



A different look at the effect of temperature on the nanofluids thermal conductivity: focus on the experimental-based models

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Abstract

This paper takes a different look at the mathematical modeling of the effective thermal conductivity of nanofluids. Most related published experimental-based mathematical models have been analyzed statistically. The sensitivity analysis showed that in the most published models, the role of nanofluids bulk temperature may be ignored. Then, we extracted a lot of data from the models in the valid ranges of variables. The next step was performing statistical analysis of the variances and means of different datasets (data populations). The results showed that changing considerably the nanofluids temperature doesn't affect considerably the nanofluids thermal conductivity. We introduced a comprehensive, simpler and more accurate correlation neglecting the nanofluids temperature to predict the effective thermal conductivity of nanofluids. Results indicated that the predicted values using the proposed correlation are in a good agreement with experimental data.

Keywords Thermal conductivity · Statistical analysis · Nanofluids · Correlation · Temperature

Introduction

The total energy consumption of the world has been increased sharply, during the past decades energy will be one of the most critical challenges in the next decades, especially for developing countries. Heat transfer scientists and engineers have concentrated on different methods to reduce the energy consumption of heat transfer phenomena. Their methods included but not limited to increase the heat transfer fluid's velocity (forced convection) [1–3], enhance the heat transfer area (like fins, heat sinks, plate heat exchangers, and so on) [4–6], fast transition from laminar flow regime to the turbulent (increasing friction factor, and etc.) [7, 8],

conjugate heat transfer and mixed convection [9–11], boiling and other phase change processes because of the high amount of latent heat (like heat pipes and phase change materials) [12, 13] and so on.

Meanwhile, some scientists believed that this challenge can be solved by increasing the relatively low thermal conductivity of conventional heat transfer fluids. Maxwell [14] thought we can experience considerable enhancement in thermophysical properties of a mixture of a fluid and dispersed metal particles compared to the base fluid. He never could set up such an experiment because of high mass value (and size) of the particles has been manufactured at the time. His idea remained only theoretical one until the last decade of the twentieth century. Choi and Eastman [15] based on recent development of nanotechnology suggested a new class of heat transfer fluid containing nanoparticles (particles generally smaller than 100 nm in dimensions). They proposed to disperse a low volume concentration of metal or oxide nanoparticles in a conventional heat transfer fluid like water, oil and ethylene glycol. They reported an anomalous enhancement in heat transfer coefficient of nanofluids compared to the base fluid [16].

Great attentions have been devoted to this new scientific horizon throughout the world and therefore an unbelievable number of scientific papers have been published during past two decades.

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Today, the nanofluids applications in different fields and industries are under serious investigations and some experimental successes have been achieved. The nanofluids applications include, but not limited to heat exchangers [17–19], impingement jets [20, 21], renewable energies [22–24], heating and tempering processes [25–30], automotive industries [31, 32], electronic cooling [33–37], lubrication [38–40], medicine [41–43], combustion [44–46], and etc.

There are two conventional approaches to simulate the nanofluids fluid flow and heat transfer characteristics named single phase [47–49] and two phase approaches [50–53]. The single phase approach assumes that a nanofluid is a homogenous mixture (suspension) of nanoparticles and base fluid. This approach ignores the different interaction between nanoparticles and liquid molecules and tends to estimate the thermophysical properties using predictive models for thermal conductivity, density, viscosity and specific heat. Thus, the single phase approach deals with the nanofluids as a whole.

Existing thermal conductivity models

Prediction of thermophysical properties of a mixture has a longer history than that of nanofluids. In 1962, Hamilton and Crosser [54] proposed one of the oldest correlation of a mixture's thermal conductivity based on a few experiments on the spheres, disks, cylinders and cubes of aluminum and balsa in silastic ($n = 3$ for spherical particles):

$$\frac{k_{\text{eff}}}{k_b} = k_b \left[\frac{k_p + (n-1)k_b - (n-1)(k_b - k_p)\varphi}{k_p + (n-1)k_b + (k_b - k_p)\varphi} \right] \quad (1)$$

At the same time, Hashin and Shtrikman [55] studied theoretically the multiphase thermophysical properties using variation approach and proposed one the first correlations on the thermal conductivity of a mixture, where k_{eff} , k_p , k_b and φ are the thermal conductivity of mixture, thermal conductivity of particles, thermal conductivity of fluid and volume concentration of particles, respectively.

$$k_f \left(1 + \frac{3\varphi(k_p - k_f)}{3k_f + (1 - \varphi)(k_p - k_f)} \right) \leq k_{\text{eff}} \leq \left(1 - \frac{3(1 - \varphi)(k_p - k_f)}{3k_p - \varphi(k_p - k_f)} \right) k_p \quad (2)$$

Jeffery [56] investigated of heat conduction of a stationary random suspension of spherical particles with a low volume concentration of particles. He extended the work of Maxwell to calculate the heat flux with by consideration of interactions between pairs of particles. Eventually, he proposed a useful correlation for thermal conductivity of a mixture:

$$k_{\text{eff}} = \left(1 + 3\beta + \left(3\beta^2 + \frac{3\beta^2}{4} + \frac{9\beta^3\alpha + 2}{16} + \frac{3\beta^4}{64} + \dots \right) \varphi^2 \right) k_b$$

$$\beta = \frac{\alpha - 1}{\alpha + 2}, \quad \alpha = \frac{k_p}{k_b} \quad (3)$$

Turian et al. [57] conducted an experimental study on the suspension of coal particles in water, oil and other liquids. They developed a simple correlation for thermal conductivity of a mixture. The results of their correlation are in good agreement with the experimental data, especially for low volume concentration of particles.

$$k_{\text{eff}} = k_p^\varphi k_b^{1-\varphi} \quad (4)$$

There are more than 70 models for predicting thermal conductivity of nanofluids. More than 37 models are developed using experimental data and results. Finally, we found 14 models predicting thermal conductivity of nanofluids including nanofluids temperature, using experimental data. Table 1 shows all published experimental models for thermal conductivity of nanofluids taking into account the nanofluids temperature.

Methodology

All of the mentioned models in Table 1 consider the thermal conductivity of nanofluids as a function of thermal conductivity of base fluid (k_b), thermal conductivity of nanoparticle (k_p), nanoparticle volume concentration (φ) and temperature (T). Two models [59, 61] additionally take into account the nanoparticle mean diameter.

Generally, it seems normal that thermal conductivity of nanofluids enhances with any increase in thermal conductivity of base fluid, thermal conductivity of nanoparticle, nanofluids temperature and any decrease in the mean diameter of nanoparticles. However, some models predict diverse and sometimes bizarre behavior of thermal conductivity of nanofluids according to different variables. For example, the thermal conductivity of nanofluids enhances usually with any increase in temperature, and conversely, some models predict the decreasing trend of thermal conductivity of nanofluids with increase in temperature. This situation is frequent, especially when the base fluid is ethylene glycol.

We decided to examine the behavior of the effective thermal conductivity against different variables appear in published models. Then, we tried to evaluate the role of each variable in effective thermal conductivity, based on published models. We guess that maybe we can neglect the effect of one variable versus the others. We think that representing the contradictions in our understanding about the role of different variables will help to attract the attention

Table 1 All published experimental models for thermal conductivity of nanofluids including nanofluids temperature

