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The proper treatment of variables in predicate logic

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Abstract In §93 of *The Principles of Mathematics*, Bertrand Russell (1903) observes that "the variable is a very complicated logical entity, by no means easy to analyze correctly". This assessment is borne out by the fact that even now we have no fully satisfactory understanding of the role of variables in a compositional semantics for first-order logic. In standard Tarskian semantics, variables are treated as meaning-bearing entities; moreover, they serve as the basic building blocks of all meanings, which are constructed out of variable assignments. But this has disquieting consequences, including Fine's antinomy of the variable and an undue dependence of meanings on language (representationalism). Here I develop an alternative, Fregean version of predicate logic that uses the traditional quantifier–variable apparatus for the expression of generality, possesses a fully compositional, non-representational semantics, and is not subject to the antinomy of the variable. The advantages of Fregean over Tarskian predicate logic are due to the former's treating variables not as meaningful lexical items, but as mere marks of punctuation, similar to parentheses. I submit that this is indeed how the variables of predicate logic should be construed.

Keywords Variables · Compositionality · Predicate logic · Binding · Frege · Tarski · Representationalism · Model-theoretic semantics

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1 Introduction

In standard, Tarski-style *syntax* for first-order logic, atomic formulas are constructed from predicate symbols and an appropriate number of variables and names. Truth-functional connectives can be used to form new formulas out of ones already constructed. From any formula already constructed, and any individual variable, a new formula can be obtained by first writing a quantifier symbol, appending the chosen variable, and then appending the original formula. For instance, we can build the atomic formula Rxa from the two-place predicate symbol R, the variable x, and the name a. Similarly, we can construct the atomic formula Px from the one-place predicate symbol P and the variable x. We can then conjoin these by means of the conjunction symbol into $(Rxa \wedge Px)$, and finally prefix this formula with the quantifier-variable combination $\exists x$ to obtain the formula $\exists x(Rxa \wedge Px)$.

In Tarskian *semantics* for first-order logic, we start from a model $\mathfrak{M}$ and a variable assignment g over $\mathfrak{M}$, let the value $\mathsf{val}(x, g, \mathfrak{M})$ of a variable x relative to g and $\mathfrak{M}$ be the object $g(x)$, and let the value $\mathsf{val}(a, g, \mathfrak{M})$ of a name a relative to g and $\mathfrak{M}$ be the interpretation $a^{\mathfrak{M}}$ of a in $\mathfrak{M}$. Then we recursively define what it means for a variable assignment to satisfy a formula in a model: An assignment g satisfies an atomic formula $Pt_1 \dots t_n$ in $\mathfrak{M}$ just in case the tuple $\langle \mathsf{val}(t_1, g, \mathfrak{M}), \dots, \mathsf{val}(t_n, g, \mathfrak{M})\rangle$ belongs to the interpretation $P^{\mathfrak{M}}$ of the predicate symbol P in $\mathfrak{M}$. A negation is satisfied by g in $\mathfrak{M}$ if and only if g fails to satisfy the negated formula in $\mathfrak{M}$, a conjunction is satisfied by g in $\mathfrak{M}$ just in case g satisfies each conjunct in $\mathfrak{M}$, and an existentially quantified formula $\exists x\phi$ is satisfied by g in $\mathfrak{M}$ if and only if there exists some x-variant of g that satisfies ϕ in $\mathfrak{M}$.

The Principle of Compositionality has been given various formulations. According to one popular gloss, it says that the meaning of a compound linguistic item is determined by the meanings of that item's immediate parts, together with the way those parts are fitted together into the compound. Another (not entirely equivalent) version requires that for every syntactic rule α there is a semantic operation r_α such that $\mu(\alpha(e_1, \dots, e_n)) = r_\alpha(\mu(e_1), \dots, \mu(e_n))$, where we write $\mu(e)$ for the meaning of the expression e.[1] As Klein and Sternefeld (2017, 65) point out:

> Compositionality is at the heart of model theoretical semantics and its application to the semantics of natural language. As has become standard practice, linguists translate a fragment of English into an intensional extension of classical predicate logic (…).

It therefore seems pressing to ascertain that classical predicate logic itself has a compositional semantics. This is not completely obvious from the usual presentations of Tarski's logical semantics in terms of the recursive definition of satisfaction, since these don't specify what the meaning of an expression *is*. Nevertheless it is widely accepted that predicate logic does have a compositional semantics. The meaning assignment for Tarski-style predicate logic that is taken to show this proceeds as follows.[2]

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[1] See e.g. Pagin and Westerstahl (2010, §3).

[2] To my knowledge, Janssen (1997) was the first to propose a compositional meaning assignment along roughly these lines, defining the meaning of a formula as the set of variable assignments that satisfy it.

Fix a model $\mathfrak{M}$ with domain M and let $\mathfrak{G}$ be the set of all variable assignments over M. All meanings to be defined will be functions whose domain is $\mathfrak{G}$.

We begin with the lexical items (i.e. the primitive meaningful expressions): The meaning $[\![P]\!]$ of an n-place predicate symbol P is the (constant) function that maps each assignment $g \in \mathfrak{G}$ to the characteristic function $\chi_{P^{\mathfrak{M}}}$ of the interpretation $P^{\mathfrak{M}} \subseteq M^n$ of P in $\mathfrak{M}$; similarly the meaning $[\![a]\!]$ of a name a is the constant function that maps each $g \in \mathfrak{G}$ to the interpretation $a^{\mathfrak{M}}$ of a in $\mathfrak{M}$. The meaning $[\![x]\!]$ of a variable x is the function that maps each variable assignment $g \in \mathfrak{G}$ to $g(x)$. The meanings $[\![\neg]\!]$, $[\![\wedge]\!]$, ... of the truth-functional connectives $\neg$, $\wedge$, ... are the constant functions that map each assignment g to the truth function corresponding to the respective connective. The meaning $[\![\exists]\!]$ of the existential quantifier is the constant function that maps any assignment to the characteristic function, relative to the power set of M, of the non-empty subsets of M.

As to the compound linguistic items, i.e. the formulas ϕ, the idea is to let the meaning $[\![\phi]\!]$ of ϕ be the function on $\mathfrak{G}$ that maps a variable assignment to the truth value t ("true") if that assignment satisfies ϕ in $\mathfrak{M}$, and to the truth value f ("false") otherwise. Given the definition of satisfaction, this yields the following recursive clauses for the assignment of meanings to formulas:

1. $[\![Pt_1 \dots t_n]\!]$ is the function that maps any assignment g to
$$[\![P]\!](g)([\![t_1]\!](g), \dots, [\![t_n]\!](g)).$$
2. $[\![\neg\phi]\!]$ is the function that maps any assignment g to
$$[\![\neg]\!](g)([\![\phi]\!](g)).$$
3. $[\![(\phi \wedge \psi)]\!]$ is the function that maps any assignment g to
$$[\![\wedge]\!](g)([\![\phi]\!](g), [\![\psi]\!](g)).$$
4. $[\![\exists x\phi]\!]$ is the function that maps any assignment g to[3]
$$[\![\exists]\!](g)(\{u \in M : \text{for some } h \sim_x g, u = [\![x]\!](h) \text{ and } [\![\phi]\!](h) = \mathsf{t}\}).$$

If we now say that the immediate parts of $Pt_1 \dots t_n$ are P as well as $t_1, \dots, t_n$, the immediate parts of $\neg\phi$ are $\neg$ and ϕ, the immediate parts of $(\phi \wedge \psi)$ are ϕ, $\wedge$, and ψ, and the immediate parts of $\exists x\phi$ are $\exists$, x, and ϕ, it seems—at first blush at least—that this meaning assignment abides by the Principle of Compositionality.[4]

Footnote 2 continued
Janssen treats the quantifier–variable pair $\exists x$ as one symbol, thus essentially relegating the role of the variable to that of an index. Below I follow the more sophisticated presentation by Zimmermann and Sternefeld (2013).

[3] As usual, $h \sim_x g$ means that h is an x-variant of g; that is, $h(y) = g(y)$ for all variables y other than x.

[4] As a matter of fact, there are two immediate (but fixable) problems here, one having to do with whether our meaning assignment really does satisfy Compositionality, the other with the plausibility of the meaning

Nevertheless there are reasons to be worried.

First there is a concern about the *representationalism* of the semantics, that is, the fact that the meanings have been constructed out of variable assignments, which in turn are constructed out of variables, i.e. bits of language (as opposed to *bona fide* model-theoretic objects, such as members of the domain of the model and relations over them). Jacobson (2003, 58) hints at this worry when she contrasts her own brand of variable-free semantics with the standard, Tarski-inspired view, according to which

> the meaning of any expression is a function from assignment functions to something else, where each assignment function in turn is a function from variable names to "normal" model-theoretic objects (such as individuals). (...) In the variable-free view, there is no role for assignment functions and hence also, of course, no role for variables.[5] The meaning of any linguistic expression is simply some normal, healthy model-theoretic object—something constructed only out of the "stuff" that any theory presumably needs: individuals, worlds, times, perhaps events, etc.

Zimmermann and Sternefeld (2013, 243) make the same point but also draw attention to a disquieting consequence. After describing Tarskian meanings (which they call *global extensions*) as "somewhat of a cheat," they elaborate:

> For other than ordinary extensions, which correspond to the objects referred to in the non-linguistic world, global extensions are language dependent in that they are functions whose domain is the set of variable assignments, which in turn are functions defined on variables [...], and hence linguistic expressions. [...] As a consequence, unless languages overlap in some of their basic means of expression (to wit, variables), no two expressions of two distinct languages could have the same global extension. This so-called *representationalism*, i.e., the fact that global extensions are language-dependent (aspects of) meaning, may be seen as a nuisance and too high a price to pay for the compositionality of variable binding.[6]

Picking up on these observations, Klein and Sternefeld (2017, 66) point out that

> [a] conceptual problem results from making assignment functions part of a compositional semantics (...). Once denotations are compositionally defined in terms of assignment functions (...), these functions become part of the ontology, with the undesirable consequence that there is more in our ontology than the simple denotations found in the standard semantics. In particular, the semantics of

Footnote 4 continued
assignment as a *meaning* assignment. We postpone discussion of these problems until we consider the meaning assignment for Fregean predicate logic in Sect. 3.

[5] As I will show, this is a *non sequitur*: We can do predicate logic *with* variables without at the same time admitting variable assignments into our semantics.

[6] See also Geurts et al. (2016, §6), who make a similar observation with respect to dynamic predicate logic: "A variable assignment is a mapping from variables to individuals, and variables are linguistic expressions, so it can hardly be claimed that assignments are non-representational entities".

a language has to refer to the variables of the language and thereby becomes language dependent.

While Jacobson's worry seems to be that Tarskian meanings violate some ideal of ontological purity, according to which meanings ought not to be contaminated by syntax (or more generally, by elements extrinsic to semantics), Zimmermann, Sternefeld, and Klein appear in addition to be concerned that Tarskian meanings fail to be language-transcendent. We'll return to these issues in Sect. 5 below.

A second reason to be dissatisfied with the standard account of the compositionality of first-order logic is Fine's (2003, 2007) so-called antinomy of the variable.

Fine asks us to consider two variables, x and y, say. Do these have the same meaning or different meanings?[7] On the one hand, it seems obvious that they have distinct meanings. Take the formula $x > y$, for example. Clearly it is not synonymous with $x > x$, but if x and y had the same meaning, it is hard to see how the two formulas could fail to be synonymous.

As one might hope, our compositional semantics for Tarskian predicate logic accounts for this difference in meaning (as long as the background model $\mathfrak{M}$ contains at least two elements), since $[\![x]\!]$ assigns a different value than $[\![y]\!]$ to any variable assignment g that doesn't map x and y to the same object.

On the other hand, there are reasons to think that x and y ought to have the same meaning. For consider the formula $x > 0$. What does x do here? It holds a place for a value from some given domain, we might say; it "indefinitely indicates" an object, to borrow Frege's turn of phrase. And that seems all there is to it. But then that's precisely what y does in $y > 0$, and the only difference between the two formulas is a typographical one, a matter of arbitrary choice, a notational rather than a semantic issue. So it seems we are driven to thinking that $x > 0$ and $y > 0$, and hence x and y, must have the same meaning.

This, then, is the antinomy of the variable in a nutshell: Assuming that variables have meanings at all (and in our Tarskian semantics, they do), any two variables ought both to have *distinct* meanings, and also the *same* meaning.[8]

The antinomy is, I take it, a central motivation for the quest, recently undertaken by linguists (Klein and Sternefeld 2017; Kracht 2011), for a compositional, *alphabetically innocent* predicate logic, alphabetic innocence requiring, roughly speaking, that any two open formulas that differ only in the indexing of the variables occurring free in them (but not in the pattern of their occurrence) be assigned the same meaning. These proposals, however, entail significant revisions to the syntax and semantics of first-order logic, as does Fine's own program of *semantic relationism* (Fine 2007).

Another prominent revisionary proposal arising out of dissatisfaction with standard Tarskian semantics is Jacobson's (e.g. 1999) project of designing a variable-free semantics of natural language. Jacobson's approach is ultimately inspired by the com-

7 Fine speaks more vaguely of "semantic role" rather than "meaning," but we will ignore this nicety here.

8 Because they find Fine's own reasoning "elusive," Pickel and Rabern (2016, §§I.1.–I.2) offer a different motivation for the antinomy, specifically for the argument that distinct variables have the same meaning. The machinery they deploy for this purpose—a theory of structured meanings and strong assumptions about the relationship between syntactic and propositional structure—goes way beyond the one Fine appeals to in his own exposition and thus makes their version of the antinomy much easier to resist.

binatory logic of Curry and Feys (1958) and the predicate–functor logic of Quine (1960), both of which abandon the traditional quantifier–variable apparatus entirely.

As we will see, neither the problem of representationalism nor the antinomy of the variable arises for Fregean predicate logic, which nevertheless abides by compositionality and retains the traditional quantifier–variable mechanism. Thus while I am sympathetic to Jacobson's project and agree with her in rejecting variable assignments as *bona fide* model-theoretic objects, I believe that Fregean predicate logic is the simplest and most conservative answer to the challenges faced by Tarskian accounts (notwithstanding the considerable ingenuity of the solutions proposed by all the authors mentioned). Moreover, since Fregean predicate logic doesn't eliminate variables but rather employs them as part of its apparatus of generality, it suggests an attractive account of the logical status of variables, an issue that variable-free accounts simply sidestep by renouncing variables altogether.

What, then, *is* Fregean predicate logic? While a complete answer will be provided in the next two sections, we can easily give a first glimpse of its distinctive syntactic features. Recall that, on a Tarskian approach, the sentence $\exists x(Rxa \wedge Px)$ is generated from the atomic formulas Rxa and Px by first forming the conjunction $(Rxa \wedge Px)$ and then prefixing it with the quantifier $\exists x$. Frege, by contrast, would build $\exists x(Rxa \wedge Px)$ in the following way: Begin with atomic formulas Rba and Pb, form the conjunction $(Rba \wedge Pb)$, erase all occurrences of the name b to obtain the compound predicate $(R\xi a \wedge P\xi)$, with ξ marking the gaps resulting from the removal of b, and finally apply quantification by prefixing this predicate with $\exists x$, simultaneously filling the gaps with x. In the Fregean approach, variables thus only ever occur in conjunction with quantifiers; there are no free occurrences of variables in any well-formed Fregean expression, and hence there is not, properly speaking, any *binding* of variables.[9]

The principal challenge in developing Fregean predicate logic is the construction of a *compositional* semantics. In fact, it has been claimed that such an undertaking is hopeless. For instance, Janssen (2011, 509; 2012, 38) suggests that no compositional semantics is available for a Frege-style predicate logic, and Fine (2003, 614; 2007, 6) argues that there is no viable, or at least, plausible, semantics for a Fregean approach to first-order syntax.[10]

Fine's worries run along the following lines.

> According to [the Fregean] approach [...] a closed quantified sentence, such as $\exists x B(x)$ is to be understood on the basis of one of its instances $B(c)$—the intuitive idea being that from an understanding of $B(c)$, we may acquire an understanding of what it is for an arbitrary individual to satisfy the condition denoted by $B(\ )$ and that, from this, we may then acquire an understanding of what it is for this condition to be satisfied by some individual or other. But although the intuitive idea behind the proposal may be clear, it is far from clear how the proposal is to be made precise. (Fine 2003, 615)

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[9] Historically, approaches to first-order syntax that are broadly Fregean in spirit have been employed e.g. by Hilbert and Bernays (1934), Lemmon (1965) and Schütte (1977).

