignorant about the answer but infers from John's demeanor that he has done better than bronze. A lot has been written about such cases and I do not have much more to say about it here. Context 2 is one in which the speaker does know the answer but is withholding information from her interlocutor. In usual discussions regarding EPI readings of *at least*, such cases have not been considered but Context 2 shows clearly that explanations solely based on ignorance on the part of the speaker cannot tell the whole story. Context 3 is one in which it is part of common ground knowledge what type of medal John won. In this context (41) is infelicitous under the EPI reading and the source of this infelicity has to be that it violates the maxim of quantity. While it is no doubt true that John has won either a silver medal or a gold medal, the maxim requires that the speaker choose the one that happens to be the case.

I should acknowledge here that the non-ignorance based EPI reading draws on the discussion of *at least* in Mendia (2016a), who provides three pieces of evidence to establish that the ignorance inference conveyed by English *at least* is pragmatic in nature: that it can be cancelled; that it can be reinforced; that it disappears in contexts where the maxim of quantity is deactivated (the examples below are from Mendia (2016a: (4)–(6)).

(43) **Cancellability**

Context: Bill has four kids. Yesterday he saw a sign at a supermarket: "Discounts for parents. To qualify you must have at least three kids." Bill reasoned as follows.

I qualify: I have four kids, so I do have **at least** three kids.

(44) **Reinforceability**

Bill has **at least** three kids, I don't know how many exactly.

(45) **Deactivation**

Context: In a game, my friend has to guess the number of marbles that I have hidden.

I know how many I have hidden and she knows that I have that information. I provide the following clue: I have at least five marbles.

↪ no ignorance about the number of marbles that I have

(43) and (44) show that the ignorance inference is cancellable and reinforceable, which are two hallmarks of conversational implicatures (Grice, 1989) and (45) shows that when the maxim of quantity is deactivated, the ignorance inference does not arise. We therefore conclude that ignorance is *not* a necessary property of the EPI reading.

A possible objection, raised by an anonymous reviewer, is that there are cases where ignorance inferences associated with *at least* seem difficult to cancel, as shown below.

Context: In order to qualify for a tax rebate, one needs to have at least three kids.

- (46) Q: Do you have three kids?  
 A: ??Yes, I have at least three kids.
- (47) Q: How many kids do you have?  
 A: #I have at least three.

I believe that this point, though relevant, does not detract from Media's fundamental claim. To see this, we note that similar facts are observed with disjunction.

- (48) Q: How many kids do you have?  
 A: #I have three or more.

Ignorance inferences associated with disjunction are standardly assumed to have the status of conversational implicatures, despite examples like (48). The issue of *whether* ignorance inferences with *at least* are cancellable, therefore, is distinct from the issue of *when* they can be cancelled. There may well be additional factors that might make them difficult to cancel. In (46)–(48), for example, the fact that the subject is the first person pronoun is an important factor, given that the speaker can reliably be taken to be knowledgeable about the number of kids they have. Furthermore, note that the example in (46) involves a cross-speaker conversation, while the one in (43) is an internal monologue. For these reasons, the soundness of (46)–(47) notwithstanding, Media's claim about the pragmatic status of ignorance inferences associated with *at least* remains valid.

Another piece of evidence in support of the position that statements with *at least* do not require ignorance comes from Westera and Brasoveanu (2014)'s observation that whether the ignorance inference arises depends primarily on whether a precise answer is requested or not.

- (49) a. A: Exactly how many students took Experimental Pragmatics?  
 B: At least ten students took Experimental Pragmatics. **Ignorance Inference**
- b. A: Did at least ten students take Experimental Pragmatics?  
 B: At least ten students took Experimental Pragmatics. **No Ignorance Inference**

The *wh*-question in (49a) requires a precise answer about the number of students who took the course while the polar question in (49b) does not. One might wonder whether the absence of the ignorance inference in (49a) somehow results from the repetition of *at least* in the question itself, and how that is to be analyzed. Although the original examples discussed in Westera and Brasoveanu (2014) may raise this issue, we can show that the lack of ignorance inference does not rely on an overt occurrence of *at least* in the prior question. Consider (50).

(50) Adam: Do you know whether Experimental Pragmatics is offered this semester?

Bill: Yes/Yeah, at least ten students have registered.

**In fact/ To be precise**, 13 students have signed up for the course.

Imagine a context where regulations require that a course can be offered if it has 10 or more registered students; Adam wants to audit Experimental Pragmatics and all he is interested in in finding out is whether the course will run, not how many students are in the course. Thus, while Adam’s question amounts to asking whether (at least) ten students have signed up for experimental pragmatics, it does not involve *at least* in the form of his question. Yet *at least* is possible in Bill’s response even though he knows the exact number of registered students.

To see why we do not get ignorance inferences in this context, let us take an explicit proposal about answers to questions, such as the one proposed in Dayal (1996, 2016).

(51)  $ANS_{Dayal}(Q)(w)$  is defined iff  $\exists p[w \in p \in Q \wedge \forall q[w \in q \in Q \rightarrow p \subseteq q]]$ .  
 When defined,  $ANS_{Dayal}(Q)(w) = \neg p[w \in p \in Q \wedge \forall q[w \in q \in Q \rightarrow p \subseteq q]]$

$ANS_{Dayal}$  is defined if and only if there is one proposition  $p$  in the denotation of the question  $Q$  (which is a set of possible answers/ propositions) such that  $p$  is true and  $p$  entails all the true propositions  $q$  in  $Q$ ; That is, when defined,  $ANS_{Dayal}$  picks out the unique maximally informative statement in  $Q$ .

In light of this, consider the statement *at least ten students registered* and relate it to two distinct discourse contexts. In context 1, we are interested in the precise number of students who registered because perhaps we have to calculate total enrollment numbers for the department. In context 2, the cut-off point is 10 for the course to be offered. In both contexts, the maximally informative answer is the one given in (52). However, the status of the other propositions is crucially different in the two contexts. In context 1, it is relevant which of the propositions in the set (53) is picked out by  $Ans(Q)$  and an existential statement over the set signals ignorance of the higher alternatives:

(52)  $\llbracket \text{How many students are enrolled} \rrbracket = \{1 \text{ student enrolled}, \dots, 15 \text{ students enrolled}, \dots\}$   
 $Ans(Q) = 13 \text{ students enrolled.}$

(53)  $SUP(C, \text{at least } 10 \text{ students are enrolled})$   
 $= \{10 \text{ students enrolled}, 11 \text{ students enrolled}, \dots, 13 \text{ students enrolled}, \dots\}$

In context 2, however, all that is needed is a divide between the propositions below 10 and the propositions at 10 or above. In this case, the existential statement that makes the cut at ten is sufficient, thus no ignorance inference arises. The situation is parallel to examples like *how many eggs are sufficient to bake a cake*, discussed by Beck and Rullmann (1999) where maximality requirements are suspended.