References	Model	Base fluid	Nanoparticle	Limitations
1	Aberoumand et al. [58]	Heat transfer oil	Ag Cu MCWNT	Volume concentration: 0–2%
	$k_{eff} = (3.9 \times 10^{-5}T - 0.0305)\varphi^2 + (0.086 - 1.6 \times 10^{-4}T)\varphi + 3.1 \times 10^{-4}T + 0.129 - 5.77 \times 10^{-6}k_p - 40 \times 10^{-4}$			
2	Fakoor Pakdaman et al. [59]	Heat transfer oil	MCWNT	Mass concentration 0–0.4% Temperature ranging from 313 to 343 K dp: 5–20 nm
	$\frac{k_{eff}}{k_b} = 1 + 304.47(1 + \varphi)^{1.36,35} \exp(-0.0217) \left(\frac{1}{d_p}\right)^{0.369} \left(\frac{T^{1.2321}}{10^{0.06387(T-180)}}\right)$			T_r is the absolute temperature ratio between nanofluids temperature and ambient temperature
3	Ahammed et al. [60]	Water	Graphene	Temperature range between 10 and 50 °C volume concentration range is 0.05–0.15%
	$\frac{k_{eff}}{k_b} = 2.5T_r^{1.032} \varphi^{0.108}$			dp: 10–50 nm Kb: 0.1–0.7
4	Patel et al. [61]	Ethylene Glycol	Al ₂ O ₃ CuO Cu Al	Volume concentration: 0.1–3% Temperature: 20–50 °C
	$k_{eff} = k_b \left(1 + 0.135 \times \left(\frac{k_p}{k_b}\right)^{0.2773} \varphi^{0.467} \left(\frac{T}{20}\right)^{0.547} \left(\frac{100}{d_p}\right)^{0.234} \right)$			Volume concentration: 1–10% Temperature: 20–50 °C
5	Li and Peterson [62]	Water	Al ₂ O ₃ CuO	
	$\frac{k_{eff}}{k_b} = 0.7644815\varphi + 0.018689T + 0.537853$, for Al ₂ O ₃ $\frac{k_{eff}}{k_b} = 3.761088\varphi + 0.017924T + 0.69266$, for CuO			
6	Hemmat Esfe et al. [63]	Water-EG (60:40)	MgO	Temperature: 20–50 °C Volume concentration: 0–3%
	$k_{eff} = 0.4 + 0.0332\varphi + 0.001017 + 0.000619\varphi T + 0.0687\varphi^3 + 0.0148\varphi^5 - 0.00218\varphi^6 - 0.0419\varphi^4 - 0.0604\varphi^2$			
7	Hemmat Esfe et al. [64]	Ethylene Glycol	Al ₂ O ₃	Volume concentration: 0.2–5% Temperature: 25–55 °C
	$\frac{k_{eff}}{k_b} = 1.04 + 5.91 \times 10^{-5}T + 0.00154T\varphi + 0.0195\varphi^2 - 0.014\varphi - 0.00253\varphi^3 - 0.000104T\varphi^2 - 0.0357 \times \sin(1.72 + 0.407\varphi^2 - 1.67\varphi)$ $\frac{k_{eff}}{k_b} = 0.999 + 9.581 \times 10^{-5}T + 0.00142T\varphi + 0.0519\varphi^2 + 0.00208\varphi^2 + 0.00208\varphi^4 - 0.00719\varphi - 0.0193\varphi^3 - 8.21 \times 10^{-5}T\varphi^2$			
8	Hemmat Esfe et al. [65]	Water-EG	Cu/TiO ₂	Volume concentration: 0.1–2% Temperature: 30–60
	$\frac{k_{eff}}{k_b} = 1.04 + 0.000589T + \frac{-0.000184}{T\varphi} \times \cos(6.11 + 0.00673T + 4.417\varphi - 0.0414 \sin(T)) - 32.5\varphi$			
9	Hemmat Esfe et al. [66]	Water	CNTs/Al ₂ O ₃	Temperature: 303–332 K Volume concentration: 0–1%
	$\frac{k_{eff}}{k_b} = \frac{-214.83+T}{346.58-106.98\varphi} + \frac{227.69}{T}$			
10	Harandi et al. [67]	Ethylene Glycol	F-MWCNTs-Fe ₃ O ₄	Temperature: 25–55 °C Volume concentration: 0.1–2.3%
	$\frac{k_{eff}}{k_b} = 1 + 0.0162\varphi^{0.7038}T^{0.6009}$			

Table 1 (continued)

References	Model	Base fluid	Nanoparticle	Limitations
11 Zadkhast et al. [68]	$\frac{k_{eff}}{k_b} = 0.907 \exp(0.36\phi^{0.3111} + 0.0009567T)$	Water	MWCNT-CuO	Temperature: 25–50 °C Volume concentration: 0.05–0.6%
12 Kakavandi and Akbari [69]	$\frac{k_{eff}}{k_b} = 0.0017 \times \phi^{0.698} \times T^{1.386} + 0.981$	Water-EG	SiC-MWCNTs	Temperature: 25–55 °C Volume concentration: 0–0.75%
13 Karimi et al. [70]	$k_{eff} = (1 + 359.28\phi \left(\frac{T}{T_{max}}\right)^{0.06418} \times (-6.58 \times 10^{-6}T^2 + 0.00187 + 0.5694))$ for Fe ₃ O ₄ $k_{eff} = (1 + 418.81\phi \left(\frac{T}{T_{max}}\right)^{0.06748} \times (-6.58 \times 10^{-6}T^2 + 0.00187 + 0.5694))$ for CoFe ₂ O ₄	Water	Fe ₃ O ₄ CoFe ₂ O ₄	Temperature: 20–60 °C Volume concentration: 0–4.8%
14 Karimpour et al. [71]	$\frac{k_{eff}}{k_b} = 0.792194 + 0.0547913\phi + 0.00998805T + 0.000730423\phi T - 0.00421237\phi^2 - 0.0000643292T^2$	Liquid paraffin	CuO	Temperature: 25–100 °C Mass concentration: 0.25–6%
15 Ranjbarzadeh et al. [72]	$\frac{k_{eff}}{k_b} = 1 + 0.4281 \left(\frac{T}{100}\right)^{1.707} \phi^{0.8449}$	Water	SiO ₂	Temperature: 25–55 °C Volume concentration: 0.1–3%
16 Keyvani et al. [73]	$\frac{k_{eff}}{k_b} = 0.9320 + 0.0673\phi + 0.0021T$	Ethylene Glycol	CeO ₂	Temperature: 25–50 °C Volume concentration: 0.25–2.5%
17 Afrand et al. [74]	$\frac{k_{eff}}{k_b} = 0.9320 + 0.0673\phi^{0.323} T^{0.245}$	Water	Fe ₃ O ₄	Temperature: 20–55 °C Volume concentration: 0.1–3%

of researchers to this challenging issue, since we highly believed that the single phase approach of nanofluids investigation is severely dependent on the predictive models. The next step is devoted to extract some easier and more accurate correlations. Figure 1 shows the algorithm has been employed to conduct this research.

Some methods should be put in the front being given before running the above-mentioned algorithm. First of all, we perform a sensitivity analysis of each proposed model to find out the most sensitive variables in the model. In fact, sensitivity analysis allows you to identify the inputs whose variations have the most impact on the key outputs.

Then, we run the published models in their validation range of variables mentioned by the authors (Table 1) to extract a couple of predicted data. Then, different datasets will be provided with different temperature. For example, when a model is valid in the temperature range of 20–40 °C, we provide five datasets (statistical populations) for using temperature of 20, 25, 30, 35 and 40 °C. It should be noted that we assumed a number of datasets statistically same when two parameters of all datasets are statistically equal: mean and variance.

The null hypothesis is all population means (or variances) are equal, while the alternative hypothesis states that at least one is different. So, when the P value is more than 0.05 (for the 0.05 level of significance), the test is unable to reject the null hypothesis. This means that despite of temperature changing in different datasets, the means (or variances) of them are statistically equal.

With this intention, we conduct statistical analysis for each model, separately. Then, if our analysis is sufficient for the hypothesis, then we move to develop a different model taking into account different variables. At the final step of the algorithm, model's the goodness of fit will be evaluated and reported.

Results and discussion

In this section, all published models (Table 1) will be investigated and analyzed, separately. The above-mentioned algorithm will be run for each model. At the end, we try to develop a simple mathematical model to predict all available experimental data. The goodness of fit of the final comprehensive model will be analyzed, statistically.

Model by Aberoumand et al. [58]

Aberoumand et al. [58] conducted an experimental study on the rheological behavior of heat transfer oil-based nanofluids and provided two correlations for effective thermal conductivity and effective viscosity of nanofluids. They dispersed Ag nanoparticles with a mean diameter of 20 nm in