[10] Janssen makes this claim with reference to predicate logic as presented in Schütte (1977). It should be noted that Schütte himself uses a quasi-substitutional semantics for quantification for which it is not obvious that it is easily amenable to a compositional reformulation.

Fine's pessimism regarding the possibility of such a semantics is, I think, predicated on a mistaken assumption as to how it would have to work. He suggests:

> A certain semantic value is to be assigned to a closed instance $B(c)$ of the existential sentence $\exists x B(x)$. Let us call it a "proposition," though without any commitment to what it is. A certain *condition* is then to be determined on the basis of this proposition. But how? We took it to be the condition denoted by the scheme $B(\ )$ which results from removing all displayed occurrences of the term c from $B(c)$. This suggests that the condition should likewise be taken to be the result of removing all corresponding occurrences of the individual denoted by c from the given proposition; indeed, we are given no other indication of how the condition might be determined. It must therefore be presupposed that there is an operation of "abstraction" which, in application to any proposition and any occurrences of an individual in that proposition, will result in a certain condition or propositional "form" in which the given occurrences of the individual have been removed. Once given such a form, we may then take the quantified sentence $\exists x B(x)$ to predicate "existence" of it. (Fine 2003, 615)

It appears that, despite his official refusal to take a stand on the nature of propositions, Fine in fact takes them to be structured entities, roughly isomorphic to the sentences expressing them, from which individuals can be removed much like names can be erased within sentences. As we shall see in the course of making the intuitive Fregean ideas precise, there is no need to posit structured propositions of this nature, or to assume an operation of abstraction that deletes constituents occurring in such propositions.[11]

The game plan for the remainder of this paper is as follows. In Sect. 2, I will present the syntax of Fregean predicate logic in detail. Section 3 is devoted to a presentation of Fregean semantics, including a compositional meaning assignment. In Sect. 4 I argue at some length that this compositional meaning assignment justifies the classification of variables as mere marks of punctuation, akin to the parentheses. The main ingredient of this argument is the observation that variables could be completely eliminated from Fregean predicate logic in favor of the graphical "bonds" once proposed by Quine as a means to indicate the dependence of argument positions on outlying quantifiers. Section 5 addresses the antinomy of the variable and the problem of representationalism. I argue that neither issue arises in the context of Fregean predicate logic. The concluding Sect. 6 reflects on the differences between Tarskian and Fregean approaches to first-order logic.

Readers interested only in the mechanics of Fregean predicate logic or the challenges of providing a compositional semantics for first-order logic may thus confine their attention to Sects. 2 and 3. Those curious about the status of variables in predicate logic are encouraged to peruse Sects. 2–4 and 6. Readers wanting to know how Fregean predicate logic escapes the antinomy of the variable should study Sects. 2 and 3 as well as subsection 5.1. Finally, anyone interested in how Fregean predicate logic avoids representationalism will want to read Sects. 2 and 3 plus subsection 5.2.

[11] See Humberstone (2000) for a thorough discussion of the difficulties involved in "abstracting" an object from a proposition.

2 Fregean syntax

We will assume the availability of an unlimited number of *predicate symbols* of each non-zero arity. These will be notated by means of the letters P, Q, and R as metavariables. The set of all predicate symbols of arity n is $\mathcal{P}^n$, and the set of all predicate symbols is $\mathcal{P}$. We will similarly assume the availability of an unlimited number of *individual constants*, or *names*, for which we'll use a, b, c, and d as metavariables, with indices if necessary. The set of all names is $\mathcal{C}$. For reasons of simplicity, we do not consider function symbols.[12]

Before we embark on defining the Fregean language $\mathcal{L}_F$, I want to point out that it will be set up in such a way as to disallow vacuous quantification. This has several reasons. One is that it is unclear whether vacuous quantification is a desirable feature or just a limiting case that may or may not be permitted depending on considerations of economy and convenience, given that it does not add to the expressive power of predicate logic. A number of authors in logic[13] have seen fit to impose prohibitions against vacuous quantification, and the standard view in linguistics seems to be that natural languages do not countenance it.[14] As it turns out, disallowing vacuous quantification motivates a definition of the construction histories of well-formed expressions that permits a particularly attractive formulation of a compositional semantics, a point to which we shall return in due course. Finally, allowing vacuous quantification would require counting as predicates sentences from which nothing has been deleted, i.e. with no gaps, which is unnatural in the Fregean context. I note, however, that the results of this paper would still hold in the presence of vacuous quantification, though some modifications would need to be made.[15]

The primitive symbols of $\mathcal{L}_F$ are

1. the members of the sets $\mathcal{P}$ and $\mathcal{C}$;
2. the sentential connectives $\neg$ and $\wedge$;
3. the existential quantifier $\exists$;
4. an unlimited stock of individual variables $\mathsf{v}_0, \mathsf{v}_1, \mathsf{v}_2, \ldots$, for which we use x, y, and z as metavariables;
5. a blank space, notated as ξ;
6. left and right parentheses.

The $\mathcal{L}_F$-sentences and $\mathcal{L}_F$-predicates, which together constitute the well-formed expressions (wfe's) of $\mathcal{L}_F$, are defined by simultaneous induction:

12 The addition of function symbols raises no new issues. It is important, however, that even in languages with compound terms Fregean predicates can be generated only by erasing a primitive name from a sentence; in other words, erasing properly compound terms must not be permitted. Frege himself does not abide by this constraint, which, from a modern perspective, creates a host of problems for his syntax. See Pickel (2010).

13 See e.g. Hilbert and Ackermann (1938, 54), Lemmon (1965, 140), Smith (2003, 245).

14 Thus May (1977, 46), Chomsky (1982, *passim*; 1995, 151), Kratzer (1995, 131), Heim and Kratzer (1998, 126–127). See Potts (2002) for a dissenting view.

15 It might be noted here that standard presentations of Tarskian predicate logic permit vacuous quantification. This is understandable insofar as the main advantage of Tarskian syntax is its context-freedom, which a prohibition against vacuous quantification would destroy (cf. van Benthem 1987 or Marsh and Partee 1987).

1. Whenever $P \in \mathcal{P}^n$ and $c_1, \ldots, c_n$ are names, $Pc_1 \ldots c_n$ is an $\mathcal{L}_F$-sentence.
2. Whenever ϕ and ψ are $\mathcal{L}_F$-sentences, so are $\neg\phi$ and $(\phi \wedge \psi)$.
3. Whenever ϕ is an $\mathcal{L}_F$-sentence, and c is a name that has at least one occurrence in ϕ, the result $\phi_c[\xi]$ of replacing all occurrences of c in ϕ with the blank space ξ is an $\mathcal{L}_F$-predicate.
4. Whenever π is an $\mathcal{L}_F$-predicate, and x is an individual variable that does not occur in π, the result $\exists x\, \pi_\xi[x]$ of simultaneously prefixing π with $\exists x$ and replacing all occurrences of ξ in π with x is an $\mathcal{L}_F$-sentence.

It is obvious that $\mathcal{L}_F$-sentences (respectively, $\mathcal{L}_F$-predicates) contain no (respectively, one or more) occurrences of ξ.

Some observations are in order. First, $\mathcal{L}_F$-wfe's do not, in general, have unique construction histories (we will use this term in an intuitive sense for the time being); in other words, they are not freely generated by the syntactic operations from the primitive symbols. For example, the $\mathcal{L}_F$-predicate $P\xi$ can be obtained from Pa by clause 3 for *any* name a whatsoever. *Prima facie*, this failure of uniqueness blocks a straightforward recursive meaning assignment to $\mathcal{L}_F$-wfe's, but we will see in the next section that it is possible to get around this.

Second, the requirement, in clause 3, that the name c actually occur in the sentence ϕ implements the prohibition against vacuous quantification.

Third, the requirement, in clause 3, that *all* occurrences of the name c must be replaced by ξ is not part of Frege's own syntax (cf. Frege 1893), in which it is permissible to delete only some, but not necessarily all, occurrences of a name from a sentence when forming a predicate.[16] While it is clear that the same expressions will be generated regardless of whether our stricter requirement is implemented, Frege's more lenient construction processes are in fact an obstacle to a compositional semantics, for one and the same Fregean expression can then have different construction histories that would confer on it different semantic values.[17]

Fourth, the requirement, in clause 4, that the variable x *not* occur in ϕ has the consequence that no Fregean wfe contains quantifiers with overlapping scopes to which the same variable is attached. This, too, is essential: Without this constraint, we could build $\exists x(Rxx \wedge \exists x \neg Rxx)$ from $(R\xi\xi \wedge \exists x \neg Rxx)$ by replacing both occurrences of ξ with x and prefixing the result with $\exists x$, but also from $(R\xi\xi \wedge \exists x \neg R\xi x)$ by replacing the three occurrences of ξ with x and prefixing with $\exists x$. It is intuitively clear,

[16] Lemmon (1965, 140) follows Frege in this.

[17] Consider the $\mathcal{L}_F$-predicate $(P\xi \wedge \neg Pd)$. According to our definition, this must have been constructed from $(Pc \wedge \neg Pd)$ for some name c other than d. But if we were allowed to replace only some occurrences of a name by a variable, this predicate could also have been obtained from $(Pd \wedge \neg Pd)$. Now the intuitive idea behind Frege's way of forming predicates is that we obtain the extension of $\phi(\xi)$ by looking at some sentence $\phi(a)$ from which $\phi(\xi)$ arises through deletion of a, and letting the reference of a vary. Clearly we will get different extensions for $(P\xi \wedge \neg Pd)$, though, depending on whether we vary the reference of c in $(Pc \wedge \neg Pd)$ or we vary the reference of d in $(Pd \wedge \neg Pd)$—we'll get the empty extension in the latter case, but not necessarily in the former. Thus the extension of the predicate would be determined in incompatible ways if we relaxed the requirement under discussion.

though, that $(R\xi\xi \wedge \exists x \neg Rxx)$ and $(R\xi\xi \wedge \exists x \neg R\xi x)$ will, in general, have different extensions.[18]

Fifth, as was remarked in Sect. 1, there is no variable *binding* in Fregean predicate logic: Variables are introduced at the same time as their quantifiers; there is nothing "free" in a Fregean wfe that could become "bound" upon introduction of a quantifier.

We next introduce as an additional level of syntax *Fregean construction histories*, or *F-construction histories* for short. These can be viewed as analogues of Montague's (1973) analysis trees or of generative grammar's logical forms—items that, in their respective frameworks, serve as the input to semantic evaluation.[19] In the investigation of Fregean predicate logic, it will be instructive to explore how meanings can be assigned to both well-formed expressions and F-construction histories.[20]

F-construction histories, then, are certain inductively defined finite sequences $\sigma = \langle s_0, \ldots, s_n \rangle$.[21] Informally, the first entry $(\sigma)_0$ of an F-construction history, which we will call its *head*, indicates the type of wfe being constructed: If the head is a predicate symbol, the generated wfe is an atomic formula; if it is the negation (conjunction) sign, the generated wfe is a negation (conjunction); if it is a name, the generated wfe is a predicate (indeed, a predicate obtained by deleting that name from a sentence); and if it is a quantifier, the generated wfe is a quantified sentence. The second-to-last entry $(\sigma)_{n-1}$, which we will also write as $N(\sigma)$, is called the *name record* and contains the names that occur in the generated wfe; it is crucial to keep such a record in order to be able to rule out vacuous quantification (and as we shall see, it also enables a particularly elegant compositional semantics). The final entry $(\sigma)_n$, called the *variable record* and alternatively written $V(\sigma)$, contains the variables occurring in the generated expression; this is important in order to obviate expressions in which quantifiers with overlapping scopes are attached to the same variable.

Officially:

1. If $P \in \mathcal{P}^n$ and $c_1, \ldots, c_n \in \mathcal{C}$, then the $(n+3)$-tuple

$$\langle P, c_1, \ldots, c_n, \{c_1, \ldots, c_n\}, \emptyset \rangle$$

is an F-construction history.

2. If σ is an F-construction history whose head is not a member of $\mathcal{C}$ (i.e. the expression generated by σ is a sentence, not a predicate), then the quadruple

$$\langle \neg, \sigma, N(\sigma), V(\sigma) \rangle$$

is an F-construction history.

[18] If, for instance, R is the identity relation, and the domain of the model under consideration has more than one element, the first predicate denotes the empty set and the second denotes the entire domain. But then we must assign incompatible truth values to $\exists x(Rxx \wedge \exists x \neg Rxx)$, for if the sentence is derived from the first predicate, it must be false, whereas if it is derived from the second, it must be true.

[19] See e.g. Pagin and Westerstahl (2010, §2) and Heim and Kratzer (1998, §3.2).

[20] Given that Fregean well-formed expressions in general have multiple construction histories, it is not entirely trivial that unique meanings can be assigned to the wfe's, though we will see in the next section that this is indeed the case.

[21] Where $\sigma = \langle s_0, \ldots, s_n \rangle$ is a finite sequence and $0 \leq i \leq n$, we let $(\sigma)_i$ be s_i.

3. If σ and τ are F-construction histories whose respective heads are not in $\mathcal{C}$, then the quintuple

$$\langle \wedge, \sigma, \tau, N(\sigma) \cup N(\tau), V(\sigma) \cup V(\tau) \rangle$$

is an F-construction history.
4. If σ is an F-construction history whose head is not a member of $\mathcal{C}$, and c is in $N(\sigma)$, then the quadruple

$$\langle c, \sigma, N(\sigma)\backslash\{c\}, V(\sigma) \rangle$$

is an F-construction history.
5. If σ is an F-construction history whose head is a member of $\mathcal{C}$, and x is any variable not in $V(\sigma)$, then the quintuple

$$\langle \exists, x, \sigma, N(\sigma), V(\sigma) \cup \{x\} \rangle$$

is an F-construction history.

From each F-construction history σ we can read out a surface string $\mathsf{yield}(\sigma)$ consisting of primitive symbols of $\mathcal{L}_F$. Technically the function yield is defined by recursion on σ, as follows.

1. $\mathsf{yield}(\langle P, c_1, \ldots, c_n, \{c_1, \ldots, c_n\}, \emptyset \rangle) = Pc_1 \ldots c_n$
2. $\mathsf{yield}(\langle \neg, \sigma, N(\sigma), V(\sigma) \rangle) = \neg\, \mathsf{yield}(\sigma)$
3. $\mathsf{yield}(\langle \wedge, \sigma, \tau, N(\sigma) \cup N(\tau), V(\sigma) \cup V(\tau) \rangle) = (\mathsf{yield}(\sigma) \wedge \mathsf{yield}(\tau))$
4. $\mathsf{yield}(\langle c, \sigma, N(\sigma)\backslash\{c\}, V(\sigma) \rangle) = (\mathsf{yield}(\sigma))_c[\xi]$
5. $\mathsf{yield}(\langle \exists, x, \sigma, N(\sigma), V(\sigma) \cup \{x\} \rangle) = \exists x\ (\mathsf{yield}(\sigma))_\xi[x]$

It is intuitively clear that the yields of F-construction histories are precisely the well-formed expressions of $\mathcal{L}_F$, that the yield of an F-construction history σ is an atomic formula (respectively negation, conjunction, predicate, quantified sentence) if and only if σ's head $(\sigma)_0$ is a predicate symbol (respectively the negation sign, the conjunction sign, a name, a quantifier), that $N(\sigma)$ contains precisely the names occurring in $\mathsf{yield}(\sigma)$, and that $V(\sigma)$ contains precisely the variables occurring in $\mathsf{yield}(\sigma)$.

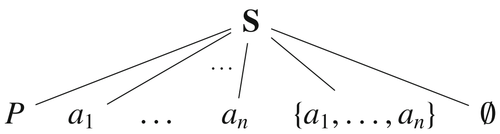
As we noted informally above, the function yield is obviously not injective, since e.g.

$$\mathsf{yield}(\langle a, \langle P, a, \{a\}, \emptyset \rangle, \emptyset, \emptyset \rangle) = P\xi = \mathsf{yield}(\langle b, \langle P, b, \{b\}, \emptyset \rangle, \emptyset, \emptyset \rangle).$$

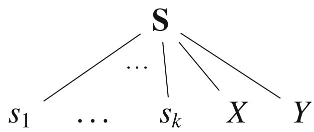
Thus we cannot uniquely associate an F-construction history with each $\mathcal{L}_F$-wfe. As we will see in the next section, however, this failure of uniqueness is semantically benign.

