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ORIGINAL RESEARCH

The counterfactual direct argument

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Abstract
Many have accepted that ordinary counterfactuals and *might* counterfactuals are duals. In this paper, I show that this thesis leads to paradoxical results when combined with a few different unorthodox yet increasingly popular theses, including the thesis that counterfactuals are strict conditionals. Given Duality and several other theses, we can quickly infer the validity of another paradoxical principle, 'The Counterfactual Direct Argument', which says that 'A > (B or C)' entails 'A > (not B > C)'. First, I provide a collapse theorem for the 'counterfactual direct argument' (CDA). The counterfactual direct argument entails the logical equivalence of the subjunctive and material conditional, given a variety of assumptions. Second, I provide a semantics that validates the counterfactual direct argument without collapse. This theory further develops extant dynamic accounts of conditionals. I give a new semantics for disjunction, on which A or B is only true in a context when A and B are both unsettled. The resulting framework validates CDA while invalidating other commonly accepted principles concerning the conditional and disjunction.

Keywords Counterfactuals · Disjunction · Semantics

1 Introduction

Alex is staring at a fair coin. After some reflection, they declare:

(1) If I had flipped the coin, it might have landed heads.

In accepting (1), Alex seems to be rejecting (2):

(2) If I had flipped the coin, it would have landed tails.

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On the basis of examples like this one, many authors have concluded that ordinary counterfactuals and *might* counterfactuals are duals.

To express duality precisely, we must take a stand on the logical form of *might* counterfactuals. Following Bennett (2003), Gillies (2010), and others, I will assume that *might* counterfactuals are derived by embedding a modal operator $\Diamond$ in the consequent of the conditional.[1] So the logical form of (1) is $\phi > \Diamond\psi$.[2] With this assumption in place, we can model Duality using the following principle:

DUALITY $\phi > \Diamond\psi \dashv\models \neg(\phi > \neg\psi)$.[3]

One argument for Duality is that it makes sense of the 'inescapable clash' involved in asserting *q if p* and *might not q if p*:

(3) ??If I were to flip a fair coin, it would land heads; but it also might land tails if I were to flip it.[4]

Another argument for Duality comes from Lewis (1973). Lewis imagines that unbeknownst to himself he does not have a penny in his pocket, and then asks us to consider an utterance of:

(4) If I had looked in my pocket, I might have found a penny.[5]

Since Lewis does not have a coin in his pocket, he would not have found a penny if he had looked. So Duality correctly predicts that (4) is false.[6]

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[1] Throughout the paper I will restrict attention to a propositional language $\mathscr{L}$, defined as follows:

Definition 1 Let $\mathscr{L}$ be a language consisting of a set $\mathcal{A}$ of atomic formulae α, α', ..., closed under the connectives $\neg, \vee, \wedge$, the indicative and subjunctive conditionals $\rightarrow$ and $>$, and the epistemic and subjunctive possibility modals $\Diamond_e$ and $\Diamond$. Say that a claim is boolean if it does not contain $\rightarrow, >, \Diamond_e, \Diamond$, or $\vee$.

Throughout, I will use the terms 'counterfactual' and 'subjunctive conditional' synonymously. But the term 'counterfactual' can be misleading, since counterfactuals need not have antecedents that are contrary to fact.

[2] This approach contrasts with Lewis (1973), where the *might* conditional is a primitive operator, and Kratzer (1986), where conditional connectives are replaced with two place modal operators. As well, it differs from the frameworks in Stalnaker (1981) and DeRose (1999), where the *might* conditional is derived by positing a possibility modal at wide scope to the conditional.

[3] Williams (2010) calls this principle 'Bennett's Hypothesis', reserving Duality for a thesis about the potentially distinct *might* conditional operator. Strictly, Duality should be restricted to cases where the antecedent is possible (see DeRose 1999, p. 408; Heller 1995, pp. 96–97 for discussion). This way, we need not commit to whether *might* conditionals or ordinary conditionals can be vacuously true, vacuously false, or undefined when their antecedents are impossible. For further discussion of whether counterpossibles pose a problem for Duality, see Zagzebski (1990) and Wierenga (1998).

[4] See DeRose (1999), who suggests a pragmatic explanation of (3) that opponents of Duality might appeal to.

[5] Lewis (1973), p. 81.

[6] Stalnaker (1981) analyzes *might* counterfactuals as an epistemic possibility claim at wide scope to an ordinary subjunctive conditional. On that theory, (4) is equivalent to the claim that for all Lewis knows, if he had looked in his pocket, he would have found a penny. This appears to be true in the example. So the falsity of (4) seems to favor Duality over the wide scope theory. For a sophisticated response on behalf of the wide scope theory, see DeRose (1994).

Duality makes a claim about negating a conditional. It is difficult to negate conditionals in English, and so difficult to directly test Duality. One way to get around this problem and test Duality is to embed conditionals under negative attitudes like *doubt*. Consider the following:

(5) Mary doubts that if John studies, he will pass.

(6) Mary believes that if John studies, he might fail.

Suppose that an agent doubts something just in case she believes its negation. Then Duality predicts that (5) and (6) are equivalent.

Another argument for Duality comes from the behavior of necessity modals in conditionals. Gillies (2010) observes that *p* and *must p* seem equivalent in the consequent of indicative conditionals. The same seems true of counterfactuals, where we can replace *must* with *had to* to create a counterfactual modal:

(7) If Mary had been at the conference, then Sally would have gone too.

(8) If Mary had been at the conference, then Sally would have had to go too.

But now suppose that *might* and *had to* are themselves duals. Then Duality follows. For (7) is equivalent to (8), which by the duality of *might* and *had to* is in turn equivalent to (9):

(9) It is not the case that: if Mary had been at the conference, then Sally might not have gone.

For all these reasons and more, Duality is commonly accepted in the conditionals literature, as many have noted:

> [Duality is] perhaps the most popular account of the relation between *might* and *would* counterfactuals. (DeRose 1999, p. 387)

> Duality is implicitly assumed by most writers on *would* counterfactuals. (Eagle 2007, p. 3)

In this paper, we will take Duality on board and argue that it leads to a variety of underappreciated and shocking consequences in the presence of various nonorthodox but increasingly popular principles. In particular, we'll see that given a strict theory of conditionals, or given a few assumptions about the interaction between possibility modals and conditionals, Duality implies that a previously undiscussed and paradoxical principle is also valid, which I call 'The Counterfactual Direct Argument'.

To dramatize this new principle, consider the following vignette. There has been a murder on the estate. The police have determined that the driver did it. But there was also a backup assassin. After much investigation, the police conclude:

(10) If the driver hadn't done it, then either the butler or the gardener would have.

Why would (10) be true? Perhaps all three employees were disgruntled, and someone was bound to snap. Whatever the reason, in this scenario (10) seems to imply (11):

(11) If the driver hadn't done it, then if the butler hadn't done it, the gardener would have.

We can represent the inference from (10) to (11) as follows:

THE COUNTERFACTUAL DIRECT ARGUMENT $\chi > (\phi \vee \psi) \models \chi > (\neg\phi > \psi)$

In this paper, we'll see that Duality leads to the validity of CDA when combined with a few different principles recently explored in the logic of conditionals. Then we'll see that the validity of CDA, and so in turn these other principles, itself has major consequences for the meaning of conditionals and disjunction. The main obstacle, I will show, is that under minimal assumptions if CDA is valid, then counterfactuals are just material conditionals. In the rest of the introduction, I will lay out a roadmap for how we will proceed.

In Sect. 2, I lay out the pressure that leads from Duality to the validity of CDA. First, CDA follows from Duality in the presence of the increasingly popular thesis that subjunctive conditionals are *strict*.[7] According to this thesis, subjunctive conditionals universally quantify over a set of possible worlds, saying that there are no such worlds where the antecedent is true and the consequent is false. More precisely, subjunctive conditionals are strict just in case the following claims are equivalent, for some reading of the counterfactual necessity modal *had to*:

(12) If the butler hadn't done it, the gardener would have.

(13) It had to have been that either the butler didn't do it, or the gardener did.

We will see below that when counterfactuals are strict, Duality implies CDA given a few other minimal assumptions.

So much the worse for strict theories of conditionals, some might say. Interestingly, however, strict theories of conditionals aren't the only route from Duality to CDA. The same result follows once we accept a few more interesting principles governing *might* counterfactuals. The crucial assumption, which we'll consider below, is that *might* counterfactuals are scopeless in a certain sense, so that the first claim below is equivalent to the second, which entails the third:

(14) If the butler hadn't done it, the gardener might have.

(15) It could have been that if the butler hadn't done it, the gardener would have.

(16) It could have been that the butler didn't do it, and the gardener did.

By the end of Sect. 2, we will have established a few interesting argumentative routes from Duality to CDA. Then in Sects. 3–4, we will see why this leads to trouble. It turns out that there are serious empirical and theoretical concerns that face any attempt to validate CDA. Imagine now that there was a single assassin, and the police haven't yet figured out who it was. They learn:

(17) Either the butler or the gardener did it.

[7] See Von Fintel (2001) and Gillies (2007).

From this, it seems to follow that:

(18) If the butler didn't do it, then the gardener did.

The inference from (17) to (18) is called the Direct Argument.[8] If it is valid, then the indicative conditional $\phi \rightarrow \psi$ collapses to the material conditional $\neg\phi \vee \psi$. Many have defended this result.[9] No one, however, defends the analogous claim for subjunctive conditionals.[10] Indeed, it seems that (17) does not entail (19):

(19) If the butler hadn't done it, then the gardener would have.

Yet holding fixed any of three plausible principles, CDA leads to this result—that the subjunctive and material conditionals are equivalent:

COLLAPSE $\neg\phi \vee \psi \dashv\vDash \phi > \psi$

To validate CDA without Collapse, one must first invalidate Vacuity, the principle that conditionally supposing a tautology ($\top$) has no effect:

VACUITY $\phi \dashv\vDash \top > \phi$

In addition, validating CDA without Collapse requires giving up Modus Ponens:

MODUS PONENS $\phi; \phi > \psi \vDash \psi$

Finally, one must also give up the rule of Disjunction Introduction:

DISJUNCTION INTRODUCTION $\phi \vDash \phi \vee \psi$

To validate CDA without Collapse requires a new theory of conditionals and disjunction, on which the above principles are invalid. So it's worth making sure that Collapse is an unacceptable result. We already saw that Collapse is on its face absurd, since (17) doesn't entail (19). Note further that, at least given Disjunction Introduction, Collapse leads to the following paradoxes of material implication:

FALSE ANTECEDENT $\neg\phi \vDash \phi > \psi$
TRUE CONSEQUENT $\psi \vDash \phi > \psi$

Some have accepted these results for the indicative conditional, relying on some sort of Gricean maneuver.[11] But this strategy is a non-starter for subjunctive conditionals. Imagine that the butler committed the crime, so that (20a) is true. But suppose further that the gardener is the victim's closest friend. Then (20b) is not merely weird to say, but also easy to outright reject:

[8] See Stalnaker (1975), Jackson (1987), Edgington (1995), Bennett (2003), Nolan (2003) and Block (2008).

[9] Here, the collapse of these conditionals amounts to their logical equivalence. See Clark (1971), Grice (1975/1989a), Lewis (1976), p. 142, Thomson (1990), Hanson (1991), Abbott (2004), Rieger (2006), Rieger (2013), Rieger (2015) and Pynn (2010).

[10] See Clark (1971), p. 37, Thomson (1990), p. 58, Hanson (1991), p. 59, Douven (2011), p. 3, Rieger (2013), p. 3164 and Rieger (2015), p. 259.

[11] See Grice (1975/1989a).

(20) a. The butler did it.
b. ??So: if the butler hadn't done it, then the gardener would have.

Moreover, the False Antecedent inference is contrary to one of the purposes of subjunctive conditionals, which are often used when the antecedent is known to be false. Imagine that A and B are debating whether the Kennedy assassination was a conspiracy.[12]

(21) A: If Oswald hadn't shot Kennedy, someone else would have.

(22) B: If Oswald hadn't shot Kennedy, no one else would have.

Intuitively, A and B are having a genuine disagreement. But if the subjunctive conditional is the material conditional, then both of them are right.

We have a dilemma. On the one hand, Duality seems to imply CDA given a variety of interesting background assumptions. On the other hand, CDA leads to an absurd result. There are a variety of ways to resolve this dilemma. First, one might reject Duality, arguing for example that *might* counterfactuals have a different form than we might have expected, in a way that undermines the arguments above. Second, one might reject either strict theories of conditionals or the controversial theories of *might*-conditional interactions that lead to CDA in the presence of Duality. These are interesting avenues of exploration, worthy of consideration. In the second half of this paper, I propose a different solution: to accept CDA while avoiding Collapse. CDA can be accepted without Collapse, as long as each of Vacuity, Modus Ponens, and Disjunction Introduction is invalid. In fact, there are interesting independent considerations for thinking these principles are invalid (Sect. 5).

To validate CDA, I will develop a dynamic semantics for subjunctive conditionals and disjunction. In this semantics, meanings are not truth conditions, but instructions for how to change a body of information (Sect. 6). A disjunction requires of an information state that it be consistent with but not entail each disjunct (Sect. 8). A subjunctive conditional requires of an information state that revising it with the antecedent creates a new body of information that contains the consequent (Sect. 7).