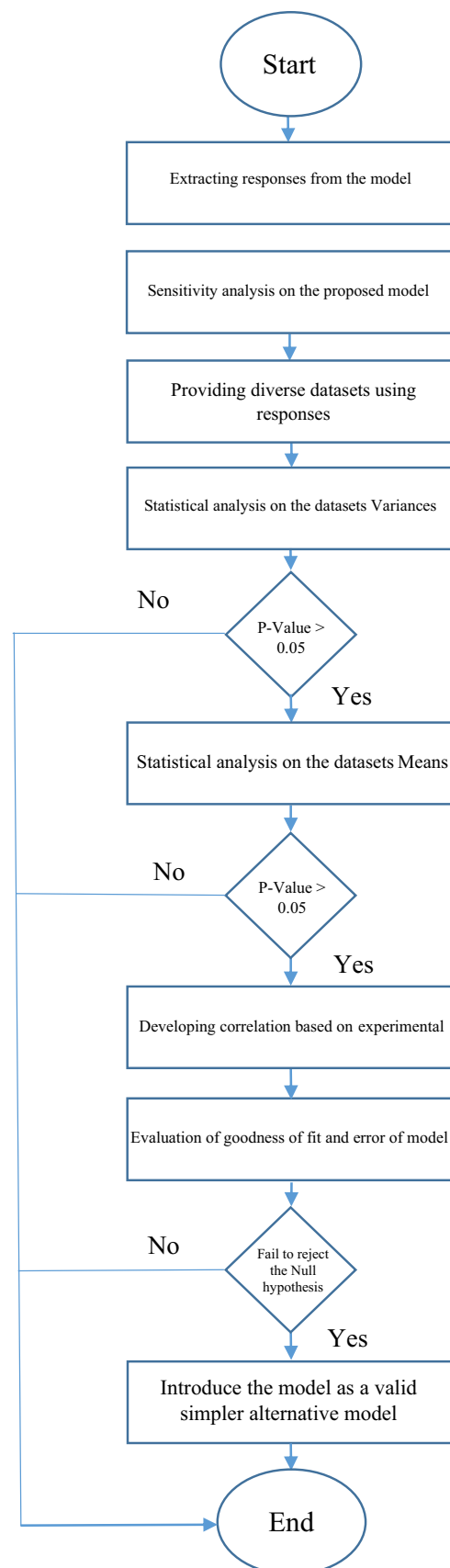


Fig. 1 The employed algorithm for statistical analysis

oil heat transfer with a range of 0–2% volume concentration. Then, they added some other experimental data about Cu and MCWNT nanoparticles to develop predictive correlation for effective thermal conductivity of nanofluids (Eq. 5).

$$k_{\text{eff}} = (3.9 \times 10^{-5}T - 0.0305)\varphi^2 + (0.086 - 1.6 \times 10^{-4}T)\varphi + 3.1 \times 10^{-4}T + 0.129 - 5.77 \times 10^{-6}k_p - 40 \times 10^{-4} \quad (5)$$

According to their model, the effective thermal conductivity decreases with any increase in the thermal conductivity of nanoparticles. Figure 2 shows the result of sensitivity analysis of their proposed model. Generally, to identify inputs whose variations have little or no effect on the response, look for inputs with a flat line. For inputs with a flat line, one can ease the requirements (tolerances) without adversely affecting the performance, which will save time and cost. The graph shows that changes in the

variation of the purple and green effects have little influence on the effective thermal conductivity. However, nanoparticle volume concentration plays a key role in the prediction of the response.

We extracted 6000 data from their model in the validation ranges of variables and categorized the responses using different temperatures. Statistical analysis gives *p* values of 0.963 and 0.332 for means and variances analysis, respectively (at the 0.95 level of significance). Therefore, there is no statistically significance difference between different datasets in different temperature. Statistically speaking, the responses are the same, despite of considerable change in temperature. It can be concluded that the nanofluids temperature doesn't play a key role in the effective thermal conductivity prediction in their model with more than 95% of probability and so we can neglect easily the role of temperature. Figure 3 shows the result of variance analysis of the model of Aberoumand et al. [58].

Fig. 2 The result of sensitivity analysis for proposed model by Aberoumand et al. [58]

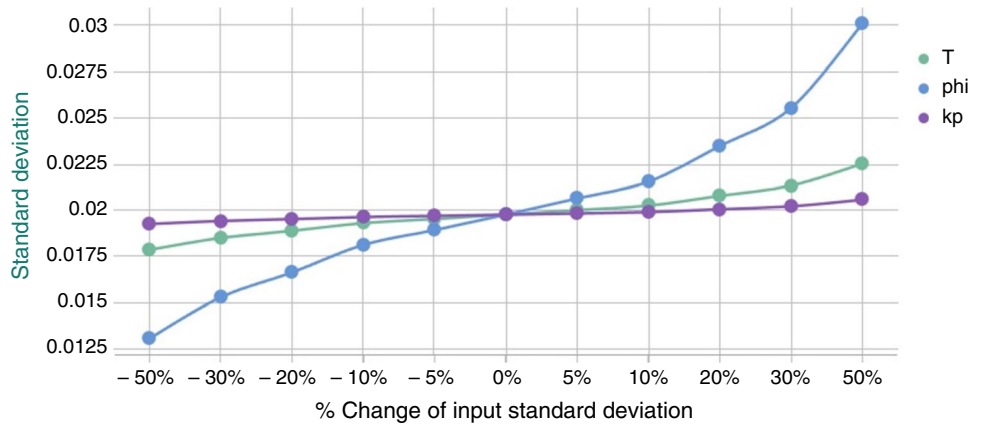
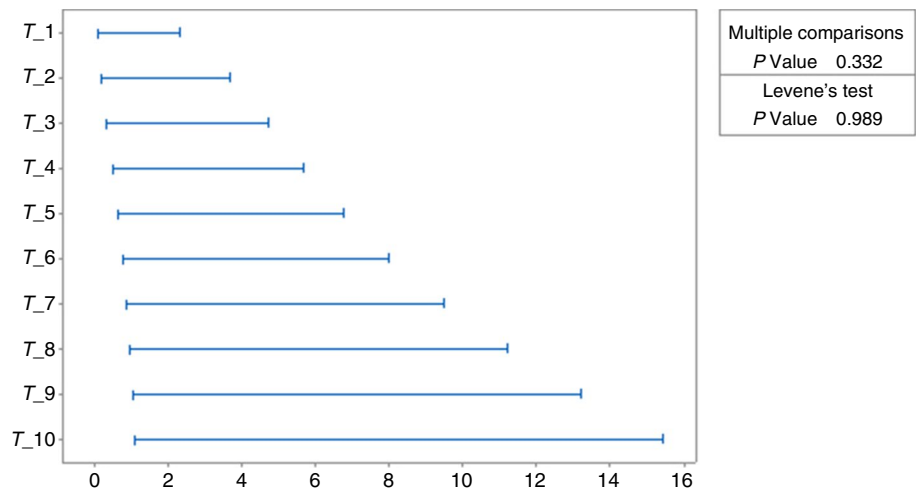


Fig. 3 The result of variance analysis of the Aberoumand et al. [58] model

Test for equal variances: $T_1; T_2; T_3; T_4; T_5; T_6; T_7; T_8; \dots$

Multiple comparison intervals for the standard deviation, $\alpha = 0.05$



Multiple comparisons
<i>P</i> Value 0.332
Levene's test
<i>P</i> Value 0.989

If intervals do not overlap, the corresponding stdevs are significantly different.

The next step is providing a correlation for effective thermal conductivity without nanofluids temperature using experimental data based on the nonlinear regression method. Equation 7 is our proposed correlation and it is shown that this correlation is simple, accurate and it also gives effective thermal conductivity equal to thermal conductivity of base fluid when the nanoparticle volume concentration is zero.

$$k_{\text{eff}} = k_b (1 + 1.25536\varphi^{0.29397}) \tag{6}$$

The average absolute error of correlation (6) from the experimental data is 3.62%. Therefore, this correlation has an appropriate goodness of fit and gives a reliable estimation of experimental data. The predicted value of effective thermal conductivity of nanofluids is approximately equal to thermal conductivity of base fluid, when the nanoparticle volume concentration is zero.

Model by Fakoor Pakdaman et al. [59]

An experimental investigation of heat transfer performance of oil-based nanofluids inside vertical helically coiled tube was conducted by Fakoor Pakdaman et al. [59]. They dispersed MWCNT nanotubes with 5–20 nm in diameter in heat transfer oil with range of 0–2% volume concentration. They introduced a correlation for effective thermal conductivity of nanofluids (Eq. 7).

$$\frac{k_{\text{eff}}}{k_b} = 1 + 304.47(1 + \varphi)^{136.35} \exp(-0.021T) \left(\frac{1}{d_p}\right)^{0.369} \left(\frac{T^{1.2321}}{10^{610.6287/(T-140)}}\right) \tag{7}$$

Figure 4 demonstrates the result of sensitivity analysis of their model. It can be concluded from the lines that the most important variables in this model are nanoparticles mean diameter and nanofluids temperature, respectively.

We extracted 8036 responses from their model in the valid ranges of variables and categorized the responses using different temperatures in seven groups. Our statistical

analysis provides *P* values of 1.000 and 1.000 for means and variances analysis, respectively (at the 0.95 level of significance). So, there is no statistically significance difference between different datasets in different temperature. Statistically speaking, the responses are the same, despite of considerable change in temperature. It can be concluded that the temperature doesn't play a key role in the effective thermal conductivity prediction in their model with more than 95% of probability and so we can neglect easily the role of temperature. Figure 5 shows the result of variance analysis of the model of Fakoor Pakdaman et al. [59].

