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RESEARCH ARTICLE

Proof-theoretic semantics for a natural language fragment

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Abstract The paper presents a proof-theoretic semantics (PTS) for a fragment of natural language, providing an alternative to the traditional model-theoretic (Montagovian) semantics (MTS), whereby meanings are truth-condition (in arbitrary models). Instead, meanings are taken as derivability-conditions in a "dedicated" natural-deduction (ND) proof-system. This semantics is effective (algorithmically decidable), adhering to the "meaning as use" paradigm, not suffering from several of the criticisms formulated by philosophers of language against MTS as a theory of meaning. In particular, Dummett's manifestation argument does not obtain, and assertions are always warranted, having grounds of assertion. The proof system is shown to satisfy Dummett's harmony property, justifying the ND rules as meaning conferring. The semantics is suitable for incorporation into computational linguistics grammars, formulated in type-logical grammar.

Keywords Proof-theoretic semantics · Natural language · Harmony · Natural deduction

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1 Introduction

In this paper, we propose a *Proof-Theoretic Semantics* (*PTS*) for a (positive) fragment E_0^+ (delineated below, and extended in the sequel) of *Natural Language (NL)* (English in this case). This semantics is intended (Francez et al. 2010) to be incorporated into actual grammars, within the framework of *Type-Logical Grammar (TLG)* (Moortgat 1997), which renders the *PTS radically lexicalized*, and allows the *PTS* to rely on its *type-driven syntax-semantics interface*. Thereby, this semantics constitutes an alternative to the traditional *model-theoretic semantics (MTS)*, originating in Montague's seminal work (Montague 1973), used in *TLG*. See (Lappin 1997) for an indication of the dominance of MTS in formal semantics of NL, and for a variety of further references to MTS treatment of various semantic issues.

We would like to stress, that this paper is mainly intended to *set the stage* for the proposed approach, focusing on properties of the system of rules itself. Subsequent work, to some of which references are provided, will focus on additional properties of the semantics itself and expand in depth on several semantic issues. There is no claim that the current paper solves any open semantic problems in MTS; however, in Sect. 6.2, we do show how PTS may cope more easily with a semantic issue with which MTS has difficulties. The motivation for pursuing PTS for NL draws on some general criticism, partly delineated below, on MTS as a way of *defining meanings* (here, for NL). By providing in detail an alternative approach overcoming (at least some of) the criticism, we prepare the ground for further research that might settle the rivalry between the two approaches.

The essence of our proposal is:

- *For sentences, replace the received approach of taking their meanings as* **truth conditions** *(in arbitrary models) by an approach taking meanings to consist of* **canonical derivability conditions** *(from suitable assumptions)*. In particular, this involves a "dedicated" proof-system in natural deduction (ND) form, on which the derivability conditions are based. In a sense, the proof system should reflect the "use" of the sentences in the fragment, and should allow recovering pre-theoretic properties of the meanings of these sentences such as entailment and assertability conditions.

An important requirement is that the ND-system should be *harmonious*, in that its rules have a certain balance between introduction and elimination, in order to qualify as conferring meaning. Two notions of harmony are shown to be satisfied by the proposed rules (see Sect. 4). The approach put forward here is different from a seemingly related one by Ranta (e.g., Ranta 1994), connecting NL constructs to Martin-Löf's constructive type-theory (MLTT). We discuss the relationship to Ranta's approach in Sect. 7.

- For *subsentential phrases*, down to lexical units (words), replace their denotations (in arbitrary models) as conferring meaning, by their *contributions* to the meanings (in our explication, derivability conditions) of sentences in which they occur. This adheres to Frege's *context principle*, made more specific by the incorporation into a TLG. This is reported in full detail elsewhere (Francez et al. 2010), and not further considered here.

To the best of our knowledge, there has been no attempt[1] to develop *PTS* as part of a grammar for *NL*. The following quotation from Schroeder-Heister 2006 (p. 525) emphasizes this lack of applicability to *NL*, the original reason for considering *PTS* to start with:

> Although the *"meaning as use"* approach has been quite prominent for half a century now and provided one of the cornerstones of philosophy of language, in particular of ordinary language philosophy, it has never become prevailing in the *formal* semantics of artificial and natural languages. In formal semantics, the *denotational* approach which starts with interpretations of singular terms and predicates, then fixes the meaning of sentences in terms of truth conditions, and finally defines logical consequence as truth preservation under all interpretations, has always dominated.

1.1 Motivating the proposal

The main motivation for pursuing PTS originates from the criticism, by several philosophers of language and logicians, about the adequacy of MTS *as a theory of meaning*, reflecting a cognitive *understanding* of meanings, notably by Dummett (e.g., Dummett 1991), Brandom (e.g., Brandom 2000) and others. It is impossible to do justice to this criticism, often expressed in books, in a paper. Still, we will try to present some of the main points of criticism, directly related to our proposal.

In attempting to devise in detail a PTS and incorporate it into the grammar of *NL*, we are not necessarily committing ourselves to the accompanying philosophical positions, such as *anti-realism* (e.g., Tennant 1987). Some of these philosophical principles have been closely scrutinized. For a recent example, see Boulter (2001), that presents a critical discussion of the anti-realism underlying the verificationist approach to meaning. Rather, our point of departure is *computational linguistics*, with its stress on *effectiveness* of its methods and theories.

The most famous criticism is Dummett's *manifestation argument*, regarding the understanding of a sentence (i.e., grasping its meaning) as involving the ability (at least in principle) to verify it, as a condition for its assertability. Trans-verificational truth is rejected since it is not reflecting a cognitive process of understanding (this is where anti-realism emerges). Since MTS cannot identify *uniquely* a model corresponding to the actual world, verifiability means *deciding*, given an arbitrary model, whether the truth-condition (constituting the MTS meaning of a sentence) obtains in the given model. In general, this task is impossible even for the simplest sentences, involving only predication, as set membership is not decidable[2] in general. It follows that *entailment*, a major concern of all formal semantics definitions, is not effective either. In our proposal, the meaning of a sentence *S* (in the fragment under consideration) constitutes its *canonical derivability conditions*

[1] Fitch (1973) provides natural deduction rules for English, some similar to the rules here, at least in spirit; however they are not claimed to confer meaning.

[2] There is no precise statement by Dummett as to what is taken as "decidable". It is plausible, at least in a computational linguistics context, to identify this notion with *effectiveness* (i.e., *algorithmic decidability*).

in a natural-deduction system modeling “use” (canonicity is explained below). Here, grasping the meaning involves having (an effective) *warrant* for assertion, in possessing a finite collection of sentences Γ (all in the fragment), from which *S* is canonically derivable in the ND-system. Deciding derivability is most often an effective process. Thus, the manifestation problem does not arise in PTS as proposed here. See also, among many other discussions of this issue, Pravitz (1978).

Another kind of criticism of MTS questions its *explanatory power*. The received wisdom regards MTS as a formalization of the relationship between language and the world. Quine (1969) relates to this view as “the museum myth”: NL expressions are stuck on objects like labels in a great museum. The claim is that no theory can succeed in directly relating language to the world. At most, language is related to some meta-language (e.g., some set-theoretical language), used to specify models and truth-conditions in them. This is true for MTS in general, but is particularly relevant to the case of NL, which is its own ultimate meta-language. See Peregrin (1997) for a discussion of this issue. One may add to this criticism also some dissatisfaction with the *ontological commitment* accompanying MTS, relating to various entities populating models: possible-worlds, events, times, degrees, kinds and many more. The kind of PTS proposed here for NL does not attempt at all to relate language to extra-linguistic elements. It is based on the notion of *derivation* of a sentence from other sentences to start with. It carries no ontological commitments, and alludes only to artifacts of the underlying natural deduction system. Thus, there are no individual entities populating some world, only individual parameters (see below), syntactic objects that participate in derivations. A related issue is the possibility of quantifying over “absolutely everything”, accompanying MTS, see Rayo and Uzquiano (1997). This problem doesn’t arise in PTS, not relying on any totalities of universes in models.

1.2 PTS for NL verses PTS for logic

There are several differences in the way the proof-theoretic semantics is conceived for NL, as compared to logic, owing to the differences between E_0^+ and traditional formal calculi for which ND-systems were proposed in logic as a basis for PTS.

- Logical calculi are *recursive*, in that each operator (connective, quantifier) is applied to (one, two or more) formulae of the calculus, to yield another formula. Thus, there is a natural notion of the *dominant* (or *main*) operator which is introduced into/eliminated from a formula. In E_0^+, on the other hand, there is no such notion (in general) of a dominant operator. In this sense, all E_0^+ sentences are *atomic* (in not having a sentence as a constituent). Furthermore, the operators are introduced as if *according to their grammatical function*; for example, ‘every’ may be introduced either into the subject or into the object of a transitive verb, or into both.
- Formal calculi are usually taken to be *semantically unambiguous*, while E_0^+ (and NL in general) is semantically ambiguous. In a PTS, the semantic ambiguity manifests itself via different derivations (from same assumptions).

This will be exemplified below by showing how traditional *quantifier scope ambiguity* manifests itself (see Sect. 3.2).

- Formal logical calculi usually have *(formal) theorems* (or *theses*), having a *proof*, i.e. a (closed) derivation from no open assumptions, namely, no assumptions that have not been discharged. In natural language (an in particular in the fragment we consider here) there are hardly any formal theorems. Typically, sentences are contingent and their derivations rely on open (undischarged) assumptions (but see Sect. 4.2). This difference has a direct influence on the conception of PTS-meanings (of sentences), and PTS-validity (of arguments, or derivations).

1.3 Proof-Theoretic Semantics for Logic

Before dwelling on our main task, PTS for NL, we exemplify the approach as applied to logic. Readers familiar with PTS for logic may skip this section. The origin of PTS for logic is in the work of Gentzen (1935), who invented the *natural deduction* (and Sequent Calculus) proof-systems for 1st-order logic (*FOL*). He hinted there, that introduction-rules (I-rules) for some logical constant (connective, quantifier) could be seen as the *definition* of that logical constant, and elimination-rules (E-rules) as uses of that definition. To understand the issues involved, consider first a simple example, the conjunction, which is the most "well-behaved" sentential connective in this respect. The main claim of PTS is that in order to understand the conjunction, there is no need to resort to truth-values and truth-tables (the standard MTS way of defining connectives). Rather, it suffices to know the following rules for introducing and eliminating it.

$$\frac{\Gamma \vdash \phi \quad \Gamma \vdash \psi}{\Gamma \vdash \phi \wedge \psi}(\wedge I) \quad \frac{\Gamma \vdash \phi \wedge \psi}{\Gamma \vdash \phi}(\wedge E_1) \frac{\Gamma \vdash \phi \wedge \psi}{\Gamma \vdash \psi}(\wedge E_2) \tag{1}$$

Thus, understanding consists of knowing that the condition to assert a conjunction is the assertability of each of the two conjuncts; as a result, one also knows that each conjunct is assertable whenever the conjunction is (more on this—below). These rules characterize completely the use of conjunction, hence its meaning by the PTS approach.