To validate CDA, we need a new theory of counterfactual revision. Current theories of revision focus on cases where one revises a body of information to include some new claim inconsistent with the original information. These theories take for granted that *consistent revision*, where the new claim is not in tension with the original, is a simple matter: add the new claim to your information, and close under logical consequence. To validate CDA without Collapse we will see that even consistent revision is quite complex. Counterfactual reasoning requires a distinctive type of suppositional reasoning. When an agent counterfactually supposes some claim ϕ, she must first suspend some of her beliefs, even some of those consistent with ϕ, and enter into a distinctive counterfactual state of mind.

[12] See Adams (1970).

2 From Duality to CDA

In this section, we'll explore two ways in which Duality leads to the validity of CDA. The first route to CDA relies on the assumption that counterfactuals are strict, so that $\phi > \psi$ is equivalent to $\Box(\neg\phi \vee \psi)$, where $\Box$ is the dual of $\Diamond$ above:

STRICTNESS $\phi > \psi \dashv\models \Box(\neg\phi \vee \psi)$

Strictness is controversial. One of the original motivations for the similarity-based semantics in Stalnaker (1968) and Lewis (1973) was to deny Antecedent Strengthening, a consequence of Strictness:

ANTECEDENT STRENGTHENING $\phi > \chi \models (\phi \wedge \psi) > \chi$

The classic arguments against Antecedent Strengthening turn on the existence of Sobel sequences, chains of counterfactuals like the following:

(23) If Sophie had gone to the parade, she would have seen Pedro dance; but of course,

(24) if Sophie had gone to the parade and been stuck behind someone tall, she would not have seen Pedro dance; and yet,

(25) if Sophie had gone to the parade on stilts and been stuck behind someone tall, she would have seen Pedro dance after all.[13]

Surprisingly, however, Von Fintel (2001) and Gillies (2007) observe that the failure of Antecedent Strengthening above is order sensitive. When the premises are presented in a different order, the discourse becomes infelicitous:

(26) If Sophie had gone to the parade and been stuck behind someone tall, she would not have seen Pedro dance; but of course,

(27) #if Sophie had gone to the parade, she would have seen Pedro dance.

This order effect calls out for explanation, and a strict analysis has the potential to provide one. On this proposal, counterfactuals are a universal quantifier over a domain, and so Antecedent Strengthening is valid. But counterfactuals also presuppose that the antecedent is consistent with the relevant domain; when this assumption is violated, accommodation is triggered and the relevant domain expands.

In addition to explaining the infelicity of 'reverse' Sobel sequences, strict analyses of conditionals have another advantage. Von Fintel (2001) observes that 'negative polarity' items are licensed in the antecedent of counterfactuals:

(28) If you had left any later, you would have missed the plane.

Following Kadmon and Landman (1993), Von Fintel (2001) suggests that NPIs are only felicitous in downward monotonic environments, where any claim ϕ can be replaced with another claim ψ which implies it. This is exactly what is claimed by Antecedent Strengthening.

Given a few more assumptions, Duality and Strictness imply CDA. First, we will rely on the fact that conditionals are upward monotonic on their second argument:

[13] See Sobel (1970).

CONSEQUENT WEAKENING If $\phi \models \psi$, then $\chi > \phi \models \chi > \psi$

Next, we will rely on the assumption that $\Box$ and $\Diamond$ are duals:

$\Box/\Diamond$ DUALITY $\Box\phi \dashv\models \neg\Diamond\neg\phi$

Third, we will assume that iterations of *might have* collapse:

AXIOM 4 $\Diamond\Diamond\phi \models \Diamond\phi$

In addition, we will assume that our counterfactual modal *had to* ($\Box$) is factive:

FACTIVITY $\Box\phi \models \phi$

Instances of Factivity, like (29), seem valid (below I use the dual of $\Box$ to illustrate the point):

(29) a. It couldn't have failed to rain yesterday.
b. So: it rained yesterday.

Finally, we will make some assumptions about the entailment relation $\models$. Most of what follows will simply require that entailment is transitive, so that:

TRANSITIVITY If $\phi \models \psi$ and $\psi \models \chi$, then $\phi \models \chi$

Given these background assumptions, Strictness and Duality imply CDA:

Fact 1 Assume Transitivity, the 4 Axiom, $\Box/\Diamond$ Duality, Consequent Weakening and Factivity. Then Duality and Strictness imply CDA.

(For proofs of major facts in what follow, see the appendix.)

We now have our first argument from Duality to CDA. For the second argument, we turn to a variety of further principles connecting possibility modals and conditionals. Our first new principle is that possibility modals are *scopeless* with respect to conditionals. To see the phenomenon, start with indicative conditionals:

(30) Maybe he told Tom, if he didn't tell Harry.

(31) Maybe if he didn't tell Harry, he told Tom.

Gillies (2018) observes that conditionals like the above are equivalent, and yet feature the possibility modal $\Diamond$ at different scopes. When we turn from indicative conditionals to counterfactuals, the same pattern emerges:

(14) If the butler hadn't done it, the gardener might have.

(15) It could have been that if the butler hadn't done it, the gardener would have.

(14) and (15) seem equivalent, and yet the possibility modal occurs at narrow scope in the first and at wide scope in the second. This gives us the following principle:

SCOPELESSNESS $\phi > \Diamond\psi \dashv\models \Diamond(\phi > \psi)$

To test Scopelessness, it's natural to check whether one can assert a claim like (14) while denying (15), or vice versa. To deny a *could have* claim like (15), we can consider *couldn't have* claims:

(32) a. The gardener might have done it if the butler hadn't.

b. ??But it couldn't have been that if the butler hadn't done it, the gardener would have.

Holding Duality fixed, denying a *might* conditional like (14) is equivalent to asserting an ordinary subjunctive conditional. So if (15) didn't entail (14), we would expect the following to be felicitous:

(33) a. It could have been that if the butler hadn't done it, the gardener would have.

b. ??But if the butler hadn't done it, the gardener wouldn't have.

While both directions of Scopelessness are plausible, only the right to left direction will be used in our argument.

So far, we have assumed that each scope combination of $\Diamond$ and $>$ is equivalent. Our next assumption will help us answer what it takes for these claims to hold. Here, it seems that each scope combination in (14) and (15) entails (16):

(16) It could have been that the butler didn't do it, and the gardener did.

Gillies (2010) has endorsed just such a principle in the case of indicatives; the same pattern also seems plausible for counterfactuals. Representing (16) with the form $\Diamond(\phi \wedge \psi)$, we reach the following principle:

IF TO AND $\phi > \Diamond\psi \models \Diamond(\phi \wedge \psi)$

We are now in the position to see another route from Duality to CDA. Duality, Scopelessness and If to And imply CDA, given a few more background assumptions.

First, we will assume that *might have* is upward monotonic:

UPWARD MONOTONICITY If $\phi \models \psi$, then $\Diamond\phi \models \Diamond\psi$

This principle is plausible for a variety of possibility modals. For example:

(34) a. Mary might have been in Manhattan.

b. So: Mary might have been in New York.

Next, we will use the left to right direction of De Morgan's Law.

DEMORGAN $\phi \vee \psi \dashv\models \neg(\neg\phi \wedge \neg\psi)$

Finally, we will assume that entailment satisfies Contraposition:

CONTRAPOSITION If $\phi \models \psi$, then $\neg\psi \models \neg\phi$

Given these assumptions and a few previously discussed principles, we can now give another derivation of CDA.

Fact 2 Assume Transitivity, Contraposition, Consequent Weakening, Upward Monotonicity for $\Diamond$, Axiom 4, and De Morgan's Law$_{lr}$. Then Duality, Scopelessness$_{rl}$, and If to And imply CDA.

We've now found that Duality implies CDA given a variety of nonclassical assumptions about counterfactuals. The next natural question at this point is whether CDA is a palatable consequence; or whether it must be rejected. In the next two sections, I will consider the case against CDA. In Sect. 3, we will consider a potential counterexample to CDA, along with a general strategy for dealing with these sorts of cases. In Sect. 4, we will see that CDA conflicts with several generally accepted properties of conditionals and disjunction.

3 A counterexample to CDA

The first challenge to CDA is empirical: there seem to be counterexamples to it. Here's one such example: imagine again that we all know that the driver committed the crime. In addition, we know that there was a backup assassin. We have narrowed down our backup suspects to the butler and the gardener. But we know that only one of the butler and the gardener were implicated in the crime. One of them was the victim's closest friend; the other was the victim's mortal enemy. In this case, it seems that the following is assertible:

(35) If the driver hadn't done it, then either the butler or the gardener would have.[14]

However, it seems inappropriate to assert:

(36) If the driver hadn't done it, then if the butler hadn't done it, the gardener would have.

This seems inappropriate because for all we know, the gardener might have been the victim's closest friend. (36) suggests that we have ruled out this possibility. Yet if (35) is true and (36) is false, then CDA is invalid.

It is unclear to me whether this is a genuine counterexample to CDA. One way to resolve the counterexample above is to appeal to scope disambiguation. This proposal denies that (35) should be analyzed as:

(35') $\chi > (\phi \vee \psi)$

Rather, (35) has the form:

(35") $(\chi > \phi) \vee (\chi > \psi)$

(35) is a disjunction of conditionals rather than a conditional disjunction.[15]

Why accept this? Note first that in this scenario, the true reading of (35) seems equivalent to:

[14] Here are a few more examples with a similar structure, from an anonymous referee:

(1) If Beatrice had bought a car, she would have bought a Porsche or a Volvo, and we all know it would have been a Porsche.

(2) If Tyrone had majored in a STEM field, he would have majored in Physics or Chemistry; I just can't remember which.

[15] See Lepore and Stone (2014) for a systematic theory of apparent cancellation in turns of disambiguation.

(37) If the driver hadn't done it, then either the butler or the gardener would have, *but I don't know which*.

I don't know which contributes a similar effect when other modals interact with disjunction. For example, consider the following 'free choice' pair (of which more later):

(38) You may have soup or salad.

(39) You may have soup or salad, but I don't know which.

(38) but not (39) suggests that the addressee is permitted to have soup. Some in the free choice literature analyze this as a scope ambiguity, with the sluice *but I don't know which* forcing disjunction to take wide scope to the modal *may*.[16] It is natural to think that the sluice *but I don't know which* has the same effect in (37), forcing disjunction to take scope over the conditional.

On the other hand, this approach to the counterexample incurs a major cost. In a classical logic for conditionals, $(\chi > \phi) \vee (\chi > \psi)$ implies $\chi > (\phi \vee \psi)$. To exploit this strategy, the defender of CDA must deny this entailment. In fact, we'll see soon enough that the defender of CDA should also deny Disjunction Introduction, which is related to the implication above.[17]

Summing up, then, I suggest that (35) does not provide a counterexample to CDA because it has a different logical form. This is shown by the felt equivalence of (35) and (37), plus the fact that (37) is obligatorily read with wide scope disjunction.[18]

[16] For more on sluicing, see Ross (1969). For its application to free choice, see Simons (2005), Aloni (2007). For concerns about this approach, see Alonso-Ovalle (2006) and Kaufmann (2016).

[17] Given the semantics developed later in this paper, blocking the inference from $(\chi > \phi) \vee (\chi > \psi)$ to $\chi > (\phi \vee \psi)$ is not straightforward. A natural idea given that theory would be to block the inference from $(\chi > \phi) \vee (\chi > \psi)$ to $(\chi > \Diamond\phi) \wedge (\chi > \Diamond\psi)$, which is in turn implied by $\chi > (\phi \vee \psi)$. Rather, $(\chi > \phi) \vee (\chi > \psi)$ would only imply $\Diamond(\chi > \phi) \wedge \Diamond(\chi > \psi)$, which would be weaker than $(\chi > \Diamond\phi) \wedge (\chi > \Diamond\psi)$.

This strategy would require violations of Scopelessness, since $\Diamond(\chi > \phi)$ would not imply $\chi > \Diamond\phi$. To get these violations, I would follow Gillies (2018), p. 30, in introducing two different possibility modals one of which requires that an information state can be updated with the prejacent without absurdity; and the other of which requires that a subset of the information state supports the prejacent. These two possibility operators generate two different kinds disjunction, and CDA violations occur when the first disjunction operator takes wide scope. To really implement this proposal semantically, however, we would also need a story where counterfactuals are updates rather than tests. A full exploration of that idea will have to wait.

[18] An anonymous referee observes that the wide scope strategy may face further trouble from *suppose* reports. Consider again our context above, where only one of the suspects actually hated the victim, but we don't know which, and now consider:

(1) Suppose that the driver hadn't done it. Then the butler or the gardener would have done it.

(2) Suppose that the driver hadn't done it. Then, if the butler hadn't done it, the gardener would have.

In this scenario, 1 seems true and 2 seems false. But it's natural to think that suppositional constructions like the above would have a similar logic to counterfactual constructions. On the other hand, it seems difficult to assign 1 the kind of wide scope form which we appealed to above. It seems that there is no way to scope disjunction above *suppose*.

Yet there may be a way to extend the wide scope strategy to these constructions. Suppose that in 1, the counterfactual force of the construction is contributed by the modal *would*, which is restricted via modal subordination by the material from the *suppose* clause. In that case, perhaps we can allow two separate forms for 1, which differ in the relative scope of disjunction and the *would* operator:

4 Collapse

Now I will turn to a more serious worry for CDA. Holding fixed either of Vacuity or Modus Ponens, CDA leads to the logical equivalence of $\phi > \psi$ and $\neg\phi \vee \psi$:

COLLAPSE $\neg\phi \vee \psi \dashv\models \phi > \psi$

Moreover, Disjunction Introduction leads to a similarly trivializing result: that several paradoxes of material implication hold within the consequent of conditionals. Ultimately, I argue that each of these other three principles should be rejected on independent grounds (Sect. 5).