Though we will not be making use of such a representation, I note that F-construction histories can alternatively be defined as labeled trees (to which we'll refer as *FCH-trees*). The item labeling the next-to-last daughter of an FCH-tree's root is called the tree's name record; the item labeling the last daughter is the tree's variable record. For convenience, we label the tree's root with either **S** (for "sentence") or **P** (for "predicate"), depending on the kind of expression being constructed. Specifically, we have the following construction rules.

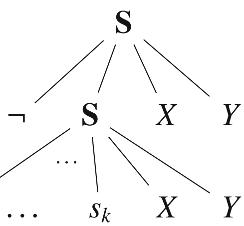
Atomic FCH-trees: Where P is an n-ary predicate symbol and a_1, …, a_n are names, the following labeled tree is a sentential FCH-tree.



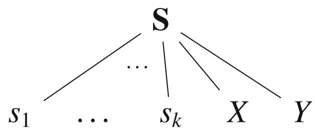
Negative FCH-trees: Where



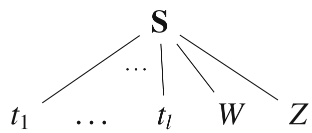
is a sentential FCH-tree with name record X and variable record Y, so is



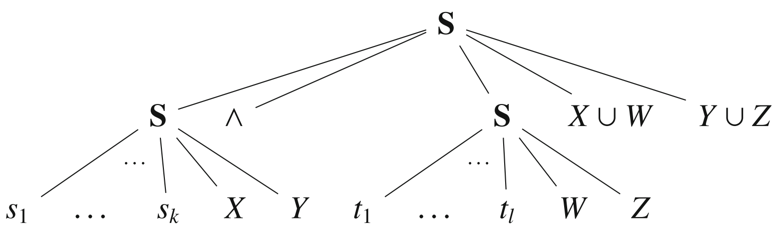
Conjunctive FCH-trees: Where



and

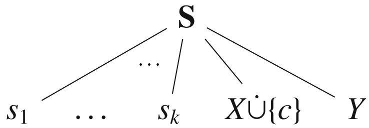


are sentential FCH-trees with name records X and W, respectively, and variable records Y and Z, respectively, the labeled tree

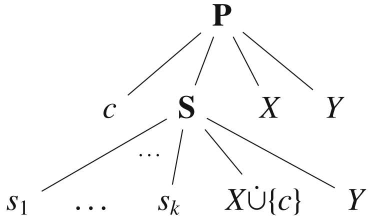


is a sentential FCH-tree.

Predicative FCH-trees: Where

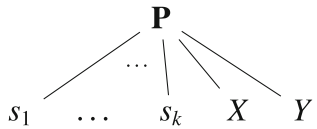


is a sentential FCH-tree whose name record can be written as the disjoint union of a set X and the singleton $\{c\}$, the labeled tree

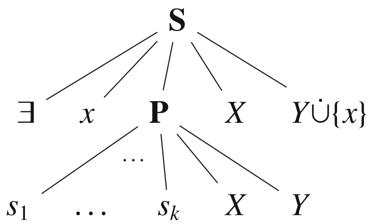


is a predicative FCH-tree.

Existential FCH-trees: Where

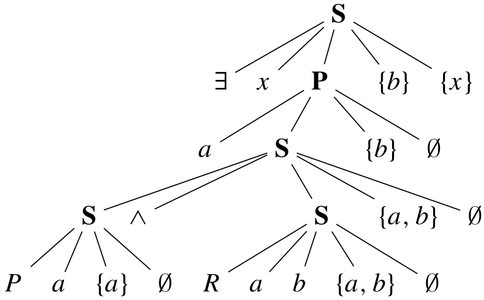


is a predicative FCH-tree of whose variable record Y the variable x is not a member, the labeled tree



is a sentential FCH-tree.

The yield function on FCH-trees can obviously be defined in much the same way as in the case of the original F-construction histories. By way of example, here is an FCH-tree that yields the $\mathcal{L}_F$-sentence $\exists x(Px \wedge Rxb)$:



It is readily apparent from the tree versions of Fregean construction histories that the names occurring in atomic sentences propagate up through FCH-trees and disappear only, if ever, at predicate-forming steps. The *prima facie* reason for letting names float to the top level of FCH-trees is that we want to exclude vacuous predicates (i.e. predicates that contain no gaps), so that at predicate-forming stages, we need to know which names are available for deletion without first constructing the yielded expression and identifying the names that occur in that string. As we shall see in the discussion of compositionality, however, there is a deeper, semantic reason to make the names occurring in the generated expression prominent at the top level.

3 Fregean semantics

A *model* $\mathfrak{M}$ is a triple $(M, (P^{\mathfrak{M}})_{P\in\mathcal{P}}, (c^{\mathfrak{M}})_{c\in\mathcal{C}})$, where $M =: \mathsf{dom}(\mathfrak{M})$, the *domain* of $\mathfrak{M}$, is a non-empty set, $(P^{\mathfrak{M}})_{P\in\mathcal{P}}$ is a family of relations over M indexed by the set $\mathcal{P}$ of predicate symbols such that, for each $P \in \mathcal{P}^n$, $P^{\mathfrak{M}} \subseteq M^n$; and $(c^{\mathfrak{M}})_{c\in\mathcal{C}}$ is a family of members of M indexed by the set $\mathcal{C}$ of names. We refer to $P^{\mathfrak{M}}$ as the *interpretation of* P in $\mathfrak{M}$, and to $c^{\mathfrak{M}}$ as the *interpretation of* c in $\mathfrak{M}$.

Where c is any name in $\mathcal{C}$, we say that a model $\mathfrak{M}$ is a c-*variant* of a model $\mathfrak{N}$, $\mathfrak{M} \sim_c \mathfrak{N}$, just in case $\mathfrak{M}$ and $\mathfrak{N}$ are exactly alike except possibly in the interpretation they assign to the name c, that is, just in case $\mathsf{dom}(\mathfrak{M}) = \mathsf{dom}(\mathfrak{N})$, $P^{\mathfrak{M}} = P^{\mathfrak{N}}$ for each $P \in \mathcal{P}$, and $d^{\mathfrak{M}} = d^{\mathfrak{N}}$ for each $d \in \mathcal{C}\backslash\{c\}$. Where $u \in M$, we let $\mathfrak{M}^u_c$ be the c-variant of $\mathfrak{M}$ that interprets c as u.

Now let the reference $[\![c]\!]^{\mathfrak{M}}$ of a name $c \in \mathcal{C}$ in a model $\mathfrak{M}$ be the $\mathfrak{M}$-interpretation $c^{\mathfrak{M}}$ of c, and let the reference $[\![P]\!]^{\mathfrak{M}}$ of an n-ary predicate symbol $P \in \mathcal{P}^n$ in a model $\mathfrak{M}$ be the characteristic function, relative to M^n, of the $\mathfrak{M}$-interpretation $P^{\mathfrak{M}}$ of P. Thus for $u_1, \ldots, u_n \in M$, $[\![P]\!]^{\mathfrak{M}}(u_1, \ldots, u_n) = \mathsf{t}$ if $\langle u_1, \ldots, u_n\rangle \in P^{\mathfrak{M}}$, and $[\![P]\!]^{\mathfrak{M}}(u_1, \ldots, u_n) = \mathsf{f}$ otherwise. Let $H_\neg$ be the unary truth function corresponding to negation, and let $H_\wedge$ be the binary truth function corresponding to conjunction. Finally let $H^M_\exists$ be the characteristic function, relative to the powerset of M, of the non-empty subsets of M.

The *reference* $[\![\sigma]\!]^{\mathfrak{M}}$ of an F-construction history σ in a model $\mathfrak{M}$ is defined by recursion on σ.

1. $[\![\langle P, c_1, \ldots, c_n, \{c_1, \ldots, c_n\}, \emptyset\rangle]\!]^{\mathfrak{M}}$ is $[\![P]\!]^{\mathfrak{M}}([\![c_1]\!]^{\mathfrak{M}}, \ldots, [\![c_n]\!]^{\mathfrak{M}})$.
2. $[\![\langle\neg, \sigma, N(\sigma), V(\sigma)\rangle]\!]^{\mathfrak{M}}$ is $H_\neg([\![\sigma]\!]^{\mathfrak{M}})$.
3. $[\![\langle\wedge, \sigma, \tau, N(\sigma) \cup N(\tau), V(\sigma) \cup V(\tau)\rangle]\!]^{\mathfrak{M}}$ is $H_\wedge([\![\sigma]\!]^{\mathfrak{M}}, [\![\tau]\!]^{\mathfrak{M}})$.
4. $[\![\langle c, \sigma, N(\sigma)\backslash\{c\}, V(\sigma)\rangle]\!]^{\mathfrak{M}}$ is the set $\{u \in M : [\![\sigma]\!]^{\mathfrak{M}^u_c} = \mathsf{t}\}$.
5. $[\![\langle\exists, x, \sigma, N(\sigma), V(\sigma) \cup \{x\}\rangle]\!]^{\mathfrak{M}}$ is $H^M_\exists([\![\sigma]\!]^{\mathfrak{M}})$.

Given that well-formed Fregean expressions, in general, have multiple construction histories, it is not immediately obvious that we can define references also for the $\mathcal{L}_F$-wfe's themselves. But it is straightforward, if somewhat tedious, to show that any two F-construction histories with the same yield will have the same reference in any given model:

Unique Interpretability: *If* $\mathsf{yield}(\sigma) = \mathsf{yield}(\tau)$, *then* $[\![\sigma]\!]^{\mathfrak{M}} = [\![\tau]\!]^{\mathfrak{M}}$.

The result is, in any case, intuitively plausible: As is easily seen, any two construction histories of a given expression ϕ must have the same structure in terms of the sequence of construction steps that have been applied; they can differ only in the choice of names that figure in the constituent atomic sentences, provided those names are deleted at corresponding predicate-forming steps. But the nature of the deleted name has no influence on the extension of the predicate so constructed. To take the simplest case, let σ be $\langle P, a, \{a\}, \emptyset\rangle$ and let τ be $\langle P, b, \{b\}, \emptyset\rangle$. Then σ yields the atomic sentence Pa and τ the atomic sentence Pb. Now $\sigma' = \langle a, \sigma, \emptyset, \emptyset\rangle$ and $\tau' = \langle b, \tau, \emptyset, \emptyset\rangle$ both yield the predicate $P\xi$ for which it doesn't matter whether we describe its extension in a model $\mathfrak{M}$ as $\{u \in M : [\![\langle P, a, \{a\}, \emptyset\rangle]\!]^{\mathfrak{M}_a^u} = \mathsf{t}\}$ or as $\{u \in M : [\![\langle P, b, \{b\}, \emptyset\rangle]\!]^{\mathfrak{M}_b^u} = \mathsf{t}\}$, since these are identical: $[\![\sigma]\!]^{\mathfrak{M}_a^u} = [\![P]\!]^{\mathfrak{M}_a^u}([\![a]\!]^{\mathfrak{M}_a^u}) = \chi_{P^{\mathfrak{M}_a^u}}(u) = \chi_{P^{\mathfrak{M}}}(u) = \chi_{P^{\mathfrak{M}_b^u}}(u) = [\![P]\!]^{\mathfrak{M}_b^u}([\![b]\!]^{\mathfrak{M}_b^u}) = [\![\tau]\!]^{\mathfrak{M}_b^u}$.

Given Unique Interpretability, yield's failure to be injective is no barrier to assigning references to $\mathcal{L}_F$-wfe's: We may let the reference $[\![\phi]\!]^{\mathfrak{M}}$ of an $\mathcal{L}_F$-wfe ϕ in a model $\mathfrak{M}$ be the reference $[\![\sigma]\!]^{\mathfrak{M}}$ of *any* F-construction history σ with $\mathsf{yield}(\sigma) = \phi$.

Alternatively, we can define the $\mathfrak{M}$-references of $\mathcal{L}_F$-wfe's directly by way of the following clauses.

1. The $\mathfrak{M}$-reference $[\![Pc_1 \ldots c_n]\!]^{\mathfrak{M}}$ of an atomic sentence of the form $Pc_1 \ldots c_n$ is $[\![P]\!]^{\mathfrak{M}}([\![c_1]\!]^{\mathfrak{M}}, \ldots, [\![c_n]\!]^{\mathfrak{M}})$.
2. The $\mathfrak{M}$-reference $[\![\neg\phi]\!]^{\mathfrak{M}}$ of a negation $\neg\phi$ is $H_\neg([\![\phi]\!]^{\mathfrak{M}})$.
3. The $\mathfrak{M}$-reference $[\![(\phi \wedge \psi)]\!]^{\mathfrak{M}}$ of a conjunction $(\phi \wedge \psi)$ is $H_\wedge([\![\phi]\!]^{\mathfrak{M}}, [\![\psi]\!]^{\mathfrak{M}})$.
4. The $\mathfrak{M}$-reference $[\![\phi_c[\xi]]\!]^{\mathfrak{M}}$ of a predicate $\phi_c[\xi]$ is the set $\{u \in M : [\![\phi]\!]^{\mathfrak{M}_c^u} = \mathsf{t}\}$.
5. The $\mathfrak{M}$-reference $[\![\exists x\pi_\xi[x]]\!]^{\mathfrak{M}}$ of an existential sentence $\exists x\pi_\xi[x]$ is $H_\exists^M([\![\pi]\!]^{\mathfrak{M}})$.

Note that, despite the invisibility of c in $\phi_c[\xi]$, $[\![\phi_c[\xi]]\!]^{\mathfrak{M}}$ is well-defined: If $\phi_c[\xi]$ equals $\psi_d[\xi]$, Unique Interpretability ensures that $\{u \in M : [\![\phi]\!]^{\mathfrak{M}_c^u} = \mathsf{t}\}$ is identical with $\{u \in M : [\![\psi]\!]^{\mathfrak{M}_d^u} = \mathsf{t}\}$.

Let us now investigate the question of the *compositionality* of Fregean semantics. By way of preparation, I want to point out and address a problem with the Tarskian meaning assignment as given in Sect. 1.

Recall that the Tarskian meaning of a formula ϕ is the function on the set of variable assignments over the background model $\mathfrak{M}$ that maps each assignment that satisfies ϕ in $\mathfrak{M}$ to t and each assignment that fails to satisfy ϕ to f. Recall, too, that a *closed* formula, or *sentence* (i.e. a formula without free occurrences of any variables), is either satisfied in $\mathfrak{M}$ by all assignments or by none. Indeed this is the basis for Tarski's definition of truth (as opposed to satisfaction by an assignment) in a model: Only closed formulas are true or false in $\mathfrak{M}$, and truth amounts to satisfaction by all assignments (equivalently, satisfaction by some assignment or other). Thus there are only two possible Tarskian meanings for a closed formula: Either the constant function mapping every assignment to t, or the constant function mapping every assignment to f. In other words, all closed formulas true in $\mathfrak{M}$ have the same meaning (to wit, the constant function with value t), and all formulas false in $\mathfrak{M}$ also have the same meaning (to wit, the constant function with value f). For example, if the interpretation $P^{\mathfrak{M}}$ of the unary predicate symbol P in the background model $\mathfrak{M}$ is empty, the sentences $\forall x Px$ and $\exists x Px$ have the same Tarskian meaning. Anyone who thinks of meanings as truth conditions must find this unsatisfactory: Whatever truth conditions might be, $\forall x Px$ and $\exists x Px$ do *not* have the same truth conditions.

We can fix this problem by introducing a dependency of meanings on models in addition to the dependency on variable assignments, say by defining the meaning of a formula ϕ to be the function that maps any model $\mathfrak{M}$ to our old meaning of ϕ when the fixed background model is $\mathfrak{M}$. Then the meaning of $\exists x Px$, for example, is no longer a constant function—it maps those models $\mathfrak{M}$ in which $P^{\mathfrak{M}} \neq \emptyset$ to the function that maps all assignments over $\mathfrak{M}$ to t, and those models $\mathfrak{M}$ in which $P^{\mathfrak{M}} = \emptyset$ to the function that maps all assignments over $\mathfrak{M}$ to f; clearly there are models of both kinds. It is then natural to think of a sentence's truth conditions as the class of those models that the formula's meaning maps to the constant function on the set of assignments over $\mathfrak{M}$ whose value is t, i.e. the class of models in which the sentence is true.