As a second example, consider (material) implication. Here, the rules introduce another feature—the use of "temporary" assumptions, *discharged* by the rule. This is indicated by square brackets and an indexing of the assumption discharged. The rules are as follows.

$$\frac{\Gamma, [\phi]_i \vdash \psi}{\Gamma \vdash \phi \rightarrow \psi}(\rightarrow I^i) \quad \frac{\Gamma \vdash \phi \rightarrow \psi \quad \Gamma \vdash \phi}{\Gamma \vdash \psi}(\rightarrow E) \tag{2}$$

Again, understanding (material) implication evades considerations of truth-values and truth-tables. Rather, it consists of using the rule of inferring (i.e., introducing) an implication by temporarily assuming its antecedent, and deriving (possibly with the help of additional, auxiliary assumptions) the consequent. As a result, ones get the familiar rule of detachment as the elimination rule, deriving consequences *from* implications.

The presentation of the rules above is in Gentzen's "Logistic"-style *ND*, with shared contexts, single succedent, set antecedent sequents $\Gamma \vdash \phi$. The formulas of the context Γ are known also as *open assumptions*, that could be closed by discharge if the derivation is part of a larger derivation. The name originates from the correspondence between ND and λ-calculus, not further elaborated here. The advantage of this style, keeping track explicitly of undischarged assumptions, can be seen in the formulation of the $(\rightarrow I)$ rule, avoiding the ellipsis in the more common formulation

$$\frac{\begin{array}{c}\phi\\ \vdots\\ \psi\end{array}}{\phi \rightarrow \psi}\,(\rightarrow I)$$

There is one important notion not explained yet, namely *canonicity*. As stated, a warrant for assertion consists of a canonical derivation: a derivation *ending with an application of an I-rule*. To see the role of canonicity, consider the following example derivation.

$$\frac{\Gamma \vdash \alpha \quad \Gamma \vdash (\alpha \rightarrow (\phi \wedge \psi))}{\Gamma \vdash \phi \wedge \psi}\,(\rightarrow E) \tag{3}$$

This is a derivation of a conjunction—but not a canonical one, as it does not end with an application of $(\wedge I)$. Thus, the conjunction here was *not* derived by its meaning! As far as this derivation is concerned, it could mean anything, e.g., disjunction. On the other hand, the following example derivation *is* according to the conjunction meaning, being canonical.

$$\frac{\dfrac{\Gamma \vdash \alpha \quad \Gamma \vdash \alpha \rightarrow \phi}{\Gamma \vdash \phi}\,(\rightarrow E) \quad \dfrac{\Gamma \vdash \beta \quad \Gamma \vdash \beta \rightarrow \psi}{\Gamma \vdash \psi}\,(\rightarrow E)}{\Gamma \vdash \phi \wedge \psi}\,(\wedge I) \tag{4}$$

We end this brief exposition of PTS for logic by considering the universal quantifier. Its standard MTS definition involves a model having a universe of discourse, an arbitrary set of elements. The truth condition for $\forall x.\phi$ is specified by quantifying (in the meta-language) over all *variable assignments* that differ from the variable-assignment at the evaluation point by assigning the variable 'x' all possible elements in the domain, and, recursively, evaluating the truth of ϕ under the resulting assignments. Thus, universal quantification in the object language is defined by universal quantification in the meta-language. In contrast, in PTS the following I-rule characterize the use of universal quantification (we skip here the corresponding E-rule).

$$\frac{\Gamma \vdash \phi}{\Gamma \vdash \forall x.\phi}\,(\forall I)\,\text{provided } x \text{ is fresh for } \Gamma \tag{5}$$

The idea behind this I-rule is that to show $\forall x.\phi$, it suffices to show ϕ from a context in which x is not mentioned—hence is not constrained, is "arbitrary". This again reflects the mathematical practice of assuming "Let x be an arbitrary ..." and then deriving ϕ, claimed to hold universally, for every x. No models, elements or alluding to truth are involved!

2 The natural deduction proof system

In this section, we show how to carry the above mentioned program of PTS to the realm of natural language.

2.1 The NL core fragment E_0^+

We start by considering the core fragment E_0^+ of English, with sentences headed by intransitive and transitive verbs, and noun phrases with a (count) noun[3] and a determiner. In addition, there is the copula. This is a typical fragment of many NLs, syntactically focusing on *subcategorization*, and semantically focusing on *predication* and *quantification*. Some typical sentences are listed[4] below.

(1) every/some girl smiles

(2) every/some girl is a student

(3) every/some girl loves every/some boy

Note the absence of *proper names*, to be added later in the paper, and *negative determiners* like no, not included here (hence the superscript '+' in the names of these positive fragments).

We refer to expressions such as every girl, some boy as *dp*s (determiner-phrases, known also as *np*s, noun-phrases). Every position that can be filled with a *dp* is a *locus of introduction* (of the quantifier corresponding to the determiner of the introduced *dp*). This is a major source of *ambiguity* in E_0^+, known as quantifier-scope ambiguity, treated below.

[3] Currently, only singular (and not plural) nouns are considered.

[4] Throughout, all NL expressions are displayed in a sans-serif font.

We note that this fragment contains only two determiners, ‘every’ and ‘some’, each treated in a sui generis way. Again, this is just setting the stage and illustrating the treatment of determiners (and quantifiers based on them). In a forthcoming paper (Ben-Avi and Francez 2011), we present a *general* treatment of determiners (and *dp*s) in PTS, providing, for example, proof-theoretic characterization of their monotonicity properties, and capturing proof-theoretically their conservativity, traditionally expressed in model-theoretic terms. Also, a deeper study of negative determiners such as ‘no’, is added to the fragment elsewhere.

2.1.1 The Extended Proof-Language L_0^+

The proof system N_0^+ is defined over a language L_0^+, extending[5] E_0^+ and schematizing over it, as well as *disambiguating* its sentences. We use $X, Y, \ldots$ to schematize over nouns[6], P, Q to schematize over intransitive verbs, and R to schematize over transitive verbs. In addition, L_0^+ incorporates a countable set **P** of *individual parameters*, ranged over by meta-variables (in boldface font) like **j, k, r**. Syntactically, parameters are also regarded as *dp*s. For simplicity, we consider is a as a single lexical unit, isa.

Schematic sentences containing occurrences of parameters, referred to as *pseudo-sentences*, only have a role in derivations *within* the proof system; even though grammatically they are *like* NL sentences, they are artifacts of inference, not of assertion. In the sequel, unless otherwise stated, we use ‘sentence’ generically both for sentences and for pseudo-sentences.

We use the meta-variable S to range over (schematic) L_0^+ sentences. For any *dp*-expression D having a quantifier, we use the notation $S[(D)_n]$ to refer to a sentence S having a *designated position* filled by D (avoiding assigning a grammatical function label to that position), where n is the *scope level* (*sl*) of the quantifier in D. In case D has no quantifier (i.e., it is a parameter), $sl = 0$. The higher the *sl*, the higher the scope. For example, $S[(\text{every } X)_1]$ refers to a sentence S with a designated occurrence of every X of the lowest scope. An example of a higher scope is $S[(\text{some } Y)_2]$, having some Y in the higher scope, as in $(\text{every } X)_1$ loves $(\text{some } Y)_2$, representing in L_0^+ the object wide-scope reading of the E_0^+ sentence every X loves some Y. Thus, following Moss (2010), we disambiguate ambiguous sentences taking part in derivations. We use the conventions that within a rule, both $S[D_1], S[D_2]$ refer to the *same* designated position in S, and when the *sl* can be unambiguously determined it is omitted. We use $r(S)$ to indicate the *rank* of S, the highest *sl* on a *dp* within S.

Pseudo-sentences are classified[7] into two groups.

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[5] There is a natural temptation to simplify the syntax of L_0^+, abstracting over some of the baroqueness of the NL syntax. For example, we might express both copular predication and intransitive verb predication in the usual FOL notation $X(\mathbf{j})$. We shall resist this temptation to emphasize the fact that we view the proof-system *directly* applied to E_0^+-sentences.

[6] Here nouns are lexical nouns only; later in the paper the language is augmented with *compound nouns*, also falling under the $X, Y, \ldots$ schematization.

[7] We prepare the ground here for extensions having more than just two positions for *dp*s.

Ground: Ground pseudo-sentences contain[8] *only parameters* in every position that can be filled by a *dp*.[9] Note that for a ground $S, r(S) = 0$.

Non-ground: Non-ground pseudo-sentences contain a *dp* with a determiner in at least one such position (but not in all).

The ground pseudo-sentences play the role of atomic sentences, and their meaning is assumed *given*, externally to the ND proof-system. The latter defines sentential meanings of non-ground pseudo-sentences (and, in particular, E_0^+-sentences), *relative* to the given meanings of ground pseudo-sentences. One way in which the meaning of ground pseudo-sentences might be given is as *substitution instances* for the parameters. For a pseudo-sentences **j** V (where V is some intransitive verb), there is a given set $\{p_{i_0}V, p_{i_1}V, \ldots\}$, where $\{p_{i_0}, p_{i_1} \ldots\} \subseteq P$. Similarly, for a pseudo-sentence **j** R **k** (where R is a transitive verb), a set $\{p_{i_0}Rp_{i_1}, \ldots\}$ is given, where $\{\langle p_{i_0}, p_{i_1}\rangle, \ldots\} \subseteq P^2$. Such substitution instances can be used as premises in N_0^+. This approach is analogous to the use of Herbrand universes in Logic Programming. A way to decide *which* values of the parameters are to be given can be found in Wieckowski (2011), where sub-atomic proof-systems are described.