Consider again the following principle:

VACUITY $\phi \dashv\models \top > \phi$

Vacuity is plausible. Lots of different theories of conditionals agree that the conditional $\phi > \psi$ involves modifying a body of information until it contains the information that ϕ. But every body of information entails $\top$. So it seems like conditionally supposing $\top$ should have no effect at all.

More concretely, the left to right direction of Vacuity follows from the And to If inference endorsed in Stalnaker (1968) and Lewis (1973):

AND TO IF $\phi \wedge \psi \models \phi > \psi$

In the Stalnaker (1968)/Lewis (1973) framework, And to If follows from the *strong centering* condition that whenever ϕ is true at w, w is the closest possible world to w where ϕ is true.[19]

Unfortunately, Vacuity and CDA lead to Collapse once we grant one more plausible assumption:

BOUNDED FROM BELOW $\phi > \psi \models \neg\phi \vee \psi$

Bounded from Below is commonly accepted. It says that the conditional $\phi > \psi$ is at least strong enough to entail the corresponding disjunction $\neg\phi \vee \psi$. Given Disjunction Introduction, Bounded from Below follows from Modus Ponens. However, since we will go on to consider theories on which Disjunction Introduction is valid, we here consider on Bounded from Below as a separate principle.

Holding fixed Bounded from Below, CDA and Vacuity lead to the logical equivalence of $\phi > \psi$ and $\neg\phi \vee \psi$:

Fact 3 Assume Transitivity. Then Vacuity, Bounded from Below, and CDA imply Collapse.

Footnote 18 continued

(3) Suppose $\neg$driver. Then $\text{would}_{\neg\text{driver}}(\text{butler}) \vee \text{would}_{\neg\text{driver}}(\text{gardener})$.

(4) Suppose $\neg$driver. Then $\text{would}_{\neg\text{driver}}(\text{butler} \vee \text{gardener})$.

On this proposal, the premise 4 would imply 2; but 3 would not have this implication. Then we can say that 3 is true in the context above, while 4 and 2 are false. (Indeed, this may be the best way of implementing a wide scope analysis of counterfactuals also.)

[19] And to If and thus Vacuity also follows from Conditional Excluded Middle:

CONDITIONAL EXCLUDED MIDDLE $\models (\phi > \psi) \vee (\phi > \neg\psi)$

In addition, Vacuity and CDA on their own entail the counterintuitive direction of Collapse:

Fact 4 Assume Transitivity. Then Vacuity and CDA imply Collapse$_{lr}$.

Proof By Vacuity, $\neg\phi \vee \psi$ implies $\top > (\neg\phi \vee \psi)$, which by CDA implies $\top > (\phi > \psi)$, which by Vacuity implies $\phi > \psi$. □

Fact 3 and Fact 4 show that on pain of Collapse, CDA is incompatible with Vacuity. But giving up Vacuity is not sufficient to avoid Collapse. Consider the following principles:

REFLEXIVITY	$\models \phi > \phi$
MODUS PONENS	$\phi; \phi > \psi \models \psi$

These principles are both plausible. Reflexivity says that an antecedent is always available for further reasoning in the consequent. Modus Ponens is the standard elimination rule for the conditional. It says that the consequent of a conditional follows from the conditional and its antecedent. But these principles and CDA also lead to Collapse.[20]

Fact 5 Assume Transitivity and Reflexivity. Then Modus Ponens and CDA imply Collapse$_{lr}$.

Proof By Reflexivity, $(\phi \vee \psi) > (\phi \vee \psi)$ is valid, and so by CDA $(\phi \vee \psi) > (\neg\phi > \psi)$ is also valid. So by Modus Ponens $\phi \vee \psi$ implies $\neg\phi > \psi$. □

While our most recent collapse result focuses on Modus Ponens, our earlier result with Vacuity invoked Bounded by Below. Modus Ponens and Bounded from Below are quite similar principles, so one might wonder whether this result is strictly stronger than the previous one. There is a crucial difference, however. Our current result relies on instances of Modus Ponens that are *right-nested*, containing a conditional in the consequent. By contrast, the Vacuity result used instances of Bounded from Below without right-nested conditionals. We can avoid the most recent result by giving up Modus Ponens for right-nested conditionals. But even once we have done this, we must still reject Vacuity in order to avoid Collapse. It is in this sense that the two results are independent.

But giving up each of Vacuity and Modus Ponens is still not sufficient to avoid the problems with Collapse. One must also abandon Disjunction Introduction. For CDA and Disjunction Introduction lead to versions of the paradoxes of material implication in the consequent of the conditional:

NESTED FALSE ANTECEDENT	$\chi > \phi \models \chi > (\neg\phi > \psi)$
NESTED TRUE CONSEQUENT	$\chi > \psi \models \chi > (\neg\phi > \psi)$

Both seem invalid:

(40) a. If the driver hadn't done it, the butler would have.
b. ??So: if the driver hadn't done it, then if the butler hadn't done it, the gardener would have.

[20] Thanks to David Etlin for this observation.

(41) a. If the driver hadn't done it, the gardener wouldn't have.

b. ??So: if the driver hadn't done it, then if the butler hadn't done it, the gardener (still) wouldn't have.

CDA and Disjunction Introduction imply both principles:

Fact 6 Assume Transitivity and Consequent Weakening. Then Disjunction Introduction and CDA imply Nested False Antecedent and Nested True Consequent.

Proof For Nested True Consequent: by Disjunction Introduction $\chi > \psi$ implies $\chi > (\neg\phi \vee \psi)$, which by CDA implies $\chi > (\phi > \psi)$. The argument is similar for Nested False Antecedent. □

To avoid paradoxical conclusions, the defender of CDA must give up each of Vacuity, Modus Ponens, and Disjunction Introduction.[21] Giving up any one principle by itself still leaves one vulnerable to paradox. In the face of these problems, a natural first reaction is to reject CDA. It seems to be the only common element. However, it turns out matters are more complex. When we take a broader look at conditionals and disjunction, we will see that there are reasons independent of CDA that each of Vacuity, Modus Ponens, and Disjunction Introduction should be rejected.

5 Avoiding collapse

In this section, we will see that each of Vacuity, Modus Ponens, and Disjunction Introduction can be challenged on grounds independent of CDA.

5.1 Vacuity

Vacuity turns out to be more controversial than it at first appears. First, in the presence of a few modest principles Vacuity leads to a more controversial claim: the 'And to If' principle that $\phi \wedge \psi$ entails $\phi > \psi$. Suppose we accept the following very restricted version of Antecedent Strengthening.

CAUTIOUS MONOTONICITY $(\phi > \chi) \wedge (\phi > \psi) \models (\phi \wedge \psi) > \chi$

Next let's suppose that logically equivalent claims are substitutable in the antecedent of conditionals:

SUBSTITUTION If $\phi \dashv\models \psi$, then $\phi > \chi \dashv\models \psi > \chi$

In the presence of these assumptions, Vacuity is equivalent to And to If.

Fact 7 Assume Transitivity, Cautious Monotonicity, and Substitution. Then Vacuity implies And to If.

[21] In particular, to avoid any instance of Collapse, Nested False Antecedent, or Nested True Consequent, one must invalidate every instance of Vacuity where the consequent is disjunctive, every instance of Modus Ponens where the consequent is a conditional, and every instance of Disjunction Introduction.

But And to If is controversial. For example, following McDermott (2007) imagine that a coin is tossed twice, and Sally bets that it will land heads both times. In this scenario (42) seems false:

(42) If at least one head had come up, Sally would have won.

But And to If implies that (42) is true in this scenario. This is just one of many counterexamples to And to If in the literature. Others have focused on cases where ϕ and ψ are irrelevant to one another, or where ψ is extremely unlikely in the presence of ϕ.[22] Since Vacuity implies And to If, all of these examples are also evidence against Vacuity.

On the other hand, many find And to If an acceptable result, and give some alternative explanation for the counterexamples above. But Vacuity can also be challenged on new, more theoretical grounds. In particular, once we accept Strictness, Vacuity becomes equivalent to the absurd thesis that anything true must have been true: that ϕ entails $\Box\phi$.

Again, Strictness is the thesis that $\phi > \psi$ is logically equivalent to $\Box(\neg\phi \vee \psi)$. Once we accept Strictness, we can define $\Box\phi$ in terms of $>$, by making it equivalent to $\top > \phi$.[23]

CONDITIONAL NECESSITY $\top > \phi \dashv\models \Box\phi$

After all, suppose that $\bot \vee \phi$ is equivalent to ϕ:

IDENTITY $\bot \vee \phi \dashv\models \phi$

In addition, assume that the K axiom is valid:

K $\Box(\neg\phi \vee \psi); \Box\phi \models \Box\psi$

Finally, assume that $\Box$ is upward monotonic:

UPWARD MONOTONICITY If $\phi \models \psi$, then $\Box\phi \models \Box\psi$

Then Conditional Necessity follows from Strictness.

Fact 8 Assume Transitivity, Identity, K, and Upward Monotonicity for $\Box$. Then Strictness implies Conditional Necessity.

Conditional Necessity looks like a plausible definition of *had to*. But in the presence of this principle, Vacuity leads to the absurd conclusion that ϕ entails $\Box\phi$:

$\Box$ INTRODUCTION $\phi \models \Box\phi$

Fact 9 Assume Transitivity. Then Conditional Necessity and Vacuity imply $\Box$ Introduction.

This conclusion is unacceptable:

[22] For discussion see among others (Lewis 1973, p. 27; Bennett 1974; Fine 1975; Penczek 1997; McDermott 2007; McGlynn 2012; He 2016).

[23] Conditional Necessity is a surprising foil to Lewis (1973), where $\Box\phi$ is equivalent to $\neg\phi > \bot$ (the contrapositive of $\top > \phi$), and $\top > \phi$ is equivalent to ϕ.

(43) a. The butler didn't do it.
b. ??So: The butler couldn't have done it.

Summarizing, we have another reason besides CDA to reject Vacuity: even if one accepted Strictness and denied Duality and CDA, one would still have to reject Vacuity in order to avoid $\Box$ Introduction. On the other hand, the opponent of CDA might be happy to reject Strictness, and so this argument is by no means conclusive.

5.2 Modus ponens

The only instances of Modus Ponens used to reach Collapse contain a conditional nested in the consequent of another conditional. So to avoid Collapse we must deny that $\phi > (\psi > \chi)$ and ϕ entail $\psi > \chi$. A variety of other work has challenged the validity of Modus Ponens for right-nested conditionals, while preserving Modus Ponens in ordinary cases. In particular, the theory in McGee (1985) invalidates Modus Ponens because it also leads to Collapse in conjunction with the following plausible principle:

IMPORT- EXPORT $(\phi \wedge \psi) > \chi \dashv\models \phi > (\psi > \chi)$

Import-Export says that conditionals like the following are equivalent:

(44) If Alex had come to the party, then if Sam had come to the party, it would have been really fun.

(45) If Alex and Sam had come to the party, then it would have been really fun.

The plausibility of Import-Export thus provides an independent line of argument against Modus Ponens in exactly the cases we need.[24]

Given that they both lead to Collapse in the presence of Modus Ponens, it's natural to wonder whether CDA and Import-Export are equivalent. There are some interesting relationships between them. For example, suppose the following principle holds.[25]

AXIOM 5 $(\phi > \neg\chi) \vee ((\phi \wedge \psi) > \chi) \dashv\models \phi > (\neg\psi \vee \chi)$

Given Axiom 5, CDA implies the left to right direction of Import-Export, which by itself is inconsistent with Modus Ponens:

Fact 10 Assume Transitivity. Then Axiom 5, Disjunction Introduction, and CDA imply Import-Export$_{lr}$.

Given that Import-Export and CDA are related, one might want to validate both of them. However, it takes some work to validate Import-Export without validating Nested False

[24] For more discussion, see Gibbard (1981), Veltman (1985), Gillies (2009), Briggs (2012), and Fitelson (2013). *Prima facie* counterexamples to Modus Ponens have also been explored when the consequent of the conditional contains deontic modals (Kolodny and MacFarlane 2010), epistemic modals (Gillies 2010), and probabilistic modals (Yalcin 2012). Not all of these authors agree that the culprit in these cases is Modus Ponens; but giving up Modus Ponens is one way to handle all the relevant cases.

[25] See Lewis (1973), p. 132.

Antecedent and True Consequent. Given Import-Export, Nested True Consequent becomes equivalent to Antecedent Strengthening, the principle that $\phi > \chi$ entails $(\phi \wedge \psi) > \chi$. In addition, given Import-Export Antecedent Strengthening implies Nested False Antecedent.

Fact 11 Assume Transitivity. Then (i) Import-Export implies that Antecedent Strengthening and Nested True Consequent are equivalent, and (ii) Consequent Weakening, Reflexivity, Import-Export and Antecedent Strengthening imply Nested False Antecedent.