The generalization regarding EPI and ignorance, then, is that English *at least* under EPI addresses the issue of informativity: ignorance inferences arise to justify the failure of providing the maximally informative unique answer. But if the context makes it clear that maximal informativity is not at issue, there is no such inference.

The facts shown above argues against semantic approaches such as Geurts and Nouwen (2007) where the ignorance inference arises because *at least* semantically encodes a covert epistemic modality. They support pragmatic approaches where the ignorance inference conveyed by *at least* is pragmatic in nature (e.g., Büring, 2008; Coppock & Brochhagen, 2013; Cummins & Katsos, 2010; Kennedy, 2015; Mayr, 2013; Mendia, 2016a; Schwarz, 2016a; Westera & Brasoveanu, 2014). It is worth pointing out, however, that the position I am taking on EPI is distinct from the pragmatic account in Biezma (2013: 17) who notes “for an [EPI] to arise it is necessary to assume that higher alternatives may be true (i.e., not to know that all higher alternatives are false)”.

(54) summarizes our discussion of the pragmatic source of the EPI reading of *at least*:

(54) **Informativity and Speaker Ignorance**

Ignorance inferences arise pragmatically to justify the failure of providing the maximally informative unique answer (when the maxim of quantity is activated).

At this point, I should note that the precise mechanism concerning how ignorance inferences of epistemic *at least* should be calculated is still an ongoing debate (see Mihoc (2019) for a recent proposal couched in the grammatical approach (Chierchia 2004, 2013; Chierchia et al., 2012; Fox, 2007, among others) and a critical review of the previous analyses). In this paper, I do not take a stand on this debate. The neo-Gricean approach adopted here illustrates one way of capturing how the ignorance implicatures of an *at least* statement. I believe that the idea that epistemic *at least* addresses the issue of informativity can be recast under the grammatical approach. I leave this line of research for another occasion.

### 2.3.2 CON without ignorance and with evaluativity

Now, let us shift our attention to the CON reading of *at least* and recall that it conveys a “settle-for-less” flavor. The goal of this section is to track the source of this intuition. We have discussed cases in Sect. 2.3.1, where a higher alternative to the prejacent was known to be true but the implicit question that the *at least* statement was answering was such that any true statement in the alternative set counted as an equally good answer. We now consider contexts in which the higher alternatives are false.

There are two sub-cases to consider. We start with one where the facts are part of common ground knowledge. His friends were hoping that Adam would win a gold medal but it has just been announced that he won the silver medal. They are obviously disappointed but one of them says to the other:

- (55) Well, at least he got a [silver]<sub>F</sub> medal.  
 a. C = {bronze, silver, gold}  
 b. Ans(what did A get) = silver

It is obvious that the statement in (55) does not answer a question based on informativity since that is already part of the Common Ground. To understand what question it does answer; let us consider the other sub-case where we have a genuine information seeking question. The following is adapted from Biezma (2013: (17)).

- (56) Tom went on a date with someone he met online. When he got home, his friend Jim asked.  
 Jim: How was your date?  
 Tom: It was ok, **at least** she was smart.

In the given context, it is clear that Jim is interested in getting the maximally informative answer about Tom's date and it is also clear that Tom would have this information. As far as the semantic content of Tom's statement is concerned, it merely asserts that the prejacent or a higher alternative is true; the pragmatics of the situation in effect narrows down the choice to the prejacent. This is what we might expect also from a statement without the superlative modifier but clearly Tom's statement does more. It provides an explanation of the direct answer to the question, namely that it was OK, and draws attention to the set of alternatives, ranked according to the properties Tom values in a date:

- (57) Great: She was tall, smart and beautiful  
 Good: She was tall and smart, or She was smart and beautiful,  
 or She was tall and beautiful  
 Ok: She was tall, or She was smart, or She was beautiful  
 Bad: She was not tall and She was not smart and She was not beautiful

As Biezma puts it, "In principle, out of context, there are many ways Tom could answer Jim's questions. Certainly, Jim is considering alternatives that would present an **evaluation** of Tom's date regarding whether she was good-looking, funny or smart (for example). Understanding the question amounts to understanding the goals of the speaker when asking the question and hence identifying what are the possible answers..." (p. 6). Note that Tom's response would have been infelicitous if it was an elaboration of a direct answer like *it was great* or *it was awful*. That is, given that the *at least* statement has to respect End-of-Scale Effects, it provides an explanation for a direct answer that targets some mid-point in the evaluative scale. Consider a slight variant of Tom's response in (56): *at least she was smart* without the first phrase *it was OK*. The *at least* statement does not provide information that Jim wants. To the extent that Tom's answer is acceptable, the explanation would have to be the same. By evoking points in the middle of the scale, Tom allows Jim to infer and accommodate the fact that the date was OK.

Returning now to the first case we considered, where the direct answer to the question *what medal did Adam win?* is already part of the common ground, the *at*

*least* statement serves the purpose of drawing attention to the mid-point of the scale. Note that the pragmatic account of CON that I am pursuing makes the prediction that the use of *at least* under the CON reading will be infelicitous or unavailable when the speaker is ignorant about the falsity of the relevant higher alternatives. For example, (58ii) is deviant in expressing Frank's concession about the medal that John won.

- (58) Emily: What medal did Adam get? Did he win a gold medal?  
 Frank: (i) No, but **at least** he won a [silver]<sub>F</sub> medal.  
 (ii) #Maybe, and/ but **at least** he won a [silver]<sub>F</sub> medal.

In providing “the settle for less” aspect of the CON reading, I have followed Biezma's lead in considering English *at least* under CON to address the issue of evaluativity: the set of answers is evaluated and ranked against the speaker's goals and the interlocutors' interests in a given discourse.