2.2 BHK-like justification of the rules

Before embarking on the actual presentation of the N_0^+ proof-system itself, we present a justification of the rules, akin to the famous BHK-justification of intuitionistic logic (see, for example, Van Dalen 1986). We mention here that in MTS there is a dependence, not often acknowledged, on the language in which truth-conditions are expressed. If this language is classical, so are the resulting meanings, and similarly for a constructive language. The justification hints at a "constructive flavor" of PTS for NL. For brevity, in this justification we do not distinguish between proof and derivation (from open assumptions), referring to both as 'proof'.

every: *Evidence transforming*: A proof of $S[(\text{every } X)]$ is a function mapping each proof of **j** isa X (for an arbitrary fresh parameter **j**) into a proof of $S[\mathbf{j}]$. For example, a proof of every X P is a function mapping each proof of **j** isa X to a proof of **j** P. A proof of every X R **r** is a function mapping each proof of **j** isa X to a proof of **j** R **r**. Similarly, a proof of **j** R every Y is a function mapping each proof of **r** isa Y to a proof of **j** R **r**.

some: *Evidence combining*: A proof of $S[(\text{some } X)]$ is a pair[10] of proofs, one of **j** isa X and the other of $S[\mathbf{j}]$, for some parameter **j**. For example, a proof of some X P is a pair of proofs, one of **j** isa X, and the other for **j** P. A proof of some X R **r** is a pair of proofs, one of **j** isa X and the other for **j** R **r**.

[8] Note that this use of 'ground' is different from the one in logic programming, where it is used for a term without any (free) variables.

[9] This definition is somewhat refined once compound nouns (with adjectives and/or relative clauses) are considered below.

[10] Strictly speaking, such a proof consists of an ordered *triple*, the first member of which is a parameter, say **j**, and the other two members are the two above-mentioned proofs. As the parameter is trivially retrievable from the pair of proofs in N_0^+, we avoid this extra pedantry.

Similarly, a proof of **j** R some Y is a pair of proofs, one of **k** isa Y and the other for **j** R **k**.

We remark in passing that when properly viewed, the fragment E_0^+ is (up to quantifier scope ambiguity, discussed below) a fragment of FOL; so it is not surprising that the ND system is related to that of FOL. However, our point here is to delineate a methodology of designing a *PTS* for NL, that can then be extended to transcend FOL, partly shown in the sequel.

2.3 The natural deduction proof-system N_0^+

The presentation is again in Gentzen's "Logistic"-style *ND*, with shared contexts, single succedent, set antecedent sequents $\Gamma \vdash S$, formed over contexts of schematic L_0^+ sentences. As is traditional, we enclose *discharged assumptions* in square brackets and index them, using the index to mark the rule-application responsible for the discharge. There are I-rules and E-rules for each determiner forming a *dp*, the latter indexed for its scope level. The ND-rules for the intransitive case are a natural adaptation of those given in Ben-Avi and Francez (2005) (for the *syllogistic fragment*).

The usual notion of (tree-shaped) derivation is assumed. We use $\mathcal{D}$ for derivations, where $\mathcal{D}^{\Gamma \vdash S}$ is a derivation of sentence $S \in L_0^+$ from context Γ. We use Γ, S for the context extending Γ with sentence S. $\mathcal{F}(\Gamma; \mathbf{j})$ means **j** is *fresh* for Γ. In the rule names, we abbreviate 'every' and 'some' to '*e*' and '*s*', respectively. The meta-rules for N_0^+ are presented in Fig. 1. In addition, the structural rule of *contraction*, namely

$$\frac{\Gamma, S', S' \vdash S}{\Gamma, S' \vdash S}\ (C)$$

is assumed, allowing multiple uses of assumptions (example below).

A word of explanation about the I-rules is due. The scope-level $r(S[j])$ is the highest scope of a quantifier already present in $S[j]$. When a new *dp* is introduced into the position currently filled by **j**, it obtains the scope level $r(S[j]) + 1$. Thereby its quantifier becomes the one with the highest scope in the resulting sentence. As for the E-rules, they always eliminate the quantifier with the highest scope. Note that the E-rules are of a format known as *generalized elimination*, relying on drawing *arbitrary consequences* from the major premise. This issue is elaborated upon in Sect. 4.1, and in a more general setting in Francez and Dyckhoff (2010).

The following is a convenient *derived* E-rule, that will be used to shorten derivations.

$$\frac{\Gamma \vdash S[(\mathsf{every}\ X)_{r(S[\mathbf{j}])+1}] \quad \Gamma \vdash \mathbf{j}\ \mathsf{isa}\ X}{\Gamma \vdash S[\mathbf{j}]}\ (e\hat{E})$$

Its derivability is shown by

$$\frac{\Gamma \vdash S[(\mathsf{every}\ X)_{r(S[\mathbf{j}])+1}] \quad \Gamma \vdash \mathbf{j}\ \mathsf{isa}\ X \quad \Gamma, [S[\mathbf{j}]]_i \vdash S[\mathbf{j}]}{\Gamma \vdash S[\mathbf{j}]}\ (eE^i)$$

$$\frac{}{\Gamma, S \vdash S}\ (Ax)$$

$$\frac{\Gamma, [\mathbf{j}\ \textsf{isa}\ X]_i \vdash S[\mathbf{j}]}{\Gamma \vdash S[(\textsf{every}\ X)_{r(S[\mathbf{j}])+1}]}\ (eI^i) \qquad \frac{\Gamma \vdash \mathbf{j}\ \textsf{isa}\ X \quad \Gamma \vdash S[\mathbf{j}]}{\Gamma \vdash S[(\textsf{some}\ X)_{r(S[\mathbf{j}])+1}]}\ (sI)$$

$$\frac{\Gamma \vdash S[(\textsf{every}\ X)_{r(S[\mathbf{j}])+1}] \quad \Gamma \vdash \mathbf{j}\ \textsf{isa}\ X \quad \Gamma, [S[\mathbf{j}]]_i \vdash S'}{\Gamma \vdash S'}\ (eE^i) \quad \frac{\Gamma \vdash S[(\textsf{some}\ X)_{r(S[\mathbf{j}])+1}] \quad \Gamma, [\mathbf{j}\ \textsf{isa}\ X]_j, [S[\mathbf{j}]]_i \vdash S'}{\Gamma \vdash S'}\ (sE^{i,j})$$

where $\mathcal{F}(\Gamma, S[\textsf{every}\ X]; \mathbf{j})$ in (eI), and $\mathcal{F}(\Gamma, S[\textsf{some}\ X], S';\ \mathbf{j})$ for (sE).

Fig. 1 The meta-rules for N_0^+

That is, we take the arbitrary consequence to be $S[\mathbf{j}]$ itself.

Lemma (*weakening*[11]) If $\Gamma \vdash S$, then $\Gamma, \Gamma' \vdash S$.

Below is an example[12] derivation establishing

$$\textsf{some}\ U\ \textsf{isa}\ X,\ (\textsf{every}\ X)_2\ R\ (\textsf{some}\ Y)_1,\ \textsf{every}\ Y\ \textsf{isa}\ Z\ \vdash\ (\textsf{some}\ U)_1\ R\ (\textsf{some}\ Z)_2$$

The derivation is

$$\frac{\dfrac{\textsf{some}\ U\ \textsf{isa}\ X \quad \dfrac{[\mathbf{k}\ \textsf{isa}\ U]_1 \quad \dfrac{(\textsf{every}\ X)_2\ R\ (\textsf{some}\ Y)_1 \quad [\mathbf{k}\ \textsf{isa}\ X]_2}{\mathbf{k}\ R\ \textsf{some}\ Y}\ (e\hat{E})}{(\textsf{some}\ U)_2\ R\ (\textsf{some}\ Y)_1}\ (sI)}{(\textsf{some}\ U)_2\ R\ (\textsf{some}\ Y)_1}\ (sE^{1,2}) \qquad \dfrac{[\textsf{some}\ U\ R\ \mathbf{j}]_3 \quad \dfrac{\textsf{every}\ Y\ \textsf{isa}\ Z \quad [\mathbf{j}\ \textsf{isa}\ Y]_4}{\mathbf{j}\ \textsf{isa}\ Z}\ (e\hat{E})}{(\textsf{some}\ U)_1\ R\ (\textsf{some}\ Z)_2}\ (sI)}{(\textsf{some}\ U)_1\ R\ (\textsf{some}\ Z)_2}\ (sE^{3,4})$$

Without the side-condition $\mathcal{F}(\Gamma, S[(\textsf{every}\ X)];\ \mathbf{j})$ on (eI), the following unwarranted derivations would be available (ignoring scope).

$$\frac{[\mathbf{j}\ \textsf{isa}\ X]_1 \quad \dfrac{\mathbf{j}\ \textsf{isa}\ Y \quad \textsf{every}\ Y\ \textsf{smiles}}{\mathbf{j}\ \textsf{smiles}}\ (e\hat{E})}{\textsf{every}\ X\ \textsf{smiles}}\ (eI^1) \qquad \frac{\dfrac{\mathbf{j}\ R\ \mathbf{j} \quad [\mathbf{j}\ \textsf{isa}\ X]_1}{(\textsf{every}\ X)_1\ R\ \mathbf{j}}\ (eI^1) \quad [\mathbf{j}\ \textsf{isa}\ X]_2}{(\textsf{every}\ X)_1\ R\ (\textsf{every}\ X)_2}\ (eI^2)$$

To see the need for the contraction, consider the following: $\mathbf{j}\ \textsf{isa}\ X,\ \ \textsf{every}\ X\ \textsf{isa}\ Y,\ \textsf{every}\ X\ \textsf{isa}\ Z\ \vdash\ \textsf{some}\ Y\ \textsf{isa}\ Z$. The assumption $\mathbf{j}\ \textsf{isa} X$ has to be used twice, to eliminate both occurrences of every.

$$\frac{\dfrac{\mathbf{j}\ \textsf{isa}\ X \quad \textsf{every}\ X\ \textsf{isa}\ Y}{\mathbf{j}\ \textsf{isa}\ Y}\ (e\hat{E}) \quad \dfrac{\mathbf{j}\ \textsf{isa}\ X \quad \textsf{every}\ X\ \textsf{isa}\ Z}{\mathbf{j}\ \textsf{isa}\ Z}\ (e\hat{E})}{\textsf{some}\ Y\ \textsf{isa}\ Z}\ (sI)$$

In PTS for logic, there is a notion of a *canonical proof*, namely a proof the last step of which is an application of an I-rule. In systems where proof-normalization obtains, every proof, i.e., *closed* derivation (with no open assumptions) can be

[11] Weakening is not really needed, and is introduced here for technical reasons, having an easier proof of the termination of the proof-search in the Sequent-Calculus (see below). Ultimately, there might be a need to remove it.