Given these observations regarding meaning assignments for Tarskian predicate logic, it won't come as a surprise that the Fregean meanings we're about to construct will likewise be functions defined on the class of all models; indeed, the Fregean meaning—or *sense*, as we will also call it, in deference to Frege—of any relevant linguistic item $\mathfrak{s}$ is going to be the function that maps any model $\mathfrak{M}$ to the reference $[\![\mathfrak{s}]\!]^{\mathfrak{M}}$ of $\mathfrak{s}$ in $\mathfrak{M}$.

We will begin by examining the assignment of senses to the well-formed expressions of $\mathcal{L}_F$; the sense assignment to F-construction histories will be considered later. Given our recipe for the construction of Fregean meanings, the sense $[\![c]\!]$ of a name c is the function that maps each model $\mathfrak{M}$ to $[\![c]\!](\mathfrak{M}) := [\![c]\!]^{\mathfrak{M}}$, i.e. the reference of c in $\mathfrak{M}$, which is just the interpretation $c^{\mathfrak{M}}$ of c in $\mathfrak{M}$. Similarly, the sense $[\![P]\!]$ of a predicate symbol $P \in \mathcal{P}^n$ is the function that maps each model $\mathfrak{M}$ to the reference $[\![P]\!](\mathfrak{M}) := [\![P]\!]^{\mathfrak{M}}$ of P in $\mathfrak{M}$, i.e. to the characteristic function of the interpretation $P^{\mathfrak{M}}$ of P in $\mathfrak{M}$. The sense $[\![\neg]\!]$ of the negation symbol is the constant function that maps each model $\mathfrak{M}$ to the reference $[\![\neg]\!](\mathfrak{M}) := [\![\neg]\!]^{\mathfrak{M}} := H_{\neg}$ of $\neg$ in $\mathfrak{M}$. Similarly the sense $[\![\wedge]\!]$ of the conjunction symbol is the constant function mapping each model $\mathfrak{M}$ to the reference $[\![\wedge]\!](\mathfrak{M}) := [\![\wedge]\!]^{\mathfrak{M}} := H_{\wedge}$ of $\wedge$ in $\mathfrak{M}$. The sense $[\![\exists]\!]$ of the existential quantifier is the function that maps each model $\mathfrak{M}$ to the reference $[\![\exists]\!](\mathfrak{M}) := [\![\exists]\!]^{\mathfrak{M}} := H_{\exists}^{M}$ of $\exists$ in $\mathfrak{M}$, i.e to the characteristic function of the set of non-empty subsets of the domain M of $\mathfrak{M}$. This completes the Fregean meaning assignment for the lexical items of $\mathcal{L}_F$. We note that the variables, the gap marker, and the parentheses are not assigned senses and thus do not count as lexical items.

For the $\mathcal{L}_F$-sentences and predicates, we have the following semantic clauses.

1. $[\![Pc_1 \ldots c_n]\!]$ is the function that maps any model $\mathfrak{M}$ to

$$[\![P]\!](\mathfrak{M})([\![c_1]\!](\mathfrak{M}), \ldots, [\![c_n]\!](\mathfrak{M})).$$

2. $[\![\neg\phi]\!]$ is the function that maps any model $\mathfrak{M}$ to

$$[\![\neg]\!](\mathfrak{M})([\![\phi]\!](\mathfrak{M})).$$

3. $[\![(\phi \wedge \psi)]\!]$ is the function that maps any model $\mathfrak{M}$ to

$$[\![\wedge]\!](\mathfrak{M})([\![\phi]\!](\mathfrak{M}), [\![\psi]\!](\mathfrak{M})).$$

4. $[\![\phi_c[\xi]]\!]$ is the function that maps any model $\mathfrak{M}$ to $\{u \in \mathsf{dom}(\mathfrak{M}) \mid [\![\phi]\!](\mathfrak{M}_c^u) = \mathsf{t}\}$.

5. $[\![\exists x\, \pi_\xi[x]]\!]$ is the function that maps any model $\mathfrak{M}$ to $[\![\exists]\!](\mathfrak{M})([\![\pi]\!](\mathfrak{M}))$.

Now is this meaning assignment compositional? Let us first consider the Principle of Compositionality in the following form:

Rule-to-Rule Compositionality
For every syntactic rule α there is a semantic operation r_α such that $[\![\alpha(e_1, \ldots, e_n)]\!] = r_\alpha([\![e_1]\!], \ldots, [\![e_n]\!])$.

With the exception of clause 4, the semantic clauses obviously abide by this form of the principle. Clause 4, however, is problematic. Let α be the syntactic rule that takes a sentence ϕ and a name c and outputs the result of deleting all occurrences of c in ϕ; i.e. $\alpha(\phi, c) = \phi_c[\xi]$. The corresponding semantic operation r_α should take as inputs the sentential sense $[\![\phi]\!]$ and the name sense $[\![c]\!]$ and output the function

$$\lambda\mathfrak{M}.\{u \in \mathsf{dom}(\mathfrak{M}) \mid [\![\phi]\!](\mathfrak{M}^u_c) = \mathsf{t}\}.$$

Unfortunately, though, while $\lambda\mathfrak{M}.\{u \in \mathsf{dom}(\mathfrak{M}) \mid [\![\phi]\!](\mathfrak{M}^u_c) = \mathsf{t}\}$ is obviously a function of $[\![\phi]\!]$, it does not seem to be a function of $[\![c]\!]$, but rather of c—that is, not of the name sense $[\![c]\!]$ but of the syntactic item c itself.

We could try to finesse the issue by relegating the role of c in the syntactic rule from an argument to an index. In other words, instead of having a single syntactic rule α that takes a sentence and a name and deletes all occurrences of the name in the sentence, we could have infinitely many rules α_c, one for each name c, such that α_c takes a sentence as argument and deletes all occurrences of c from it. That seems like cheating, however, as it means obscuring the fact that deletion of c and deletion of d are instances of the same kind of syntactic operation; the theory would be treating α_c and α_d as completely unrelated rules.

A parallel problem arises in the context of Tarskian predicate logic, where the quantifier clause relies on the relation of x-variance between assignments, a relation that makes explicit reference to a variable *qua* syntactic item. For the Tarskian case, Zimmermann and Sternefeld (2013, 242) offer a fix that can be adapted to our Fregean setting: Observe that, if c and d are distinct names, there is a model $\mathfrak{M}$ for which $[\![c]\!](\mathfrak{M}) = c^{\mathfrak{M}} \neq d^{\mathfrak{M}} = [\![d]\!](\mathfrak{M})$, so if we let const be the function defined on senses of names that maps any such sense $\mathfrak{s}$ to the unique name c such that, for all $\mathfrak{M}$, $\mathfrak{s}(\mathfrak{M}) = c^{\mathfrak{M}}$, we can replace mention of c in clause 4 by mention of $\mathsf{const}([\![c]\!])$. Then $[\![\phi_c[\xi]]\!](\mathfrak{M})$ can be written as

$$\{u \in \mathsf{dom}(\mathfrak{M}) \mid [\![\phi]\!](\mathfrak{M}^u_{\mathsf{const}([\![c]\!])}) = \mathsf{t}\},$$

and the sense of the $\mathcal{L}_F$-predicate $\phi_c[\xi]$ is now expressed as a function exclusively of the senses of ϕ and of c.

At this point, we have accomplished what Fine suggests isn't viable. From a grasp of the sense of a sentence $B(c)$ we obtain a grasp of the sense of the predicate $B(\xi)$, or in Fine's notation, $B(\ )$, by realizing that the association of the name c with its referent $c^{\mathfrak{M}}$ is a merely conventional matter and figuring out how we would compute the reference of $B(c)$ if c had any of its other possible references in M. The set of those possible references of c which make $B(c)$ true then serves us as the reference of $B(\xi)$. Finally, we predicate "existence" of $B(\xi)$ by means of $\exists x B(x)$. Note, too, that we have accomplished this entirely within the resources of truth-conditional semantics; we did not need to posit propositions as structured entities from which constituents could be removed in a way akin to the removal of a name from a sentence.

I suggested earlier that it would be worthwhile to explore the consequences of letting F-construction histories, rather than $\mathcal{L}_F$-wfe's, be the bearers of meaning. Apart from the fact that this strategy is well established in formal semantics, there are a number of substantive advantages to taking this route. One of them is that our sense assignment to the sentences and predicates of $\mathcal{L}_F$ is somewhat difficult to assess in terms of a variant formulation of Compositionality that I'll call *Constituent Compositionality*, for want of a better term.

Constituent Compositionality
The meaning of a compound linguistic item is a function of the meanings of its immediate constituents and the manner in which they are put together in the compound.

This version of the principle presupposes that there is a unique set of linguistic items that we can identify as *the* constituents of a given compound. This raises an immediate problem with $\mathcal{L}_F$-predicates, for what are the immediate constituents of $(Ra\xi \wedge P\xi)$, for example? The pair consisting of the name b and the sentence $(Rab \wedge Pb)$ has as good a claim as the pair consisting of the name c and the sentence $(Rac \wedge Pc)$, as well as infinitely many other such pairs. Relatedly, whichever pair or pairs we want to pick, we'll have to count as a constituent a name that doesn't even occur in the predicate—perhaps not an insuperable difficulty, but certainly an oddity.

Another reason for wanting to explore assigning meanings to Fregean construction histories relates to our invoking the syntax-valued function const above. Its purpose was, the reader will recall, to eliminate the dependency of predicative senses on syntactic items. But why should the semantic machinery contain a syntax-valued function at all? Of course we don't actually need to introduce such a function explicitly.[22] We could write $[\![\phi_c[\xi]]\!](\mathfrak{M})$ as

$$\{u \in M : \text{for some } \beta \in \mathcal{C} \text{ such that for all models } \mathfrak{K}, [\![\beta]\!](\mathfrak{K}) = [\![c]\!](\mathfrak{K}), [\![\phi]\!](\mathfrak{M}^u_\beta) = \mathsf{t}\},$$

which eliminates mention of const in favor of quantification over the set $\mathcal{C}$. But that doesn't really settle the matter. One might well object to having to quantify over a syntactic category (in this case, $\mathcal{C}$) in order to specify a meaning assignment. The compositional apparatus, it might seem, should really be autonomous from the syntax.[23] We should note, however, that quantification over syntactic categories in our definition of senses is more pervasive than we have so far made out: The model $\mathfrak{M}^u_c$ is defined in terms of c-variance, and as soon as we invoke c-variance (never mind how we effect reference to c), we're quantifying over the members of both $\mathcal{C}$ and $\mathcal{P}$. After

22 Thanks to Ede Zimmermann for pointing this out.

23 This concern applies equally to the Zimmermann–Sternefeld trick when applied in the Tarskian context.

all, by definition, models $\mathfrak{M}$ and $\mathfrak{N}$ are c-variants if, and only if, $\mathsf{dom}(\mathfrak{M}) = \mathsf{dom}(\mathfrak{N})$, *for every* $a \in \mathcal{C}\backslash\{c\}$, $a^{\mathfrak{M}} = a^{\mathfrak{N}}$, and *for every* $P \in \mathcal{P}$, $P^{\mathfrak{M}} = P^{\mathfrak{N}}$.[24]

Let's take these observations as sufficient motivation to examine the assignment of senses to F-construction histories. Aiming for satisfaction of Constituent Compositionality, let's say that the immediate constituents of an F-construction history of the form $\langle s_0, \ldots s_n \rangle$ are its entries s_0, …, s_n.[25] Let's further say that the construction history is *atomic* if s_0 is a predicate symbol, *negative* if s_0 is the negation sign, *conjunctive* if s_0 is the conjunction symbol, *predicative* if s_0 is a name, and *existential* if s_0 is the existential quantifier. We then have to show that the sense of an F-construction history $\langle s_0, \ldots s_n \rangle$ is a function of the senses of s_0, …, s_n and of the kind of construction history it is.

As we said above, the Fregean sense of any relevant linguistic item $\mathfrak{s}$ is the function on the class of all models that maps a model $\mathfrak{M}$ to the Fregean reference $[\![\mathfrak{s}]\!]^{\mathfrak{M}}$ of $\mathfrak{s}$ in $\mathfrak{M}$. We've already explained what this means for the primitive meaning-bearing symbols of $\mathcal{L}_F$, so it remains to consider the senses of the F-construction histories. These turn out to be subject to the following recursive clauses.

1. If τ is an F-construction history $\langle P, c_1, \ldots, c_n, \{c_1, \ldots, c_n\}, \emptyset \rangle$, its sense $[\![\tau]\!]$ is the function that maps any model $\mathfrak{M}$ to the result

$$[\![\tau]\!](\mathfrak{M}) := [\![P]\!](\mathfrak{M})([\![c_1]\!](\mathfrak{M}), \ldots, [\![c_n]\!](\mathfrak{M}))$$

   of applying the value at $\mathfrak{M}$ of the sense $[\![P]\!]$ of P to the sequence of values at $\mathfrak{M}$ of each of the senses $[\![c_1]\!]$, …, $[\![c_n]\!]$ of $c_1, \ldots, c_n$, respectively.
2. If τ is an F-construction history $\langle \neg, \sigma, N(\sigma), V(\sigma) \rangle$, its sense $[\![\tau]\!]$ is the function mapping any $\mathfrak{M}$ to the result

$$[\![\tau]\!](\mathfrak{M}) := [\![\neg]\!](\mathfrak{M})([\![\sigma]\!](\mathfrak{M}))$$

   of applying the value $[\![\neg]\!](\mathfrak{M})$ of the sense $[\![\neg]\!]$ of $\neg$ at $\mathfrak{M}$ to the value $[\![\sigma]\!](\mathfrak{M})$ of the sense $[\![\sigma]\!]$ of σ at the argument $\mathfrak{M}$.
3. If τ is an F-construction history $\langle \wedge, \sigma, \rho, N(\sigma) \cup N(\rho), V(\sigma) \cup V(\rho) \rangle$, its sense $[\![\tau]\!]$ is the function that maps any model $\mathfrak{M}$ to the result

$$[\![\tau]\!](\mathfrak{M}) := [\![\wedge]\!](\mathfrak{M})([\![\sigma]\!](\mathfrak{M}), [\![\rho]\!](\mathfrak{M}))$$

   of applying the value $[\![\wedge]\!](\mathfrak{M})$ of the sense $[\![\wedge]\!]$ of $\wedge$ at $\mathfrak{M}$ to the ordered pair consisting of the values at $\mathfrak{M}$ of the senses $[\![\sigma]\!]$ and $[\![\rho]\!]$ of σ and ρ, respectively.

[24] Similarly the Tarskian must already quantify over the set of variables as soon as x-variance between variable assignments comes in (never mind how reference to x itself is effected), since $g \sim_x h$ if, and only if, $g(y) = h(y)$ *for every* $y \in V\backslash\{x\}$.

[25] Since we encode the names occurring in an $\mathcal{L}_F$-wfe within its construction history σ as a finite *set* $N(\sigma)$, we need to say what the meaning of a finite set X of names is: It will be the finite set of the meanings of the members of X. Note that the set $V(\sigma)$ of variables occurring in the wfe constructed by σ, while being a syntactic constituent of σ, will not be assigned any meaning; its function is, as it were, purely syntactic. We will have occasion to return to this point below.

4. If τ is an F-construction history $\langle c, \sigma, N(\sigma)\backslash\{c\}, V(\sigma)\rangle$, its sense $[\![\tau]\!]$ is the function that maps any $\mathfrak{M}$ to

$$[\![\tau]\!](\mathfrak{M}) := \{u \in M \ : [\![\sigma]\!](\mathfrak{M}^u_{\mathsf{const}([\![c]\!])}) = \mathsf{t}\}.$$

5. If τ is an F-construction history $\langle \exists, x, \sigma, N(\sigma), V(\sigma) \cup \{x\}\rangle$, its sense $[\![\tau]\!]$ is the function that maps any model $\mathfrak{M}$ to the result

$$[\![\tau]\!](\mathfrak{M}) := ([\![\exists]\!](\mathfrak{M}))([\![\sigma]\!](\mathfrak{M}))$$

of applying the value at $\mathfrak{M}$ of the sense $[\![\exists]\!]$ of $\exists$ to the value at $\mathfrak{M}$ of the sense $[\![\sigma]\!]$ of σ.