### (59) Evaluativity and Speaker Concession

The “settle-for-less” flavor arises when **(a)** the set of answers is evaluated and ranked against the speaker's goals and the interlocutors' interests in a given discourse; and **(b)** the relevant higher alternatives are known to be false.

A point I have tried to highlight is that the falsity of the higher alternatives does *not* have to be part of the common ground. As the dating context makes clear, the common ground does not even have to include any of the propositions in the set of alternatives to the *at least* statement; but by using such a statement, the speaker simultaneously introduces the set of alternatives and the fact that the alternatives higher than the prejacent are false. That is, it is a necessary condition for the CON reading that the speaker should be in a position to know the truth/ falsity of the relevant alternatives but not that it be part of the common ground knowledge, as long as it can be accommodated.<sup>18</sup> The facts discussed here establish that the pragmatics of CON addresses the issue of evaluativity rather than informativity.

As a final example, let's look at (60), which shows that under the CON reading of *at least*, the falsity of the higher alternative may not be part of the common ground of the interlocutors in a given discourse, in contrast to what we have seen in (55).

<sup>18</sup> An anonymous reviewer suggests that the CON reading may convey a kind of partial satisfaction. According to the reviewer, in the mentioned dating scenario, as far as the speaker is sure that the person under discussion is tall, he/ she can comfort himself/ herself and say ‘At least she is tall’, even without having more information. First, to my understanding, the feeling of partial satisfaction is essentially the “settle-for-less” flavor in the terminology of Nakanishi & Rullmann (2009), which has been identified as the hallmark of the CON reading. Second, if the CON reading of *at least* signals the speaker's partial satisfaction (by the reviewer's terminology), it in turn means that the speaker is *not fully satisfied* in the given situation. More importantly, by hearing the utterance “At least she is tall”, the hearer is entitled to ask in more details about which part the speaker is not satisfied; crucially, that very piece of information is known by the speaker (i.e., this is the part where the relevant higher alternatives are known to be false by the speaker in a given discourse).



Suppose that Adam, Bill and Chris, and Danny are the relevant individuals in the discourse.<sup>19</sup>

- (60) A: In the end, who did John invite? Did he invite everyone?  
 B: No, but **at least** he invited [Adam and Bill]<sub>F</sub>.  
 C: **Hey, wait a minute!** You mean he didn't invite even Chris?

In (60), speaker A explicitly raises an issue concerning whether the content of the prejacent (i.e., *John invited Adam and Bill*) is true and speaker B's assertion serves as a proposal to settle the raised issue. Crucially, speaker C is entitled to use the phrase "hey, wait a minute" (Shannon, 1976; von Stechow, 2004) and convey their surprise at the fact that the propositional content of the higher alternative (i.e., *John invited Adam, Bill and Chris*) is false. Examples like (60) illustrate that the requirement of the concessive reading that the higher alternatives are known to be false in a given discourse should be **speaker-oriented**, rather than necessarily part of the common ground; see (55)).

To sum up, under the CON reading of *at least*, the falsity of the higher alternatives is a crucial factor. The falsity of the higher alternatives may or may not be part of the common ground; but crucially, it must be known by the speaker (i.e., it is speaker-oriented) and easy for the addressee to accommodate. The fact that the prejacent is entailed under the CON reading is derived from the proposed semantics of *at least* (i.e., requiring that either the prejacent or its higher alternative is true) coupled with the pragmatic condition that the higher alternatives are known to be false. The "settle-for-less" flavor of the CON reading arises from the fact that the asserted situation is neither the best (i.e., because the higher alternatives are false) nor the worst (i.e., because the prejacent is true and there are some lower-ranked propositions in *C*, the alternative set of the prejacent), given the evaluativity in the discourse.

## 2.4 Section summary

In this section I have explicated how the semantics of *at least* provided in Sect. 2.1 interacts with the pragmatics of informativity and evaluativity to deliver a range of readings: an ignorance-based EPI reading and a non-ignorance based EPI reading and an evaluativity based CON reading.

As indicated in Sect. 1, the proposal took Biezma (2013) as its starting point and while preserving some of its core features has cast it in terms that give semantic content to the superlative morphology. And in doing so, provides a distinction between the two end-of-scale effects that Biezma's theory does not address. Most significantly, it preserves the view that there is only one lexical item that leads to two distinct readings based on different pragmatic conditions.

To conclude, while aspects of these pragmatic explanations draw on the earlier literature on *at least*, it provides a new synthesis of those accounts, capturing readings that in earlier accounts were at odds with each other, or had not been sufficiently factored into the explanations.

<sup>19</sup> An anonymous reviewer asks how speaker A knows that it is a case of concessive *at least* in speaker B's assertion. There is one important linguistic cue here. The position of *at least* provides a syntactic cue: EPI is never available in a clause-initial position (see N&R and Sect. 3.1).

### 3 Cross-linguistic variation in EPI-CON

This section discusses implications of the current analysis of English *at least*, and other such items by extension, by briefly addressing the following four issues: syntactic restrictions on the distribution of EPI and CON; cross-linguistic variation in the availability of the EPI-CON ambiguity; the connection between the morpho-syntax and the semantics of SMs; and the connection between the proposed account of *at least* and that of the SM *at most*.

#### 3.1 Syntactic restrictions on EPI and CON

An issue we have not focused on so far are distributional restrictions on EPI and CON readings, though we indicated at several points that EPI but not CON is available with prenominal *at least*, and CON but not EPI is available with clause-initial *at least*. (61) presents the full picture of available readings for English (N&R 2009: slide 6) and (62) characterizes the distribution in Mandarin, where I use *zhi-shao* for illustration though the same facts hold for *qima* and *zui-shao*.