[12] As is common in ND-presentation, in actual examples we suppress the Γ, using only the succedent, to save space; note that Γ is easily recoverable in such small examples.

reduced to a canonical proof (of the same conclusion). Here, in NL-PTS, we are mainly interested in *open* derivations, having open assumptions. We extend the notion of canonicity to open derivations, and take (see Sect. 3) them to contribute to sentential meanings. However, not every open derivation can be reduced to a canonical one (ending with an application of an I-rule). Following is an example of such a derivation, establishing some X isa $Y \vdash$ some Y isa X.

$$\dfrac{\text{some } X \text{ isa } Y \quad \dfrac{[\mathbf{j} \text{ isa } Y]_1 \quad [\mathbf{j} \text{ isa } X]_2}{\text{some } Y \text{ isa } X}\,(sI)}{\text{some } Y \text{ isa } X}\,(sE^{1,2})$$

This derivation, ending with an application of the E-rule (sE), is not canonical. It has *no* canonical counterpart. An alleged canonical counterpart might be

$$\dfrac{\dfrac{\text{some } X \text{ isa } Y \quad \dfrac{[\mathbf{j} \text{ isa } X]_1 \quad [\mathbf{j} \text{ isa } Y]_2}{\mathbf{j} \text{ isa } Y}}{\mathbf{j} \text{ isa } Y}\,(sE^{1,2}) \quad \dfrac{\text{some } X \text{ isa } Y \quad \dfrac{[\mathbf{j} \text{ isa } X]_1 \; [\mathbf{j} \text{ isa } Y]_2}{\mathbf{j} \text{ isa } X}}{\mathbf{j} \text{ isa } X}\,(sE^{1,2})}{\text{some } Y \text{ isa } X}\,(sI)$$

However, this derivation violates the freshness condition on **j** for the conclusion of (sE).

We use $\vdash^c$ for canonical derivability, and $\mathcal{D}^{\Gamma \vdash^c S}$ for a canonical derivation of S from (open) assumptions Γ. Furthermore, $[\![S]\!]^c_\Gamma$ denotes the (possibly empty) collection of all canonical derivations of S from Γ.

3 The sentential proof-theoretic meaning

In the discussions of PTS in logic, it is usually stated that '*the ND-rules* determine *the meanings (of the connectives/quantifiers)*'. However, there is no *explicit* denotational meaning[13] defined (*proof-theoretic*, not *model-theoretic*, denotation). In other words, there is no explicit definition of the result of this determination. Thus, one cannot express claims of the form 'the meaning of S has this or that property', or generalizations about all meanings, involving quantification over meanings. In particular, if one wants to apply Frege's context principle to those PTS-meanings, and derive meanings for subsentential phrases (including lexical words) as *contributions to sentential meanings*, such an explication is needed (see Francez and Ben-Avi 2011 and Francez et al. 2010).

We take here the PTS-meaning of an E_0^+ sentence S, and also of an L_0^+ non-ground pseudo-sentence S, to be the function from contexts Γ returning the collection of all the canonical derivations in N_0^+ of S from Γ. Recall that for a ground L_0^+ pseudo-sentence S, its meaning is assumed *given*, and the meaning of E_0^+ sentences, as well as non-ground L_0^+ pseudo-sentences, is defined *relative to the given meanings of ground sentences*.

In accordance with many views in the philosophy of language, every derivation in the meaning of a sentence S can be viewed as providing $G[\![S]\!]$, *grounds of asserting* S (recall that ground pseudo-sentences are not used for making any

[13] Known also as the *semantic value*.

assertion, as they are not part of the natural language, only of the extension to a language for defining meanings by derivations). Semantic equivalence of sentences is based on equality of meaning (and *not* interderivability). In addition, a weaker semantic equivalence is based on equality of grounds of assertion.

Definition *(PTS-meaning, equivalence, grounds):*

1. For a sentence S, or a non-ground pseudo-sentence S, in L_0^+:

$$[\![S]\!]^{PTS}_{L_0^+} =^{df.} \lambda\Gamma.[\![S]\!]^c_\Gamma \quad G[\![S]\!] =^{df.} \{\Gamma | \Gamma \vdash^c S\}$$

where:

(a) For S a sentence in E_0^+, Γ consists of E_0^+-sentences only. Parameters are not "observable" in grounds for assertion.

(b) For S a pseudo-sentence in L_0^+, Γ may also contain pseudo-sentences with parameters.

Recall again that the meanings of ground sentences *is given* (possibly extra-linguistically), and meaning for E_0^+ is defined *relative* to those ground pseudo-sentence meanings.

2. For S_1, S_2 in L_0^+,

(a) $S_1 \equiv_m S_2$ iff $[\![S_1]\!]^{PTS}_{L_0^+} = [\![S_2]\!]^{PTS}_{L_0^+}$.

(b) $S_1 \equiv_g S_2$ iff $G^{PTS}_{L_0^+}[\![S_1]\!] = G^{PTS}_{L_0^+}[\![S_2]\!]$.

When it is clear which language is meant, the subscript L_0^+ will be omitted, as well as the *PTS* superscript.

We do not dwell further here on the induced equivalences. As for the grounds of assertion, a member $\Gamma \in G[\![S]\!]$ can be seen, in Dummett's terms, as a *warrant* for the assertion of S (by a speaker). Being in possession of Γ, and of a canonical derivation of S from Γ, are a justification of the proper assertion of S. There are various ways of viewing "possession" of Γ. It may reflect the knowledge (or belief) of the speaker, or some non-linguistic (e.g., visual) observation.

The main formal property of meanings (under this definition) is the overcoming (at least for the fragment considered) of the manifestation argument against MTS: asserting a sentence S is based on (algorithmically) decidable grounds (see Sect. 4.3). A speaker in possession of Γ *can decide* whether $\Gamma \vdash^c S$. Some properties of meanings of specific sentences are discussed below (in particular, see Sect. 3.1), showing that the proposed rules of N_0^+ do fit our pre-theoretic concept of the use of the E_0^+ sentences. Clearly, further research is needed to determine the limits (in terms of the size of the fragment captured) of this effective approach to meaning. Additional properties of PTS-meanings, and some additional problems of MTS-meanings not arising in the proof-theoretic setup, are discussed below.

3.1 A note on quantification in NL: the meaning of every and of nouns

There is an important consequence of the every introduction (meta-) rule regarding a difference between PTS (as proposed here) and (current proposals for) MTS.

In standard MTS, the meanings of the two complements of every (one a noun, the other a verb-phrase) have *the same semantic type*; namely, they are both predicates (of type (e, t)), having arbitrary subsets of the domain of individual elements as their extensions in Henkin models. In the truth-conditions assigned to $S[(\text{every } X)]$, both complements have similar roles. Thus, if $\forall x.P(x) \Longrightarrow Q(x)$ is semantically well-formed, so is $\forall x.Q(x) \Longrightarrow P(x)$. This does not predict the asymmetry of the roles of the nominal predicate and the verbal predicate (noted already in Barwise and Cooper 1981), and the *semantic* anomaly of (*) every smiles girl, or (*) every loves Rachel smiles, leaving them to be ruled out by the syntax.

On the other hand, our (eI) I-rule[14] is *asymmetric* w.r.t. the types of its two arguments[15]. The discharged assumption is that of a nominal predicate $(\text{j isa } X)$, and its role in the rule *cannot* be filled by a predicate originating in an intransitive verb (or a verb-phrase)! Thus the semantic abnormality of (*) every smiles girl is predicted by the rule. So, our PTS makes finer type-distinctions among predicates, according to their semantic contribution to sentential meaning. This does not imply that no finer MTS can be proposed, incorporating such a distinction; however, it is not obvious on what to base such a refinement within a model-theoretic denotation-based type-system.

The role of a noun X in $S[(\text{every } X)]$ is *not* that of a predicate. Rather, it *determines (or specifies) the domain of quantification*! The notion of a universal domain of quantification, consisting of "property-less" objects, may well be an artifact of MTS (following the use of FOL in mathematics), not really needed[16] for conferring meaning on natural language sentences with localized quantification. In that, the view here of universal quantification in NL, as *evidence-transforming*, is different *both* from the model-theoretic view as ranging over some external domain of objects, and from the PTS view in logic, by means of ranging over all substitution instances.

We mention in passing that in Hebrew, for example, since present tense forms of verbs are analogous to nouns (or adjectives), the analogue of (*) every smiles girl *is* semantically well-formed[17]; thus, different rules should be used for a PTS of a similar fragment of Hebrew. This fact also demonstrates that the Montagovian semantic type system is too coarse.

3.2 Interlude: semantic ambiguity

In order to better understand the PTS of E_0^+, consider one of its well-known features: *quantifier scope ambiguity*. The following E_0^+ sentences are usually attributed to two readings each, with the following FOL expressions of their respective truth-conditions in model-theoretic semantics.

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[14] The (sI) rule is also, in a less apparent way, asymmetric.

[15] This is made more explicit in Francez et al. (2010) and Ben-Avi and Francez (2011), where the lexical meanings of determiners are discussed, together with a proof-theoretic interpretation of semantic types.

[16] Even when a *dp* like 'everything' is considered, unrestricted quantification is not used, as languages provide nouns such as 'thing', whose extension in MTS should be the whole domain of quantification.

[17] As noted by a referee of this journal, the proper rendering every smiles girl into English is by means of a relative clause, e.g., every thing that smiles is a girl., or by every smiler is a girl.

(4) Every girl loves some boy

(5) Some girl loves every boy

Consider sentence (4).

Subject wide-scope (sws): $\forall x.\mathbf{girl}(x) \rightarrow \exists y.\mathbf{boy}(y) \wedge \mathbf{love}(x, y)$
Subject narrow-scope (sns): $\exists y.\mathbf{boy}(y) \wedge \forall x.\mathbf{girl}(x) \rightarrow \mathbf{love}(x, y)$

In our PTS, the difference in meanings reflects itself by the two readings having *different grounds of assertion*. This is manifested in derivations by different *order of introduction* of the subject and object *dp*s.

Subject wide-scope (sws):

$$\cfrac{\cfrac{\cfrac{\cfrac{[\mathbf{r}\ \text{isa girl}]_i}{\mathcal{D}_1}}{\mathbf{r}\ \text{loves}\ \mathbf{j}} \quad \cfrac{\mathcal{D}_2}{\mathbf{j}\ \text{isa boy}}}{\mathbf{r}\ \text{loves (some boy)}_1}(sI)}{\text{(every girl)}_2\ \text{loves (some boy)}_1}(eI^i)$$

Subject narrow-scope (sns):

$$\cfrac{\cfrac{\cfrac{\cfrac{[\mathbf{r}\ \text{isa girl}]_i}{\mathcal{D}_1}}{\mathbf{r}\ \text{loves}\ \mathbf{j}}}{\text{(every girl)}_1\ \text{loves}\ \mathbf{j}}(eI^i) \quad \cfrac{\mathcal{D}_2}{\mathbf{j}\ \text{isa boy}}}{\text{(every girl)}_1\ \text{loves (some boy)}_2}(sI)$$

Note that there is no way to introduce a *dp* with a narrow-scope where the *dp* with the wider-scope has already been introduced. In the N_0^+ calculus, only disambiguated sentences participate.