It is now obvious that our sense assignment to F-construction histories abides by Constituent Compositionality. Moreover, every constituent is, in a reasonable sense, actually present in the compound of which it is a constituent.[26]

It remains to address the worry about invoking the syntax-valued function const, or alternatively, quantification over syntactic categories, in the predicative semantic clause. As a first step, recall the familiar result in Tarskian predicate logic, variously called the *local determination lemma* or the *coincidence lemma*, according to which any two variable assignments that agree on the variables occurring free in a given formula either both satisfy or both fail to satisfy the formula. The analogous result in Fregean predicate logic is that, if σ is an F-construction history and the models $\mathfrak{M}$ and $\mathfrak{N}$ have the same domain, agree on the interpretation of every predicate symbol in $\mathcal{P}$, and also agree on the interpretation of all the names in $N(\sigma)$, the reference of σ in $\mathfrak{M}$ is the same as the reference of σ in $\mathfrak{N}$. If we say that models $\mathfrak{M}$ and $\mathfrak{N}$ are *name-variants*, $\mathfrak{M} \sim_{\mathcal{C}} \mathfrak{N}$, just in case they have the same domain and agree on the interpretations of all predicate symbols in $\mathcal{P}$, we can restate this fact as follows: If $\mathfrak{M} \sim_{\mathcal{C}} \mathfrak{N}$, and σ is an F-construction history such that $[\![c]\!]^{\mathfrak{M}} = [\![c]\!]^{\mathfrak{N}}$ for each name $c \in N(\sigma)$, then $[\![\sigma]\!]^{\mathfrak{M}} = [\![\sigma]\!]^{\mathfrak{N}}$.

Thus we may reformulate the semantic clause for predicative F-construction histories as follows:

4′. If τ is an F-construction history $\langle c, \sigma, N(\sigma)\backslash\{c\}, V(\sigma)\rangle$, its sense $[\![\tau]\!]$ is the function that maps any model $\mathfrak{M}$ to the set of all $u \in M$ such that, for some $\mathfrak{N} \sim_{\mathcal{C}} \mathfrak{M}$,
   - $[\![\sigma]\!](\mathfrak{N}) = \mathsf{t}$;
   - $u = [\![c]\!](\mathfrak{N})$; and
   - $\gamma(\mathfrak{N}) = \gamma(\mathfrak{M})$ for each element $\gamma \in [\![N(\tau)]\!]$.[27]

In this formulation, we see that $[\![\tau]\!]$ is a function of the senses $[\![N(\tau)]\!]$, $[\![\sigma]\!]$, and $[\![c]\!]$ of τ's immediate constituents $N(\tau)$, σ, and c. Note, too, that there is no reference to a particular syntactic item, no use of a syntax-valued function, and no quantification over the members of $\mathcal{C}$, in clause 4′.

[26] Our F-construction histories are set-theoretic constructs rather than expressions, so we can't rightly say that the constituents are *visible* in the construction histories.

[27] The final bullet point is equivalent to the condition that $[\![b]\!](\mathfrak{N}) = [\![b]\!](\mathfrak{M})$ for each name $b \in N(\tau)$.

We have not, however, eliminated quantification over syntactic categories entirely. To be sure, we no longer need to quantify over names, but our definition of name-variance still involves quantification over the set of predicate symbols, for $\mathfrak{M}$ and $\mathfrak{N}$ are name-variants just in case $\mathsf{dom}(\mathfrak{M}) = \mathsf{dom}(\mathfrak{N})$ and $P^{\mathfrak{M}} = P^{\mathfrak{N}}$ *for every* $P \in \mathcal{P}$. Thus quantification over syntax has been reduced, but not eliminated.

We can do better. Recall that the reason we were able to eliminate quantification over $\mathcal{C}$ in the move from clause 4 to clause 4′ was that an F-construction history contains a record of all the names relevant to its semantic evaluation: Instead of asking for c-variance, it was enough to ask, in 4′, for agreement on the interpretation of the names explicitly listed in the name record (as well as agreement on all predicate symbols, and domain identity). But actually, requiring agreement on the interpretation of *all* predicate symbols is overkill: Two models with the same domain will agree on a sentence's truth value as soon as they agree on the interpretations of the names and the predicate symbols *occurring in the sentence*. Now recall that the F-construction histories carry a name record in order to prevent vacuous first-order quantification. If we take first-order logic to be a fragment of higher-order logic (as Frege certainly did), we should take precautions also against vacuous *second-order* quantification. For this purpose, we will need the F-construction histories σ to also carry a *predicate record* $P(\sigma)$ that lists all the primitive predicate symbols occurring in the yielded expressions, so that we know which predicate symbols are available for deletion when creating higher-order predicates.

It follows that, if we were to revise[28] our definition of F-construction histories in such a way that they also contain a predicate record, we could reformulate clause 4′ in a way that completely eliminates any reference to syntax, even quantification over the members of syntactic categories:

4″. If τ is an F-construction history $\langle c, \sigma, N(\sigma)\backslash\{c\}, P(\sigma), V(\sigma)\rangle$, its sense $[\![\tau]\!]$ is the function that maps any model $\mathfrak{M}$ to the set of all $u \in M$ such that, for some model $\mathfrak{N}$,
- $\mathsf{dom}(\mathfrak{M}) = \mathsf{dom}(\mathfrak{N})$;
- $[\![\sigma]\!](\mathfrak{N}) = \mathsf{t}$;
- $u = [\![c]\!](\mathfrak{N})$;
- $\gamma(\mathfrak{N}) = \gamma(\mathfrak{M})$ for each element $\gamma \in [\![N(\tau)]\!]$; and
- $\Pi(\mathfrak{N}) = \Pi(\mathfrak{M})$ for each element $\Pi \in [\![P(\tau)]\!]$.

While, for reasons of simplicity, we won't officially integrate predicate records into our F-construction histories, I propose that this is the way to ensure a fully satisfactory, syntax-independent, compositional meaning assignment for predicate logic.[29]

[28] The necessary changes are obvious: Call the final entry in a history the variable record, the next-to-last entry the predicate record, and the second-to-last entry the name record. Atomic histories now have the form $\langle P, c_1, \ldots, c_n, \{c_1, \ldots, c_n\}, \{P\}, \emptyset\rangle$; in every syntactic construction step other than conjunction we simply carry along the predicate record, and in the conjunction step we take the union of the conjuncts' predicate records to be the conjunctive history's predicate record. Nothing else needs to be done as long as we don't venture into higher-order logic.

[29] Analogous moves could be made for Tarski-style predicate logic, as long as Tarskian construction histories are defined in such a way as to keep track both of the individual variables and of the predicate variables occurring free in the yielded formula.

4 The role of variables

The $\mathcal{L}_F$-variables are semantically idle in the sense that they aren't assigned lexical meanings. They do, however, play a syntactic role in that they serve to disambiguate an expression's construction history—the variable pattern in $\exists x \forall y Rxy$ tells us that the predicate to which non-emptiness is being attributed is $\forall y R\xi y$, while that in $\exists x \forall y Ryx$ indicates that being instantiated is attributed to the predicate $\forall y Ry\xi$. In this the variables resemble the parentheses, which do not have lexical meanings either but likewise serve to disambiguate an expression's construction history—the bracketing pattern in $((\phi \wedge \psi) \vee \chi)$ tells us that $\phi \wedge \psi$ is being disjoined with χ, whereas the bracketing pattern in $(\phi \wedge (\psi \vee \chi))$ indicates that ϕ is being conjoined with $\psi \vee \chi$. Moreover, just as variables only ever enter wfe's together with a quantifier, parentheses only ever enter wfe's together with a binary propositional connective.

The variables differ from the parentheses, however, in that they show up as constituents of F-construction histories, into which they are introduced via the existential quantifier clause. This difference might make one hesitate to categorize variables with the parentheses as mere devices of punctuation. Let's ponder for a moment, however, what would happen if we omitted all mention of variables from F-construction histories.

In other words, let's consider, instead of F-construction histories, the Q-construction histories defined below.[30] It is obvious that we could assign meanings to Q-construction histories in just the same way as we did to F-construction histories, so if we cared only about the histories, not the generated expressions, we could jettison variables altogether.[31]

1. If $P \in \mathcal{P}^n$ and $c_1, \ldots, c_n \in \mathcal{C}$, then the $(n+2)$-tuple

$$\langle P, c_1, \ldots, c_n, \{c_1, \ldots, c_n\} \rangle$$

   is a Q-construction history. (Its sense is, as before, the function mapping any model $\mathfrak{M}$ to $[\![P]\!](\mathfrak{M})([\![c_1]\!](\mathfrak{M}), \ldots, [\![c_n]\!](\mathfrak{M}))$.)
2. If σ is a Q-construction history whose head is not a member of $\mathcal{C}$, then the triple

$$\langle \neg, \sigma, N(\sigma) \rangle$$

   is a Q-construction history. (Its sense is, as before, the function mapping any $\mathfrak{M}$ to $[\![\neg]\!](\mathfrak{M})([\![\sigma]\!](\mathfrak{M}))$.)
3. If σ and ρ are Q-construction histories neither of whose heads is a name, then the quadruple

$$\langle \wedge, \sigma, \rho, N(\sigma) \cup N(\rho) \rangle$$

[30] In the context of Q-construction histories, we let $N(\sigma)$ be the *final* entry in the finite sequence σ, for obvious reasons.

[31] For each step in the inductive generation of a Q-construction history, we indicate in parentheses the corresponding clause for the compositional semantics to make it obvious that the meaning assignments can easily be made, in a compositional fashion, even for these construction histories stripped of all information regarding variables.

is a Q-construction history. (Its sense is, as before, the function mapping any model $\mathfrak{M}$ to $[\![\wedge]\!](\mathfrak{M})([\![\sigma]\!](\mathfrak{M}), [\![\rho]\!](\mathfrak{M}))$.)

4. If σ is a Q-construction history whose head is not a name, and c is in $N(\sigma)$, then the triple

$$\langle c, \sigma, N(\sigma)\backslash\{c\}\rangle$$

is a Q-construction history. (Its sense is, as before, the function mapping any $\mathfrak{M}$ to the set of all $u \in M$ such that $[\![\sigma]\!](\mathfrak{M}^u_{\mathsf{const}([\![c]\!])}) = \mathsf{t}$.)

5. If σ is a Q-construction history whose head is a member of $\mathcal{C}$, then the triple

$$\langle \exists, \sigma, N(\sigma)\rangle$$

is a Q-construction history. (Its sense is, as before, the function mapping any $\mathfrak{M}$ to $([\![\exists]\!](\mathfrak{M}))([\![\sigma]\!](\mathfrak{M}))$.)

Interestingly, the Q-construction histories generate precisely the well-formed expressions of a first-order logical notation once suggested in passing by Quine (1940, 70) and adopted as the official notation of *Éléments de Mathématique* by Bourbaki (1954, chapter I, §1).[32]

Informally speaking, the idea is to establish the link between a quantifier occurrence and the argument places of predicates into which it reaches, not by means of occurrences of the same variable, but by drawing a curved line, or *bond*, from the quantifier occurrence to those argument places. Thus, for instance, what both Tarski and Frege would write as

$$(\forall x\, Rax \wedge \neg\forall x\, Rxa)$$

is rendered in Quinean bond notation as

$$(\forall\, Ra\bullet \wedge \neg\forall\, R\bullet a)\,,$$

what Tarski and Frege write as

$$\forall x\, \forall y\, Rxy$$

becomes

$$\forall\forall R\bullet\bullet,$$

and Tarski's and Frege's

$$\forall y\, \forall x\, Rxy$$

[32] *Éléments de Mathématique* uses Hilbert's ε-operator rather than the usual quantifiers, but its "variable-binding" notation is otherwise Quine's. Other logicians who have noted the attractions of this notation include Tennant (1978, 13–15), Kaplan (1986, 244), and Smith (2003, 215). In linguistics, Evans (1977) and Higginbotham (1983) have employed versions of the same idea, calling it *chaining* and *linking*, respectively.

turns into

$$\overset{\frown}{\forall}\overset{\frown}{\forall} R \bullet\bullet.$$

Let us now make these informal suggestions more precise by defining the well-formed expressions of the Quinean bond language $\mathcal{L}_Q$. The *primitive symbols* of $\mathcal{L}_Q$ are those of $\mathcal{L}_F$ other than the variables, plus curved lines, henceforth called *bonds*, as well as the argument place marker $\bullet$. The $\mathcal{L}_Q$-sentences and $\mathcal{L}_Q$-predicates, which together constitute the well-formed Quinean expressions (the $\mathcal{L}_Q$*-wfe's* for short), are generated by a simultaneous inductive definition as follows.

1. Whenever $P \in \mathcal{P}^n$ and $c_1, \ldots, c_n$ are names, $Pc_1 \ldots c_n$ is an $\mathcal{L}_Q$-sentence.
2. Whenever ϕ and ψ are $\mathcal{L}_Q$-sentences, so are $\neg\phi$ and $(\phi \wedge \psi)$.
3. Whenever ϕ is an $\mathcal{L}_Q$-sentence, and c is a name that has at least one occurrence in ϕ, the result $\phi_c[\xi]$ of replacing all occurrences of c in ϕ with the blank space ξ is an $\mathcal{L}_Q$-predicate.
4. Whenever π is an $\mathcal{L}_Q$-predicate, the result $\mathsf{ex}(\pi)$ of prefixing π with $\exists$, replacing each occurrence of ξ in π with $\bullet$, and drawing a bond from the new prefix $\exists$ to each of the new occurrences of $\bullet$ is an $\mathcal{L}_Q$-sentence.

The $\mathcal{L}_Q$-wfe's are generated by the Q-construction histories in the obvious way. That is, we recursively define a function yield from the Q-construction histories to the $\mathcal{L}_Q$-wfe's as follows.

1. $\mathsf{yield}(\langle P, c_1, \ldots, c_n, \{c_1, \ldots, c_n\}\rangle) = Pc_1 \ldots c_n$
2. $\mathsf{yield}(\langle \neg, \sigma, N(\sigma)\rangle) = \neg\, \mathsf{yield}(\sigma)$
3. $\mathsf{yield}(\langle \wedge, \sigma, \tau, N(\sigma) \cup N(\tau)\rangle) = (\mathsf{yield}(\sigma) \wedge \mathsf{yield}(\tau))$
4. $\mathsf{yield}(\langle c, \sigma, N(\sigma)\backslash\{c\}\rangle) = (\mathsf{yield}(\sigma))_c[\xi]$
5. $\mathsf{yield}(\langle \exists, \sigma, N(\sigma)\rangle) = \mathsf{ex}(\mathsf{yield}(\sigma))$

By way of example, if $R \in \mathcal{P}^3$, we can make the Q-construction history

$$\langle R, a, b, a, \{a, b\}\rangle,$$

for which

$$\mathsf{yield}(\langle R, a, b, a, \{a, b\}\rangle) = Raba.$$

From $\langle R, a, b, a, \{a, b\}\rangle$ we obtain the Q-construction history

$$\langle b, \langle R, a, b, a, \{a, b\}\rangle, \{a\}\rangle,$$

whose yield is the $\mathcal{L}_Q$-predicate

$$Ra\xi a.$$

We may go on to generate the Q-construction history

$$\langle \exists, \langle b, \langle R, a, b, a, \{a, b\}\rangle, \{a\}\rangle, \{a\}\rangle,$$

which yields

$$\exists\, Ra\bullet a.$$

Then we can construct the Q-construction history

$$\langle \neg, \langle \exists, \langle b, \langle R, a, b, a, \{a, b\}\rangle, \{a\}\rangle, \{a\}\rangle, \{a\}\rangle,$$

with yield

$$\neg\exists\, Ra\bullet a.$$

Now we move on to the Q-construction history

$$\langle a, \langle \neg, \langle \exists, \langle b, \langle R, a, b, a, \{a, b\}\rangle, \{a\}\rangle, \{a\}\rangle, \{a\}\rangle, \emptyset\rangle,$$

whose yield is

$$\neg\exists\, R\xi\bullet\xi.$$

Finally we can generate the Q-construction history

$$\langle \exists, \langle a, \langle \neg, \langle \exists, \langle b, \langle R, a, b, a, \{a, b\}\rangle, \{a\}\rangle, \{a\}\rangle, \{a\}\rangle, \emptyset\rangle, \emptyset\rangle,$$

with yield

$$\exists\,\neg\exists R\bullet\bullet\bullet.$$

It seems abundantly clear that the bonds of $\mathcal{L}_Q$ are mere devices of punctuation: Like the parentheses, they are not lexical items, and again like the parentheses, they do not show up in the Q-construction histories, the compound bearers of meaning. Since the variables of $\mathcal{L}_F$ are nothing but a replacement for the Quinean bonds, they too are mere punctuation, their presence in F-construction histories being due to a less economical, though typographically more convenient, scheme of achieving unique interpretability—to wit, the scheme of eliminating bonds by marking their beginnings and ends with equiform variables.