This last fact will help structure our inquiry in what follows. In recent years, Antecedent Strengthening has been resuscitated by the dynamic strict conditionals in Von Fintel (2001) and Gillies (2007). These theories validate Antecedent Strengthening, but invalidate Import-Export. By contrast, the semantics in Starr (2014) validates Import-Export, but invalidates Antecedent Strengthening. We can understand both theories as attempts to avoid the paradoxical results in Fact 11.

We have seen that there are powerful reasons independent of CDA to reject Vacuity and Modus Ponens. Now let's turn to Disjunction Introduction.

5.3 Disjunction introduction

As many have recently emphasized, Disjunction Introduction is in tension with the principle of Free Choice permission. Free Choice permission is a phenomenon where disjunction operates like conjunction when combining with possibility operators.[26]

FREE CHOICE $\Diamond(\phi \vee \psi) \models \Diamond\phi$

This inference seems valid for a variety of different possibility modals:

(46) a. Mary might be in New York or Los Angeles.
b. So: Mary might be in New York.

(47) a. Mary may[/ is permitted to] go to New York or Los Angeles.
b. So: Mary may[/ is permitted to] go to New York.

Like CDA, Free Choice is incompatible with Disjunction Introduction. For let's assume again that $\Diamond$ is upward monotonic, so that whenever ϕ is possible any consequence of ϕ is also possible. Given this assumption, Free Choice and Disjunction Introduction lead to the equivalence of any two possibility claims:

EXPLOSION $\Diamond\phi \models \Diamond\psi$

Fact 12 (Kamp). Assume Transitivity and Upward Monotonicity of $\Diamond$. Then Free Choice and Disjunction Introduction imply Explosion.

[26] For semantic accounts of free choice, see among others (Asher and Bonevac 2005; Aher 2012; Aloni 2007; Barker 2010; Ciardelli et al. 2009; Charlow 2015; Fusco 2015; Geurts 2005; Roelofsen 2013; Simons 2005; Starr 2016; Willer 2017; Zimmermann 2000).

For suppose $\Diamond\phi$. Then by Disjunction Introduction and Upward Monotonicity, $\Diamond(\phi \vee \psi)$. So by Free Choice, $\Diamond\psi$. Thus Free Choice provides independent reason from CDA to give up Disjunction Introduction.

Free Choice also has an analogue in the case of necessity modals:

ROSS'S PRINCIPLE $\Box(\phi \vee \psi) \models \Diamond\phi$

Most semantic accounts of Free Choice also validate this principle. Ross's Principle is somewhat similar to what we will ultimately require of conditionals. If we think of conditionals as a type of necessity modal, then Ross's Principle corresponds to the requirement that $\chi > (\phi \vee \psi)$ implies $\chi > \Diamond\phi$ and $\chi > \Diamond\psi$. This is a short step from the requirement we will ultimately derive from our semantics for disjunction, that $\chi > (\phi \vee \psi)$ implies $\chi > \Diamond\neg\phi$.

Of course, some respond to the tension above by denying the semantic validity of Free Choice, and offering a pragmatic account.[27] In this paper, I won't directly argue against the various pragmatic accounts of Free Choice. One place to look for concerns about various pragmatic analyses is a growing body of literature suggesting that free choice differs from scalar implicature with respect to processing time (Chemla and Bott 2014) and acquisition (Tieu et al. 2016). But my purpose here is simply to explore the best package of views consistent with the validity of CDA, so I will put this debate aside in what follows.

I have argued that three commonly accepted principles about conditionals and disjunction, each inconsistent with CDA, should be rejected on independent grounds. Below, I develop a semantics that explains why principles like CDA, Import-Export, and Free Choice are valid, and also explains why Vacuity, Modus Ponens, and Disjunction Introduction fail.

6 An introduction to update semantics

To explain why principles like Vacuity, Modus Ponens, and Disjunction Introduction fail, and to understand why principles like CDA, Import-Export, and Free Choice hold, we will need to depart from a truth-conditional conception of meaning, and instead think about meaning dynamically. On this framework, every sentence in our language can be understood as imposing a condition on some body of information.

The particular theory we will pursue is implemented in update semantics, a type of dynamic semantics.[28,29] According to dynamic semantics, the meaning of a sentence is not its truth conditions. Rather, the meaning of a sentence is its ability to change the context in which it is said—its *context change potential*.

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[27] For pragmatic accounts of free choice, see among others: Alonso-Ovalle (2006), Fox (2007), Franke (2011), Klinedinst (2007), Kratzer and Shimoyama (2002), Romoli and Santorio (2017), and Schulz (2005).

[28] See Stalnaker (1973), Karttunen (1974), Heim (1982), Heim (1983), Veltman (1985), Groenendijk and Stokhof (1990), and Groenendijk and Stokhof (1991); and many others.

[29] In what follows we will develop a dynamic strict semantics instead of relying on a static strict conditional. One reason for this, proved in Kaufmann and Kaufmann (2015), is that dynamic strict conditionals can validate Import Export without some of the trivializing results of static strict conditionals. This latter theory can only validate Import Export if the underlying accessibility relation is shift identical, so that whenever wRv, v can access exactly itself.

We will see that while ordinary, non-modal claims impose a condition on the common ground, subjunctive conditionals impose a condition on a larger set of possibilities. The big picture idea is that counterfactual revision involves entering a distinctive counterfactual state of mind, in which some of our ordinary beliefs (even those consistent with what we revise with) are momentarily suspended. For this reason, both Vacuity and Modus Ponens fail. For example, while ϕ requires the common ground (the epistemic possibilities) to contain only ϕ worlds, $\top > \phi$ requires an even larger set of worlds (the subjunctive possibilities) to entail ϕ. This means that counterfactual revision differs from ordinary learning even in cases of consistent revision (cases where what we are counterfactually revising with is compatible with the common ground). We will also be able to understand Free Choice and the failure of Disjunction Introduction in terms of the information dependence of disjunction. On the theory that follows, disjunctions don't simply require a body of information to be made up of worlds where one disjunct is true. In addition, they require that the body of information be consistent with but not entail either disjunct. Disjunction Introduction will fail because the truth of ϕ does not guarantee that this further condition is satisfied. This further unsettledness condition will also be essential to validating Free Choice and CDA.

To give an update semantics, we will need two things. First, we will need a definition of contexts. Second, we will need an update function, which assigns a context change potential to each sentence in our language. Our model of context marks our first departure from more traditional forms of update semantics. The semantics in Veltman (1996) treats a context as a set of possible worlds. However, we want to allow subjunctive conditionals to operate on a wider body of information than indicative claims. So we will let a context contain two different sets of possibilities, e and s: e tracks the epistemic possibilities, while s, a superset of e, tracks the subjunctive possibilities. For our purposes, it won't matter exactly how the possibilities in s are constructed; but one option is that they are determined by the laws of nature.[30]

Definition 2 A possible world w is a function from the set of atomic formulae $\mathcal{A}$ to $\{0,1\}$. W is the set of all possible worlds. A context σ is a pair $\langle e_\sigma, s_\sigma \rangle$, where $e_\sigma \subseteq s_\sigma \subseteq W$. e_σ is the set of *epistemic possibilities* of σ; s_σ is the set of *subjunctive possibilities* of σ. The trivial context, $\top$, is $\langle W, W \rangle$. The absurd context, $\bot$, is $\langle \emptyset, \emptyset \rangle$.

Once we have a representation of context, we can then recursively define sentence meanings with $[\cdot]$, a function that assigns each sentence a context change potential.

Definition 3 A context change potential is a function from the set of contexts into the set of contexts. An update function $[\cdot]$ is a function from $\mathscr{L}$ to context change potentials. The result of applying the function $[\phi]$ to σ is $\sigma[\phi]$.

Footnote 29 continued
On the other hand, the results in Kaufmann and Kaufmann (2015) are proved in a bivalent setting, without truth value gaps. Once truth value gaps are allowed, an anonymous referee observes, we have the option of Strawson validating Import-Export, so that it is truth preserving whenever defined. Perhaps there is a way to validate both Import-Export and CDA in this setting without trivializing accessibility.

[30] For more on the connection between laws and counterfactuals, see Pollock (1976). See Veltman (2005) for a structurally similar model of contexts, combined with a different semantics for the conditional. See Kaufmann (2000) for an alternative model of contexts on which they contain multiple bodies of information.

In update semantics, meanings are more fine grained than truth conditions. But it is still possible to define an analogue of truth in this framework. A body of information σ *supports* a sentence ϕ just in case σ is a fixed point of $[\phi]$: just in case adding ϕ to σ has no effect on σ.

Definition 4 σ supports ϕ ($\sigma \models \phi$) iff $\sigma[\phi] = \sigma$.

The semantics of the conditional will involve the idea of the consequent being supported in contexts that are changed to include the antecedent. In addition, we will ultimately define entailment in terms of preservation of support.

We now have the tools we need to give a semantics for our language. Before turning to conditionals and disjunction, let's introduce some standard dynamic meanings for the other parts of our language.[31] For atomic formulae, negation, and conjunction, we will basically follow Veltman (1996). An atomic formula α updates σ by narrowing the epistemic possibilities e_σ down to the set of worlds where α is true. α leaves the subjunctive possibilities s_σ unchanged:

Definition 5 $\sigma[\alpha] = \langle\{w \in e_\sigma \mid w(\alpha) = 1\}, s_\sigma\rangle$

A negation $\neg\phi$ has the opposite effect on σ as ϕ.[32]

Definition 6 $\sigma[\neg\phi] = \langle e_\sigma - e_{\sigma[\phi]}, s_\sigma\rangle$

Next, a conjunction $\phi \wedge \psi$ updates the context with ϕ, and then updates the result with ψ:

Definition 7 $\sigma[\phi \wedge \psi] = \sigma[\phi][\psi]$

So far, we have integrated some standard dynamic meanings for atoms, negation, and conjunction into our new definition of contexts. In the next section, we will put our definition of context to use by giving a new semantics for subjunctive conditionals.

7 Subjunctive conditionals

With these preliminaries out of the way, we can turn to the subjunctive conditional. Here, I take my inspiration from Ramsey (1927):

> If two people are arguing 'If p, then q?'...they are adding p hypothetically to their stock of knowledge and arguing on that basis about q.

Following work by Von Fintel (2001), Veltman (2005), Gillies (2007), and Starr (2014), I propose a dynamic treatment of subjunctive conditionals. I suggest that the conditional $\phi > \psi$ tests a context σ to see whether updating with ϕ *in a certain way* creates

[31] Officially, Definitions 5, 6, 7, 8, 9, 13, 14, and 15 constitute one recursive definition of $[\cdot]$.

[32] The semantics for negation only operates on the epistemic possibilities. This will nonetheless allow negation to embed sentences that operate on the subjunctive possibilities, because in the semantics to follow any operation that is sensitive to the subjunctive possibilities will always either return the original context or will return the empty set of epistemic possibilities.

a context that supports ψ. If the test is passed, then the context supports $\phi > \psi$. Otherwise, updating the context with $\phi > \psi$ produces the absurd state $\perp$.

To make this semantics precise, we need a theory of updating σ with ϕ in the right kind of way. Subjunctive conditionals do not simply explore what happens when we change σ with $[\phi]$. Rather, we need a second kind of update function: $*$. I will call this function a *revision* operation. Subjunctive conditionals test a context to see whether revising with the antecedent creates a context in which updating with the consequent has no effect.

Definition 8 $\sigma[\phi > \psi] = \begin{cases} \sigma & \text{if } \sigma * \phi \models \psi \\ \perp & \text{otherwise} \end{cases}$

This proposal contrasts interestingly with the dynamic account of the indicative conditional ($\rightarrow$) proposed in Gillies (2004). On that proposal, $\phi \rightarrow \psi$ tests a context to see whether *updating* the context with ϕ creates a context that supports ψ:

Definition 9 $\sigma[\phi \rightarrow \psi] = \begin{cases} \sigma & \text{if } \sigma[\phi] \models \psi \\ \perp & \text{otherwise} \end{cases}$

Interestingly, this semantics for indicative conditionals validates an indicative analogue of CDA: $\chi \rightarrow (\phi \vee \psi) \models \chi \rightarrow (\neg\phi \rightarrow \psi)$. Moreover, this semantics for indicatives validates the indicative analogue of Collapse when disjunction is understood classically, because $\phi \rightarrow \psi$ is supported just in case the epistemic possibilities contain no $\phi \wedge \neg\psi$ worlds. In the case of indicatives, the badness of Collapse can be explained away, since in this hyperintensional semantics indicatives and disjunctions behave differently when embedded under higher operators such as negation. However, in the case of subjunctive conditionals we will look for a semantics that invalidates Collapse, since as we saw earlier such a result is much harder to stomach for counterfactuals.

Indicative and subjunctive conditionals perform the same structural test, except that the former involves updating with the antecedent, while the latter involves revising with the antecedent. To get predictions about subjunctive conditionals, we now need to supply a theory of revision. There are many available options.[33] In order to validate CDA, however, we will not need to specify how revision works in every case. We will only need to say how to revise a context with ϕ when the context is itself consistent with ϕ. This is quite surprising; most work on revision focuses on cases of adding ϕ to a body of information inconsistent with ϕ. Consistent revision has widely been thought to amount simply to updating. To validate CDA, however, this won't do. We will need a more complex story about revision, even in the apparently 'easier' case where revision is consistent.

Our own revision function $*$ will be defined in terms of $[\cdot]$. But consistent revision will not simply be a matter of updating with $[\cdot]$. Rather, the key idea will be that $*$ performs the update associated with $[\cdot]$, but on the subjunctive possibilities (s_σ) rather than the epistemic ones (e_σ).