This way of capturing the source of quantifier-scope ambiguity, by means of order of application of I-rule, is a major element in the PTS tool-box. We shall encounter another use of this tool below, in the treatment of opaque transitive verbs. Its usefulness is a direct result of the way I-rules take part in conferring meaning. Obviously, this tool is unavailable to MTS.

The central pre-theoretic relationship between the two readings is the entailment[18] present here in the form

$$\text{(every girl)}_1\ \text{loves (some boy)}_2 \vdash \text{(every girl)}_2\ \text{loves (some boy)}_1$$

as shown by the following derivation.

$$\cfrac{\cfrac{\cfrac{[\mathbf{j}\ \text{isa}\ X]_1 \quad [\text{(every}\ X)_1\ R\ \mathbf{r}]_2}{\mathbf{j}\ R\ \mathbf{r}}(e\hat{E}) \quad [\mathbf{r}\ \text{isa}\ Y]_3}{\mathbf{j}\ R\ \text{(some}\ Y)_1}(sI) \quad \text{(every}\ X)_1\ R\ \text{(some}\ Y)_2}{\cfrac{\mathbf{j}\ R\ \text{(some}\ Y)_1}{\text{(every}\ X)_2\ R\ \text{(some}\ Y)_1}(eI^1)}(sE^{2,3})$$

[18] A more general treatment of truth and entailment among sentences is deferred to Ben-Avi and Francez (2011), where truth under Γ is captured as non-emptiness of the grounds for assertion (for any given Γ).

Of course, in the other direction

$$(\mathsf{every}\ X)_2\ R\ (\mathsf{some}\ Y)_1 \nvdash (\mathsf{every}\ X)_1\ R\ (\mathsf{some}\ Y)_2$$

Clearly, no such proviso of explicit quantifier scope indication is needed in case both subject and object have *the same determiner* (either both are ‘every’ or both are ‘some’), as the two reading in this case are (weakly) equivalent. This can be seen as follows. Assume $\Gamma \vdash^c (\mathsf{every}\ X)_2\ R\ (\mathsf{every}\ Y)_1$. The derivation has the form

$$\dfrac{\dfrac{\Gamma, [\mathbf{j}\ \mathsf{isa}\ X]_1, [\mathbf{r}\ \mathsf{isa}\ Y]_2 \vdash\ \mathbf{j}\ R\ \mathbf{r}}{\Gamma, [\mathbf{j}\ \mathsf{isa}\ X]_1 \vdash \mathbf{j}\ R\ (\mathsf{every}\ Y)_1}(eI^2)}{\Gamma \vdash (\mathsf{every}\ X)_2\ R\ (\mathsf{every}\ Y)_1}(eI^1)$$

Therefore, the following is an available derivation for $\Gamma \vdash^c (\mathsf{every}\ X)_1\ R\ (\mathsf{every}\ Y)_2$.

$$\dfrac{\dfrac{\Gamma, [\mathbf{j}\ \mathsf{isa}\ X]_1, [\mathbf{r}\ \mathsf{isa}\ Y]_2 \vdash\ \mathbf{j}\ R\ \mathbf{r}}{\Gamma, [\mathbf{r}\ \mathsf{isa}\ Y]_2 \vdash (\mathsf{every}\ X)_1 R\ \mathbf{r}}(eI^1)}{\Gamma \vdash (\mathsf{every}\ X)_1\ R\ (\mathsf{every}\ Y)_2}(eI^2)$$

The other direction, as well as the some—some case, are shown similarly.

We mention in passing that for the proof-theoretic treatment of transitive verbs in Nishihara et al. (1990), the whole issue of characterizing ambiguity proof-theoretically is avoided by positing that existential *dp*s *always* have wide scope.

3.3 Further properties of sentential meanings

In this section we delineate several additional properties of proof-theoretic sentential meanings, in addition to the characterization of universally quantified sentences, and to the novel identification of the origin of quantifier scope ambiguity, both discussed above.

Unity of the proposition: This problem, which has its origin in antiquity (see http://en.wikipedia.org/wiki/Unity-of-the-proposition for a description and further references) can be stated as follows:

(q) *what distinguishes a sentence from a mere list of words?*

In model-theoretic semantics, when the question is posed, say, regarding Mary smiles, or Mary loves John, the question is what “glues together” the meanings of the words, denotations of certain kinds, to produce a truth-value. The words Mary, loves and John are stipulated by MTS to have the denotations they have independently of any state-of-affairs (fact).

According to the PTS view, the answer to (q) is: sentences, in contrast to lists of words, *have proofs (derivations from other sentences)*! Sentences do not derive their meanings from meanings of the words of which they consist—rather, from their canonical derivations in the ND-system of rules. It is words that have meanings derived from their contribution to sentences in which they occur. This is further elaborated upon, including the technicalities, in Francez et al.(2010). Those canonical derivations form the “glue” endowing the proposition its unity of meaning. This view answers also the question what distinguishes any two true sentences: having different meanings, their truth is established on different grounds. This observation may prove important to the study of intensionality.

No logical form: By this way of defining sentential meanings, we do not allude to any “logical form” of the sentence, differing from its surface form.

4 Properties of $\mathbf{N}_0^+$

4.1 Harmony

As mentioned already, the origin of *PTS* for logic is in the work of Gentzen (1935), who invented the *natural deduction* proof-system for *FOL*. He hinted there that I-rules could be seen as the *definition* of the logical constant serving as the main connective, while the E-rules are nothing more than *consequences of this definition*. This was later refined into the *Inversion Principle* by Prawitz (1965), which shows how the I-rules determine the E-rules. The I-rules were taken as a determination of *the meaning* of the logical constant under consideration, instead of the model-theoretic interpretation, that appeals to *truth in a model* (e.g., the well-known truth-tables for the propositional logic case).

However, in view of Prior’s (1960) attack on this approach, by presenting a connective ‘*tonk*’, whose I-rule was that of a disjunction, while its E-rule was that of conjunction, trivializing the whole deductive theory by rendering every two propositions inter-derivable, it became apparent that not every combination of ND-rules can serve as a basis for *PTS*.

The notion of *harmony* of the ND-rules (Dummett 1991), taken in a broad sense to express a certain balance between E-rules and I-rules (absent from the *tonk* rules) became a serious contender for an appropriateness condition for ND-rules to serve as a basis for a *PTS*. See Read (2000, 2008) for a critical discussion of *tonk*’s disharmony.

We consider the following two harmony notions, and show that N_0^+ satisfies both.

General-Elimination (GE) harmony: By this approach, in order to be harmonious, an E-rule has to have some *specific form*, depending on the corresponding I-rules. This form is known as *generalized E-rules*, and was considered by von Plato (2001) as having a better relationship to cut-free sequent-calculus derivations. Such an E-rule allows drawing an *arbitrary conclusion*, provided it is derivable from the premisses of the corresponding I-rule(s).

This form guarantees that the inversion-principle obtains, and leads to the availability of *proof-reduction*, the elimination of a detour caused by an introduction immediately followed by an elimination. This underlies *proof normalization*, and also constitutes a requirement of intrinsic harmony (see below). Proof-normalization (in its strong version) requires that there is no possibility of an infinite sequence of such reductions (see Restall (2010) for a general discussion of the role of normalization in PTS). In Francez and

Dyckhoff (2010), we show that a rule-form generalizing a proposal by Read (2008) guarantees the availability of the required reduction. *All the E-rules in* N_0^+ *are of this generalized-elimination form, hence* N_0^+ *is GE-harmonious.*

Local Intrinsic harmony: Here, in order to be harmonious, no constraints on the form of the E-rules are imposed, but they have to stand in a certain relationship to the I-rules, to directly reflect the required balance among them. We consider here a specific proposal by Pfenning and Davies (2001), based on two properties known as *local soundness* and *local completeness*.

- *Local Soundness*: Every introduction followed *directly* by an elimination can be reduced. This shows that the elimination-rules are not *too strong* w.r.t. the I-rules.
- *Local Completeness*: There is a way to eliminate and to reintroduce, recovering the original formula. This process is called *expansion*. Since 'reintroduce' might allude to *ordering* of the applications of I-rules/E-rules within the reconstructed derivation, we prefer the following order-neutral formulation:
  Every derivation of a formula ϕ with principal operator δ *can be* expanded to one containing an application of an E-rule of δ, and applications of all I-rules of δ each with conclusion ϕ.
  This shows that the E-rules are not *too weak* w.r.t. the I-rules.

In the case of logic, introduction and elimination are of a top-level operator. Here, they refer to the introduction of a *dp* into every allowable position (and any scope level), and elimination from the same position.

We show local intrinsic harmony (in the above sense) for N_0^+, even though Francez and Dyckhoff (2010) shows this follows from the form of the rules. We do, however, omit showing the reductions/expansions for the extensions of the fragment presented below.

every: This is a natural adaptation of the analogous properties of the syllogistic fragment of Ben-Avi and Francez (2005).

Local soundness:

$$\dfrac{\dfrac{\dfrac{\dfrac{[\mathbf{j}\ \mathsf{isa}\ X]_i}{\mathcal{D}_1}}{S[\mathbf{j}]}}{S[(\mathsf{every}\ X)]}(eI^i) \quad \begin{matrix}\mathcal{D}_2\\ \mathbf{k}\ \mathsf{isa}\ X\end{matrix} \quad \dfrac{\dfrac{[S[\mathbf{k}]]_j}{\mathcal{D}_3}}{S'}}{S'}(eE^j) \quad \leadsto_e \quad \begin{matrix}\mathcal{D}_2\\ \mathcal{D}_1[\mathbf{k}\ \mathsf{isa}\ X/\mathbf{j}\ \mathsf{isa}\ X, \mathbf{k}/\mathbf{j}]\\ S[\mathbf{k}]\\ \mathcal{D}_3\\ S'\end{matrix}$$

Here $\begin{matrix}\mathcal{D}_2\\ \mathcal{D}_1[\mathbf{k}\ \mathsf{isa}\ X/\mathbf{j}\ \mathsf{isa}\ X, \mathbf{k}/\mathbf{j}]\end{matrix}$ denotes a derivation in which every instance of use of the assumption $\mathbf{j}$ isa X is replaced by the derivation $\begin{matrix}\mathcal{D}_2\\ \mathbf{k}\ \mathsf{isa}\ X\end{matrix}$ of its variant $\mathbf{k}$ isa X. Since $\mathbf{j}$ is fresh for the assumptions on which $\mathcal{D}_1$ depends, the replacement of $\mathbf{j}$ by $\mathbf{k}$ is permissible.