Then, we try to propose a correlation to predict the effective thermal conductivity of nanofluids, neglecting the role of temperature using nonlinear regression method. Equation 8 is our proposed correlation with an average absolute error of 2.41%.

$$k_{\text{eff}} = k_b \left(1 + \left(\frac{2.73465\varphi}{d_p} \right)^{0.270822} \right) \tag{8}$$

Therefore, this correlation has an excellent goodness of fit and gives a high reliable estimation of response. Our proposed equation (Eq. 8) gives effective thermal conductivity of 0.1235 W m K⁻¹, when the nanoparticle volume concentration is zero. This is equal to thermal conductivity of the base fluid. It means that when there is no any nanoparticle

in the liquid, we have the pure base fluid.

Model by Ahammed et al. [60]

Ahammed et al. [60] investigated heat transfer and fluid flow of grapheme—water nanofluids, experimentally implementing a transient hot wire technique at temperatures below and

Fig. 4 The result of sensitivity analysis for proposed model by Fakoor Pakdaman et al. [59]

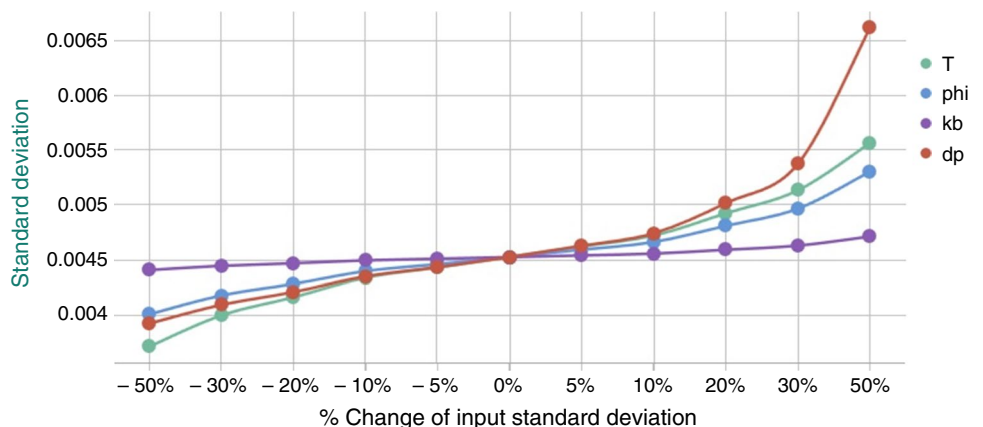
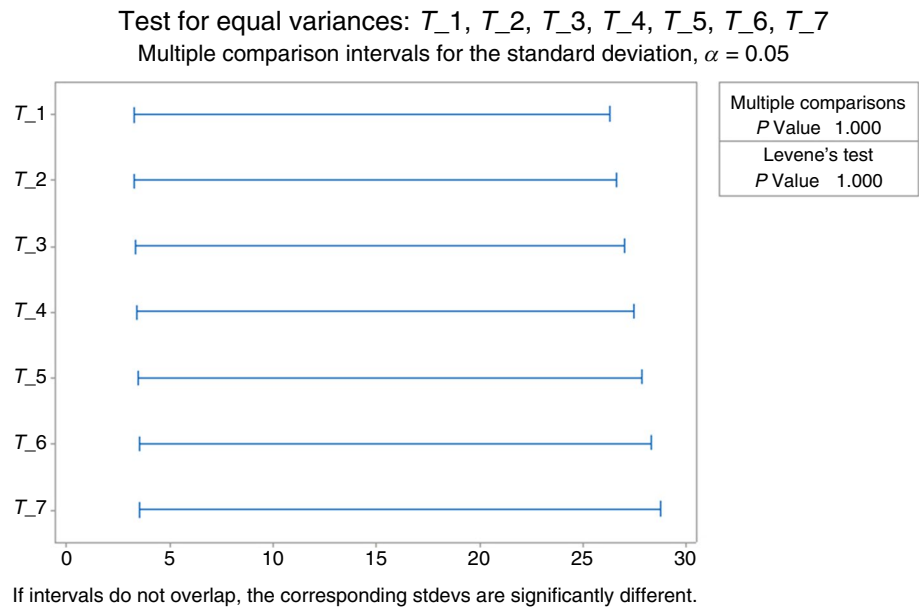


Fig. 5 The result of variance analysis of the Fakoor Pakdaman et al. [59] model



above ambient conditions ranging from 10 to 50 °C. The author mentioned that the valid range of nanoparticle volume concentration is between 0.05 to 0.15%, and an empirical correlation has been developed in form of Eq. 9 (which is dimensionally consistent). It shown that the nanofluids temperature plays an important role in prediction of the effective thermal conductivity of nanofluids.

$$\frac{k_{\text{eff}}}{k_b} = 2.5T_r^{1.032}\varphi^{0.108} \quad (9)$$

Nevertheless, we decided to investigate the case in details. So, we extracted 243 responses in different values of variables using their model and categorized them in nine groups for different nanofluids temperature. The statistical analysis for mean and variance gives p values of 0.772 and 0.762, respectively (Appendix, Figs. 1–2). Therefore, there is no statistical significant difference between nine groups. It means that changing considerably temperature doesn't affect the mean and variances of different datasets. So, one may develop a correlation neglecting the nanofluids temperature. We followed such an approach and Eq. 10 has been resulted.

$$k_{\text{eff}} = k_b(1 + \varphi^{0.698441}) \quad (10)$$

The average absolute error of this correlation from experimental data is 3.31%. So, it can be concluded that the proposed simple correlation is accurate, reliable and well-fitted.

Model by Patel et al. [61]

A series of experiments have been conducted to measure thermal conductivity of different oxide and metal-based nanofluids by Patel et al. [61]. They dispersed silver,

alumina, copper and aluminum nanoparticles in water, ethylene glycol and transformer oil with volume concentration in range of 0.1–3% using sonication method. The mean diameter of nanoparticles was in range of 10–50 nm, and the minimum and maximum nanofluids temperature were 20 and 50 °C, respectively. They also proposed a correlation for prediction of effective thermal conductivity of nanofluids (Eq. 11).

$$k_{\text{eff}} = k_b \left(1 + 0.135 \times \left(\frac{k_p}{k_b} \right)^{0.273} \varphi^{0.467} \left(\frac{T}{20} \right)^{0.547} \left(\frac{100}{d_p} \right)^{0.234} \right) \quad (11)$$

Results show that most and least affective variables in the proposed model based on a sensitivity analysis are thermal conductivity of the base fluid and nanofluids temperature, respectively (Appendix, Fig. 3).

For detailed investigation, we extracted 105,847 responses from their model within the valid ranges of variables and created seven unique datasets using different nanofluids temperatures. The statistical analysis shows that the p values for the means and variances are 0 and 0.938, respectively. Although the p value for means is zero, and therefore, this test fails to prove the null hypothesis, we decided to compare the values of responses in different datasets. It should be noted that the average of response values for datasets with temperature of 20, 25, 30, 35, 40, 45 and 50 °C, are 0.49772, 0.50430, 0.51031, 0.51588, 0.52110, 0.52604, and 0.53072, respectively. This means that when we increase the nanofluids temperature from 20 to 25 °C, we experience only 1.32% enhancement in effective thermal conductivity. Increasing temperature from 45 to 50 °C gives only 0.89% enhancement in effective thermal conductivity. Therefore, we can

conclude that there is no significant difference between these seven groups of response (Appendix, Fig. 4).

We developed a simple correlation for prediction the effective thermal conductivity of nanofluids neglecting the role of nanofluids temperature, employing nonlinear regression method. Equation 12 shows our proposed correlation.

$$k_{\text{eff}} = k_b \left(1 + \frac{0.232706 \varphi^{0.402458} \times k_p}{d_p} \right) \quad (12)$$

The average absolute error from the experimental data is 3.44%. It can be concluded that Eq. 12 is accurate, reliable and well-fitted.