5 The challenges

As mentioned in Sect. 1, two main reasons for dissatisfaction with the standard meaning assignment for Tarskian predicate logic are its representationalism and the antinomy of the variable. In this section, we investigate whether Fregean predicate logic is subject to the same objections.

5.1 Fine's antinomy

Fine (2003; 2007, chapter 1), it will be recalled from Sect. 1, argues that the meanings of distinct variables must both be distinct and identical. But an obvious response suggests itself. After all, it is a crucial presupposition of the antinomy that variables *have* meanings in the first place.[33] This is, as we've seen, unwarranted: Fregean predicate logic has a fully compositional semantics without attributing any kind of meaning to variables, so the antinomy *of the variable* simply cannot arise there. Indeed, it cannot even be formulated, since expressions such as "$x > 0$" and "$y > 0$" aren't well-formed in $\mathcal{L}_F$.

Perhaps, however, we need to be a little more open-minded about the shape Fine's antinomy might take in the context of Fregean predicate logic. To be sure, it doesn't work with variables, but the roles played by variables in Tarskian predicate logic are, as it were, distributed among three kinds of symbols in $\mathcal{L}_F$: besides the variables themselves, the members of $\mathcal{C}$ and the gap marker ξ. Can we reconstruct an antinomy for one of these other categories of signs?

Let's consider names first. Names clearly have meanings, so the move we made with respect to Fregean variables does not apply. Do distinct names c and d ever have the same meaning? According to our Fregean[34] semantics, the answer is in the negative, since there will be models that interpret c and d by distinct objects, and hence the function that maps each model to its interpretation of c is distinct from the function that maps each model to its interpretation of d.[35] Note, however, that we could easily accommodate distinct but sense-identical names a and b by restricting the domain of the sense function to the class of models that interpret a and b by the same object, or in other words, by making the equation $a = b$ a meaning postulate.[36]

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33 Fine is fully aware of this assumption: "In stating the antinomy of the variable, we have presupposed that variables *have* a semantic role; and it might be thought that this is the root cause of our difficulties" (Fine 2007, 12).

34 Tarskian semantics, incidentally, gives the same answer, as long as Tarskian meanings are defined as functions taking not just variable assignments, but also models as arguments—as they should; see Sect. 3.

35 A reviewer has raised the question whether this isn't an unwelcome consequence for Fregeans. I see no reason to think that it should be. Frege certainly held that distinct *compound* expressions can express the same sense (and this is borne out by our formal Fregean semantics), but I am not aware of any place where he suggests that this should be true of primitive expressions as well. In fact it seems overwhelmingly likely that Frege would have objected to introducing into a "logically perfect" language distinct primitive symbols with the same sense. On this topic see also (May 2006), especially p. 118.

36 Some care is required, though. Suppose we make a and b sense-identical by restricting the model space to models that interpret a and b by the same object. Suppose $\mathfrak{M}$ is such a model. Then the only a-variant of $\mathfrak{M}$ in the restricted model space is $\mathfrak{M}$ itself, because any other a-variant interprets a and b differently. Now consider the predicate $R\xi b$, which we can obtain from Rcb by deleting c or (presumably) from Rab by deleting a. If we compute $[\![R\xi b]\!]^{\mathfrak{M}}$ as $\{u \in M \mid \text{for some } \mathfrak{N} \sim_c \mathfrak{M}, u = c^{\mathfrak{N}} \text{ and } [\![Rcb]\!]^{\mathfrak{N}} = \mathfrak{t}\}$, we obtain $[\![R\xi b]\!]^{\mathfrak{M}} = \{u \in M \mid \langle u, b^{\mathfrak{M}}\rangle \in R^{\mathfrak{M}}\}$. But if we compute $[\![R\xi b]\!]^{\mathfrak{M}}$ as $\{u \in M \mid \text{for some } \mathfrak{N} \sim_a \mathfrak{M}, u = a^{\mathfrak{N}} \text{ and } [\![Rab]\!]^{\mathfrak{N}} = \mathfrak{t}\}$, we obtain either $[\![R\xi b]\!]^{\mathfrak{M}} = \{a^{\mathfrak{M}}\}$, if $[\![Rab]\!]^{\mathfrak{M}} = \mathfrak{t}$, or else $[\![R\xi b]\!]^{\mathfrak{M}} = \emptyset$. So in general $[\![R\xi b]\!]^{\mathfrak{M}}$ isn't well defined. The simplest way to circumvent this problem is to require, for any two names a_0 and a_1 intended to be sense-identical, that they be treated as the same name syntactically—in other words, a_i must not be deleted from a sentence without a_{1-i} also being deleted, and a_i's occurring in an expression also counts as a_{1-i} occurring in the expression. Alternatively, we could restrict the domain

Both of these features—the general non-synonymy of distinct Fregean names and the possibility of accommodating synonymous names via meaning postulates—seem to be in accord with our intuitions. Surely the general case with two names is that they have distinct meanings; after all, two names typically have distinct references even in the intended model. At the same time it is possible for a linguistic community to introduce a new name as a mere variant of an extant one, for example, the nickname *Marty* in addition to the original name *Martin*. These names will have the same senses because the nickname has been introduced with the understanding that it abbreviates the name *Martin*.

Fine's own discussion (2007, 46–47) of the apocryphal story about Carl Hempel's being called *Peter* upon his arrival in Princeton actually illustrates how well these features of Fregean semantics accord with our intuitions. Fine considers two scenarios: In one, Hempel's fellow philosophers decide to use *Peter* as a variant of *Carl*, in the other, Hempel is re-christened *Peter* shortly after moving to Princeton. As Fine points out, there is an intuitive difference between the meanings of the statement *Carl is Peter* in each of the two scenarios. In the first, the linguistic community has effectively stipulated that *Peter* be used as a synonym of *Carl*, so that a member of the community is in a position to know that the statement is true merely on the basis of linguistic competence. No knowledge of facts about the intended model is required. The statement can be recognized as true, Fregeans will say, by noting the sense-identity of *Carl* and *Peter*, which has been effected by the community's insisting that every model interpret the new name *Peter* by whatever individual interprets *Carl*. The intended model needn't be inspected in order to know the identity. But in the second case, no linguistic convention ties the sense of the name *Peter* to the sense of the name *Carl*; rather, the two names are tied, by the baptismal act, to the same reference *in the intended model*. So to know that the identity statement is true, it is not enough, in the second scenario, to know the (distinct!) senses of the names. We must also have knowledge of facts specific to the intended model.

Fine himself calls the kind of knowledge required in the first scenario (that is, knowledge of senses), semantic facts in the narrow sense, and the kind of knowledge required in the second (that is, knowledge of references in a particular model), semantic facts in the broad sense. But regardless of terminology, we clearly want the semantics of names to reflect the intuitions that a given pair of distinct names is generally used non-synonymously but can under special circumstances be stipulated to be synonyms. Our Fregean semantics does just that. So since there does not seem to be any good reason to expect that distinct names should, absent an explicit stipulation to that effect, have the same senses, it is unlikely that any analog of the antinomy of the variable can be generated for names in $\mathcal{L}_F$.

If we cannot refashion the antinomy of the variable as an antinomy of the name, what about an antinomy of the gap marker? This seems even more hopeless. For one thing, like the variables, the gap marker is not assigned any meaning in Fregean semantics. Moreover, there is only one gap marker, so even if it had a meaning it would

of the senses to the class of all models in which a and b are interpreted identically, but allow quantification over the full model space when looking for a-variants in the semantic clause for predicates.

make no sense to ask whether any *two* gap markers have the same meaning or distinct meanings.

But perhaps this last argument is too quick. Indeed Pickel and Rabern (2016, §II.2) claim that a Fregean predicate logic would fall prey to an antinomy of the gap marker. Let us consider their argument.

Pickel and Rabern argue that Frege needs a means to distinguish a gappy expression $(\ ) \leq (\ )$ in which both gaps are to be filled by the same symbol—a monadic predicate—from a gappy expression $(\ ) \leq (\ )$ in which the gaps may be filled with distinct symbols (a dyadic predicate), and that once such a means is introduced the antinomy of the variable will raise its ugly head again. They contend that

> (…) if Frege were to introduce marks capable of typographically distinguishing between these predicates, then that mark would need its own semantic significance, *which in this context means designation*. (Pickel and Rabern 2016, 151; emphasis added)

A quick but ultimately unsatisfactory reply would be to point out that we don't have any need for compound dyadic predicates in the development of $\mathcal{L}_F$, since Fregean predicates only serve as arguments to the standard existential and universal quantifiers, which each carry a single variable that fills the gaps marked by ξ. To be sure, we have *primitive* predicates in $\mathcal{L}_F$ that have two and more argument places, but as Dummett (1973, 27–33) has pointed out in his classic exposition of Frege's treatment of quantification, attaching gap markers to primitive predicates, as Frege (1893) admittedly does, is an unnecessary feature of the *Grundgesetze* syntax. This is borne out by the fact that we had no need to do this in the system developed in Sect. 2. In other words, as long as we restrict our attention strictly to first-order logic, we will never even encounter the issue Pickel and Rabern raise.

We cannot rest content with this answer, however, for it is predicated on too narrow a view of first-order logic. Suppose, for example, we wanted to introduce a new primitive universal-existential quantifier Ξ that attaches to two variables simultaneously and has the meaning of the $\forall\exists$ combination: $\Xi xy\, \phi(x, y)$ is to mean $\forall x \exists y \phi(x, y)$. Such a dyadic quantifier would have to take as its arguments properly dyadic predicates of the kind Pickel and Rabern urge us to consider.[37]

A moment's reflection reveals, however, that we can accommodate dyadic quantifiers and dyadic compound predicates in a Fregean setting without running up against a version of Fine's antinomy. On the syntax side, we add the following two clauses to the inductive definition of $\mathcal{L}_F$-wfe's:

- If ϕ is an $\mathcal{L}_F$-sentence and c and d are distinct names that both occur in ϕ, then the result $\phi_{c,d}[\xi, \zeta]$ of simultaneously substituting ξ for c and ζ for d in ϕ is a dyadic $\mathcal{L}_F$-predicate.
- If π is a dyadic $\mathcal{L}_F$-predicate and x and y are distinct variables neither of which occurs in π, then the result $\Xi xy\, \pi_{\xi,\zeta}[x, y]$ of simultaneously substituting x for ξ and y for ζ in π and prefixing with Ξxy is an $\mathcal{L}_F$-sentence.

[37] See again Dummett (1973, 29–30).

This necessitates the addition of the following two clauses to the inductive definition of the F-construction histories:

- If σ is an F-construction history whose head is not a name, and c and d are in σ's name record $N(\sigma)$, then

$$\langle c, d, \sigma, N(\sigma)\backslash\{c, d\}, V(\sigma)\rangle$$

  is an F-construction history.
- If σ is an F-construction history whose head $(\sigma)_0$ is a name and whose next constituent $(\sigma)_1$ is also a name, and x and y are distinct variables not in $V(\sigma)$, then

$$\langle \Xi, x, y, \sigma, N(\sigma), V(\sigma) \cup \{x, y\}\rangle$$

  is an F-construction history.[38]

Needless to say, we will then define

$$\mathsf{yield}(\langle c, d, \sigma, N(\sigma)\backslash\{c, d\}, V(\sigma)\rangle) = (\mathsf{yield}(\sigma))_{c,d}[\xi, \zeta]$$

and

$$\mathsf{yield}(\langle \Xi, x, y, \sigma, N(\sigma), V(\sigma) \cup \{x, y\}\rangle) = \Xi xy\, (\mathsf{yield}(\sigma))_{\xi,\zeta}[x, y].$$

On the semantics side, we first need to assign a lexical meaning $[\![\Xi]\!]$ to Ξ. We'll say that $[\![\Xi]\!]$ is the function that maps a model $\mathfrak{M}$ to $[\![\Xi]\!](\mathfrak{M}) := [\![\Xi]\!]^{\mathfrak{M}}$, which is the function that maps a subset X of M^2 to t if for every $u \in M$ there is a $v \in M$ such that $\langle u, v\rangle \in X$, and to f otherwise. A model $\mathfrak{N}$ is a c, d-variant of a model $\mathfrak{M}$, $\mathfrak{N} \sim_{c,d} \mathfrak{M}$, just in case $\mathfrak{N}$ and $\mathfrak{M}$ are identical except perhaps for the respective interpretations of c and d.

It is then straightforward to extend the meaning assignment:

- $[\![\langle c, d, \sigma, N(\sigma)\backslash\{c, d\}, V(\sigma)\rangle]\!]$ is the function that maps a model $\mathfrak{M}$ to the reference $[\![\langle c, d, \sigma, N(\sigma)\backslash\{c, d\}, V(\sigma)\rangle]\!]^{\mathfrak{M}}$ of $\langle c, d, \sigma, N(\sigma)\backslash\{c, d\}, V(\sigma)\rangle$ in $\mathfrak{M}$, this being the set of pairs[39]

$$\{\langle u, v\rangle \in M^2 : \text{ for some } \mathfrak{N} \sim_{c,d} \mathfrak{M}, [\![c]\!](\mathfrak{N}) = u, [\![d]\!](\mathfrak{N}) = v, [\![\sigma]\!](\mathfrak{N}) = \mathsf{t}\}.$$

- $[\![\langle \Xi, x, y, \sigma, N(\sigma), V(\sigma) \cup \{x, y\}\rangle]\!]$ is the function that maps any given model $\mathfrak{M}$ to the reference $[\![\langle \Xi, x, y, \sigma, N(\sigma), V(\sigma) \cup \{x, y\}\rangle]\!]^{\mathfrak{M}}$ of the F-construction history $\langle \Xi, x, y, \sigma, N(\sigma), V(\sigma) \cup \{x, y\}\rangle$ in $\mathfrak{M}$, this being $[\![\Xi]\!](\mathfrak{M})([\![\sigma]\!](\mathfrak{M}))$.

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[38] To be pedantic, we would also have to reformulate the ordinary existential quantifier clause as follows: "If σ is an F-construction history whose head is a name *but for which* $(\sigma)_1$ *is not a name*, and x is a variable not in $V(\sigma)$, $\langle \exists, x, \sigma, N(\sigma), V(\sigma) \cup \{x\}\rangle$ is also an F-construction history."

[39] Of course we can replace mention of the names c and d by mention of $\mathsf{const}([\![c]\!])$ and $\mathsf{const}([\![d]\!])$ as in §3 above.

It is obvious that neither ξ nor ζ has been assigned a meaning; *pace* Pickel and Rabern, the significance of the gap markers is *not* founded on designation. Consequently there is no antinomy of the gap marker either.

We should note that, even if one ignores the gap markers' lack of meaning, the antinomy as presented by Fine cannot get off the ground, for while an expression such as $\xi > 0$ is certainly well-formed, $\zeta > 0$ is not: There are no well-formed expressions of the extended Fregean language in which ζ occurs without ξ. Accordingly there is not even a temptation to think that ξ and ζ should be freely interchangeable, or that $\xi > 0$ should mean the same as $\zeta > 0$. It is the syntactic function of ξ to mark, as it were, first argument places, and that of ζ, to mark second argument places, in compound dyadic relations.

This last observation also bears upon the question of alphabetic innocence. For one way to argue that standard Tarskian predicate logic is not alphabetically innocent is to point out that the formulas $x < y$ and $w < z$—or, for that matter, $y < x$—differ only in their respective choices of variables, which is a purely conventional matter, so that they ought to have the same meaning; yet on the standard compositional meaning assignment they don't. One might be tempted to construct a parallel conundrum for Fregean predicate logic, for both $\xi < \zeta$ and $\zeta < \xi$ are indeed well-formed relational predicates of the extended Fregean language. Clearly the senses of $\xi < \zeta$ and $\zeta < \xi$ are distinct, for the former's is the function mapping any model $\mathfrak{M}$ to the set of pairs $\langle u, v\rangle \in M^2$ such that $u <^{\mathfrak{M}} v$ whereas the latter's is the function mapping any model $\mathfrak{M}$ to the set of pairs $\langle u, v\rangle \in M^2$ such that $v <^{\mathfrak{M}} u$.