- (61) a. Mary won **at least** a [silver]<sub>F</sub> medal.  $\sqrt{\text{EPI}}$ , #CON  
 b. Mary **at least** won a [silver]<sub>F</sub> medal.  $\sqrt{\text{EPI}}$ ,  $\sqrt{\text{CON}}$   
 c. **At least** Mary won a [silver]<sub>F</sub> medal. #EPI,  $\sqrt{\text{CON}}$
- (62) a. Liubei    xie-le    **zhi-shao** [san]<sub>F</sub>-ben-xiaoshuo.  $\sqrt{\text{EPI}}$ , #CON  
 Liubei    write-ASP    at least    three-CL-novel  
 ‘Liubei wrote at least three novels.’  
 b. Liubei    **zhi-shao**    xie-le    [san]<sub>F</sub>-ben-xiaoshuo.  $\sqrt{\text{EPI}}$ ,  $\sqrt{\text{CON}}$   
 Liubei    at least    write-ASP    three-CL-novel  
 ‘Liubei at least wrote three novels.’  
 c. **Zhi-shao**    Liubei    xie-le    [san]<sub>F</sub>-ben-xiaoshuo. #EPI,  $\sqrt{\text{CON}}$   
 At least    Liubei    write-ASP    three-CL-novel  
 ‘At least Liubei wrote three novels.’

It is worth pointing out that these syntactic restrictions speak against any purely pragmatic account of the ambiguity. It is unclear, for example, how Biezma’s analysis could explain it. To be fair though, Biezma takes CON to be available for prenominal *at least* in English and provides (63) as a support for her position:

- (63) The track and field coaches are looking at the statistics and discussing the results of the last competition.  
 Coach 1: The competition was awful.  
 Coach 2: Yes, but Mary won **at least** that gold medal [pointing at the statistics data].

The native speakers that I have consulted do not agree with Biezma’s judgment. Given this, and the fact that similar distributional restrictions are also observed in Mandarin, I side with N&R’s observation for English that CON is not available when SMs appear in a prenominal position and that EPI may not be available at the clause-initial position.

I will briefly sketch one way to make sense of these distributional facts. I suggest that concessive *at least* requires its quantificational domain to be (minimally) propositional; this hypothesis, formulated in (64), is based on the idea that when the speaker makes an assertion with concessive *at least*, what’s evaluated is a set of different “cases” or “circumstances” (in semantic terms, *propositions*; or equivalently, *sets of worlds/ situations*).

- (64) **The Quantificational Domain Hypothesis**  
 The quantificational domain of concessive *at least* must be propositional (i.e., a set of propositional alternatives).

Now, let’s see what it buys us. In terms of Rooth (1992), the relation between the quantificational domain of a focus operator, the denotation of a question, and the focus value of an answer to the question can be understood as follows.

- (65) a.  $[[Q]]^o \subseteq [[Ans]]^f$                       the question-answer congruence  
 b.  $C = [[Q]]^o$                                       the anaphoric domain restrictor *C*

What (65) requires is that the denotation of a question be a subset of the focus value of the answer; and the quantificational domain of a focus operator be contextually restricted by the question under discussion. With these assumptions, the availability of CON at preverbal and clause initial position can be explained:

- (66) **The availability of CON** (at preverbal position)  
 a. The LF of (61b):  $[_{IP} \text{ at least}(C) [_{vP} [_{vP} \text{ Mary won a [silver]}_F \text{ medal}] \sim C]]$ <sup>20</sup>  
 b.  $C = \{\text{Mary won a gold medal, Mary won a silver medal, Mary won a bronze medal}\}$

- (67) **The availability of CON** (at sentence-initial position)  
 a. The LF of (61c):  $[_{IP} \text{ at least}(C) [_{IP} [_{IP} \text{ Mary won a [silver]}_F \text{ medal}] \sim C]]$   
 b.  $C = \{\text{Mary won a gold medal, Mary won a silver medal, Mary won a bronze medal}\}$

The next task is to see why CON is not available in prenominal position. I suggest that this is because of the quantificational domain, which in this position is a set of generalized quantifiers over individuals:

<sup>20</sup> In this paper, I assume (a) *at least* is adjoined to vP in the case of the preverbal *at least*; (b) the subject is generated at Spec, vP and thus vP is propositional (Kratzer 1996). For simplicity, I further assume that the subject reconstructs back to its base position at Spec, vP, for interpretative purposes at LF. Nothing crucial hinges on the assumption of the reconstruction, however.

(68) **The unavailability of CON** (at prenominal position)

- a. The LF of (61a):  $[_{DP} \text{ at least}(C)[_{DP} [_{DP} a [\text{silver}]_F \text{ medal}] \sim C] \lambda z [Mary \text{ won } z]]$   
 b.  $C = \{a \text{ gold medal, a silver medal, a bronze medal}\}$

The view that the relevant question denotation here must refer to a set of generalized quantifiers (rather than simply a set of propositions), draws on Jacobson (2016) and Xiang (2016). An interesting prediction that we can make here is that fragment answers are predicted to lack CON readings and we can see below that this prediction is indeed borne out:

- (69) Context. There are three individuals in the discourse: Adam, Bill and Chris.  
 Emily: Who did John invite?  
 Frank: **At least** [Adam and Bill]<sub>F</sub>  $\sqrt{\text{EPI, \#CON}}$

In (69), Frank's utterance conveys an ignorance inference about whether John invited all the three individuals: Adam, Bill and Chris. In contrast, Frank's utterance **cannot** be understood as conveying a concessive inference: although John didn't invite all the three individuals, he invited Adam and Bill. The quantificational domain hypothesis suggests that the concessive reading is missing in short answers because the relevant quantificational domain is a set of generalized quantifiers, rather than a set of propositions.<sup>21</sup>

We have illustrated the puzzle with CON readings, but the distribution of EPI, which holds the mirror image to CON, is also puzzling. EPI readings seem to be ruled out with sentence-initial *at least*. We can see this in (70) below, a variant of the example in (61c):<sup>22</sup>

- (70) **\#At least** John likes [Mary]<sub>F</sub>. Intended: **EPI**

Note that similar restrictions have been observed for English *only* (e.g., *\#Only John invited [Adam and Bill]<sub>F</sub>*). Here, *only* necessarily must associate with the subject *John*, it cannot associate at a distance with *Adam and Bill*. This, as far as I know, has not been adequately explained in the literature on this topic. While I cannot add to our understanding of this restriction, the parallelism between the two phenomena that I note here suggests that whatever explains the pattern for one will also account for the other.

Before concluding this section, it should be noted that there is some variation with regard to distributional restrictions across languages. According to Japanese speakers that I have consulted, *sukunaku-tomo* in Japanese, for example, seems to show no distributional restriction on the availability of the two readings.

<sup>21</sup> Note that the semantic underpinning of fragment answers is what delivers the restriction on CON readings. Therefore, the proposal here does not depend on a particular approach to fragment answers. See Weir (2014) and references cited there for further discussion of fragment answers.