[33] Veltman (2005) offers an operation that is sensitive to fine-grained features of individual possible worlds. Gillies (2007) relies on updating systems of spheres. While our own semantics is in some ways similar to that in Gillies (2007), we depart from slightly from it by constructing *might* counterfactuals out of a counterfactual operator which takes scope over a *might* operator, rather than taking them as primitive. Finally, Starr (2014) offers an analysis in terms of Stalnaker (1968)'s selection functions.

7.1 Consistent revision axioms

To understand revision, we will start axiomatically. Rather than looking at any particular revision operation, we will instead look at a variety of structural properties that revision might have (Sect. 7.1). After this, we will find a particular revision operator that satisfies these axioms (Sect. 7.2). In what follows, we will focus especially on the phenomenon of consistent revision. A theory of consistent revision models counterfactuals whose antecedents are consistent with the context. This in turn is a model of how we, as rational agents, make suppositions that are consistent with our beliefs.

To state our axioms, we need to be able to say when one context contains at least as much information as another. Let's use $\sqsubseteq$ to model this relation, so that $\sigma \sqsubseteq \sigma'$ just in case σ is at least as strong as σ'. In what follows we will let $\sqsubseteq$ focus on non-modal claims alone. Since non-modal claims only operate on the epistemic possibilities, we will say that $\sigma \sqsubseteq \sigma'$ just in case e_σ is a subset of $e_{\sigma'}$:

Definition 10 $\sigma \sqsubseteq \sigma'$ iff $e_\sigma \subseteq e_{\sigma'}$

When $\sigma \sqsubseteq \sigma'$, σ contains at least as much information as σ', supporting at least as many boolean claims as σ'.

With a definition of $\sqsubseteq$ in place, we can now state a series of axioms that might govern $*$. We will start by reviewing the principles of the most popular theory of revision—the AGM revision axioms.[34] While most of these axioms are plausible, I will argue that one of them (Vacuity), governing the revision of a context with claims consistent with it, must be replaced. I replace this axiom with a pair of weaker axioms governing consistent revision. These axioms reduce the problem of consistent revision to the problem of revising with the tautology, and then constrain this latter process. The key idea is that when an agent supposes a tautology, she momentarily gives up some of her current beliefs. Counterfactual supposition is a different, weaker state of mind than belief.

The first AGM axiom, Closure, requires that revision is closed under logical consequence.[35]

CLOSURE $\sigma * \phi$ is a context

Since contexts are logically closed, whenever one revises with ϕ, any consequence of ϕ is also learned. Given our semantics for conditionals, Closure corresponds to the principle that the consequents of conditionals agglomerate, so that $\chi > \phi$ and $\chi > \psi$ entail $\chi > (\phi \wedge \psi)$.

The next axiom, Success, says that revision with ϕ is a way of supposing that ϕ.

SUCCESS $\sigma * \phi \models \phi$

Whenever one revises with ϕ, one enters a state that supports ϕ, so that updating with ϕ has no further effect. Success corresponds to the Reflexivity principle that $\models \phi > \phi$.

[34] See Alchourrón et al. (1985). The positive theory below also differs from other accounts of belief revision in the literature (Katzuno and Mendelzon 1992), especially in the potential for revision with a tautological claim to change the belief state.

[35] Each of the axioms below is restricted to cases where ϕ and ψ are boolean, containing no modals.

The third axiom, Consistency, says that revising a consistent context with a consistent claim produces another consistent context.[36]

CONSISTENCY If $\phi \not\models \bot$ & $\sigma \neq \bot$, then $\sigma * \phi \neq \bot$

The Consistency axiom distinguishes revision from ordinary updating. When one updates a consistent context with some consistent claim that has already been ruled out by the context, the result is absurd. However, in the case of revision the effect is different. Rather than becoming absurd, the new context will remove some of the information in the original context, to make room for the new claim.

The next axiom, Extensionality, says that logically equivalent sentences induce the same revision on any context.

EXTENSIONALITY If $\phi \dashv\models \psi$, then $\sigma * \phi = \sigma * \psi$

Given our semantics, this corresponds to the Substitution principle, which says that logically equivalent sentences are substitutable in the antecedents of conditionals.[37]

Finally, Inclusion says that updating always contributes at least as much information as revision.

INCLUSION $\sigma[\phi] \sqsubseteq \sigma * \phi$

In the semantics above, this is equivalent to the claim that the subjunctive conditional entails its indicative counterpart for any boolean consequent. For suppose that $\sigma[\phi]$ always contains at least as much information as $\sigma * \phi$. Then any boolean claim ψ supported by $\sigma * \phi$ will be supported by $\sigma[\phi]$, and so $\sigma > \psi$ will entail $\phi \rightarrow \psi$. For this reason, Inclusion also implies that the subjunctive conditional satisfies Modus Ponens for boolean sentences.

The AGM theory of revision consists of all the above axioms, plus one extra assumption: the Vacuity axiom. The Vacuity axiom characterizes consistent revision. It says that whenever ϕ is compatible with a context, revising with ϕ contributes as much information as updating with ϕ.

VACUITY If $\sigma \not\models \neg\phi$, then $\sigma * \phi \sqsubseteq \sigma[\phi]$

Together with Inclusion, this implies that consistent revision is identical to updating. Counterfactual supposition will consistently add to an agent's current beliefs, whenever this can be done without contradiction.

While most of the axioms above are quite natural, I will now argue that Vacuity should be rejected. We need a new theory of consistent revision. Given our semantics, the Vacuity revision axiom entails the Vacuity principle we considered earlier, that ϕ entails $\top > \phi$. After all, since any context σ is consistent with $\top$, Vacuity implies that $\sigma * \top$ always contains at least as much information as $\sigma[\top]$. But $\sigma[\top]$ is just σ; so any non-modal claim that holds in σ holds in $\sigma * \top$. So $\phi \models \top > \phi$.[38] However,

[36] This is analogous to the Activity principle for conditionals discussed in Gillies (2010).

[37] Extensionality can fail when ϕ and ψ are not both boolean. For example, in Veltman (1996) ϕ and $\Box\phi$ are logically equivalent but have different meanings. In the theory of revision below, $*\phi$ and $*\Box\phi$ are different operations.

[38] Indeed, previous theories that interpret the conditional in terms of belief revision (Gardenförs 1988; Levi 1996) have predicted that ϕ entails $\top > \phi$.

we saw earlier that this principle and CDA lead to the collapse of the subjunctive and material conditionals. So if we want to validate CDA, we must reject the Vacuity revision axiom.

There are also reasons independent of CDA to give up the Vacuity axiom. To sharpen our intuitions about both Vacuity and the Inclusion axiom above, let's return to the Oswald sentences.[39] (48) seems true, while (49) seems bizarre:

(48) If Oswald didn't kill Kennedy, someone else did.

(49) If Oswald hadn't killed Kennedy, someone else would have.

Given Inclusion, subjunctives like (49) are at least as strong as their corresponding indicatives like (48). One might at first think that subjunctives do not entail indicatives, since it seems like (50) and (48) are both true:

(50) If Oswald hadn't killed Kennedy, no one else would have.

However, I think this impulse should be rejected. It is commonly thought that indicative conditionals presuppose the epistemic possibility of their antecedent.[40] But it is bizarre to acknowledge the possibility that Oswald didn't kill Kennedy while asserting both (48) and (50).

According to Vacuity, whenever we accept both an indicative like (48) and the epistemic possibility of its antecedent, we are forced to accept the corresponding subjunctive (49). This principle clashes with the idea that indicative conditionals presuppose the epistemic possibility of their antecedent. Together, these claims imply that whenever an indicative conditional is defined, it entails the corresponding subjunctive conditional.

We have now seen that there are reasons to reject the Vacuity revision axiom, whether one accepts CDA or not. So we need new axioms for consistent revision. I propose that consistent revision is like updating, except that it involves an additional component: before updating, one first accesses a broader body of information (the counterfactual state of mind). To access this broader body of information, one simply revises with the tautology $\top$. So consistent revision is not equivalent to updating with ϕ, but rather to first revising with $\top$ and then updating with ϕ:

RESTRICTED VACUITY If $\sigma[\phi] \neq \bot$, then $\sigma * \phi \sqsubseteq (\sigma * \top)[\phi]$

Restricted Vacuity reduces the problem of consistent revision to the problem of tautological revision. This principle will do much of the work below in validating CDA.[41]

All that's left to characterize consistent revision is to give a theory of revision with the tautology. Again, revising with $\top$ models the process of withdrawing some of the information in the common ground, to reflect the broader set of subjunctive possibilities. That revision with $\top$ has some sort of weakening effect is guaranteed

[39] See Adams (1970).

[40] For discussion, see Stalnaker (1975), Karttunen and Peters (1979), von Fintel (1998), Gillies (2004), Gillies (2009), Leahy (2011), and Starr (2014).

[41] Although Restricted Vacuity is weaker than Vacuity it is still substantive. For example, the theory in Starr (2014) validates Inclusion, but not Restricted Vacuity (nonetheless, it still predicts that $\phi \models \top > \phi$).

by what we have said so far. Inclusion guarantees that revising σ with $\top$ produces a context with no more information than σ itself. Since we are requiring tautological revision to have some effect, it must remove some of the information in σ.

To reach a principled theory of revising with the tautology, I propose one more constraint. Revision with $\top$ will have no effect other than allowing one to enter the counterfactual state of mind. This effect is achieved whenever one performs any kind of revision. So revising with ϕ is equivalent to revising with $\top$ and also revising with ϕ:

IDEMPOTENCE $\sigma * \phi = \sigma * \top * \phi = \sigma * \phi * \top$

Summing up, I propose to replace Vacuity with two axioms. Restricted Vacuity reduces consistent revision to tautological revision plus updating. Idempotence constrains tautological revision to be well behaved, so that it has no effect after one has started revising.

We have now reviewed a variety of structural conditions on revision. Once we have the right semantics for disjunction, we will see that any revision function satisfying these constraints will validate CDA. This is only interesting, however, if it is possible for a revision function to satisfy all these axioms. I now show that these axioms are satisfied by an intuitive theory of revision.

7.2 A theory of consistent revision

The basic idea is simple. Consistent revision is a two step process. To counterfactually suppose ϕ, one must first enter into the counterfactual state of mind. This requires turning one's attention from the epistemic possibilities to the subjunctive possibilities, and is accomplished by revising with the tautology. Once one's attention is directed at the right body of information, one then updates this information with ϕ. To implement this, we factorize $*$ into two operations. First, $*$ accesses the counterfactual state of mind in σ; then $*$ applies $[\cdot]$ to this new context.

The first step is to define operators that allow us to move freely between the epistemic and subjunctive information in a context. One operator, $\uparrow$, replaces the epistemic possibilities in a context with the subjunctive possibilities. The other operator, $\downarrow$, replaces the subjunctive possibilities with the epistemic ones.

Definition 11 $\uparrow(\sigma) = \langle e_{\sigma\uparrow}, s_{\sigma\uparrow}\rangle$, where $e_{\sigma\uparrow} = s_\sigma = s_{\sigma\uparrow}$.
$\downarrow(\sigma) = \langle e_{\sigma\downarrow}, s_{\sigma\downarrow}\rangle$, where $e_{\sigma\downarrow} = e_\sigma = s_{\sigma\downarrow}$.

Now we define revision in terms of $\uparrow$, $\downarrow$, and $[\cdot]$. To revise a context σ with ϕ, one first overwrites σ's epistemic with σ's subjunctive possibilities to reach $\uparrow(\sigma)$. Then one updates the result with ϕ, to reach $\uparrow(\sigma)[\phi]$. For example, when ϕ is atomic, $[\phi]$ narrows down the epistemic possibilities of σ to the ϕ worlds. By contrast, $*\phi$ will narrow down the subjunctive possibilities of σ to the ϕ worlds by first moving the subjunctive possibilities of σ over to the epistemic slot and then applying $[\phi]$.

However, things are a bit more complicated. First, we cannot simply let $\sigma * \phi$ be $\uparrow(\sigma)[\phi]$. This would mean that the subjunctive possibilities of $\sigma * \phi$ would be the same as σ's. But this would make trouble for interpreting nested counterfactuals like $\phi > (\psi > \chi)$, which should explore the result of updating the subjunctive possibilities with first ϕ and then ψ. To model this, our revision operator takes the new epistemic possibilities of $\uparrow(\sigma)[\phi]$, and copies them over to the subjunctive possibilities with $\downarrow$.

Second, since we are limiting attention to consistent revision, it will be crucial that revision only be defined when it can be performed without crash. So if $*\phi$ would create the absurd state, it is undefined. This will give us some measure of nonmonotonicity when it comes to revision. When the definedness condition on $*\phi$ is not satisfied, we may *accomodate* this condition by expanding our horizon of subjunctive possibilities.[42] This would allow us to explain why $\sigma * \phi$ may support χ even though $\sigma * \phi \wedge \psi$ does not: $\sigma * \phi \wedge \psi$ may induce accommodation to a wider set of subjunctive possibilities, where χ need not hold.

Definition 12 $\sigma * \phi$ is defined only if $\uparrow(\sigma)[\phi] \neq \bot$.
If defined, $\sigma * \phi = \downarrow(\uparrow(\sigma)[\phi])$.