Importantly, however, we have no reason to think of the difference between $\xi < \zeta$ and $\zeta < \xi$ as being "merely conventional" in the same sense that, in the Tarskian setting, the difference between $x < y$ and $y < x$ looks "merely conventional". The Tarskian conventionality consists in the fact that syntax gives us no reason to treat any one variable differently than any other, so that it would indeed be arbitrary if we required a semantics to make $x < y$ stand for the less-than relation and $y < x$ for the greater-than relation. But Fregean syntax *does* give us ample reason to treat ξ differently than ζ; first because ξ may occur in a Fregean well-formed expression without ζ also occurring, but not the other way round, and second, because the first variable following Ξ must replace the occurrences of ξ, whereas the second such variable must replace the occurrences of ζ, in the relational predicate to which Ξ is applied. There is, that is, an intrinsic syntactic asymmetry between ξ and ζ, so that $\xi < \zeta$ and $\zeta < \xi$ cannot plausibly be considered mere notational variants of each other.[40] In other words, we cannot generate mere notational variants of open Fregean expressions—(relational) predicates—by permuting gap markers: Fregean predicate logic is thus trivially alphabetically innocent.

40 There is a different, metaphysical issue here that it is crucial not to confuse with the requirement of alphabetic innocence, namely the question whether a non-symmetric relation should be considered distinct from its converse (Williamson 1985; Fine 2000). I take this to be a deep and interesting question, but by adopting a set-theoretical metalanguage for our model theory, we have made the decision to simulate binary relations as sets of ordered pairs and have thereby already answered in the affirmative the metaphysical question whether the less-than relation $\{\langle u, v\rangle \in \omega^2 \mid u < v\}$ is different from the greater-than relation $\{\langle u, v\rangle \in \omega^2 \mid v < u\}$.

5.2 Representationalism

In the introduction, we noted objections to the standard compositional semantics for Tarski-style predicate logic that are based on Tarskian meanings being ultimately constructed out of the variables themselves, that is, on the representationalism of the semantics. More specifically, we distinguished between an "ontological purity" concern that arises from the contamination of semantic values with non-semantic, especially syntactic, items (even more especially, variables), and a "language transcendence" concern that points to the impossibility of two Tarskian languages with distinct sets of variables sharing even a single meaning.

If we want to compare Tarskian and Fregean predicate logic with respect to worries arising from representationalism, there are at least three lines of inquiry to pursue. First, and most concretely, we'll want to know wether it is possible for two Fregean languages with distinct sets of *variables* to express the same meanings (we already know that this is not the case for Tarskian predicate logic). This addresses the question in how far the two approaches are impacted by language transcendence issues with respect to variables in particular. Second, we should ask whether there are other possible sources (besides variables) of language-dependence of Tarskian and Fregean meanings, and whether, where either or both semantics are affected, we can remove this language dependence by reformulating them. Third, we need to investigate how ontologically pure the two semantics (possibly reformulated for language independence) are, that is, to what extent their respective meanings are constructed out of material that is extraneous to semantics.

The first point is easily disposed of: Any two Fregean languages with the same names and predicate symbols express the same meanings, regardless of their sets of variables. For let the language $\mathcal{L}'$ be exactly like $\mathcal{L}_F$, except that, instead of the variables $\mathsf{v}_0, \mathsf{v}_1, \mathsf{v}_2, \ldots$, $\mathcal{L}'$ uses the variables $\mathsf{w}_0, \mathsf{w}_1, \mathsf{w}_2, \ldots$ Then of course the two languages will have different construction histories and different expressions. However, simple inspection of the definitions reveals that the *senses* assigned to the construction histories and well-formed expressions of the two languages will be exactly the same, since variables simply do not contribute to senses.[41]

Thus with respect to variables, Tarskian meanings are language-dependent, while Fregean meanings are not. But what about names and predicate symbols? Recall that models assign interpretations to the members of $\mathcal{C}$ and of $\mathcal{P}$; that is, models contain interpretation functions defined on sets of linguistic entities. But Fregean and (as argued in Sect. 3 above) Tarskian meanings are functions on the class of models, so if two languages, whether Fregean or Tarskian, differ in their names or predicate symbols, and accordingly do not have the same models, they cannot express the same meanings.

[41] More precisely, for any sentence ϕ of either language, let X be the finite set of variables occurring in ϕ; then for any one-one function f from X into the set of variables of the other language, the result of replacing each occurrence of a member x of X in ϕ by an occurrence of $f(x)$ is a sentence of the other language that has the same sense as ϕ. For example, the $\mathcal{L}_F$-sentence $\exists \mathsf{v}_0(Pc\mathsf{v}_0 \wedge \forall \mathsf{v}_1 R\mathsf{v}_0\mathsf{v}_1)$ is assigned the same function from models to truth values as its sense as the $\mathcal{L}'$-sentence $\exists \mathsf{w}_{47}(Pc\mathsf{w}_{47} \wedge \forall \mathsf{w}_{16} R\mathsf{w}_{47}\mathsf{w}_{16})$.

Now this wouldn't be particularly alarming if we were interested *only* in formal languages, for there isn't much *prima facie* pressure to think that distinct formal languages should be able to express the same abstract meanings. But of course the reason we're interested in these formal languages is that they can, when suitably interpreted, go proxy for natural languages. Natural languages, however, are *translatable* into one another, which suggests that they do in fact manage to express the *same* meanings. So the language-transcendence worry that arises from building meanings out of syntax is, more precisely, this: How can *intertranslatable* languages that differ in their relevant syntactic building blocks nevertheless express the same meanings?

We will investigate this issue with respect to Fregean languages first. So let's suppose that $\{\mathcal{L}_i \mid i \in I\}$ is the set of all relevant Fregean languages, where each $\mathcal{L}_i$ has $\mathcal{C}_i$ as its set of names and $\mathcal{P}_i$ as its set of predicate symbols.[42] Suppose further that, for distinct i and j, the sets $\mathcal{C}_i, \mathcal{C}_j, \mathcal{P}_i, \mathcal{P}_j$ are pairwise disjoint.

The assumption that any two Fregean languages are intertranslatable is cashed out formally as the existence of a *system of dictionaries* for $(\mathcal{L}_i)_{i \in I}$, by which we mean a family $\varphi = (\varphi_i^j)_{i,j \in I}$, doubly indexed by I, of functions φ_i^j from $\mathcal{C}_i \cup \mathcal{P}_i$ to $\mathcal{C}_j \cup \mathcal{P}_j$ with the following properties:

(a) φ_i^j maps $\mathcal{C}_i$ one-one onto $\mathcal{C}_j$ and $\mathcal{P}_i$ one-one onto $\mathcal{P}_j$.
(b) If $P \in \mathcal{P}_i$ is n-ary, so is $\varphi_i^j(P) \in \mathcal{P}_j$.
(c) φ_i^i is the identity function on $\mathcal{C}_i \cup \mathcal{P}_i$.
(d) $\varphi_j^k \circ \varphi_i^j = \varphi_i^k$.

Clauses (a) and (b) ensure that names are translated as names, and n-ary predicates are translated as n-ary predicates; moreover, by clause (a) each φ_i^j is a bijection. Clause (c) means that a language is translated into itself by changing nothing. Clause (d) formally captures the idea that, if we first look up the French translation of an English expression in an English–French dictionary, and then look up the German translation of that French expression in a French–German dictionary, we end up with the same German expression as if we had looked up the German translation of the original English expression in an English–German dictionary. Together with (a), clauses (c) and (d) imply that φ_j^i is the inverse of φ_i^j.

Given any $i \in I$, $c \in \mathcal{C}_i$, and $P \in \mathcal{P}_i$, let $[c]$ be $\{\varphi_i^j(c) \mid j \in I\}$, and let $[P]$ be $\{\varphi_i^j(P) \mid j \in I\}$. So $[c]$ is the set consisting of c and all of its translations into some $\mathcal{L}_j$, and similarly for $[P]$. We call $[c]$ the translational equivalence class of c, and similarly $[P]$ the translational equivalence class of P.[43] Note that, by condition (b), all members of $[P]$ have the same arity, which we will therefore also call the arity of $[P]$. Let $\mathbf{C}$ be the set $\{[c] \mid c \in \bigcup_{i \in I} \mathcal{C}_i\}$ and let $\mathbf{P}$ be the set $\{[P] \mid P \in \bigcup_{i \in I} \mathcal{P}_i\}$.

The existence of these translational equivalence classes, guaranteed by the system of dictionaries φ, now allows us to deparochialize our models (and hence our meanings), that is, to make them independent of any particular language $\mathcal{L}_i$.

42 We can leave it open whether distinct $\mathcal{L}_i$ have the same or distinct sets of variables, since we already know that the choice of variables does not affect Fregean senses.

43 Here names c and d in $\bigcup_{i \in I} \mathcal{C}_i$ are equivalent in the requisite sense just in case there are $i, j \in I$ such that $\varphi_i^j(c) = d$, and similarly for predicate symbols.

A *schmodel* $\mathfrak{S}$ is a triple $(S, (\pi^{\mathfrak{S}})_{\pi\in\mathbf{P}}, (\gamma^{\mathfrak{S}})_{\gamma\in\mathbf{C}})$, where S, the *domain* of $\mathfrak{S}$, is a non-empty set, $(\pi^{\mathfrak{S}})_{\pi\in\mathbf{P}}$ is a family of relations over S indexed by the set $\mathbf{P}$ of translational equivalence classes of predicates such that, whenever $\pi \in \mathbf{P}$ is n-ary, $\pi^{\mathfrak{S}} \subseteq M^n$; and $(\gamma^{\mathfrak{S}})_{\gamma\in\mathbf{C}}$ is a family of members of S indexed by the set $\mathbf{C}$ of translational equivalence classes of names. We refer to $\pi^{\mathfrak{S}}$ as the *interpretation of* π in $\mathfrak{S}$, and to $\gamma^{\mathfrak{S}}$ as the *interpretation of* γ in $\mathfrak{S}$. Where $\gamma \in \mathbf{C}$, schmodels $\mathfrak{S}$ and $\mathfrak{S}'$ are γ-variants, $\mathfrak{S} \sim_\gamma \mathfrak{S}'$, just in case $\mathfrak{S}$ and $\mathfrak{S}'$ are identical except possibly in their interpretation of γ. For $u \in S$, $\mathfrak{S}^u_\gamma$ is the γ-variant of $\mathfrak{S}$ that interprets γ as u.

Note that for each $i \in I$, a schmodel $\mathfrak{S} = (S, (\pi^{\mathfrak{S}})_{\pi\in\mathbf{P}}, (\gamma^{\mathfrak{S}})_{\gamma\in\mathbf{C}})$ uniquely determines an $\mathcal{L}_i$-model $\mathfrak{S}_i = (S_i, (P^{\mathfrak{S}_i})_{P\in\mathcal{P}_i}, (c^{\mathfrak{S}_i})_{c\in\mathcal{C}_i})$, which we might call the i-th projection of $\mathfrak{S}$: Simply let $S_i = S$, $P^{\mathfrak{S}_i} = [P]^{\mathfrak{S}}$, and $c^{\mathfrak{S}_i} = [c]^{\mathfrak{S}}$. Moreover, for each $i \in I$, every $\mathcal{L}_i$-model $\mathfrak{M}$ uniquely determines a schmodel $\mathfrak{S}^{\mathfrak{M}}$ of which it is the i-th projection: If $\mathfrak{M}$ is $(M, (P^{\mathfrak{M}})_{P\in\mathcal{P}_i}, (c^{\mathfrak{M}})_{c\in\mathcal{C}_i})$, let $\mathfrak{S}^{\mathfrak{M}}$ be $(M, (\pi^{\mathfrak{S}^{\mathfrak{M}}})_{\pi\in\mathbf{P}}, (\gamma^{\mathfrak{S}^{\mathfrak{M}}})_{\gamma\in\mathbf{C}})$, where $\pi^{\mathfrak{S}^{\mathfrak{M}}}$ is $P^{\mathfrak{M}}$ for the unique $P \in \mathcal{P}_i$ for which $\pi = [P]$, and $\gamma^{\mathfrak{S}^{\mathfrak{M}}}$ is $c^{\mathfrak{M}}$ for the unique $c \in \mathcal{C}_i$ for which $\gamma = [c]$.[44]

We can easily formulate the Fregean semantics for each individual language $\mathcal{L}_i$ in terms of schmodels $\mathfrak{S} = (S, (\pi^{\mathfrak{S}})_{\pi\in\mathbf{P}}, (\gamma^{\mathfrak{S}})_{\gamma\in\mathbf{C}})$: The schmeference $[\![c]\!]^{\mathfrak{S}}$ of a name $c \in \mathcal{C}_i$ relative to the schmodel $\mathfrak{S}$ is the interpretation $[c]^{\mathfrak{S}}$ of $[c] \in \mathbf{C}$ in $\mathfrak{S}$ (which is also the interpretation of c in the i-th projection of $\mathfrak{S}$), and the schmeference $[\![P]\!]^{\mathfrak{S}}$ of a predicate $P \in \mathcal{P}_i$ relative to the schmodel $\mathfrak{S}$ is the characteristic function of the interpretation $[P]^{\mathfrak{S}}$ of $[P] \in \mathbf{P}$ in $\mathfrak{S}$ (in other words, of the interpretation of P in the i-th projection of $\mathfrak{S}$). The schmeferences of $\neg$, $\wedge$, and $\exists$ relative to schmodel $\mathfrak{S}$ are, as in the case of models, the truth functions $H_\neg$ and $H_\wedge$ and the characteristic function $H^M_\exists$, relative to the powerset of S, of the non-empty subsets of S.

The schmeferences relative to a schmodel of the $\mathcal{L}_i$-construction histories and the $\mathcal{L}_i$-well-formed expressions can be defined essentially as for the original Fregean language $\mathcal{L}_F$.[45] Schmeferences relative to $\mathfrak{S}$ are thus identical to the respective references relative to the i-th projection of $\mathfrak{S}$. Finally, the schmense of an $\mathcal{L}_i$-construction history or an $\mathcal{L}_i$-well-formed expression is just the function that maps each schmodel to the schmeference of the construction history or expression relative to that schmodel. Given the one-one correspondence between schmodels and $\mathcal{L}_i$-models, as well as the identity of schmeferences in schmodels and references in their i-th projections, we see that it makes no difference to the logical relations between well-formed $\mathcal{L}_i$-expressions whether we formulate their semantics in terms of models or in terms of schmodels.

However, the building blocks of the *schmenses* of each of the languages $\mathcal{L}_i$ are the schmodels, and schmodels are not, like models, language-dependent, for each schmodel interprets not just a single language, but all of them simultaneously. Accordingly, any two languages $\mathcal{L}_i$ and $\mathcal{L}_j$ express exactly the same schmenses. So the move

44 $\pi^{\mathfrak{S}^{\mathfrak{M}}}$ is well defined because whenever $[P] = [Q]$ for $P, Q \in \mathcal{P}_i$, we must have that $Q = \varphi^j_i(P)$ for some $i, j \in I$ by definition of $[P]$ and $[Q]$, but then since $\mathcal{P}_i$ and $\mathcal{P}_j$ are by assumption disjoint whenever $i \neq j$ we must have $i = j$ and hence $Q = \varphi^i_i(P) = P$. Similarly for $\gamma^{\mathfrak{S}^{\mathfrak{M}}}$.

45 For example, the schmeference $[\![\pi]\!]^{\mathfrak{S}}$ of an $\mathcal{L}_i$-predicate π in a schmodel $\mathfrak{S}$ can be defined as the set $\{u \in S \mid \text{for some } \mathfrak{S}' \sim_{[c]} \mathfrak{S}, u = [\![c]\!]^{\mathfrak{S}'} \text{ and } [\![\pi_\xi[c]]\!]^{\mathfrak{S}'} = \mathsf{t}\}$, where c is any name in $\mathcal{C}_i$ that doesn't occur in π.

from models to schmodels completely eliminates any language-dependence of intertranslatable families of Fregean languages.

Before we return to the Tarskian side of things, let us ask how ontologically pure the schmenses are, that is, let us ask whether they are built out of material that is intrinsically semantic. This is admittedly a somewhat vague question, but I am inclined to think that the answer is yes. The *models*, building blocks of senses, contain functions defined on names and predicates, which are syntactic items and as such extrinsic to semantics. The *schmodels*, however, instead contain functions defined on *translational equivalence classes* of names and of predicates, and translation is arguably a semantic phenomenon—the translational equivalence classes simply encode an equivalence in lexical meaning. One might worry that these translational equivalence classes are themselves constructed out of bits of syntax in that they contain the names and predicate symbols of particular languages as elements. But this is really an artifact of the set-theoretic machinery we're deploying at the meta-level. Friends of Fregean abstraction could simply replace these equivalence classes with the abstracts generated by the following abstraction principle, where μ and ν range over $\bigcup_{i\in I}\mathcal{C}_i \cup \bigcup_{i\in I}\mathcal{P}_i$:

$$@(\mu) = @(\nu) \leftrightarrow \exists i, j \in I : \varphi_i^j(\mu) = \nu.$$

There is then no obvious reason to think that these abstracts are literally constructed out of names and predicate symbols. If this is correct, our schmenses are indeed made exclusively of intrinsically semantic material.