<sup>22</sup> An anonymous reviewer wonders whether the CON reading of *at least* is relevant to speech acts. While this is an interesting line of thought, I am more inclined to treat *at least* as a focus adverb, under either reading. Furthermore, the restriction on *only* discussed below provides additional support for it.

- (71) a. **Sukunaku-tomo** John-wa [san]<sub>F</sub>-ko ringo-o kat-ta √EPI, √CON  
 few-even.if John-TOP three-CL apple-ACC buy-PAST  
 ‘At least John bought three apples.’  
 b. John-wa **sukunaku-tomo** [san]<sub>F</sub>-ko ringo-o kat-ta √EPI, √CON  
 John- TOP few-even.if three-CL apple-ACC buy-PAST

It is not clear to me why distributional restrictions of the kind we have been discussing are not attested in every language. I must leave it as an open question for now.

### 3.2 EPI-CON: one without the other

We have been focusing on the persistent ambiguity of SMs with respect to EPI and CON, but there are languages that employ distinct lexical items to convey either one or the other reading. We see in (72) and (73), for example, that German *wenigstens* admits CON but not EPI and *mindestens* admits EPI but not CON. *Wenigstens* in (73) has the implausible implication that more casualties are better than less, something we expect from CON:

- (72) Maria hat kein Gold gewonnen, aber **wenigstens** / **#mindestens** Silber.  
 Maria has no gold won but at.least at.least silver  
 ‘Maria didn’t win gold, but at least she won silver.’

- (73) Bei dem Unfall gab es **mindestens** / **#wenigstens** fünf Tote.  
 at the accident gave it at.least at.least five casualties  
 ‘There were at least five casualties in the accident.’

German is not the only language with dedicated items lexicalizing one of the two readings. According to Grosz (2011), similar lexical differentiation of the two meanings holds in Romanian, Polish and Finish. Facts such as these may seem to pose a challenge to the position I have argued for in this paper, but this conclusion would be hasty.

We can see, first of all, that there is no parametric variation between languages having or not having items that are ambiguous between EPI and CON. There are many languages that have items that are ambiguous in the relevant way as well as items that are not. English is a case in point. English *at least* demonstrates the ambiguity, but the related expression *at the very least* conveys only EPI.

- (74) a. Numeral Scales (a contextual ranking:  $4 > 3 > 2$ )  
**At the very least**, John wrote [three]<sub>F</sub> novels. √EPI, #CON  
 b. Plurality Scales (a contextual ranking:  $adam \oplus bill > adam \oplus chris > adam$ )  
**At the very least**, John hired [Adam and Bill]<sub>F</sub>. √EPI, #CON  
 c. Lexical Scales (a contextual ranking:  $gold\ medal > silver\ medal > bronze\ medal$ )  
**At the very least**, John got a [silver]<sub>F</sub> medal. √EPI, #CON

- d. Pragmatic Scales (a contextual ranking: *cherries*  
> *apples* > *bananas*)

At the very least, John bought [apples]<sub>F</sub>.  $\sqrt{\text{EPI}}$ , #CON

There are other languages besides English that show similar variation. Grosz (2011: 578) suggests that Russian *xotja* is a lexical item exclusively conveying CON. According to my consultants, Russian *po krajnej mere* does demonstrate the ambiguity, as shown below.<sup>23</sup> Furthermore, the availability of the two meanings in Russian *po krajnej mere* is also sensitive to the syntactic position, similar to what was discussed in Sect. 3.1.

(75) Russian *po krajnej mere*

- a. **po krajnej mere** Sasha kupil Tri yabloka. #EPI,  $\sqrt{\text{CON}}$   
by extreme Measure Sasha Boughtthree apples  
'At least Sasha bought three apples.'
- b. Sasha **po krajnej mere** Kupil tri yabloka. #EPI,  $\sqrt{\text{CON}}$   
Sasha by extreme measure bought three apples.  
'Sasha at least bought three apples.'
- c. Sasha kupil **po krajnej mere** tri yabloka.  $\sqrt{\text{EPI}}$ , #CON  
Sasha bought y extreme measure three apples.  
'Sasha bought at least three apples.'

Finally, Japanese also belongs in this set. Although Japanese *sukunaku-tomo* discussed in Sect. 3.1 demonstrates the ambiguity between EPI and CON, another expression *semete* displays only CON.<sup>24</sup>

<sup>23</sup> There are five possible lexical items in Russian that can be translated as *at least*: *kak minimum*, *po men'sej mere*, *po krajnej mere*, *hot'a by* and *hot'*. The first three demonstrate the ambiguity (though they all seem to be subject to the distributional restriction, similar to English *at least*). The expression *xotja* reported in Grosz (2011) seems morphologically related to the latter two, but it is not clear whether they are the same.

<sup>24</sup> Japanese *semete*, unlike concessive *at least* in English, cannot occur in a plain declarative sentence, as in (i).

- (i) \*Semete John-wa 3-ko ringo-o kat-ta.  
Semete John-TOP 3-CL apple-ACC buy-PAST  
'John at least bought three apples.'

As shown in (iv), Japanese *semete* is strongly preferred in a desiderative sentence, i. e., sentences expressing the speaker's wishes or needs—it almost always appears with *-tai* 'want' in declarative sentences.

- (ii) Semete 3-ko ringo-o kai-tai.  
Semete 3-CL apple-ACC buy-want  
'I want to at least buy three apples.'

(76) **Japanese** *semete*: #EPI,  $\sqrt{\text{CON}}$

- a. Hyaku-ten-ga    tore-naku-temo,   **semete**    kyuujuut-ten-wa   tori-tai.  
     100-point-NOM   get-NEG-even.if   **semete**    ninety-point-TOP   get-want  
     ‘Even if I can’t get 100 points, I want to at least get 90 points.’
- b. **Semete**            is-syuukan-wa    nihon-o    ryokoo-si-tai  
     semete            one-week-TOP    Japan-ACC   travel-do-want  
     ‘At least, I want to travel in Japan for one week.’