Models by Li and Peterson [62]

Li and Peterson [62] introduced the first correlation for effective thermal conductivity of nanofluids including the nanofluids temperature based on experimental data. They synthesized two categories of nanofluids in nanoparticles volume concentration in range of 2–10%: Al_2O_3 -water and CuO-water. The mean diameters of alumina and copper oxide were around 36 and 20 nm, respectively. The experiments have been performed in nanofluids bulk temperature of 27.5–34.7 °C. Generally, they observed significant enhancement in thermal conductivity up to 52% compared to the base fluid. Finally, they developed two correlations for prediction of effective thermal conductivity (Eqs. 13 and 14).

$$\frac{k_{\text{eff}}}{k_b} = 0.7644815 \varphi + 0.018689 T + 0.537853, \quad \text{for } \text{Al}_2\text{O}_3 \quad (13)$$

$$\frac{k_{\text{eff}}}{k_b} = 3.761088 \varphi + 0.017924 T + 0.69266, \quad \text{for } \text{CuO} \quad (14)$$

The investigations showed that nanofluids temperature doesn't play a key role in prediction of the effective thermal conductivity of nanofluids (Appendix, Fig. 5). For more investigation, we extracted 1862 responses from their correlations and categorized them into seven unique datasets (for each correlation) with different temperature. Statistical analysis gives p values of 0.736 and 0.927 for means and variances, respectively for Eq. 13 and 0.755 and 0.925 for means and variances, respectively for Eq. 14. Therefore, there is no statistical meaningful difference between the means and variances of different datasets (Appendix, Fig. 6). We tried to develop two simple correlations for effective thermal conductivity, neglecting the nanofluids bulk temperature. Equations 15 and 16 show our proposed correlation for alumina–water and copper oxide–water nanofluids.

$$k_{\text{eff}} = k_b(1 + 0.81874\varphi^{0.56}) \quad \text{for } \text{Al}_2\text{O}_3 \quad (15)$$

$$k_{\text{eff}} = k_b(1 + \varphi^{0.255643}) \quad \text{for } \text{CuO} \quad (16)$$

Equations 15 and 16 give predicted values with average absolute error of 4.88% and 2.79% from experimental data. Therefore, these two simple correlations are valid, accurate and well-fitted.

Model by Hemmat Esfe et al. [63]

An experimental study has been performed on the thermal conductivity of MgO-based nanofluids by Hemmat Esfe et al. [63]. They dispersed 40 nm MgO nanoparticles in a binary mixture of water and ethylene glycol (60:40) with volume concentration in range of 0.1–3%, using an ultrasonic homogenizer. The minimum and maximum nanofluids bulk temperature were 20 and 50 °C, respectively. They proposed a correlation for effective thermal conductivity (Eq. 17), using experimental data.

$$k_{\text{eff}} = 0.4 + 0.0332\varphi + 0.00101T + 0.000619\varphi T + 0.0687\varphi^3 + 0.0148\varphi^5 - 0.00218\varphi^6 - 0.0419\varphi^4 - 0.0604\varphi^2 \quad (17)$$

Figure 7 of Appendix represents the result of sensitivity analysis of their proposed model, indicating that the nanofluids temperature doesn't affect considerably the effective thermal conductivity of nanofluids. For detailed investigation, we extracted 784 responses from their model in validation ranges of variables and categorized them in seven unique datasets with different temperatures. The statistical analysis gives p values of 0.995 and 0.927 for means and variances, respectively. Therefore, the test fails to reject the null hypothesis and there is no statistical significant difference between these seven groups, despite of changing considerable the nanofluids bulk temperature. We tried to develop a simple correlation neglecting the bulk nanofluids temperature using experimental data and nonlinear regression method (Eq. 18). The average absolute error of the proposed correlation from experimental data is 3.5%.

$$k_{\text{eff}} = k_b(1 + 15.9106\varphi^{1.26896}) \quad (18)$$

Models by Hemmat Esfe et al. [64]

Hemmat Esfe et al. [64] conducted an experimental study on thermal conductivity of alumina-ethylene glycol nanofluids. The experiments performed at temperature ranging from 24 to 50 °C while volume concentration up to 5%. They proposed two correlations for prediction the effective thermal conductivity of nanofluids (Eqs. 19 and 21).

$$\frac{k_{\text{eff}}}{k_b} = 1.04 + 5.91 \times 10^{-5}T + 0.00154T\varphi + 0.0195\varphi^2 - 0.014\varphi - 0.00253\varphi^3 - 0.000104T\varphi^2 - 0.0357 \times \sin(1.72 + 0.407\varphi^2 - 1.67\varphi) \quad (19)$$

$$\frac{k_{\text{eff}}}{k_b} = 0.999 + 9.581 \times 10^{-5}T + 0.00142T\varphi + 0.0519\varphi^2 + 0.00208\varphi^2 + 0.00208\varphi^4 - 0.00719\varphi - 0.0193\varphi^3 - 8.21 \times 10^{-5}T\varphi^2 \quad (20)$$

The result of the sensitivity analysis shows that the role of changing temperature on the effective thermal conductivity is negligible (Appendix, Fig. 8). We extracted 2548 responses from these two correlations in their valid range of performance and categorized them in seven unique datasets (seven datasets for each correlation). Statistical analysis for means and variances gives p values of 0.971 and 1.000, respectively, for Eq. 19 and 0.880 and 1.000, respectively, for Eq. 20. Therefore, it can be concluded that there is no statistical significant difference between datasets.

For more investigation, we tried to develop a simpler correlation neglecting the nanofluids bulk temperature using experimental data and nonlinear regression model (Eq. 21). This correlation predicts effective thermal conductivity of nanofluids (covering all 14 datasets) with average absolute error of 1.69%. Figs. It is clear that this simple correlation is accurate, reliable and well-fitted (Appendix, Figs. 9–10).

$$k_{\text{eff}} = k_b(1 + 7.05149\varphi) \quad (21)$$

Model by Hemmat Esfe et al. [65]

An interesting study on the thermal conductivity of hybrid nanofluids has been performed by Hemmat Esfe et al. [65]. They used experimental data about dispersion of Cu and TiO₂ nanoparticles with mean diameter 70 and 40 nm in a binary water-ethylene glycol (60:40) base fluid with volume concentration of 0.1–2%. They implemented artificial neural network (ANN) to correlate the result in a mathematical model. They considered temperature and volume concentration as input layers and relative thermal conductivity (the proportion of effective thermal conductivity of nanofluids to thermal conductivity of base fluid) as output layer. Finally, they developed a trigonometric correlation:

$$\frac{k_{\text{eff}}}{k_b} = 1.04 + 0.000589T + \frac{-0.000184}{T\varphi} \times \cos(6.11 + 0.00673T + 4.41T\varphi - 0.0414 \sin(T)) - 32.5\varphi \quad (22)$$

The result of sensitivity analysis shows that the most and least affective variables on the prediction of the effective

thermal conductivity of nanofluids based on their model are nanoparticles volume concentration and nanofluids temperature, respectively (Appendix, Fig. 11).

For a detailed investigation, we extracted 980 values from their model in mentioned ranges of variables and then categorized them onto seven unique datasets with different temperatures. The statistical analysis gives p values of 0.999 and 0.976 for means and variances of the model, respectively. This test fails to reject the null hypothesis, and therefore, there is no statistical significant difference between these seven datasets. This means that temperature doesn't affect considerably the responses, despite changing considerably temperature. We implemented a nonlinear regression method to propose Eq. 23 with an average absolute error of 3.86%. So, the proposed correlation is accurate, reliable and well-fitted.

$$k_{\text{eff}} = k_b(1 + \varphi^{0.30409}) \quad (23)$$

Model by Hemmat Esfe et al. [66]

Hemmat Esfe et al. [66] dispersed carbon nanotubes with mean average of 5–15 nm and alumina nanoparticles with mean diameter of 20 nm into water using an ultrasonic vibrator and a magnetic mixer to measure thermal conductivity of nanofluids, experimentally. Experiments were conducted with various solid volume concentrations of 0.02, 0.04, 0.1, 0.2, 0.4, 0.8 and 1.0% and various fluid temperatures of 303, 314, 323 and 332 K. Measured data reveal that the thermal conductivity of nanofluids highly depends on the solid volume concentration. They finally proposed Eq. 24 to predict the effective thermal conductivity of nanofluids. Results of sensitivity analysis indicate that the nanofluids temperature is the least affective variable on the prediction of the effective thermal conductivity (Appendix, Fig. 12).

$$\frac{k_{\text{eff}}}{k_b} = \frac{-214.83 + T}{346.58 - 106.98\varphi} + \frac{227.69}{T} \quad (24)$$

We extracted 1029 different responses from their model within their valid ranges of variables and then categorized them into seven unique datasets. The statistical analysis gives p values of 1.000 and 1.000 for mean and variances, respectively. This means that there is no statistical significant difference between seven datasets. So, changing considerably temperature doesn't affect considerably responses. We proposed a simple correlation using the nonlinear regression method to predict effective thermal conductivity based on experimental data. Equation 25 gives an average absolute error of 1.11%. So, the proposed correlation is very accurate and well-fitted.

$$k_{\text{eff}} = k_b(1 + 5.72977\varphi^{0.806027}) \quad (25)$$

Model by Harandi et al. [67]

Harandi et al. [67] conducted an experimental study on thermal conductivity of hybrid nanofluids and considered the effect of nanofluids bulk temperature and volume concentration. They dispersed F-MWCNT-Fe₃O₄ nanoparticles in ethylene glycol using an ultrasonic processor. The experiments were carried out for solid volume concentration range of 0 to 2.3% in temperatures ranging from 25 to 50 °C. They finally developed a correlation (Eq. 26) using their experimental data to predict effective thermal conductivity.

$$\frac{k_{\text{eff}}}{k_b} = 1 + 0.0162 \varphi^{0.7038} T^{0.6009} \quad (26)$$

We conducted a sensitivity analysis on their proposed model and concluded that the role of nanofluids temperature on the response can be neglected (Appendix, Fig. 13).