Local completeness:

$$\begin{array}{c}\mathcal{D}\\ S[(\textsf{every X})]\end{array} \rightsquigarrow_e \dfrac{\dfrac{\begin{array}{c}\mathcal{D}\\ S[(\textsf{every X})]\end{array} \quad [\mathbf{j}\ \textsf{isa X}]_1 \quad [S[\mathbf{j}]]_2}{S[\mathbf{j}]}(eE^2)}{S[(\textsf{every}\ X)]}(eI^1)$$

some: *Local soundness*:

$$\dfrac{\dfrac{\begin{array}{cc}\mathcal{D}_1 & \mathcal{D}_2\\ \mathbf{j}\ \textsf{isa}\ X & S[\mathbf{j}]\end{array}}{S[(\textsf{some}\ X)]}(sI) \quad \begin{array}{c}[\mathbf{k}\ \textsf{isa}\ X]_1 \quad [S[\mathbf{k}]]_2\\ \mathcal{D}_3\\ S'\end{array}}{S'}(sE^{1,2}) \rightsquigarrow_r \begin{array}{c}\mathcal{D}_1 \quad \mathcal{D}_2\\ \mathbf{j}\ \textsf{isa}\ X \quad S[\mathbf{j}]\\ \mathcal{D}_3[\mathbf{j}/\mathbf{k}]\\ S'\end{array}$$

Again, a fresh **k** has been replaced.

Local completeness:

$$\begin{array}{c}\mathcal{D}\\ S[(\textsf{some X})]\end{array} \rightsquigarrow_e \dfrac{\begin{array}{c}\mathcal{D}\\ S[(\textsf{some X})]\end{array} \quad \dfrac{[\mathbf{j}\ \textsf{isa}\ X]_1 \quad [S[\mathbf{j}]]_2}{S[(\textsf{some}\ X)]}(sI)}{S[(\textsf{some}\ X)]}(sE^{1,2})$$

There are also other views of harmony, e.g., based on a *conservative extension* of the theory of the introduced operator Belnap (1962).

4.2 Closed derivations

A derivation of $\Gamma \vdash S$ is *closed* iff $\Gamma = \emptyset$. In logic, as already mentioned above, closed derivations are a central topic, determining the *(formal) theorems* of the logic. In particular, for bivalent logics, they induce the (syntactic) notions *tautology* and *contradiction*.

In L_0^+, in the absence of negation and *negative determiners* (like *no*), there is no natural notion of a contradiction. Furthermore, the only "positive" closed derivation in N_0^+ is for sentences of the form every X isa X. The closed derivation is shown below.

$$\dfrac{[\mathbf{j}\ \textsf{isa}\ X]_1 \vdash \mathbf{j}\ \textsf{isa}\ X}{\vdash \textsf{every}\ X\ \textsf{isa}\ X}(eI^1)$$

In particular, note that $\nvdash$some X isa X.

4.3 Decidability of N_0^+ derivability

We now attend to the issue of *decidability* of derivability in N_0^+. The positive result provided here makes *PTS*-based meaning effective for L_0^+. Figure 2 displays a sequent-calculus SC_0^+ for L_0^+, easily shown equivalent to N_0^+ (in having the same provable sequents). The rules are arranged in the usual way of *L*-rules (introduction

$$\frac{}{\Gamma, S \vdash S}\ (ID)$$

$$\frac{\Gamma, \mathbf{j}\ \mathsf{isa}\ X, S[(\mathsf{every}\ X)_{r(S[\mathbf{j}])+1}], S[\mathbf{j}] \vdash S'}{\Gamma, \mathbf{j}\ \mathsf{isa}\ X, S[(\mathsf{every}\ X)_{r(S[\mathbf{j}])+1}] \vdash S'}\ (Le) \qquad \frac{\Gamma, \mathbf{j}\ \mathsf{isa}\ X \vdash S[\mathbf{j}]}{\Gamma \vdash S[(\mathsf{every}\ X)_{r(S[\mathbf{j}])+1}]}\ (Re)$$

$$\frac{\Gamma, \mathbf{j}\ \mathsf{isa}\ X, S[\mathbf{j}] \vdash S'}{\Gamma, S[(\mathsf{some}\ X)_{r(S[\mathbf{j}])+1}] \vdash S'}\ (Ls) \qquad \frac{\Gamma \vdash \mathbf{j}\ \mathsf{isa}\ X \quad \Gamma \vdash S[\mathbf{j}]}{\Gamma \vdash S[(\mathsf{some}\ X)_{r(S[\mathbf{j}])+1}]}\ (Rs)$$

where $\mathbf{j}$ is fresh in Re and Ls.

Fig. 2 A sequent-calculus SC_0^+ for L_0^+

in the antecedent) and *R*-rules (introduction in the succedent). The following claims are routinely established for SC_0^+.

- The structural rules of *weakening* (W) and *contraction* (C) are admissible.
- (Cut) is admissible.

The existence of a terminating proof-search procedure follows. The essence of the proof is as follows. First, observe that for all rules except (Le), the premise is simpler than the conclusion. Secondly, for (Le), even though $S[(\mathsf{every}\ X)_{r(S[\mathbf{j}])+1}]$ is retained in the premiss (causing non-simplification), the rule is applicable only with $\mathbf{j}$ isa X already in the context Γ. So, this rule is applicable only finitely often, as Γ is finite, and every rule that may contribute $\mathbf{j}$ isa X to the context is itself only finitely often applicable.

5 Extending the fragment

In this section, we consider some simple extensions of E_0^+ (and the induced extension of L_0^+). The first one adds[19] proper names, and the other two are related to extending the notion of noun. In E_0^+, we had only primitive nouns. We now consider two forms of *compound noun*: one formed by adding *(intersective) adjectives* and the other by adding *relative clauses*. In both cases, in the corresponding extensions of N_0^+, we let X, Y schematize over compound nouns also in the original rules. Note that pseudo-sentences including adjectives and/or relative clauses, even if they only have parameters in *dp* positions, do not count anymore as ground, since they are derived via I-rules. For example, **j** isa beautiful girl and **j** loves **k** who smiles are not ground.

[19] This is different from the role of names in Moss (2010); the names there are our parameters, while Moss (2010) has no proper names provided by the NL fragment itself.

5.1 Adding proper names

In this section, we extend E_0^+ with *proper names* occurring in *dp* positions. Typical sentences are:

(6) Rachel is a girl
(7) Rachel smiles
(8) Rachel loves every/some boy
(9) every boy loves Rachel

Proper names are strictly distinct from parameters in the way they function in the proof-system, as explained below. We retain the name E_0^+ for this (minor) extension. In L_0^+, let proper names be schematized by *N*, and add pseudo-sentences of the forms

(10) $\mathbf{j}$ is N, N is $\mathbf{j}$
(11) $\mathbf{j}$ is $\mathbf{k}$, N is M

Note that pseudo-sentences having a proper name in any *dp*-position *are not ground!*

First, we add I-rules and E-rules for is (a disguised identity). We adopt a version of the rules in Read (2004).

$$\frac{\Gamma, [S[\mathbf{j}]]_1 \vdash S[\mathbf{k}]}{\Gamma \vdash \mathbf{j} \text{ is } \mathbf{k}}\,(isI^1) \quad \frac{\Gamma \vdash \mathbf{j} \text{ is } \mathbf{k} \quad \Gamma \vdash S[\mathbf{j}]\ \Gamma, [S[\mathbf{k}]]_1 \vdash S'}{S'}\,(isE^1)$$

where S does not occur in Γ.

From these, we can derive rules for reflexivity $(is-refl)$, symmetry $(is-sym)$ and transitivity $(is-tr)$. For shortening the presentation of derivations, combinations of these rules are still referred to as applications of (isE).

Next, we incorporate I-rules and E-rules of proper names into *dp*-positions, letting names function similarly as determiner-headed *dp*s, fitting their model-theoretic semantic view as generalized quantifiers (here viewed proof-theoretically). In the *MTS*, where an intransitive verb has the predicate type (e, t), a proper name has the *GQ*-type $((e, t), t)$. So, it is not the meaning of the verb applied to the meaning of a proper name; rather, the meaning of a proper name is applied to the meaning of the verb.

$$\frac{\Gamma \vdash \mathbf{j} \text{ is } N \quad \Gamma \vdash S[\mathbf{j}]}{\Gamma \vdash S[N]}\,(nI) \quad \frac{\Gamma \vdash S[N]\ \Gamma, [\mathbf{j} \text{ is } N]_1, [S[\mathbf{j}]]_2 \vdash S'}{\Gamma \vdash S'}\,(nE^{1,2}), \quad \mathbf{j} \text{ fresh for } \Gamma$$

Below are two example derivations.

Rachel isa girl, every girl smiles $\vdash$ Rachel smiles: Note that Rachel is not a parameter, and $(e\hat{E})$ is not *directly* applicable.

$$\cfrac{\text{Rachel isa girl} \quad \cfrac{[\mathbf{r} \text{ is Rachel}]_1 \quad \cfrac{[\mathbf{r} \text{ isa girl}]_2 \quad \text{every girl smiles}}{\mathbf{r} \text{ smiles}}\,(e\hat{E})}{\text{Rachel smiles}}\,(nI)}{\text{Rachel smiles}}\,(nE^{1,2})$$

Rachel isa girl, Rachel smiles $\vdash$ some girl smiles: Again, since Rachel is not a parameter, (sI) is not *directly* applicable.

$$\dfrac{\text{Rachel isa girl} \quad \dfrac{\text{Rachel smiles} \quad \dfrac{\dfrac{\dfrac{[\mathbf{r}_1 \text{ is Rachel}]_1 \quad [\mathbf{r}_2 \text{ is Rachel}]_3}{\mathbf{r}_1 \text{ is } \mathbf{r}_2}(isE) \quad [\mathbf{r}_1 \text{ isa girl}]_2}{\mathbf{r}_2 \text{ isa girl}}(isE) \quad [\mathbf{r}_2 \text{ smiles}]_4}{\text{some girl smiles}}(sI)}{\text{some girl smiles}}(nE^{3,4})}{\text{some girl smiles}}(nE^{1,2})$$

The corresponding extension to the sequent calculus SC_0^+ consists of the following rules.

$$\frac{\Gamma, \mathbf{j}\ is\ N, S[\mathbf{j}] \vdash S'}{\Gamma, S[N] \vdash S'}(Ln) \quad \frac{\Gamma \vdash \mathbf{j}\ is\ N \quad \Gamma \vdash S[\mathbf{j}]}{\Gamma \vdash S[N]}(Rn)$$

5.2 Adding adjectives

We augment E_0^+ with sentences containing *adjectives*, schematized by A. We consider here only what is known in model-theoretic semantics as *intersective adjectives*. Typical sentences are:

(12) Rachel is a beautiful girl/clever beautiful girl/clever beautiful red-headed girl

(13) Rachel/every girl/some/girl is beautiful

(14) Rachel/every beautiful girl/some beautiful girl smiles

(15) Rachel/every beautiful girl/some beautiful girl loves Jacob/every clever boy/ some clever boy

A noun preceded by an adjective is again a (compound) noun (the syntax is treated more precisely once the grammar is presented, as in Francez et al.(2010)). Denote this extension by $E_{0,adj}^+$. Recall that in the N_0^+ rules, the noun schematization should be taken over compound nouns too. Note that $E_{0,adj}^+$ is no longer finite, as an unbounded number of adjectives may precede a noun.