What do we learn from this cross-linguistic picture? Clearly, a unified account of those expressions that show the EPI-CON ambiguity, such as I have proposed for English *at least*, Mandarin *zhi-shao* and their correspondents mentioned in Sect. 1 is desirable. But this does *not* rule out the possibility of lexical distinction in other items. That is, the *pragmatic* conditions of EPI or CON discussed in this paper may be lexically encoded as *semantic* conditions for other expressions, along the lines of N&R’s proposal. That is, though N&R’s proposal may not be suitable for those expressions demonstrating the EPI-CON ambiguity, it may very well explain the linguistic behavior of items lexicalizing one of the two meanings. And finally it is worth explicitly noting that this is not a parametric choice that languages need to make; it is possible for both options to be part of the grammar of a single language.

**3.3 Degree morphology and the semantics of EPI-CON**

As suggested in Sect. 2, the superlative component SUP in our semantics of *at least* introduces the scalarity of SMs, which plays an important role in explaining some key semantic properties of *at least* (e.g., the two end-of-scale effects, TSE and BSE) but do we have any independent motivation for introducing the superlative component? The answer to this question leads us to a very interesting observation regarding the morpho-semantic profile of superlative modifiers: these focus adverbs involve quantity adjectives (Q-adjectives) and degree morphemes in their morphological makeup in many languages. Take English *at least* for example, the same component *least* is shared in SMs and quantity superlatives.

- (77) a. Adam bought at **least** [three]<sub>F</sub> apples.    Superlative Modifier
- b. Adam drank the **least** amount of water.   Quantity Superlative

The same pattern is replicated in many genetically-unrelated languages, such as Mandarin (*zui-shao*), Indonesian (*paling-sedikit*), Turkish and Vietnamese, where each item is made up of the superlative morpheme combining with a morpheme meaning ‘little’. Below, the bolded part indicates Q-adjectives and the underlined part the superlative morpheme: Indonesian paling-sedikit ‘at least’, Mandarin zui-shao ‘at least’, Turkish enaz ‘at least’ or enaz-in-dan ‘at least’, and Vietnamese ít-nhất ‘at least’. What’s even more striking, but perhaps not surprising at this point, is that when used as an SMs these expressions all demonstrate the familiar EPI-CON ambiguity shown. Of course, a scalar focus adverb does not entail the morphological involvement of gradable adjectives or degree morphemes (cf. English *only* and

*even*). However, given the pervasiveness of such morphology a connection with semantics, as in the current account, is desirable.

Using Mandarin for illustration we present a quick view of how the two meanings connect. (78) is an example of superlative modifiers and (79) an example of quantity superlatives. Crucially, the same expression *zui-shao* is involved, thus apparently ambiguous between SMs and quantity superlatives. The expression *zui-shao* morphologically involves the quantity adjective *shao* ‘little’ and the superlative morpheme *zui*. As we have seen in Sect. 1, *zui-shao* as a SM shows the familiar EPI-CON ambiguity (though admittedly, out of blue, the epistemic reading is more salient in (78)).

(78) Liubei **zui-shao** mai-le [san]<sub>F</sub>-ke-pinguo. **Superlative Modifiers**  
 Linbei SUP-little buy-ASP three-CL-apple  
 ‘Liubei at least bought three apples.’

(79) Liubei mai de pinguo **zui-shao**. **Quantity Superlatives**  
 Liubei buy MOD apple SUP-little  
 ‘The apples that Liubei bought are fewer than the ones that anyone else did.’

According to the proposed semantics in Sect. 2.1, the truth-conditions of (78) are given in (80), which essentially conveys that there is one proposition  $\gamma$  in the domain exclusively consisting of the prejacent ( $\lambda w$ . *Liubei bought three apples in w*) and its higher alternatives (e.g.,  $\lambda w$ . *Liubei bought four apples in w*; etc.) such that the proposition is true.

(80)  $\llbracket(78)\rrbracket^w, c = 1$  iff  $\exists \gamma[\gamma \in \text{SUP}(C, \alpha) \wedge \gamma_w]$   
 where  $C = \{ \dots; \lambda w$ . *Liubei bought two apples in w*;  $\lambda w$ . *Liubei bought three apples in w*;  
 $\lambda w$ . *Liubei bought four apples in w*; ...etc. }  
 $\text{SUP}(C, \alpha) = \lambda \beta: \beta \in C. \beta \neq \lambda w$ . *Liubei bought three apples in w*  
 $\rightarrow \mu_c(\beta) > \mu_c(\lambda w$ . *Liubei bought three apples in w*)  
 $= \{ \lambda w$ . *Liubei bought three apples in w*;  $\lambda w$ . *Liubei bought four apples in w*;  
 ...etc. }

Assuming the analysis of superlatives in Heim (1999), the truth-conditions of (79) are shown in (81).

(81)  $\llbracket(79)\rrbracket = 1$  iff  $\forall y[y \in C \wedge y \neq \text{Liubei} \rightarrow \max(\lambda d. y \text{ bought } d\text{-many apples})$   
 $> \max(\lambda d. \text{Liubei bought } d\text{-many apples})$   
 where  $C = \{ \text{Adam, Bill, Chris, Liubei, etc.} \}$

The common semantic core of SMs and superlatives lies in the superlative component: in the case of quantity superlatives (degree constructions), the comparison relation holds between the individuals in the domain  $C$  with respect to a particular gradable dimension (i.e., cardinality); in the case of SMs (focus adverbs), scalarity is



introduced, where the ranking of alternatives is understood as a comparison relation between the alternatives induced by information focus along some dimension. Along these lines, both SMs and quantity superlatives actually share the same bounding property due to the common superlative component: in the case of SMs, the preajcent, reminiscent of the role of a comparative standard, serves as the lower bound in the restricted set of focus alternatives (i.e.,  $SUP(C, \alpha)$ ); in the case of quantity superlatives, the quantity of apples Liubei bought is clearly the lower bound (i.e., the fewest) among the quantities of apples that any other relevant individual bought.

There are, of course, important issues that arise regarding exactly how Q-adjectives and superlatives should be analyzed, but to go into them in detail is beyond the scope of this paper. Readers are referred to Coppock (2016), Mihoc (2019) and Chen (2023) for different proposals attempting to connect the semantics of SMs with their morphological components. The reason for introducing quantity superlatives here is to point to independent morphological evidence in the grammar of natural language for the superlative-based semantics for *at least* proposed in Sect. 2.1.