One might worry at this point that our theory is incomplete, since it doesn't give a story about how exactly accommodation works (the kind of story suppled in Von Fintel (2001) and Gillies (2007)). Surprisingly, however, we'll be able to explain CDA in what follows without any such story. Soon, we'll introduce a new semantics for disjunction, which secures the result that whenever $\chi > (\phi \vee \psi)$ is defined, $\sigma * \chi$ can be consistently updated with $\neg\phi$. This allows our explanation of CDA to focus on the case of consistent revision.

Here's an example of how our revision operator works: start with three possible worlds, w, v, and u, where α is true at w and u, but false at v; and where β is true at v and u but false at w. Let $e_\sigma = \{w, v\}$ and let $s_\sigma = \{w, v, u\}$. In a traditional theory of consistent revision, $\sigma * \alpha$ could be $\langle\{w\}, s_\sigma\rangle$. Since the epistemic possibilities are consistent with α, revision with α would simply amount to narrowing down the epistemic possibilities to w, the only epistemic possibility where α is true. $\sigma * \alpha$ would support $\neg\beta$, since the remaining epistemic possibilities after revising with α would all be $\neg\beta$ worlds. By contrast, on the current theory $\sigma * \alpha$ is $\langle\{w, u\}, \{w, u\}\rangle$. Crucially, the possibility u that has been stored in the subjunctive possibilities is now an epistemic possibility in this revised context. Here, $\sigma * \alpha$ leaves unsettled the question of whether β, since we have introduced a new possibility, u, where α and β both hold.

Generalizing from this case, our theory of revision has several illuminating properties. First, it explains the relationship between revision and update. Both functions perform the same basic operation; but they perform this operation on different bodies of information. $[\cdot]$ operates on the epistemic possibilities, while $*$ operates on the subjunctive possibilities.

Second, this theory explains why revision with a tautology has an effect. Revision with the tautology simply allows one to enter the counterfactual state of mind, transferring s_σ to e_σ:

Fact 13 $\sigma * \top = \langle s_\sigma, s_\sigma\rangle$

In other words, tautological revision is identical to our overwrite operation $\uparrow$. The reason that revising with the tautology changes the underlying state is that revision

[42] See Von Fintel (2001), Gillies (2007).

cares about a different body of information than updating. But revising with a tautology is the weakest possible revision, since it doesn't actually narrow down any of the subjunctive possibilities.

Finally, this theory of revision satisfies the restricted AGM axioms we constructed in the previous section.[43]

Fact 14 $*$ satisfies the restricted AGM revision axioms.

This shows that restricted AGM revision is not simply an ad hoc weakening of ordinary AGM revision. Rather, we have an intuitive theory of revision that fits perfectly with our restricted axioms. This theory of revision models the big picture idea that counterfactual supposition is a distinctive mental state, not reducible to some operation on the agent's particular beliefs. For example, imagine that the subjunctive possibilities for an agent are determined by the agent's conception of the laws of nature. The agent may have many particular beliefs that are stronger than what she takes the laws of nature to be. When one counterfactually supposes that ϕ, one first removes from her belief set any beliefs stronger than what she takes the laws of nature to be. One then updates the resulting body of information with ϕ. The meaning of the subjunctive conditional is an abstract representation of this process, exploring what happens to a body of information when this sort of revision is performed.

8 Disjunction

We are halfway to validating CDA. All that's left is a theory of disjunction. We saw above that CDA and Disjunction Introduction together lead to some paradoxical results. So we need a semantics for disjunction that gives up Disjunction Introduction. In addition, we want to validate the Free Choice inference that $\Diamond(\phi \vee \psi)$ entails $\Diamond\phi$, since this inference is already inconsistent with Disjunction Introduction.

Here's the big picture idea. The disjunction $\phi \vee \psi$ doesn't simply narrow down a context to the set of worlds where one of ϕ or ψ is true. In addition, $\phi \vee \psi$ is only defined in a context when the context leaves both ϕ and ψ unsettled, so that each claim and their negations are possible in the context.

More precisely, I propose to analyze disjunction using the epistemic possibility operator *might* ($\Diamond_e$) from the data semantics developed in Veltman (1985) and recently integrated into update semantics in Gillies (2018). On this theory, $\Diamond_e\phi$ tests a context σ to see that some strengthening of it supports ϕ. That is, it tests whether there is some way to consistently strengthen the information σ to support ϕ. If so, the context remains the same; otherwise, the context becomes absurd.[44]

[43] $*$ is not defined for cases of inconsistent revision. For this reason, some instances of the restricted AGM axioms are undefined. However, in the appendix I show how to extend $*$ straightforwardly in cases of undefinedness so that all the restricted AGM axioms are always defined and satisfied.

[44] This possibility operator is different than the one in update semantics (Veltman 1996), where $\Diamond\phi$ tests σ to see whether $\sigma[\phi]$ is not absurd. That semantics would have quite similar effects to the data semantics above, except in two respects (of which more later). First, the possibility operator from data semantics above validates Scopelessness, while the possibility operator from update semantics does not. Second, the possibility operator from data semantics validates a wide scope version of Free Choice (that $\Diamond\phi \vee \Diamond\psi$ entails $\Diamond\phi$), while the possibility operator from update semantics does not.

Definition 13 $\sigma[\Diamond_e\phi] = \begin{cases} \sigma & \text{if } \exists\sigma' \neq \bot : \sigma' \sqsubseteq \sigma \;\&\; \sigma' \models \phi \\ \bot & \text{otherwise} \end{cases}$

We can then make our subjunctive possibility operator $\Diamond\phi$ equivalent to $\top > \Diamond_e\phi$, operating on the subjunctive possibilities by applying the test above to $\uparrow(\sigma)$.

Definition 14 $\sigma[\Diamond\phi] = \begin{cases} \sigma & \text{if } \exists\sigma' \neq \bot : \sigma' \sqsubseteq \uparrow(\sigma) \;\&\; \sigma' \models \phi \\ \bot & \text{otherwise} \end{cases}$

In addition, this semantics for possibility modals allows us to define necessity modals as the duals of possibility modals. On the resulting theory, σ will support $\Box_e\phi$ just in case every way of strengthening the information in σ supports ϕ.

Now we can analyze disjunction using this possibility operator. Say that a claim ϕ is *unsettled* in σ just in case $\Diamond\phi$ and $\Diamond\neg\phi$ both hold in σ. Then $\phi \vee \psi$ requires that both ϕ and ψ are unsettled.[45] More precisely, I claim that $\phi \vee \psi$ *presupposes* the unsettledness of ϕ and ψ. Following a tradition from Heim (1983), Beaver (2001), and others, let's treat presuppositions as definedness conditions on update. So ϕ presupposes ψ just in case $\sigma[\phi]$ is defined only if $\sigma \models \psi$.

$\phi \vee \psi$ is defined in σ only if ϕ and ψ are unsettled in σ. When defined, $\phi \vee \psi$ updates the epistemic possibilities of σ to the union of updating with ϕ and updating with ψ:

Definition 15 $\sigma[\phi \vee \psi]$ is defined only if: $\sigma \models \Diamond_e\phi$, $\sigma \models \Diamond_e\neg\phi$, $\sigma \models \Diamond_e\psi$, and $\sigma \models \Diamond_e\neg\psi$.
If defined, $\sigma[\phi \vee \psi] = \langle e_{\sigma[\phi]} \cup e_{\sigma[\psi]}, s_\sigma \rangle$.

At this point, it is worth pausing to clarify the status of the definedness conditions in play both here and in the case of counterfactuals. With Gillies (2007), it may be worth distinguishing the definedness failures here from ordinary cases of presupposition failure. Failures of definedness here may not trigger the same kinds of infelicity. For example, Gillies (2007) observes that entertainability presuppositions in general fail the *hey, wait a minute test*:

(51) If Hans had come to the party, we would have run out of punch.

(52) ??Hey, wait a minute. I had no idea that Hans might have come.

(53) If Hans had come to the party, he might have run into Anna.

(54) ??Hey, wait a minute. I had no idea that Hans might have run into Anna.

What is important for our purpose is that in the case of counterfactuals and with disjunction, the failure of definedness triggers accommodation via domain expansion, and that there is a notion of entailment that is sensitive to this kind of definedness failure.[46]

45 Unsettledness presuppositions have also been suggested for desire verbs like *wish* (Heim 1992) and *glad* (Von Fintel 1999), as well as for epistemic modals (von Fintel and Gillies 2010).

46 At this point, it is natural to wonder *why* entertainability presuppositions appear easier to accommodate than ordinary presuppositions. Here, one natural strategy would appeal to the failure of persistence for possibility claims within update semantics. When an ordinary presupposition is undefined, accommodating

In addition, it is worth clarifying the connection between the kinds of possibilities in play and the knowledge of conversational participants. It can be acceptable to utter a disjunction even while conceding that the true disjunct is known:

(55) We all know who stole the jewels, but don't say it: it was either Robbie or Julia.

In these cases, the possibilities in the context set come apart from the common knowledge of the conversational participants. While the participants know who stole the jewels, they pretend that both options are live. It is this pretense that is modeled by the 'epistemic' possibilities of the context.[47]

Finally, its worth clarifying the extent to which this semantics departs from a classical one. Of most interest might be that this semantics invalidates one direction of De Morgan's Law: $\neg(\neg\phi \wedge \neg\psi)$ no longer implies $\phi \vee \psi$, since only the conclusion of the argument carries with it a requirement that ϕ and ψ be unsettled. On the other hand, we'll see in the next section that there is a way of introducing a more classical notion of entailment within this system where this principle holds.

This concludes our semantics. In the final section, I will apply the semantics to the problem we started with. To do so, we will need to find the right definition of entailment.

9 Entailment

In dynamic semantics, it is common to define entailment as preservation of support.[48]

Definition 16 Γ entails δ ($\Gamma \models \delta$) iff for every context σ, if for every $\gamma \in \Gamma$ $\sigma[\gamma]$ is defined and $\sigma \models \gamma$, then $\sigma \models \delta$.

Given this definition of validity, our semantics validates CDA.

Fact 15 $\chi > (\phi \vee \psi) \models \chi > (\neg\phi > \psi)$

Interestingly, we can show that CDA is valid without appealing to our particular theory of revision. All we need to assume is that our revision operator satisfies Idempotence and Restricted Vacuity.

Footnote 46 continued
that information requires shrinking the domain. By contrast, when an entertainability presupposition is undefined, accommodating that information requires expanding the domain. Perhaps the asymmetry in the ease of accommodation of these two types of presupposition follows from a more general asymmetry regarding whether conversational participants are more willing to add or subtract information from a shared body of information.

[47] See Stalnaker (2014) for a similar proposal:

'The common ground is what is presumed to be common knowledge, and normally one presumes that something is common knowledge when one believes that it is. But in some cases, it may serve the purposes of the conversation to engage in some mutually recognized pretense, or to carry on a conversation within the scope of some mutually recognized supposition.' (§2.3)

[48] Suppose we operated with the more dynamic update-to-test notion of entailment (see van Benthem 1996). Then we would say that $\gamma_1; \ldots; \gamma_n$ entails δ just in case for any σ where $\sigma[\gamma_1]\ldots[\gamma_n]$ is defined, $\sigma[\gamma_1]\ldots[\gamma_n] \models \delta$. For more discussion of this idea, see Willer (2014), p. 12.

Proof Suppose that $\chi > (\phi \vee \psi)$ is defined at σ, and that σ supports $\chi > (\phi \vee \psi)$. Since $\chi > (\phi \vee \psi)$ is defined, $\sigma * \chi$ supports $\Diamond\neg\phi$, and hence $\sigma * \chi$ can be consistently updated with $\neg\phi$. So revising $\sigma * \chi$ with $\neg\phi$ is consistent. Further, since $\sigma * \chi$ supports $\phi \vee \psi$ we know that the result of updating $\sigma * \chi$ with $\neg\phi$ supports ψ. Next, by Idempotence we can add in an occurrence of revising with $\top$, to get that the result of updating $\sigma * \chi * \top$ with $\neg\phi$ supports ψ. Now applying Restricted Vacuity we know that $\sigma * \chi * \neg\phi$ supports ψ. So $\sigma * \chi$ supports $\neg\phi > \psi$, and σ supports $\chi > (\neg\phi > \psi)$. □

While our theory validates CDA, it does not lead to Collapse:

Fact 16 $\phi \vee \psi \not\models \neg\phi > \psi$

Here, it's useful to distinguish two things: (i) the structural condition a sentence imposes on a set of worlds; and (ii) the set of worlds that condition is imposed upon. $\phi \vee \psi$ requires of a set of worlds that every world makes true either ϕ or ψ, that it contains a $\neg\phi$ world, and more. This is exactly what is required by $\neg\phi > \psi$. Crucially, however, $\neg\phi > \psi$ requires more worlds to satisfy this condition than $\phi \vee \psi$ does. So suppose that the epistemic possibilities are all either ϕ or ψ worlds, while the subjunctive possibilities include some $\neg\phi \wedge \neg\psi$ worlds. In this case $\phi \vee \psi$ is supported while $\neg\phi > \psi$ is not.