We extracted 828 responses from their model in valid range of variables and then categorized them into six unique datasets. The statistical analysis gives *p* values of 0.997 and 1.000 for mean and variances of seven unique datasets. So, temperature doesn't play a key role in prediction of effective thermal conductivity in their model. We proposed a simple correlation based on experimental data using the nonlinear regression method (Eq. 27).

$$k_{\text{eff}} = k_b(1 - 0.526495\varphi^{-0.0539237}) \quad (27)$$

The average absolute error of the proposed correlation is 2.7%. It can be concluded that the proposed correlation is accurate and well-fitted.

Model by Zadkhast et al. [68]

An another experimental study on the thermal conductivity of hybrid nanofluids has been performed by Zadkhast et al. [68]. They used an ultrasonic processor to disperse MWCNTs and CuO nanoparticles into water with volume concentration range of 0.05–0.6%. All thermal conductivity measurements are repeated three times in the range of 25–50 °C. A hot water bath is used to stabilize the temperature at 25, 30, 35, 40, 45 and 50 °C, during the measurements. They developed a mathematical model (Eq. 28) to predict the effective thermal conductivity of hybrid nanofluids using experimental data. It can be concluded that the most and least affective variables on the prediction of the effective thermal conductivity are the thermal conductivity of the base fluid and the nanofluids temperature (Appendix, Fig. 14).

$$\frac{k_{\text{eff}}}{k_b} = 0.907 \exp(0.36\varphi^{0.3111} + 0.000956T) \quad (28)$$

We extracted 432 responses from their model within their valid ranges and categorized them into six unique datasets with different temperatures. Statistical analysis gives *p* values of 0.065 and 1.000 for means and variances, respectively. Therefore, there is no statistical significant difference between datasets. We tried to propose a simple correlation using the nonlinear regression model based on experimental data. Equation 29 gives an average absolute error of 4.71%. So, the proposed correlation is accurate and well-fitted.

$$k_{\text{eff}} = k_b(1 + 0.574519\varphi) \quad (29)$$

Model by Kakavandi and Akbari [69]

Kakavandi and Akbari [69] measured thermal conductivity of SiC-MWCNT/water-EG hybrid nanofluids, experimentally. The volume concentration was in range of 0–0.75%. They finally introduced a correlation to predict the effective thermal conductivity of hybrid nanofluids (Eq. 30).

$$\frac{k_{\text{eff}}}{k_b} = 0.0017 \times \varphi^{0.698} \times T^{1.386} + 0.981 \quad (30)$$

We performed a sensitivity analysis on their proposed model, indicating that the nanofluids temperature doesn't play a key role in the prediction of the effective thermal conductivity. We tried to extract as much as possible responses from their model and categorized 576 responses into six unique datasets. Statistical analysis gives *p* values of 0.998 and 0.632 for means and variances of these datasets, respectively. Therefore, the test failed to reject the null hypothesis, and so, temperature doesn't affect considerably the response. We proposed a simple correlation neglecting the role of temperature using the nonlinear regression model based on experimental data with average absolute error of 6.27% from experimental data. Since the proposed correlation (Eq. 31) is accurate and well-fitted enough.

$$k_{\text{eff}} = k_b(1 + 39842\varphi^{2.61172}) \quad (31)$$

Model by Karimi et al. [70]

An experimental investigation of thermal conductivity of Fe₃O₄ and CoFe₂O₄—water magnetic nanofluids has been performed by Karimi et al. [70]. They synthesized nanoparticles using the co-precipitation method and the X-ray diffraction, transmission electronic microscopy and vibration sample magnetometer are implemented to characterize the structure, size and magnetic properties of nanoparticles. They measured thermal conductivity of nanofluids

with volume concentration in range of 0 to 4.8% and finally developed two correlations to predict effective thermal conductivity of different nanofluids (Eqs. 32 and 33).

$$k_{\text{eff}} = \left(1 + 359.28\varphi \left(\frac{T}{T_{\text{max}}}\right)\right)^{0.06418} \times (-6.58 \times 10^{-6}T^2 + 0.0018T + 0.5694) \text{ for Fe}_3\text{O}_4 \quad (32)$$

$$k_{\text{eff}} = \left(1 + 418.81\varphi \left(\frac{T}{T_{\text{max}}}\right)\right)^{0.06748} \times (-6.58 \times 10^{-6}T^2 + 0.0018T + 0.5694) \text{ for CoFe}_2\text{O}_4 \quad (33)$$

The results demonstrate that the most and least affective variables on the response are thermal conductivity of the base fluid and nanofluids temperature, respectively (Appendix, Fig. 15).

We extracted 4851 responses from these two models and categorized them into nine unique datasets. The statistical analysis of these responses gives p values of 0.982 and 0.930 for means and variances. It means that changing considerably temperature doesn't affect considerably responses. We proposed a simple correlation for prediction of effective thermal conductivity of nanofluids using the nonlinear regression method based on experimental data (Eq. 34).

$$k_{\text{eff}} = k_b(1 + 0.148349\varphi^{0.0614858}) \quad (34)$$

This correlation gives an average absolute error of 3.06% from experimental data. It can be concluded that this correlation is accurate and well-fitted.

Model by Karimipour et al. [71]

Karimipour et al. [71] measured thermal conductivity of CuO/liquid paraffin nanofluids in different range of volume concentration (0.25 to 6 mass%) and bulk temperature (25 to 100 °C), experimentally. They introduced a mathematical model to predict effective thermal conductivity of nanofluids (Eq. 35). The sensitivity analysis shows highest sensitivity on the thermal conductivity of the base fluid.

$$\frac{k_{\text{eff}}}{k_b} = 0.792194 + 0.0547913\varphi + 0.00998805T + 0.000730423\varphi T - 0.00421237\varphi^2 - 0.0000643292T^2 \quad (35)$$

This case needs more investigations; therefore, we extracted 6144 responses from their model in valid ranges of variables and categorized them into 16 unique datasets. Our statistical analysis gives p values of 1.000 and 0.980 for means and variances of these datasets. Therefore, nanofluids temperature doesn't play a vital role in estimation of effective thermal conductivity (Appendix, Fig. 16).

We proposed a simple correlation (Eq. 36), neglecting the role of bulk nanofluids temperature using the nonlinear regression method based on experimental data with an average absolute error of 7.34%. It can be concluded that the proposed correlation is accurate enough and well-fitted.

$$k_{\text{eff}} = k_b(1 + 2.13017\varphi^{0.500216}) \quad (36)$$

Model by Ranjbarzadeh et al. [72]

Ranjbarzadeh et al. [72] conducted an experimental study on the water–silica nanofluids with an eco-friendly method of nanoparticles production, recently. Their temperature and nanoparticle volume concentration ranges are 20–55 °C and 0.1 to 3%, respectively. They proposed a correlation using Levenberg–Marquardt algorithm, based on experimental data (Eq. 37):

$$\frac{k_{\text{eff}}}{k_b} = 1 + 0.4281 \left(\frac{T}{100}\right)^{1.707} \varphi^{0.8449} \quad (37)$$

It can be concluded that thermal conductivity of the base fluid is the most affective variable in effective thermal conductivity prediction and the response isn't much sensitive to the nanofluids temperature (Appendix, Fig. 17). For further investigation, we extracted 441 different responses from their correlation in above-mentioned range of variables and categorized them into seven unique datasets with different temperatures. We computed p values of 0.998 and 0.960 for means and variances, respectively, using statistical analysis. It means that when we change considerably the nanofluids temperature, the responses don't change significantly (Appendix, Fig. 18). We introduced a simple correlation using the nonlinear regression method, using experimental data (Eq. 38). This correlation predicts effective thermal conductivity of nanofluids with an average absolute error of 3.44%. It can be concluded that the proposed correlation is a simple, accurate and well-fitted.

$$k_{\text{eff}} = k_b(1 + 5.02743\varphi^{0.867218}) \quad (38)$$

Model by Keyvani et al. [73]

Keyvani et al. [73] conducted an experimental study on the measurement of cerium (CeO₂)-ethylene glycol nanofluids, probably for the first time. The samples were made in volume concentration range of 0.25–2.5% using a two-step method. Measurements were done for all samples at temperatures ranging from 25 to 50 °C. They eventually proposed a new correlation to predict the effective thermal conductivity of nanofluids (Eq. 39):

$$\frac{k_{\text{eff}}}{k_b} = 0.9320 + 0.0673\varphi + 0.0021T \quad (39)$$

As a result of the investigations, the response is a high sensitive function of the nanoparticle volume concentration and relatively insensitive to the nanofluids temperature (Appendix, Fig. 19).