There are two questions that naturally arise at this point: How does a superlative-based analysis fare with expressions that demonstrate the EPI-CON ambiguity and bear superlative morphology, while lacking the use of quantity superlatives, such as Mandarin *zhi-shao*? How does a superlative-based semantics of SMs fare with those expressions that lack degree morphology while showing the EPI-CON ambiguity, such as Mandarin *qima* ‘at minimum’ and English *at minimum*?

According to my consultants, *zhi-shao* does not seem to have the use of quantity superlatives in Modern Chinese. However, it is worth emphasizing that whether a given expression can be syntactically used as a quantity superlative is not a precondition for a superlative-based semantics; instead, it is the morphological makeup that is crucial. For example, the English expression *at least* cannot be used as a quantity superlative either, but this does not change the fact that it has morphology associated with superlatives and quantity adjectives (i.e., *least*). Morphologically speaking, *zhi-shao* also involves the superlative morpheme (in Archaic Chinese) *zhi* and the quantity adjective *shao* ‘little, few’. The only morphological difference between *zhi-shao* and *zui-shao* concerns the status of the superlative morpheme (i.e., *zhi* vs. *zui*) in Modern Chinese. This may in turn explain why *zhi-shao* is not used as a quantity superlative in Modern Chinese but the online dictionary and search engine provides the following Chinese expressions where the word *zhi* conveys superlative meaning: *zhi-guan-zhongyao* ‘the most important’, *zhi-cheng* ‘the sincerest’, *zhi-gao-dian* ‘the highest point’, *zhi-gao-wu-shang* ‘the highest’, *zhi-jiao* ‘the closest friend’, *zhi-qin* ‘the closest relatives/ the family’. Although the expressions mentioned are somehow fossilized in Modern Chinese, it can be seen very clearly that the superlative meaning comes from the morpheme *zhi*. There may be independent historical reasons why *zhi-shao* does not have the use of quantity superlatives in Modern Chinese (e.g., the archaic nature of *zhi*), despite its morphological makeup. For reasons of space, a detailed study of the differences between *zui-shao* and *zhi-shao* is left for future research.<sup>25</sup>

<sup>25</sup> I am very grateful to an anonymous reviewer for drawing my attention to the distinction between *zui-shao* and *zhi-shao*, with respect to the use of quantity superlatives in Mandarin Chinese.

Let us shift our attention now to the second question: how a superlative-based semantics of SMs meshes with those expressions that lack degree morphology while showing the EPI-CON ambiguity, such as Mandarin *qima* ‘at minimum’ and English *at minimum*. From a cross-linguistic perspective, it is clear that there are two sources of expressions leading to the EPI-CON ambiguity, those that involve degree morphology and a Q-adjective and those that display the EPI-CON ambiguity but do not involve degree morphology or a Q-adjective. We have two analytical options at hand: one option is to pursue a strong version and claim that all expressions which display the EPI-CON ambiguity have a superlative-based semantics. Or we may pursue a weaker version of the theory: the superlative-based semantics suggested in this paper applies to only those expressions of SMs employing the superlative strategy.<sup>26</sup> I leave for future research to determine whether the strong or the weak version of the superlative-based semantics is better for those expressions lacking degree morphology while showing the EPI-CON ambiguity.

It is also worth noting that there are expressions that show EPI-CON ambiguity but lack a connection to superlatives in their morphology.<sup>27</sup>

(82) Q-adjectives plus even.if. (e.g., Korean and Japanese)

- |    |                       |            |          |
|----|-----------------------|------------|----------|
| a. | <b>sukunaku</b> -tomo | ‘at least’ | Japanese |
|    | few-even.if           |            |          |
| b. | <b>cek</b> -eto       | ‘at least’ | Korean   |
|    | few-even.if           |            |          |

(83) Q-adjectives plus comparatives (e.g., Magahi, Hindi, Russian; among others)

- |            |      |            |            |                  |
|------------|------|------------|------------|------------------|
| <b>kam</b> | se   | <b>kam</b> | ‘at least’ | Magahi and Hindi |
| less       | than | less       |            |                  |

While these expressions do not have superlative morphology, they do involve Q-adjectives, as indicated by the bolded part above. There are many issues that arise with respect to the map between morphology and semantics that we see in these examples. In fact, morphology tied to Q-adjectives and degree morphology can also be seen in SMs that have only one of the two readings, for example, German *mindestens* and *wenigstens*. The fundamental question underlying all these cases is the connection between the two meanings EPI and CON, Q-adjectives and degree morphology. This study has suggested some plausible ways of thinking about this issue but clearly more work needs to be done in this domain.

<sup>26</sup> I thank an anonymous reviewer for bringing up the issue of the EPI-CON ambiguity in the absence of degree morphology and quantity adjective. It is worth noting in this connection that Greenberg (2016, 2018) argues for a gradability-based semantics for English *even*, even though *even* does not involve any degree morphology or gradable adjective.

<sup>27</sup> Ihara & Mizutani (2020) recently proposes a decompositional analysis of Japanese *sukunakutomo* ‘at least’ in connection with its morphological makeup in concessive conditionals. Interested readers are referred to their paper for more details.

### 3.4 The case of *at most*

Although previous studies on the EPI-CON ambiguity (e.g., N&R and Biezma, 2013) exclusively focus on *at least*, another superlative modifier *at most* seems to also demonstrate the same ambiguity. Resembling the case of *at least* (see Sect. 3.1), in (84), where the maximally informative unique answer is requested (i.e., the maxim of quantity is active), speaker B’s assertion with *at most* leads to an ignorance inference that she does not know exactly how many apples John ate.<sup>28</sup>

- (84) A: How many apples did John eat?  
 B: He ate **at most** [three]<sub>F</sub> apples.

Again, resembling the case of *at least*, *at most* also displays CON. For example, Cohen and Krifka (2014: (112)) report the sentence in (85), where *at most* conveys that nothing better than being centrally located can be said about the hotel.

- (85) This is a bad hotel; **at most**, it’s centrally located.