To validate CDA without Collapse, the theory invalidates Vacuity, Modus Ponens, and Disjunction Introduction. In the case of Vacuity and Modus Ponens, the key is to again distinguish the epistemic and subjunctive possibilities:

Fact 17 $\phi \not\models \top > \phi$

Suppose that the epistemic possibilities are all ϕ worlds, while the subjunctive possibilities contain some extra $\neg\phi$ worlds. Here, counterfactually supposing the tautology has an effect, introducing the $\neg\phi$ worlds. In this derived context, $\uparrow(\sigma)$, ϕ is not supported. Counterfactual supposition has an effect, even in the case of the tautology, because it accesses a broader body of information.

Similarly, Modus Ponens fails for right-nested conditionals:

Fact 18 $\phi; \phi > (\psi > \chi) \not\models \psi > \chi$

Suppose the epistemic possibilities are all ϕ worlds. Suppose further that the subjunctive possibilities are made up of some ϕ worlds and some $\neg\phi$ worlds. At all the ϕ worlds where ψ holds, χ also holds; but at some $\neg\phi$ worlds where ψ holds, χ doesn't. In these cases, Modus Ponens will fail. σ supports ϕ, since the epistemic possibilities are all ϕ worlds. Further, it supports $\phi > (\psi > \chi)$, since once we revise with ϕ, thereby shrinking the subjunctive possibilities down, the only remaining ψ worlds are χ worlds. However, σ does not support $\psi > \chi$, since the original subjunctive possibilities contain $\neg\phi$ worlds where ψ and $\neg\chi$ both hold. Since the epistemic and subjunctive possibilities are different, σ can support ϕ without filtering out the $\neg\phi$ worlds from the subjunctive possibilities.

We saw above that these right-nested instances of Modus Ponens must be rejected once we accept CDA, or else we would reach Collapse. On the other hand, our semantics is a conservative rejection of Modus Ponens. Modus Ponens continues to hold when ϕ and ψ are themselves boolean:

Fact 19 $\phi; \phi > \psi \models \psi$ where ψ is boolean

After all, suppose ψ is boolean. Suppose $\sigma \models \phi$ and $\sigma \models \phi > \psi$. Then $\sigma[\phi] \neq \bot$, and $\sigma * \phi \models \psi$. So by Inclusion, $\sigma[\phi] \models \psi$. Since $\sigma[\phi] = \sigma$, this means $\sigma \models \psi$.

There is another sense in which our semantics is a conservative rejection of Modus Ponens. We saw above that another reason to reject right-nested Modus Ponens besides CDA was to validate Import-Export. Our semantics does just this:

Fact 20 $(\phi \wedge \psi) > \chi \mathrel{=\!\!|\!\models} \phi > (\psi > \chi)$

Import-Export holds whenever $(\phi \wedge \psi) > \chi$ is defined. For in this case, $\sigma * (\phi \wedge \psi)$ contains a χ world, and hence $\phi > (\psi > \chi)$ is defined. In this case, revision is just a matter of updating the subjunctive possibilities with the antecedent. Updating with $\phi \wedge \psi$ is equivalent to updating first with ϕ and then updating with ψ.

Finally, the semantics invalidates Disjunction Introduction.

Fact 21 $\phi \not\models \phi \vee \psi$

Here, the reason is that disjunctions presuppose unsettledness. So whenever a context supports ϕ, the conclusion $\phi \vee \psi$ is undefined. So there's no context that supports both ϕ and $\phi \vee \psi$.

Again, the semantics invalidates Disjunction Introduction for the right reasons. In addition to validating CDA, the semantics also validates Free Choice, which we saw above is inconsistent with Disjunction Introduction:

Fact 22 $\Diamond(\phi \vee \psi) \models \Diamond\phi$

Free Choice is valid because disjunctions presuppose that each disjunct is possible. So updating σ with $\Diamond(\phi \vee \psi)$ is only defined in the first place if σ supports $\Diamond\phi$.

This approach to free choice is structurally analogous to Zimmermann (2000), who also posits a semantic connection between disjunction and possibility. Yet the current semantics offers two advantages over Zimmermann (2000). First, Zimmermann (2000) must assume that Free Choice only occurs when disjunction takes wide scope. However, I validate Free Choice when $\vee$ occurs at either scope. That is: not only does $\Diamond(\phi \vee \psi)$ entail $\Diamond\phi$; we also have that $\Diamond\phi \vee \Diamond\psi$ entails $\Diamond\phi$.[49]

Second, Zimmermann (2000) faces problems when it comes to free choice for other flavors of modality.[50] For example, Willer (2015) observes a free choice effect in *might* counterfactuals:

(56) a. If Mary had not gone to Pisa, she might have gone to Lisbon or Rome.
b. So: if Mary had not gone to Pisa, she might have to Lisbon.

[49] Our theory of disjunction also gives a new, semantic explanation of Hurford's Constraint, the principle that it is inappropriate to assert a disjunction whose disjuncts are ordered by strength (Hurford 1974; Gazdar 1979; Chierchia et al. 2009; Meyer 2014, 2015). The above semantics predicts that any such disjunction is infelicitous. For whenever $\phi \models \psi$ or $\psi \models \psi$, the sentence $\phi \vee \psi$ is not supported by any context.

[50] See Geurts (2005) for an extension of Zimmermann (2000) that generalizes to other flavors of modality, but not to narrow scope disjunction.

Unlike Zimmermann (2000), where disjunctions essentially involve epistemic possibility, our theory of disjunction says that disjunctions presuppose that each disjunct is possible in the disjunction's local context. When a disjunction occurs in the consequent of a conditional, the disjunction is evaluated against the subjunctive possibilities. So $\chi > \Diamond(\phi \vee \psi)$ is defined in σ only if $\sigma * \chi$ is compatible with ϕ, which in turn requires that $\sigma * \chi$ support $\Diamond\phi$. This requires that $\chi > \Diamond\phi$.[51]

Since the semantics invalidates Disjunction Introduction, it also avoids the nested paradoxes of implication:

Fact 23 $\chi > \phi \not\models \chi > (\neg\phi > \psi)$

Fact 24 $\chi > \psi \not\models \chi > (\neg\phi > \psi)$

$\chi > \phi$ being supported requires that $\sigma * \chi$ contain only ϕ worlds. But in this case $\chi > (\neg\phi > \psi)$ is not defined. Similarly, $\chi > \psi$ being defined does not require that $\chi > (\neg\phi > \psi)$ is defined, since $\sigma * \chi$ may contain only ϕ worlds.

For similar reasons, Antecedent Strengthening is invalid on this theory.

Fact 25 $\phi > \chi \not\models (\phi \wedge \psi) > \chi$

For suppose that the subjunctive possibilities contain ϕ worlds, but no $\phi \wedge \psi$. Then $\phi > \chi$ can be supported while $\phi \wedge \psi$ is undefined.[52,53]

Our theory makes just the predictions we want. We have validated CDA while avoiding the collapse of the subjunctive and material conditional. To do so, we have given up Vacuity, Modus Ponens, and Disjunction Introduction. But in doing so, we have managed to validate Import-Export and Free Choice, traditional barriers to Modus Ponens and Disjunction Introduction.[54]

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[51] Similarly, our semantics predicts Free Choice is valid for deontic modals, provided that deontic modals overwrite the epistemic possibilities with the deontic possibilities. However, one outstanding problem for our semantics is in the behavior of Free Choice under negation, where *Mary can't have soup or salad* entails *Mary can't have soup and Mary can't have salad*. For discussion, see Alonso-Ovalle (2006), Willer (2015), and Starr (2016). For an attempt to derive this last entailment as an implicature, see Barker (2010).

[52] While our semantics invalidates Antecedent Strengthening, it validates Simplification of Disjunctive Antecedents, the principle that $(\phi \vee \psi) > \chi$ entails $(\phi > \chi) \wedge (\psi > \chi)$. Since disjunctions presuppose unsettledness, $(\phi \vee \psi) > \chi$ is defined only if there are subjunctive possibilities where ϕ and where ψ, in which case $(\phi > \chi) \wedge (\psi > \chi)$ is also defined. See Alonso-Ovalle (2006) for discussion.

[53] These last points differentiate our semantics from the restrictor theory of conditionals developed in Kratzer (1986). On that theory, conditional sentences are not represented through a special conditional operator $>$. Rather, *if* clauses restrict (sometimes covert) modal operators. Nested *if* clauses are interpreted as conjunctive restrictions. CDA is then equivalent to the claim that $\Box(\chi)(\phi \vee \psi)$ entails $\Box(\chi \wedge \neg\phi)(\psi)$. Interestingly, this only follows given Antecedent Strengthening (that $\Box(\phi)(\chi)$ implies $\Box(\phi \wedge \psi)(\chi)$). By contrast, the semantics above validates CDA without Antecedent Strengthening.

On the other hand, if the restrictor theory takes on board the theory of disjunction above, it can also validate CDA without Antecedent Strengthening. For then the restrictor theory may predict that $\Box(\chi)(\phi \vee \psi)$ presupposes $\Diamond(\chi)(\neg\phi)$. In that case, CDA will follow from a weakening of Antecedent Strengthening, on which $\Box(\phi)(\chi)$ and $\Diamond(\phi)(\psi)$ implies $\Box(\phi \wedge \psi)(\chi)$. This weakened form of Antecedent Strengthening follows from Axiom 5, discussed above.

[54] The definition of entailment above differs from an alternative—Strawson entailment—explored in Strawson (1952), Von Fintel (1999), and Von Fintel (2001), an argument is Strawson valid just in case whenever the premises *and the conclusion* are defined and the premises are supported, the conclusion is also supported.

10 Conclusion

In this paper, we've explored a simple pattern of entailment: CDA. First, we saw that CDA follows from the duality of *might* and *would* counterfactuals given several

Footnote 54 continued

This definition contrasts interestingly with the definition above. First, this definition quantifies over strictly fewer contexts than the previous one. So any argument that is valid is also Strawson valid. Thus CDA, Import-Export, and Free Choice are all Strawson valid. However, some inferences that are invalid are nonetheless Strawson valid. For example, Disjunction Introduction is vacuously Strawson valid since there is no context where ϕ is supported and $\phi \vee \psi$ is defined (Strawson validity avoids the Free Choice collapse result in Fact by denying Transitivity). For similar reasons, the semantics above predicts that Nested False Antecedent, Nested True Consequent, and Antecedent Strengthening are Strawson valid.

Interestingly, the major premises we used to support CDA—Scopelessness, If to And, and Strictness—are all Strawson valid.

Fact 26 $\phi > \psi \dashv\models \neg(\phi > \Diamond\neg\psi)$ where ϕ and ψ are boolean.

Suppose updating with both $\phi > \psi$ and $\neg(\phi > \Diamond\neg\psi)$ is defined in σ. Then σ's subjunctive possibilities contain a ϕ world. σ supports $\phi > \psi$ just in case every subjunctive ϕ world is a ψ world, which holds just in case it is not true that some subjunctive ϕ world is a $\neg\psi$ world, which is the condition imposed by $\neg(\phi > \Diamond\neg\psi)$.
Duality also holds when the consequent is itself a conditional. Suppose that χ, ϕ, and ψ are all boolean and that updating with both $\chi > (\phi > \psi)$ and $\neg(\chi > \Diamond\neg(\phi > \psi))$ is defined. σ then supports $\chi > (\phi > \psi)$ just in case every subjunctive $\chi \wedge \phi$ world is a ψ world. This holds just in case it is not true that there is some subset of subjunctive χ worlds where some ϕ world is not a ψ world, which is the condition required by $\neg(\chi > \Diamond\neg(\phi > \psi))$.

Fact 27 $\Diamond(\phi > \psi) \dashv\models \phi > \Diamond\psi$ where ϕ and ψ are boolean.

Suppose updating σ with both $\Diamond(\phi > \psi)$ and $\phi > \Diamond\psi$ is defined. σ supports $\Diamond(\phi > \psi)$ just in case σ's subjunctive possibilities contain a ϕ world, and there is some set of subjunctive ϕ worlds that are all ψ worlds. This is equivalent to the condition that the set of all ϕ worlds in σ contain some ψ worlds, which in turn is the condition imposed by $\phi > \Diamond\psi$.

Fact 28 $\phi > \Diamond\psi \models \Diamond(\phi \wedge \psi)$ where ϕ and ψ are boolean.

Suppose updating σ with $\phi > \Diamond\psi$ and $\Diamond(\phi \wedge \psi)$ is defined. Then σ contains a subjunctive ϕ world. σ supports $\phi > \Diamond\psi$ just in case some subjunctive ϕ world is a ψ world, which holds just in case there is some subjunctive $\phi \wedge \psi$ world in σ, which is the condition required for σ to support $\Diamond(\phi \wedge \psi)$.

Fact 29 $\phi > \psi \dashv\models \Box(\neg\phi \vee \psi)$ where ϕ and ψ are boolean.

Suppose updating σ with $\phi > \psi$ and $\Box(\neg\phi \vee \psi)$ is defined. Then σ's subjunctive possibilities contain a ϕ world, and are unsettled with respect to each of ϕ and ψ. σ supports $\phi > \psi$ just in case every subjunctive ϕ world is a ψ world; this is equivalent to the condition that every subjunctive possibility in σ is either a $\neg\phi$ world or a ψ world, which is what it takes for σ to support $\Box(\neg\phi \vee \psi)$.

Summarizing then, CDA is an important sense weaker than some of the principles we used to justify it, like Strictness. CDA is both valid and Strawson valid, while Strictness is merely Strawson valid. So CDA is a principle that should be popular not only for proponents of strict conditionals, but also to others.