For further investigation, we extracted 360 responses from their model within the valid range of variables and categorized them into six unique datasets. The statistical analysis of the means and variances gives 0.997 and 0.984, respectively. This shows that there is no significant difference between datasets. We proposed a simpler correlation neglecting the nanofluids temperature using experimental data (Eq. 40) with an average absolute error of 5.66% from experimental values.

$$k_{\text{eff}} = k_b(1 + 556.762\varphi^{2.28623}) \quad (40)$$

Model by Afrand et al. [74]

An experimental study on thermal conductivity of Fe_3O_4 -water nanofluids has been performed by Afrand et al. [74] with the nanoparticle volume concentrations of 0.1%, 0.2%, 0.4%, 1%, 2% and 3% under temperatures ranging from 20 °C to 55 °C. They introduced a new correlation for prediction of effective thermal conductivity of nanofluids (Eq. 41).

$$\frac{k_{\text{eff}}}{k_b} = 0.9320 + 0.0673\varphi^{0.323}T^{0.245} \quad (41)$$

It is obvious from the statistical considerations (Appendix, Fig. 20) that the most and least affective variables on the effective thermal conductivity are the nanoparticle volume concentration and nanofluids temperature, respectively.

We extracted 1920 responses from their model and categorized them into eight unique datasets. Our statistical analysis gives p values of 0.937 and 0.926 for means and variances of datasets, respectively. We tried to propose a replacing simpler correlation ignoring the role of nanofluids temperature. The proposed correlation gives an average absolute error of 5.98%.

$$k_{\text{eff}} = k_b(1 + 2.42534\varphi^{0.241448}) \quad (42)$$

Table 2 lists published experimental model for effective thermal conductivity of nanofluids considering the nanofluids bulk temperature. P values for means and variances are listed based on statistical analysis. The proposed replacing correlation (neglecting the role of temperature) for each model is shown in a separate column. The average absolute error and the maximum deviation for each correlation also

are mentioned. All proposed correlations have been proved simple accurate, reliable and well-fitted.

A comprehensive correlation

To propose a comprehensive correlation, we need to take a statistical look at the result of sensitivity analysis of published models. As it shown in Table 3, the nanofluids temperature is the most frequent variable with title of “Least Affective Variable”. Since we ignored the role of nanofluids temperature and used 603 experimental data extracted from 10 published experimental papers (all mentioned papers except those related to hybrid nanofluids) to correlate a comprehensive mathematical model (Eq. 43) which had been derived employing the nonlinear regression method to predict effective thermal conductivity of nanofluids:

$$k_{\text{eff}} = k_b \left(1 + 1.9647 \varphi^{0.410499} k_{\text{pr}}^{0.169384} d_{\text{pr}}^{-0.244673} \right) \quad (43)$$

$$k_{\text{pr}} = \frac{k_p}{k_{\text{p,ref}}}, \quad d_{\text{pr}} = \frac{d_p}{d_{\text{p,ref}}}$$

where k_b , k_{pr} , φ and d_{pr} are thermal conductivity of base fluid (W m K^{-1}), thermal conductivity ratio of nanoparticles (W m K^{-1}), nanoparticle volume concentration and mean diameter ratio of nanoparticles (nm). Thermal conductivity ratio of nanoparticles is assumed thermal conductivity ($k_{\text{p,ref}}$), which is 1500 (W m K^{-1}) for MWCNTs. Mean diameter ratio of nanoparticles also is the proportion of mean diameter of used nanoparticles to the reference value of mean diameter ($d_{\text{p,ref}} = 10$). Equation 39 is dimensionless and its average absolute error of proposed compared to the experimental data is 5.39%. This correlation is much simpler than the other correlations involving trigonometric (sinus and cosine), exponential, logarithmic, and so on. Furthermore, it covers all experimental measured data in volume concentrations ranging from 0 to 4% and mean diameter of nanoparticles ranging from 10 to 150 nm. Figure 6 demonstrates the fitness of predicted values versus experimental data. Figure 7 also shows the accuracy of the proposed correlation. Table 4 shows the predicted values of effective thermal conductivity versus the experimental data and their deviations. Two last columns list the correlated values by the models provided by the other researchers and their deviations. As shown, our proposed correlation gives more accurate values. It finally can be concluded that the proposed correlation is comprehensive, reliable in mentioned range of variables, simpler than the other correlations, accurate enough and well-fitted on the experimental values. Therefore, it recommended to use this correlation in prediction of nanofluids thermal conductivity in studies employing the single-phase approach.

Table 2 The summary of statistical analysis on the published models and their proposed replacing correlations

References	Model	P value for means	P value for vari-ances	Proposed correlation	Average error (%)	Maximum error (%)
1	Aberoumand et al. [58]	0.963	0.332	$k_{\text{eff}} = k_b(1 + 1.25536\varphi^{0.29397})$ $+ 3.1 \times 10^{-4}T + 0.129 - 5.77 \times 10^{-6}k_p - 40 \times 10^{-4}T\varphi$	3.62	8.82
2	Fakoor Pakdaman et al. [59]	1.000	1.000	$k_{\text{eff}} = 1 + 304.47(1 + \varphi)^{136.35} \exp(-0.021T) \left(\frac{1}{d_p}\right)^{0.369} \left(\frac{T^{1.2331}}{10^{0.0287(T-140)}}\right)$	2.41	14.04
3	Ahamed et al. [60]	0.772	0.762	$\frac{k_{\text{eff}}}{k_b} = 2.5T^{1.032} \varphi^{0.108}$	3.31	7.37
4	Patel et al. [61]	0.000	0.938	$k_{\text{eff}} = k_b \left(1 + 0.135 \times \left(\frac{k_p}{k_b}\right)^{0.273} \varphi^{0.467} \left(\frac{T}{20}\right)^{0.547} \left(\frac{100}{d_p}\right)^{0.234}\right)$	3.44	10.68
5	Li & Peterson [62]	0.736 0.755	0.927 0.925	$\frac{k_{\text{eff}}}{k_b} = 0.7644815\varphi + 0.018689T + 0.537853$, for Al_2O_3 $k_{\text{eff}} = 3.761088\varphi + 0.017924T + 0.69266$, for CuO	4.88 2.79	11.41 7.74
6	Hemmat Esfe et al. [63]	0.995	0.927	$k_{\text{eff}} = 0.4 + 0.0332\varphi + 0.001017T + 0.000619\varphi T + 0.0687\varphi^3$ $+ 0.0148\varphi^5 - 0.00218\varphi^6 - 0.0419\varphi^4 - 0.0604\varphi^2$	3.5	10.26
7	Hemmat Esfe et al. [64]	0.971 0.880	1.000 1.000	$\frac{k_{\text{eff}}}{k_b} = 1.04 + 5.91 \times 10^{-5}T + 0.00154T\varphi + 0.0195\varphi^2$ $- 0.014\varphi - 0.00253\varphi^3 - 0.000104T\varphi^2 - 0.0357$ $\times \sin(1.72 + 0.407\varphi^2 - 1.67\varphi)$	1.69	6.84
8	Hemmat Esfe et al. [65]	0.999	0.976	$\frac{k_{\text{eff}}}{k_b} = 0.999 + 9.581 \times 10^{-5}T + 0.00142T\varphi + 0.0519\varphi^2$ $+ 0.00208\varphi^2 + 0.00208\varphi^4 - 0.00719\varphi - 0.0193\varphi^3$ $- 8.21 \times 10^{-5}T\varphi^2$	3.86	8.53
9	Hemmat Esfe et al. [66]	1.000	1.000	$\frac{k_{\text{eff}}}{k_b} = 1.04 + 0.000589T + \frac{-0.000184}{T\varphi}$ $\times \cos(6.11 + 0.00673T + 4.41T\varphi - 0.0414 \sin(T)) - 32.5\varphi$	1.11	3.43
10	Harandi et al. [67]	0.997	1.000	$\frac{k_{\text{eff}}}{k_b} = \frac{-214.83+T}{346.58-106.98\varphi} + \frac{227.69}{T}$ $= 1 + 0.0162\varphi^{0.7038}T^{0.6009}$	2.70	7.59
11	Zadkhasht et al. [68]	0.065	1.000	$\frac{k_{\text{eff}}}{k_b} = 0.907 \exp(0.36\varphi^{0.3111} + 0.0009567T)$	4.71	7.20
12	Kakavandi and Akbari [69]	0.998	0.632	$\frac{k_{\text{eff}}}{k_b} = 0.0017 \times \varphi^{0.698} \times T^{1.386} + 0.981$	6.27	32.51