It is worth emphasizing that just as in the case of concessive *at least*, the speaker is **not** ignorant when asserting (85) with *at most*. The parallel between *at least* and *at most* with respect to the EPI-CON ambiguity thus crucially reinforces our proposal that the two readings should be treated as variants, at least for the expressions that are ambiguous: the epistemic reading addresses the issue of the informativity and the concessive reading addresses the issue of evaluativity. To complete our discussion, we suggest the unified entry of *at most* in (86).<sup>29</sup>

$$(86) \llbracket at\ most \rrbracket^{w,c} = \lambda\alpha_{\langle s, t \rangle} . \neg\exists\gamma[\gamma_w \wedge \gamma \notin SUP(C, \alpha)],$$

$$\text{where } SUP(C, \alpha) = \lambda\beta: \beta \in C. \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta)$$

<sup>28</sup> The same observation applies to disjunction, where in subsequent discourse, the content of each disjunct must be (epistemically) available to the speaker.

- (i) Context: Speaker B knows that John read Hamlet yesterday.  
 A: What did John read yesterday?  
 B: #John read Hamlet or Macbeth.
- (ii) Context: Speaker B knows that John read Macbeth yesterday.  
 A: What did John read yesterday?  
 B: #John read Hamlet or Macbeth.

<sup>29</sup> For cases where the prejacent is not a proposition, I suggest the following entry, which can be obtained by the Geach Rule (Jacobson, 1999). See Coppock and Brochhagen (2013: (21)) for a similar proposal for *only*.

$$(i) \llbracket at\ most \rrbracket^{w,c} = \lambda\alpha_{\langle a, \langle s, t \rangle \rangle} \cdot \lambda\beta_{\langle a \rangle} \neg\exists\gamma[\gamma \notin SUP(C, \alpha(\beta)) \wedge \gamma_w]$$

According to (86), the assertion with *at most*  $\alpha$  is true iff there is no proposition  $\gamma$  that is true (i.e.,  $\gamma_w$ ) and ranked above the prejacent  $\alpha$  (i.e.,  $\gamma \notin \text{SUP}(C, \alpha)$ ). Note that the superlative component  $\text{SUP}(C, \alpha)$  denotes a set consisting of only the prejacent and those propositions ranked below the prejacent (i.e., the prejacent is the upper bound). This in turn means that any alternative propositions ranked higher than the prejacent would be false. Put differently, the proposed semantics of *at most* amounts to saying that the prejacent is the highest ranked proposition that can be true, which crucially gives the required semantic flexibility allowing the pragmatic conditions of EPI and CON to connect to the discourse.

Let us end this section by witnessing that the same EPI-CON ambiguity repeats itself with the superlative modifier *zui-duo* ‘at most’ in Mandarin. In (87), speaker B’s assertion with *zui-duo*, as an answer to speaker A’s question, conveys an ignorance inference that speaker B does not know exactly how many apples *Liubei* has bought (i.e., an EPI reading). By contrast, (88), due to Yu Cao (p.c.), is a complaint about a given house; crucially, the speaker is not ignorant about the higher alternatives and the use of *zuiduo* conveys an evaluative meaning (cf. (87)).

- (87) A: Liubei daodi mai-le duoshao pinguo?  
 Linbei exactly buy-ASP how.many apple  
 ‘Exactly how many apples did Liubei buy?’  
 B: Liubei mai-le **zui-duo** [san]<sub>F</sub>-ke-pinguo.  
 Linbei buy-ASP SUP-much three-CL-apple  
 ‘Liubei bought at most three apples.’

- (88) Zhe-fangzi suiran bu-da, danshi buju ye hai heli.  
 This-house although not-big but organization also still acceptable  
**Zuiduo** keren lai de shihou, ni shui-xia shafa;  
 At most guest come DE when you sleep-PRT couch  
 Qita ye mei shenme buhao le.  
 others also not what bad SFP  
 ‘Although this house is not big, its organization is still acceptable. At most/ in the worst-case scenario, you need to sleep on the couch when guests come and stay; except for that, nothing is bad.’

One key difference between (85) and (88) is that the evaluation (on which the ranking of alternatives is based) is reversed: in (85) the mentioned property (i.e., being -centrally-located) is the best feature among the relevant alternatives under evaluation, while in (88) the mentioned property (i.e., sleeping on the couch when guests come and stay overnight) is the worst feature. I thank a reviewer for drawing my attention to this difference.

Last but not the least, as we have seen in the case of the Mandarin superlative modifier *zui-shao* ‘at least’ in Sect. 3.3, the expression *zui-duo* ‘at most’ similarly involves the quantity adjective *duo* ‘much’ and the superlative morpheme *zui* in its morphological makeup. Crucially, resembling *zui-shao*, the same expression *zui-duo*

is again ambiguous between an SM meaning and a quantity superlative, as illustrated in (89) and (90).

(89) Liubei **zui-duo** mai-le [san]<sub>F</sub>-ke-pinguo. **Superlative Modifiers**  
 Linbei SUP-much buy-ASP three-CL-apple  
 'Liubei at most bought three apples.'

(90) Liubei mai de pinguo **zui-duo**. **Quantity Superlatives**  
 Liubei buy MOD apple SUP-much  
 'The apples that Liubei bought are more than the ones that anyone else did.'

The Mandarin and English examples discussed here jointly reinforce the key point of this paper: that the cross-linguistic pervasiveness of the EPI-CON ambiguity in SMs requires a unified analysis and that their morphological makeup cannot be taken as accidental.

## 4 Conclusions

The ignorance and concession readings of SMs have been discussed in various studies, but not always together. And when they have been considered together, they have often been seen as a case of lexical homophony. In this paper I have argued that taking a cross-linguistic perspective suggests an intrinsic connection between the two readings. Focusing on lexical items in two unrelated languages, *at least* in English and *zui-shao* in Mandarin, I have made an explicit proposal for a single superlative-based semantics and articulated under which pragmatic conditions the perceived ambiguity arises. This perspective is informed not only by the same ambiguity repeatedly occurring in a number of genetically unrelated languages but also by the same ambiguity being manifested by multiple lexical items within a single language. Finally, I presented evidence that having an expression showing the EPI-CON ambiguity does not exclude the possibility of having another expression conveying EPI or CON exclusively, suggesting that the locus of variation concerning the EPI-CON ambiguity lies at the level of lexical items, rather than at the level of languages. That is, the claims for ambiguity for the superlative modifiers like English *at least* is consistent with the possibility that the *pragmatic* conditions that give rise to EPI or CON readings may be equally well be lexically encoded *semantically* in the case of certain other expressions.

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