Another interesting feature of Strawson validity is that it allows us to validate CDA without relying on the presuppositions of disjunction. So imagine that disjunction behaved classically, simply taking the union of updating with each conjunct. On the resulting view, CDA would remain valid, Disjunction Introduction would now be nonvacuously Strawson valid, and yet Collapse would still be avoided. As well, Free Choice would be invalid. The resulting theory might appeal to those who are skeptical of semantic accounts of Free Choice, and willing to embrace the nested paradoxes of material implication, while still wanting to avoid Collapse.

nonclassical but tempting principles. Next, however, we saw that this is paradoxical, since CDA itself requires major revisions to our understanding of conditionals and disjunction. In the face of this problem, we could either give up some of our starting points, or accept CDA and avoid its consequences. In the second half of this paper, I developed one form of this latter kind of solution, developing a semantics that can validate CDA along with Duality, Strictness, and Scopelessness, while invalidating Modus Ponens, Vacuity, and Disjunction Introduction in order to avoid Collapse.

A natural question, though, is how the opponent of CDA should best develop their theory. Here, the following strategy might be best. It turns out that given a pair of plausible principles, CDA can be treated as an implicature. First, suppose that we accept Import-Export. Second, suppose that we accept a restricted form of Antecedent Strengthening:

STRENGTHENING WITH A POSSIBILITY $\phi > \chi ; \phi > \Diamond\psi \models (\phi \wedge \psi) > \chi$

Together, these two principles imply that $\chi > (\phi \vee \psi)$ and $\chi > \Diamond\neg\phi$ imply $\chi > (\neg\phi > \psi)$. For from Strengthening with a Possibility, $\chi > (\phi \vee \psi)$, and $\chi > \Diamond\neg\phi$, we can infer $(\chi \wedge \neg\phi) > (\phi \vee \psi)$, which in turn implies $(\chi \wedge \neg\phi) > \psi$ given a few classical background assumptions. Then from Import-Export we can infer $\chi > (\neg\phi > \psi)$. So the defender of Import-Export and Strengthening with a Possibility can explain the apparent plausibility of CDA as arising from a natural implicature: that $\chi > (\phi \vee \psi)$ implicates $\chi > \Diamond\neg\phi$. This last inference is predicted from standard methods of implicature calculation, such as those in Sauerland (2004).

While this is a promising strategy for explaining the potential appeal of CDA, it is not without its problems. Besides the controversy surrounding Import-Export, Strengthening with a Possibility is not without its critics. For example, the principle is rejected by Pollock (1976), Kratzer (1981), and Boylan and Schultheis (2017). As always, the final status of CDA must await a full accounting of the various costs and benefits of the various logical principles discussed above.[55]

Appendix

Fact 1 Assume Transitivity, the 4 Axiom, $\Box/\Diamond$ Duality, Consequent Weakening and Factivity. Then Strictness and Duality imply CDA.

Proof Here it is useful to start by proving that Duality implies that $\Box\phi$ and ϕ are substitutable in the consequent of conditionals:

$\Box$ SUBSTITUTION $\phi > \psi \dashv\models \phi > \Box\psi$

Lemma 1 *Assume Transitivity, the 4 Axiom, $\Box/\Diamond$ Duality, Consequent Weakening and Factivity. Then Duality implies $\Box$ Substitution.*

For the left to right direction, we can first apply Duality to establish that $\phi > \psi \models \neg(\phi > \Diamond\neg\psi)$. By 4 and Consequent Weakening, $\neg(\phi > \Diamond\neg\psi) \models \neg(\phi >$

[55] Thanks to Sam Carter, David Etlin, Thony Gillies, Alex Kocurek, Ernie Lepore, and the audience of the 2016 Pacific APA.

$\Diamond\Diamond\neg\psi$). By another application of Duality, $\neg(\phi > \Diamond\Diamond\neg\psi) \models \phi > \neg\Diamond\neg\psi$. Finally, by $\Box/\Diamond$ Duality and Consequent Weakening, $\phi > \neg\Diamond\neg\psi \models \phi > \Box\psi$. Applying the transitivity of $\models$, we have that $\phi > \psi \models \phi > \Box\psi$. The right to left direction follows immediately from Factivity and Consequent Weakening.

So Duality, combined with some minimal assumptions, gets us the substitutability of $\Box\phi$ and ϕ in the consequent of conditionals.[56] From Lemma 1, we can then prove:

Lemma 2 *Assume Transitivity and Consequent Weakening. Then Strictness and $\Box$ Substitution imply CDA.*

By $\Box$ Substitution, $\chi > (\phi \vee \psi) \models \chi > \Box(\phi \vee \psi)$. Now we apply Strictness and Consequent Weakening to the consequent, to establish that $\chi > \Box(\phi \vee \psi) \models \chi > (\neg\phi > \psi)$. Since entailment is transitive, this means that $\chi > (\phi \vee \psi) \models \chi > (\neg\phi > \psi)$ □

Fact 2 Assume Transitivity, Contraposition, Consequent Weakening, Upward Monotonicity for $\Diamond$, Axiom 4, and De Morgan's Law$_{lr}$. Then Duality, Scopelessness$_{rl}$, and If to And imply CDA.

Proof By Duality $\neg(\chi > (\neg\phi > \psi))$ entails $\chi > \Diamond(\neg(\neg\phi > \psi))$. Applying Duality to the consequent of this latter conditional, via Upward Monotonicity for $\Diamond$ and Consequent Weakening, we reach $\chi > \Diamond(\neg\phi > \Diamond\neg\psi)$. This last entails $\chi > \Diamond\Diamond(\neg\phi > \neg\psi)$ by Scopelessness and Consequent Weakening. By applying 4 and Consequent Weakening to the consequent, we know that $\chi > \Diamond\Diamond(\neg\phi > \neg\psi)$ entails $\chi > \Diamond(\neg\phi > \neg\psi)$. Now applying If to And to the consequent, and applying Consequent Weakening, we have that $\chi > \Diamond(\neg\phi > \neg\psi)$ entails $\chi > \Diamond(\neg\phi \wedge \neg\psi)$. Applying Duality to the whole conditional, we get that $\chi > \Diamond(\neg\phi \wedge \neg\psi)$ entails $\neg(\chi > \neg(\neg\phi \wedge \neg\psi))$. Finally, applying De Morgan's Law and Consequent Weakening: $\neg(\chi > \neg(\neg\phi \wedge \neg\psi))$ entails $\neg(\chi > \phi \vee \psi)$. Now we can apply Transitivity to this chain to reach the result that $\neg(\chi > (\neg\phi > \psi))$ entails $\neg(\chi > \phi \vee \psi)$. By Contraposition, we reach CDA. □

Fact 3 Assume Transitivity. Then Vacuity, Bounded from Below, and CDA imply Collapse.

Proof Follows immediately from: □

Fact 4 Assume Transitivity. Then Vacuity and CDA imply Collapse$_{lr}$.

Proof By Vacuity, $\neg\phi \vee \psi$ entails $\top > (\neg\phi \vee \psi)$. But by CDA this conditional entails $\top > (\phi > \psi)$, which by Vacuity entails $\phi > \psi$. Applying the transitivity of entailment, $\neg\phi \vee \psi$ entails $\phi > \psi$. □

[56] This fact is somewhat surprising from the perspective of Lewis (1973), who endorses Duality but not $\Box$ Substitution. This is possible because Lewis does not factorize *might* conditionals into a conditional and a possibility modal, but rather introduces a primitive *might* conditional operator. For this reason, the 4 axiom cannot be applied directly to his *might* conditionals.

Fact 5 Assume Transitivity and Reflexivity. Then Modus Ponens and CDA imply Collapse$_{lr}$.

Proof By Reflexivity, $\phi \vee \psi > \phi \vee \psi$ is valid. But by CDA, this last claim entails $\phi \vee \psi > (\neg\phi > \psi)$, which is therefore valid. So by Modus Ponens $\phi \vee \psi$ entails $\neg\phi > \psi$. □

Fact 6 Assume Transitivity and Consequent Weakening. Then Disjunction Introduction and CDA imply Nested False Antecedent and Nested True Consequent.

Proof For Nested False Antecedent, we can first use Disjunction Introduction and Consequent Weakening to establish that $\chi > \phi$ entails $\chi > (\phi \vee \psi)$. Then by CDA it follows that $\chi > \phi$ entails $\chi > (\neg\phi > \psi)$. For Nested True Consequent, Disjunction Introduction and Consequent Weakening entail that $\chi > \psi$ entails $\chi > (\phi \vee \psi)$, which by CDA again entails $\chi > (\neg\phi > \psi)$. □

Fact 7 Assume Transitivity, Cautious Monotonicity, and Substitution. Then Vacuity implies And to If.

Proof By Vacuity, $\phi \wedge \psi$ entails $\top > \psi$ and $\top > \phi$; which by Cautious Monotonicity entails $\top \wedge \phi > \psi$; which by Substitution entails $\phi > \psi$. □

Fact 8 Assume Transitivity, Identity, K, and Upward Monotonicity for $\Box$. Then Strictness implies Conditional Necessity.

Proof By Strictness, $\top > \phi \mathrel{=\!\!|\!\!\models} \Box(\bot \vee \phi)$. By K and $\models \Box\top$, we have that $\Box(\bot \vee \phi) \models \Box\phi$. By the Upward Monotonicity of $\Box$ and the fact that $\phi \models \bot \vee \phi$, we have that $\Box\phi \models \Box(\bot \vee \phi)$. So by the transitivity of $\models$, $\top > \phi \mathrel{=\!\!|\!\!\models} \Box\phi$. □

Fact 9 Assume Transitivity. Then Conditional Necessity and Vacuity imply $\Box$ Introduction.

Proof By Vacuity, $\phi \models \top > \phi$. By Conditional Necessity, $\top > \phi \models \Box\phi$. So by the transitivity of $\models$, $\phi \models \Box\phi$. □

Fact 10 Assume Transitivity. Then Axiom 5, Disjunction Introduction, and CDA imply Import-Export$_{lr}$.

Proof By Disjunction Introduction, $(\phi \wedge \psi) > \chi \models (\phi > \neg\chi) \vee (\phi \wedge \psi > \chi)$. So by Axiom 5 and the transitivity of $\models$, $(\phi \wedge \psi) > \chi \models \phi > (\neg\psi > \chi)$. So by CDA and the transitivity of $\models$, $(\phi \wedge \psi) > \chi \models \phi > (\psi > \chi)$. □

Fact 11 Assume Transitivity. Then (i) Import-Export implies that Antecedent Strengthening and Nested True Consequent are equivalent, and (ii) Consequent Weakening, Reflexivity, Import-Export and Antecedent Strengthening imply Nested False Antecedent.

Proof For the left to right direction of (i), Antecedent Strengthening guarantees that $\chi > \psi$ entails $\chi \wedge \neg\phi > \psi$. By Import-Export, this last that $\chi > (\neg\phi > \psi)$. For the right to left direction of (i), Nested True Consequent gives us that $\phi > \chi$ entails $\phi > (\psi > \chi)$, which by Import Export entails $(\phi \wedge \psi) > \chi$. For (ii), Antecedent Strengthening gives us that $\chi > \phi$ entails $(\chi \wedge \neg\phi) > \phi$, which by Import Export entails $\chi > (\neg\phi > \phi)$. This last, by Consequent Weakening and Reflexivity, entails $\chi > (\neg\phi > \psi)$. □

Fact 12 $*$ satisfies the restricted AGM revision axioms.

Proof Let's start with the new axioms: Restricted Vacuity, and Idempotence. Suppose that $\sigma[\phi] \neq \bot$. Then $\sigma * \phi$ is defined and $\sigma * \phi = \downarrow (\uparrow (\sigma)[\phi])$. Then $\sigma * \phi$ is defined and $\sigma * \phi = \downarrow (\uparrow (\sigma)[\phi]) \sqsubseteq (\sigma * \top)[\phi]$. This requires the epistemic possibilities in $\downarrow (\uparrow (\sigma)[\phi])$ to be a subset of those in $(\sigma * \top)[\phi]$. The former set of worlds is simply the subjunctive possibilities in σ where ϕ is true. But this is the same set as the epistemic possibilities of $(\sigma * \top)[\phi]$. That covers Restricted Vacuity; Idempotence follows trivially from Fact 13.

Validating the other axioms requires us to say what happens when $\sigma * \phi$ is undefined. Really, what we can show here is that there is a conservative extension of $*$ that satisfies the other axioms. In particular, let's extend $*$ with a standard account of how counterfactual revision works when $\sigma[\phi] \neq \bot$. Building on Von Fintel (2001) and Gillies (2007), we can let revision *expand the modal horizon* in these cases, expanding σ to a new context whose subjunctive possibilities are a superset of σ's, and then applying $*\phi$ to this new context:

Definition 17 Let $\leq_s$ be an ordering over supersets of s by inclusion. Let $f(\sigma, \phi)$ be the pair of e_σ and the smallest set ordered by $\leq_s$ consistent with $\{w \mid \langle\{w\}, \{w\}\rangle \models \phi\}$. Whenever $\sigma * \phi$ is undefined, let the accommodation of σ with ϕ is $f(\sigma, \phi)$. Let $*_f$ be a revision operator just like $*$ except that whenever $*$ is undefined, $\sigma *_f \phi$ is the result of applying $*\phi$ to the accommodation of σ with ϕ.

Closure, Success, Consistency, Extensionality, and Inclusion are satisfied by $*_f$. □

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