We augment N_0^+ with the following ND-rules for adjectives.

$$\frac{\Gamma \vdash \mathbf{j} \text{ isa } X \quad \Gamma \vdash \mathbf{j} \text{ is } A}{\Gamma \vdash \mathbf{j} \text{ isa } A\ X}(adjI) \quad \frac{\Gamma \vdash \mathbf{j} \text{ isa } A\ X \quad \Gamma, [\mathbf{j} \text{ isa } X]_1, [\mathbf{j} \text{ is } A]_2 \vdash S'}{\Gamma \vdash S'}(adjE^{1,2})$$

Let the resulting system be $N_{0,adj}^+$.

Again, we can obtain the following *derived* elimination rules, used to shorten presentations of example derivations.

$$\frac{\Gamma \vdash \mathbf{j} \text{ isa } A\ X}{\Gamma \vdash \mathbf{j} \text{ isa } X}(adj\hat{E}_1) \quad \frac{\Gamma \vdash \mathbf{j} \text{ isa } A\ X}{\Gamma \vdash \mathbf{j} \text{ is } A}(adj\hat{E}_2)$$

Note that the intersectivity here is manifested by the rules themselves (embodying an "invisible" conjunctive operator), at the sentential level. These rules induce intersectivity as a lexical property of (some) adjectives by the way lexical meanings are extracted from sentential meanings, as shown in Francez et al. (2010).

The following sequent, the corresponding entailment of which is often taken as the definition of intersective adjectives, is derivable in $N_{0,adj}^+$:

$$\textbf{j} \text{ isa } A\ X,\ \textbf{j} \text{ isa } Y \vdash \textbf{j} \text{ isa } A\ Y$$

as shown by

$$\frac{\textbf{j} \text{ isa } Y \quad \dfrac{\textbf{j} \text{ isa } A\ X}{\textbf{j} \text{ is } A}\,(adj\ \hat{E}^2)}{\textbf{j} \text{ isa } A\ Y}\,(adj\ I)$$

As an example of derivations using the rules for adjectives, consider the following derivation for

$$\textbf{j} \text{ loves every girl} \vdash \textbf{j} \text{ loves every beautiful girl}$$

In model-theoretic semantics terminology, the corresponding entailment is a witness to the *downward monotonicity* of the meaning of every in its second argument. We use an obvious schematization.

$$\frac{\dfrac{\textbf{j}\ R \text{ every } Y \quad \dfrac{[\textbf{r} \text{ isa } A\ Y]_1}{\textbf{r} \text{ isa } Y}\,(adj\ \hat{E})}{\textbf{j}\ R\ r}\,(e\hat{E})}{\textbf{j}\ R \text{ every } A\ Y}\,(eI^1)$$

A proof-theoretic reconstruction of monotonicity is presented in Ben-Avi and Francez (2011).

Under this definition of the meaning of intersective adjectives, such adjectives are also *extensional*, in the sense of satisfying the following entailment: every X isa $Y \vdash$ every $A\ X$ isa $A\ Y$, as shown by the following derivation:

$$\frac{\dfrac{\dfrac{\text{every } X \text{ isa } Y \quad \dfrac{[\textbf{j} \text{ isa } A\ X]_1}{\textbf{j} \text{ isa } X}\,(adj\ \hat{E}_1)}{\textbf{j} \text{ isa } Y}\,(e\hat{E}) \quad \dfrac{[\textbf{j} \text{ isa } A\ X]_1}{\textbf{j} \text{ is } A}\,(adj\ \hat{E}_2)}{\textbf{j} \text{ isa } A\ Y}\,(adj\ I)}{\text{every } A\ X \text{ isa } A\ Y}\,(eI^1)$$

Decidability of derivability remains intact, by adding to SC_0^+ the following two rules, obtaining thereby a sequent-calculus $SC_{0,adj}^+$ for $L_{0,adj}^+$.

$$\frac{\Gamma, \textbf{j}\ is\ A, \textbf{j} \text{ isa } X \vdash S'}{\Gamma, \textbf{j} \text{ isa } A\ X \vdash S'}\,(Ladj) \qquad \frac{\Gamma \vdash \textbf{j} \text{ is } A \quad \Gamma \vdash \textbf{j} \text{ isa } X}{\Gamma \vdash \textbf{j} \text{ isa } A\ X}\,(Radj)$$

5.3 Adding relative clauses

We next add relative clauses (*rc*s) to the fragment. This fragment transcends the locality of subcategorization in E_0^+, in having *long-distance dependencies*. We refer to this (still positive) fragment as E_1^+. Typical sentences include the following.

(16) Jacob/every boy/some boy loves every/some girl who(m) smiles/loves every flower/Rachel loves

(17) Rachel/every girl/some girl is a girl who loves Jacob/every boy

(18) Jacob loves every girl who loves every boy who smiles (nested relative clause)

So, girl who smiles and girl who loves every boy are compound nouns. We treat the case of the relative pronoun somewhat loosely, in the form of who(m), abbreviating either who or whom, as required. Note that E_1^+, by its nesting of *rc*s, expands the stock of available positions for *dp*-introduction/elimination. Thus, in (18), 'everboy who smiles' is the object of the relative clause modifying the object of the matrix clause. In addition, new scope relationships arise among the multitude of *dp*s present in E_1^+ sentences. Island conditions, preventing some of the scopal relationships, are ignored here.

The corresponding ND-system N_1^+ extends N_0^+ by adding the following I-rules and E-rules. For their formulation, we extend the distinguished position notation with $S[-]$, indicating that the position is *unfilled*. For example, loves every girl and every girl loves have their subject and object *dp* positions, respectively, unfilled.

$$\frac{\Gamma \vdash \mathbf{j} \text{ isa } X \quad \Gamma \vdash S[\mathbf{j}]}{\Gamma \vdash \mathbf{j} \text{ isa } X \text{ who } S[-]} (relI) \quad \frac{\Gamma \vdash \mathbf{j} \text{ isa } X \text{ who} S[-] \quad \Gamma, [\mathbf{j} \text{ isa } X]_1, [S[\mathbf{j}]]_2 S'}{\Gamma \vdash S'} (relE^{1,2}),\ \mathbf{j} \text{ fresh}$$

The simplified elimination-rules are:

$$\frac{\Gamma \vdash \mathbf{j} \text{ isa } X \text{ who } S[-]}{\Gamma \vdash \mathbf{j} \text{ isa } X} (rel\hat{E})_1 \quad \frac{\Gamma \vdash \mathbf{j} \text{ isa } X \text{ who } S[-]}{\Gamma \vdash S[\mathbf{j}]} (rel\hat{E})_2$$

As an example of a derivation in this fragment, consider

$$\text{some girl who smiles sings} \vdash_{N_1^+} \text{some girl sings}$$

exhibiting the model-theoretical *upward monotonicity* of some in its first argument.

$$\frac{\text{some } X \text{ who } P_1\ P_2 \qquad \dfrac{\dfrac{[\mathbf{r} \text{ isa } X \text{ who } P_1]_1}{\mathbf{r} \text{ isa } X} (rel\hat{E})_1 \quad [\mathbf{r}\ P_2]_2}{\text{some } X\ P_2} (sI)}{\text{some } X\ P_2} (sE^{1,2})$$

Similarly, the following witness of the downward monotonicity of every (in its first argument) can be derived.

$$\text{every girl sings} \vdash_{N_1^+} \text{every girl who smiles sings}$$

$$\frac{\dfrac{\text{every girl sings} \qquad \dfrac{[\mathbf{j} \text{ isa girl who smiles}]_1}{\mathbf{j} \text{ isa girl}} (rel\hat{E}_1)}{\mathbf{j} \text{ sings}} (e\hat{E})}{\text{every girl who smiles sings}} (eI^1)$$

Once again, decidability of derivability is shown by means of the following additional sequent-calculus rules, added to SC_0^+, to form SC_1^+.

$$\frac{\Gamma, \mathbf{j} \text{ isa } X, S[\mathbf{j}] \vdash S'}{\Gamma, \mathbf{j} \text{ isa } X \text{ who} S[-] \vdash S'} (Lrel) \quad \frac{\Gamma \vdash \mathbf{j} \text{ isa } X\ \Gamma \vdash S[\mathbf{j}]}{\Gamma \vdash \mathbf{j} \text{ isa } X \vdash \text{who } S[-]} (Rrel)$$

6 Intensional transitive verbs with notional objects

6.1 Introduction

The purpose of this section is to present a semantic problem, notorious for its difficult specification of model-theoretic truth-conditions for sentences exhibiting it (e.g., Zimmermann 1993; Moltmann 1997, 2008, to appear — draft on her homepage), which will be shown to be more manageable in formulating its meaning proof-theoretically. *Transitive intensional verbs*, known also as *opaque verbs*, are verbs like need, look for and want: they take *dp*s as complements that display a special, intensional interpretation, differing from the extensional transitive verbs in E_1^+. Consider, for example,

$$\text{Jacob needs a sheep.} \tag{6}$$

in contrast to its extensional counterpart

$$\text{Jacob milks a sheep.} \tag{7}$$

While in (7) Jacob milks *some specific sheep*, there is no specific sheep Jacob necessarily needs in one reading of (6), though there is also a reading of (6) where he does need a specific sheep. Note that this ambiguity in the meaning of sentences like (6) is not a problem of quantifier scope ambiguity: the sentence contains just one quantifier. This issue has been noticed already in mediaeval times, where Buridan considered

$$\text{Debeo tibi equum, namely, I owe you a horse} \tag{8}$$

A sharpening of the problem occurs in sentences like

$$\text{Jacob seeks a unicorn} \tag{9}$$

where unicorns need not exist at all in order to be sought (evading the debatable issue of non-existent objects). We mention that other quantifiers can stand in the object position of such verbs: one may look for all or most unicorns, still without looking for any specific one. We focus here on the existential quantifier.