What about Tarskian predicate logic? Clearly any representationalism that is due to the dependence of Tarskian meanings on *models* (in that models contain interpretation functions for names and predicate symbols) can be resolved along the lines we just developed for Fregean predicate logic, that is, by replacing models with schmodels. But there is an additional complication in the Tarskian case in that the meanings are constructed not only out of models (which we might replace with schmodels) but also out of variable assignments. The replacement of models by schmodels does nothing to make Tarskian meanings independent of the choice of variables, so *prima facie* Tarskian predicate logic seems to be stuck with both language dependence and ontological impurity.

Perhaps that's too quick. Let's see whether we can, by pushing the techniques just developed for the Fregean case, tame the variables as a specifically Tarskian source of representationalism.

Since we already know how to eliminate any language-dependence that is due to names or predicate symbols, we'll stick throughout with the original, fixed sets $\mathcal{C}$ of names and $\mathcal{P}$ of predicates. Suppose that for each $i \in I$, V_i is a countably infinite set disjoint from $\mathcal{C}$ and from $\mathcal{P}$, and that, for distinct i and j in I, V_i and V_j are disjoint. For $i \in I$, let $\mathcal{L}^i$ be the Tarskian language whose set of variables is V_i. Let $\varphi = (\varphi_i^j)_{i,j\in I}$ be a family, doubly indexed by I, of bijections $\varphi_i^j : V_i \to V_j$ with the following properties:

(a) φ_i^i is the identity function on V_i.

(b) $\varphi_j^k \circ \varphi_i^j = \varphi_i^k$.

For later reference, let's call such families *systems of variable-set bijections*.

Systems of variable-set bijections are to variables what systems of dictionaries are to names and predicate symbols. There is one intuitive difference, however, that will be significant later: Names and predicate symbols correspond to the names and predicates of natural languages, and real-life dictionaries provide translations for the names and predicates of natural languages. They do not, of course, do this at random; which German words a dictionary pairs with which English words is empirically constrained: Next to *table* we find *Tisch*, not *Fisch*, because German speakers use the former, not the latter, for the communicative purposes for which English speakers use *table*. There is, therefore, some justification for assuming that for any two intertranslatable languages, there is a unique, canonical dictionary covering their lexical expressions (here, names and predicates), and this is an assumption we did indeed make when we showed how to make Fregean meanings independent of particular languages.

The variables of our formal languages, however, presumably correspond to things like traces, or indices on pronouns, in natural languages. These are invisible to the lexicographer in the field who composes dictionaries. Consequently there is no empirical constraint on the English translation of the French trace f_8; the English trace t_3 is just as eligible as t_{143}. There simply isn't a fact of the matter whether f_8 translates as t_3 or as t_{143}.[46] That is to say, even for intertranslatable languages, there is no canonical translation of their respective traces and pronoun indices.[47] We shall return to this observation shortly.

For $x \in V_i$, let $[x]_\varphi$ be $\{\varphi_i^j(x) \mid j \in I\}$, and let $\mathbf{V}_\varphi$ be $\{[x]_\varphi \mid x \in \bigcup_{i \in I} V_i\}$. We'll call the members of $\mathbf{V}_\varphi$ the φ-variables. Given a model $\mathfrak{M}$, a φ-variable assignment is a mapping from $\mathbf{V}_\varphi$ into the domain of $\mathfrak{M}$. Where $x \in \bigcup_{i \in I} V_i$, we say that the φ-variable assignments G and H are x-variants of each other, $G \sim_x H$, if G and H agree on all φ-variables except possibly $[x]_\varphi$.

We can now assign φ-meanings to the well-formed formulas of each $\mathcal{L}^i$ in pretty much the same way we did in Sect. 1, except that we'll use φ-variable assignments instead of ordinary, language-specific variable assignments. So fix a model $\mathfrak{M}$ and let $\mathfrak{G}_\varphi$ be the set of all φ-variable assignments in $\mathfrak{M}$.[48] The φ-meaning $[\![P]\!]_\varphi$ of an n-place predicate symbol $P \in \mathcal{P}$ is the constant function that maps each φ-variable assignment to the characteristic function of the interpretation $P^{\mathfrak{M}}$ of P in $\mathfrak{M}$. The φ-meaning $[\![a]\!]_\varphi$ of a name $a \in \mathcal{C}$ is the constant function that maps each φ-variable assignment to the interpretation $a^{\mathfrak{M}}$ of a in $\mathfrak{M}$. The φ-meaning $[\![x]\!]_\varphi$ of an $\mathcal{L}^i$-variable x is the function that maps each φ-variable assignment G to $G([x]_\varphi)$. The φ-meanings of $\neg$, $\wedge$, and $\exists$ are the constant functions that map any φ-assignment to $H_\neg$, $H_\wedge$ and $H_\exists^M$, respectively. The φ-meanings of $\mathcal{L}^i$-formulas are defined as expected.[49] As is

46 In fact, there is no way of knowing whether English and French have the same or distinct classes of traces or, for that matter, whether distinct speakers of English employ the same or distinct classes of traces.

47 If you are tempted to think that there's something canonical about translating each French trace f_i as the English trace t_i, imagine a language whose traces τ_q are indexed not by the natural numbers but by, say, the rationals. What is the natural number index on the English trace that $\tau_{3.853}$ canonically translates to?

48 To simplify exposition, we will for the time being ignore the necessity, discussed in §3, of making Tarskian meanings dependent on models.

49 In particular, for the quantifier case, $[\![\exists x \phi]\!]_\varphi$ is the function that maps any φ-variable assignment G to $[\![\exists]\!]_\varphi(G)(\{u \in M \mid \text{ for some } H \sim_x G, u = [\![x]\!]_\varphi(H) \text{ and } [\![\phi]\!]_\varphi(H) = \mathsf{t}\})$.

easy to see, the same φ-meanings are expressible in $\mathcal{L}^i$ and $\mathcal{L}^j$: For any $\mathcal{L}^i$-formula ϕ, the φ-meaning it has in $\mathcal{L}^i$ is expressed in $\mathcal{L}^j$ by the $\mathcal{L}^j$-formula that arises from ϕ if one replaces each variable occurring in ϕ by its φ_i^j-image. The φ-meanings are thus independent of any particular language $\mathcal{L}_i$.

Alas, the φ-meanings *are* dependent on φ, the system of variable-set bijections—change the system, and you change the meanings. And there is no canonical φ that we could just fix on. But this arguably means we haven't made any progress on the language-independence front after all. For any particular φ, we've replaced the individual languages $\mathcal{L}^i$ made up, *inter alia*, of the names in $\mathcal{C}$, the predicates in $\mathcal{P}$, and the variables in V_i, by a new superlanguage $\mathcal{L}^\varphi$, made up of the names in $\mathcal{C}$, the predicates in $\mathcal{P}$, and the φ-variables in $\mathbf{V}_\varphi$, and we've simply applied the old meaning construction to the superlanguage $\mathcal{L}^\varphi$. But given the existence of a multitude of systems of variable-set bijections φ, we're now looking at a multitude of superlanguages $\mathcal{L}^\varphi$ with different sets $\mathbf{V}_\varphi$ of φ-variables, so that the meanings constructed for a superlanguage $\mathcal{L}^\varphi$ will be disjoint from the meanings constructed for a different superlanguage $\mathcal{L}^{\varphi'}$. The transcendency problem has caught up with us, one level up.

We can put the point more metaphysically as follows: As long as φ and φ' are both systems of variable-set bijections, the φ-meanings have as good a claim to being the true Tarskian meanings as do the φ'-meanings; by the Principle of Sufficient Reason, the φ-meanings are the true Tarskian meanings if and only if the φ'-meanings are. Since they can't both be the true meanings, neither of them is.[50]

As a last-ditch effort, we might propose to define the true Tarskian meanings—the *Meanings*, let's say—as functions mapping systems of variable-set bijections φ (and, strictly speaking, models $\mathfrak{M}$) to the φ-meanings relative to $\mathfrak{M}$. Thus, for example, the Meaning of the $\mathcal{L}^i$-formula $\exists x \phi$ would be the function that maps each system of variable-set bijections φ (and each model $\mathfrak{M}$) to the function that maps any φ-assignment G over M to the result of applying $H_\exists^M$ to the set

$$\left\{u \in M \mid \text{ for some } H \sim_x G, u = [\![x]\!]_\varphi(H) \text{ and } [\![\phi]\!]_\varphi(H) = \mathsf{t}\right\}.$$

Since we've now abstracted from φ by making it an additional argument, the Meanings are no longer dependent on the choice of a system of variable-set bijections.

Does this mean that we've banished the specter of representationalism from Tarskian predicate logic? I think not. For one thing, the Meanings score no better on ontological purity than the original Tarskian meanings, for they are built out of variable-set bijections and thus, ultimately, out of bits of syntax (namely the variables of all the $\mathcal{L}^i$), which was deemed objectionable in our original Tarskian meanings.

Moreover, the Meanings are hard to justify intuitively as playing the role of meanings. If meanings are creatures that enable us to identify the reference of an expression, relative to some factual background, it seems plausible that meanings take models (or schmodels) as arguments (as our original Fregean and Tarskian meanings do), since models (or schmodels) formally represent factual backgrounds. Perhaps it is also intuitively plausible to think that meanings take ("salient") variable assignments as

[50] See Benacerraf (1965) for another application of this principle.

arguments (as our original Tarskian meanings do); such is, in any case, often assumed without much argument.[51] But why should meanings take a system of variable-set bijections as argument? And which such system should the Meaning be applied to if we want to find out what the reference of an expression is? I suspect that there are no good answers to these questions. If this is correct, there is no way of delivering Tarskian predicate logic from the evils of representationalism.

6 Conclusion

What have we accomplished? We've provided a compositional, non-representational semantics for a Frege-style predicate logic that is immune to the antinomy of the variable and that, while it eschews variable assignments as does e.g. Quine's predicate–functor logic, nevertheless retains the familiar quantifier–variable mechanism for the expression of generality. This shows conclusively that the somewhat mysterious phenomenon of variable "binding" is an artifact of a particular way of presenting first-order logic—in Fregean predicate logic, there simply is no "binding" of items that previously were "free".

We also saw that the reason why Fregean predicate logic escapes the problems of its Tarskian cousin is that it manages to get by without assigning lexical meanings to variables. Why exactly does Tarski-style first-order logic, but not Fregean predicate logic, invite the idea that variables have meanings? The answer, I submit, has to do with the number of logical roles played by variables within the well-formed expressions of the respective systems. In Fregean wfe's, the variables perform a single function, namely that of punctuating quantified sentences. This is clearly a role the variables play in Tarskian predicate logic as well. But only *bound* occurrences of variables function in this way, and since Tarskian predicate logic allows free occurrences of variables in its well-formed expressions, it obviously countenances a second role for variables, and this second role is essentially one of reference (the reference of a variable being provided by an externally given variable assignment).

We thus have a case of overloading a single character here; within well-formed expressions, the letter "x" sometimes functions like a name, and sometimes like a punctuation mark. This is not in itself problematic—after all, the context always makes clear which function is accorded to a variable occurrence—but it can become so if the conceptual distinctions between the different roles are not clearly observed. As Wittgenstein (1922) points out:

> TLP 3.323: In the language of everyday life it very often happens that the same word signifies in two different ways—and therefore belongs to two different symbols (…)
> TLP 3.324: Thus there easily arise the most fundamental confusions (of which the whole of philosophy is full).

[51] Usually one thinks more generally of *contexts* as constituting these additional arguments, but in simple cases the context just reduces to a variable assignment.

Fine's antinomy of the variable is, I think, a case in point, even if its setting is not the "language of everyday life" but a formal one: The existence of referential uses of variables makes them seem very much like names, which invites the idea that they should be treated as lexical items with full-fledged meanings. Indeed, against the background of a variable assignment g, a variable x on its free occurrences really just *is* a name, and its reference is its assigned value $g(x)$. As such, x and y in general have different references—to wit, $g(x)$ and $g(y)$—and therefore they must have different meanings.

Now because we use these same characters, which are meaningful when occurring free, also within expressions of generality—such as $(\exists x)\ (x > 0)$ and $(\exists y)\ (y > 0)$—we are seduced into thinking that, when they occur in this role, they must still have meaning. And since, within expressions of generality, one variable is essentially as good as any other, we are further seduced into thinking that any two variables must have the *same* meaning.

Fregean predicate logic completely avoids overloading one character with several logical roles; or, to put it in Tractarian terms, it avoids using one sign within several symbols. In particular, the variables are never anything but punctuation marks and are, as such, devoid of lexical meaning. The availability of the Fregean approach therefore enables us to realize, and thereby dissolve, this conceptual confusion engendered by Tarskian syntax. To quote the *Tractatus* one last time:

> TLP 3.325: In order to avoid these errors, we must employ a symbolism which excludes them, by not applying the same sign in different symbols (…)

Given these advantages of the Fregean over the Tarskian approach, we have, I think, given a definitive answer to the question whether variables in predicate logic, broadly understood, carry meaning: They do not. Rather, their function is exhausted by providing punctuation in well-formed expressions in much the same way parentheses do.

Finally, our results provide some perspective to the evaluation of the relative merits of Frege's and Tarski's accomplishments in the foundations of quantification theory. There is no doubt that, by modern standards, the details of the syntax developed by Frege in *Grundgesetze* leave a lot to be desired, as Pickel (2010) correctly points out, but in the light of our investigation it strikes me as going too far to claim that "what has been heralded as Frege's greatest innovation—his theory of iterated quantification—was, in fact, unsatisfactory" (Pickel 2010, 272); the more so when it is contrasted with Tarski's achievement in the following way:

> The Tarskian method for constructing sentences and its corresponding semantics were genuine innovations. The development required a complete rethinking of the semantic properties of sentences. In order to think about how one sentence contributes to the semantic value of a sentence that contains it, Tarski was forced out of thinking that the semantic contribution a sentence makes to a sentence that contains it (even in an extensional context) is its truth-value. The fundamental semantic property of a sentence, Tarski realized, is not that it has a certain truth-value, but that it is satisfied by certain sequences. I will not here defend the success of Tarski's project. I want to note that it is different from and a significant

improvement over Frege's treatment of the semantics of quantification (Pickel 2010, 272).

Of course Frege held that the contribution a *sentence* ϕ makes to the truth value of a sentence $\psi(\phi)$ that contains it is ϕ's truth value—because it is. Frege's great insight, one that is easily lost sight of in the Tarskian framework, was that a quantified sentence such as $\exists x(Px \wedge Rax)$ does not contain a "sentence" $(Px \wedge Rax)$ (which, in Fregean syntax, isn't even well-formed), but is rather composed of the *predicate* $(P\xi \wedge Ra\xi)$, the quantifier $\exists$, and punctuation $x \dots x \dots x \dots$ Moreover, as the viability of the Fregean framework for quantification theory shows, the satisfaction of a formula by a variable assignment (or, alternatively, if the variables are indexed by the natural numbers, by a sequence of objects) is *not* a fundamental semantic property, but rather an artifact of doing predicate logic Tarski-style.[52] I therefore see no basis for the claim that Tarski's approach to predicate logic constitutes "a significant improvement over Frege's treatment of quantification," notwithstanding Tarski's numerous and deep contributions to logic otherwise.[53]

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52 That is, both Tarskian and Fregean semantics require a function assigning objects to names and relations to predicate symbols (an interpretation function), but only Tarskian approaches require a *second* function assigning objects to variables (a variable assignment). The involvement of variable assignments in the semantics is therefore an artifact of Tarskian approaches.

53 I find myself largely in agreement here with Partee (2013, 120–121): "Frege [...] developed the syntax and semantics of quantifiers [...] And in a sense he did it more compositionally than Tarski. In Tarski's semantics for quantified sentences, standard in logic textbooks, the quantifier symbols ∀ and ∃ are not themselves given a semantic interpretation. They are treated *syncategorematically*: we are given semantic interpretation rules for formulas *containing* quantifiers. Tarski's semantics is thus not strictly compositional. [...] Frege treated the quantifier symbols as categorematic, standing for certain second-order objects [...]".

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