There is no consensus regarding the truth-conditions of opaque sentences; not even regarding the semantic type of the object *dp* of the intensional verb, e.g., a quantifier (Zimmermana 1993), a property (Moltmann to appear), or a minimal situation (Moltmann 1997). There is also an indirect interpretation via *decomposition* of the intensional verb (Larson 2001). Another approach (Forbes 2000) appeals to Davidsonian event semantics and its thematic roles.

Our aim here is not to present a *full* proof-theoretical account of opacity; just enough of it to allow contrast with model-theoretical interpretation.

The meaning of sentences with opaque verbs exhibits an *upward monotonicity* in the object argument (Zimmermann 2006), as exhibited by the following instance of the general inference involved.

$$\frac{\textsf{Jacob needs a white sheep}}{\textsf{jacob needs a sheep}} \qquad (10)$$

We leave the full, general treatment of monotonicity to another paper, but show here how our proposed proof-theoretic meaning supports specific instances of the appropriate inference pattern. By this, we show that PTS has advantages not only by evading the general methodological criticism as delineated in the introduction, but also providing better solutions to some semantic problems, found hard for MTS treatment.

6.2 A proof-theoretic meaning for opacity

The basic step is to augment the proof-language with another family of parameters referred to as *notional parameters*, in addition to the individual parameters used so far. Recall that such an extension, being proof artifacts and not used in assertions, do not carry any ontological commitments, in contrast to the various entities assumed by MTS. The purpose of those additional notional parameters is to allow the *dp* Some *X to be introduced in a different way*, not assuming any individual parameter being an *X*, thus exhibiting also a reading escaping specificity. Recall that I-rules were shown before as explaining quantifier-scope ambiguity. I-rules constitute a major tool available in PTS, with strong explanatory power.

Let *B* be a countable set, disjoint from *P*, of *notional parameters*, ranged over by (possibly indexed) meta-variables **n,m**. We need an analog of the predication **j** isa *X*, associating a notional parameter with *the property of being an X*. We thus extend the proof-language *L* with ground pseudo-sentences of the form **n** is being a(n) *X*. As not all transitive verbs admit notional arguments, we let $\hat{R}$ range over those which do. For an opaque verb $\hat{R}$, we have the ground pseudo-sentence $\mathbf{j}\ \hat{R}\ \mathbf{n}$, $\hat{R}$-relating the individual parameter **j** to the notional object **n**. It is important to observe that the following are *ill-formed*:

$$\mathbf{j}\ \textsf{is being a(n)}\ X,\ \ \mathbf{n}\ \textsf{isa}\ X$$

We now can formulate the following I-rule, introducing some *X* notionally.

$$\frac{\Gamma \vdash \mathbf{n}\ \textsf{is being a(n)}\ X \quad \Gamma \vdash \mathbf{j}\hat{R}\mathbf{n}}{\Gamma \vdash \mathbf{j}\ \hat{R}\ \textsf{some}\ X}\ (s_n I) \qquad (11)$$

Note that the I-rule enforces the introduction of the notional object some *X* only at the lowest scope level. Note also that there is no specific individual parameter assumed by the rule to be an *X*; this is in contrast to the introduction of some *X*, that has **j** isa *X* as a premise, generating the specific reading. The harmonious E-rule is

$$\frac{\Gamma \vdash \mathbf{j}\ \hat{R}\ \textsf{some}\ X \quad \Gamma, [\mathbf{n}\ \textsf{is a(n)}\ X]_i, [\mathbf{j}\ \hat{R}\ \mathbf{n}]_j \vdash S}{\Gamma \vdash S} (s_n E^{i,j}) \tag{12}$$

6.2.1 *Monotonicity of opacity*

We now establish the instance of monotonicity inference exhibiting by (11). First, we introduce I/E rules for notional objects expressed via intersective adjectives, like **n** is being white. We assume here that the intersectivity of intersective adjectives holds also under the scope of opacity[20]. Thus, in addition to the inference in (11), we also assume the inference in (13).

$$\frac{\textsf{Jacob needs a white sheep}}{\textsf{Jacob needs a white (thing)}} \tag{13}$$

The ND-rules are the following.

$$\frac{\Gamma \vdash \mathbf{n}\ \textsf{is being a(n)}\ X \quad \Gamma \vdash \mathbf{n}\ \textsf{is being}\ A}{\Gamma \vdash \mathbf{n}\ \textsf{is being a(n)}\ A\ X} (a_n I) \tag{14}$$

$$\frac{\Gamma \vdash \mathbf{n}\ \textsf{is being a(n)}\ A\ X \quad \Gamma, [\mathbf{n}\ \textsf{is being a(n)}\ X]_i, [\mathbf{n}\ \textsf{is being}\ A]_j \vdash S}{\Gamma \vdash S} (a_n E^{i,j}) \tag{15}$$

From the generalized elimination rule we once more obtain the following two simplified derived E-rules.

$$\frac{\Gamma \vdash \mathbf{n}\ \textsf{is being a(n) A X}}{\Gamma \vdash \mathbf{n}\ \textsf{is being a(n)}\ X} (a_n \hat{E}_1) \tag{16}$$

$$\frac{\Gamma \vdash \mathbf{n}\ \textsf{is being a(n)}\ A\ X}{\Gamma \vdash \mathbf{n}\ \textsf{is being}\ A} (a_n \hat{E}_2) \tag{17}$$

We now show the monotonicity exhibited by

$$\mathbf{j}\ \hat{R}\ \textsf{some}\ A\ X \vdash \mathbf{j}\ \hat{R}\ \textsf{some}\ X \tag{18}$$

[20] As far as we are aware of, this issue is not discussed in the literature. There are also opinions denying the monotonicity of opacity. A way to block it might be prohibiting intersectivity under the scope of opacity. We deal here only with the case where monotonicity *is* assumed to hold.

The derivation is

$$\dfrac{\mathbf{j}\ \hat{R}\ \text{some}\ A\ X \qquad \dfrac{[\mathbf{j}\ \hat{R}\ \mathbf{n}]_1 \qquad \dfrac{[\mathbf{n}\ \text{is being a(n)}\ A\ X]_2}{\mathbf{n}\ \text{is being a(n)}\ X}\ (a_n\hat{E}_1)}{\mathbf{j}\ \hat{R}\ \text{some}\ X}\ (s_nI)}{\mathbf{j}\ \hat{R}\ \text{some}\ X}\ (s_nE^{1,2})$$

We end the the discussion by mentioning that while our fragment does not contain *coordination* (neither *np*-coordination nor sentential one), this approach promises smooth interface to an extended fragment that does contain coordination, treating the wealth of examples in Richard (2001).

7 Conclusions and relation to other work

In this paper, we have provided a revisionary view of semantics, replacing truth-conditions (in arbitrary models) by *assertability conditions* based on canonical derivability in a “dedicated” natural-deduction proof-system. The assignment of proof-theoretical meanings to NL-sentences (and to subsentential phrases), is, to the best of our knowledge, completely new. Basing meaning on I-rules provides a very powerful tool. By manipulating properties of application of such rules, we have provided a novel view of (the source of) quantifier-scope ambiguity and non-specifity of objects of opaque transitive verbs. Our approach escapes Dummett’s manifestation-argument, a major criticism of the adequacy of model-theoretical semantics as a theory of meaning.

There is a vast literature on the use of proof-theory *in deriving meanings*; however, the derived meanings are all model-theoretic. Besides the traditional meaning derivation in TLG, relying on the Curry–Howard correspondence, there is also a similar approach in LFG, called ‘glue’, using linear logic for the derivations. There are also approaches like de Groote and Retoré (1996), that read off meanings from proof-nets instead of derivations. In all these approaches, the common theme is that some proof-theoretic object is used for *deriving* meanings, and does not *constitute* the meaning. The latter is usually formulated in some (extension of a) λ-calculus, the terms of which are interpreted model-theoretically, e.g., in Henkin models.

As we mentioned in the introduction, there is a body of work also going under the title of PTS for sentential meanings, based on constructive type-theory (MLTT), which is clearly related, but, we believe, different than our approach to PTS. One obvious difference is that our PTS is *direct*, using a dedicated proof-system, in contrast to the indirectness of translating to constructive type-theory and using its logic as the proof-system. By our approach, NL sentences directly contribute to meanings of other sentences, and directly serve as grounds for assertion.

Furthermore, in our view of the MLTT-based work, it is still a model-theoretic semantics, but one *constrained by proof-theoretic constraints*—constructivism in this case. This is most evident in the treatment, according to this approach, of nouns and intransitive verbs. Both are viewed as *denoting* sets of objects (from some universe of discourse), serving as arguments to forming dependent types. This can be seen in Ranta (2004a), where an implementation of Montague grammar in *Grammatical Framework (GF)* is proposed Ranta (2004b). Use is made of type **ent** (entities in a model) to define the meaning of nouns and intransitive verbs, and first arguments of Π and Σ.

This difference is clearly seen from the following quotation (Hinzen 2000, p. 284), from a work dedicated to using constructive type-theory for semantics:

> ... Truth, which is not epistemic on the intuitive understanding of it, is here not replaced by an epistemic notion, nor are truth-conditions replaced by assertability conditions. It is rather that we *add* to our semantic analysis of language a further proof-theoretic layer, taking the latter to give us a richer picture of the truth-conditional content of a statement or discourse.

In another work (Fernando 2001), dedicated mainly to the analysis of *presupposition* by using a PTS based on Ranta's approach using MLTT, the author says (p. 17):

> Put in very general terms, the present work is an attempt to introduce a proof-theoretic interpretation of type theory to Montague semantics, while retaining model-theoretic interpretations.

Ultimately, it may turn out that the approaches have more in common than we currently perceive. Future extensions of the fragment may reveal more differences, or highlight more similarities.

Our PTS *does not provide a "real" lexical semantics*! Only (proof-theoretic) type information is provided for words expressing predicates (nouns, verbs, adjectives). This is the same as TLG's coarse lexical semantics definition in model-theoretic terms, where a predicate may obtain *arbitrary* extensions in Henkin models. Thus, cry and smile may end up synonyms, disjoint, one implying the other, identically false/true, or anything else. Neither is the knowledge that red implies colored incorporated in this coarse lexical semantics of adjectives. However, *determiners* do receive a specific lexical meaning.

The semantics, again like model-theoretic TLG-based semantics, does not handle *selectional restrictions*. For example, every chair smiles will get its meaning similarly to every girl smiles. One way to handle this is to import into the proof-system non-logical axioms describing *ontologies* (see Ben-Avi and Francez (2004)).

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