Table 2 (continued)

References	Model	P value for means	P value for variances	Proposed correlation	Average error (%)	Maximum error (%)
13 Karimi et al. [70]	$k_{\text{eff}} = (1 + 359.28\varphi \left(\frac{T}{T_{\text{max}}}\right)^{0.06418} \times (-6.58 \times 10^{-6}T^2 + 0.0018T + 0.5694) \text{ for Fe}_3\text{O}_4$ $k_{\text{eff}} = (1 + 418.81\varphi \left(\frac{T}{T_{\text{max}}}\right)^{0.06748} \times (-6.58 \times 10^{-6}T^2 + 0.0018T + 0.5694) \text{ for CoFe}_2\text{O}_4$	0.982	0.930	$k_{\text{eff}} = k_b(1 + 0.148349\varphi^{0.0614858})$	3.06	9.21
14 Karimipour et al. [71]	$\frac{k_{\text{eff}}}{k_b} = 0.792194 + 0.0547913\varphi + 0.00998805T + 0.000730423\varphi T - 0.00421237\varphi^2 - 0.0000643292T^2$	1.000	0.980	$k_{\text{eff}} = k_b(1 + 2.13017\varphi^{0.500216})$	7.34	19.13
15 Ranjbarzadeh et al. [72]	$\frac{k_{\text{eff}}}{k_b} = 1 + 0.4281 \left(\frac{T}{100}\right)^{1.707} \varphi^{0.8449}$	0.998	0.960	$k_{\text{eff}} = k_b(1 + 5.02743\varphi^{0.867218})$	3.44	10.24
16 Keyvani et al. [73]	$\frac{k_{\text{eff}}}{k_b} = 0.9320 + 0.0673\varphi + 0.0021T$	0.997	0.984	$k_{\text{eff}} = k_b(1 + 556.762\varphi^{2.28623})$	5.66	13.65
17 Afrand et al. [74]	$\frac{k_{\text{eff}}}{k_b} = 0.9320 + 0.0673\varphi^{0.323}T^{0.245}$	0.937	0.926	$k_{\text{eff}} = k_b(1 + 2.42534\varphi^{0.241448})$	5.98	14.16

Table 3 The summary of sensitivity analysis of the published models

References	Most affective variable	Least affective variable
Aberoumand et al. [58]	Volume concentration	Thermal conductivity of nanoparticles
Fakoor Pakdaman et al. [59]	Mean diameter of nanoparticles	Thermal conductivity of the base fluid
Ahammed et al. [60]	Nanofluids temperature	Thermal conductivity of the base fluid, volume concentration
Patel et al. [61]	Thermal conductivity of the base fluid	Nanofluids temperature, thermal conductivity of nanoparticles
Li and Peterson [62]	Volume concentration	Nanofluids temperature, thermal conductivity of the base fluid
Hemmat Esfe et al. [63]	Volume concentration	Nanofluids temperature
Hemmat Esfe et al. [64]	Volume concentration	Nanofluids temperature
Hemmat Esfe et al. [65]	Volume concentration	Nanofluids temperature
Hemmat Esfe et al. [66]	Volume concentration	Nanofluids temperature
Harandi et al. [67]	Thermal conductivity of the base fluid	Nanofluids temperature, volume concentration
Zadkhast et al. [68]	Thermal conductivity of the base fluid	Nanofluids temperature
Kakavandi and Akbari [69]	Thermal conductivity of the base fluid	Nanofluids temperature
Karimi et al. [70]	Thermal conductivity of the base fluid	Nanofluids temperature
Karimipour et al. [71]	Volume concentration	Thermal Conductivity of the Base Fluid
Ranjbarzadeh et al. [72]	Thermal conductivity of the base fluid	Nanofluids temperature, volume concentration
Keyvani et al. [73]	Volume concentration	Nanofluids temperature
Afrand et al. [74]	Volume concentration	Nanofluids temperature

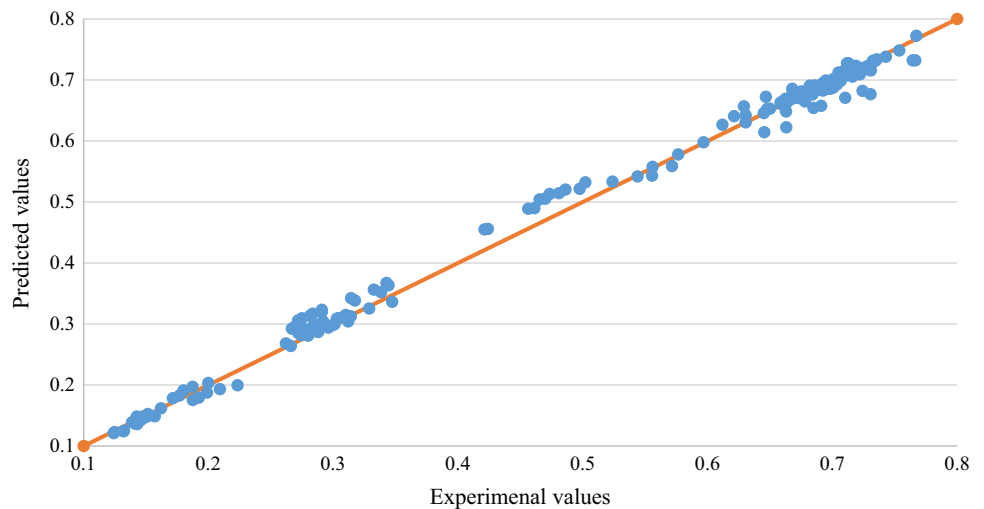
Fig. 6 The fitness of responses predicted by the proposed correlation on the experimental values

Table 4 The predicted values of effective thermal conductivity (by our proposed correlation and the other correlations) versus the experimental data and their deviations

Ref	Temperature/ ^o C	Nanoparticle volume concentration	Nanoparticle thermal conductivity /W mK ⁻¹	Base fluid thermal conductivity/W mK ⁻¹	Nanoparticle mean diameter/nm	Measured thermal conductivity/W mK ⁻¹	Predicted thermal conductivity/W mK ⁻¹	Deviation (%)	Correlated thermal conductivity/W mK ⁻¹ by Ref	Deviation/%
Aberoumand et al. [58]	60	0	403	0.1228	20	0.12505	0.12280	-1.80	0.14483	15.82
	70	0	403	0.1214	20	0.12407	0.12140	-2.15	0.14793	19.23
	40	0.0012	403	0.125	20	0.14108	0.13702	-2.87	0.13871	-1.68
	50	0.0012	403	0.124	20	0.14304	0.13592	-4.97	0.14178	-0.88
	40	0.0036	403	0.125	20	0.14500	0.14387	-0.78	0.13899	-4.15
	50	0.0036	403	0.124	20	0.14615	0.14272	-2.34	0.14209	-2.78
	40	0.0072	403	0.125	20	0.15269	0.15008	-1.71	0.13930	-8.77
	50	0.0072	403	0.124	20	0.15710	0.14888	-5.23	0.14234	-9.40
	10	0	1500	0.578	20	0.57627	0.57800	0.30	-	-
	Ahammed et al. [60]	20	0	1500	0.598	20	0.59661	0.59800	0.23	-
10		0.0005	1500	0.578	20	0.61186	0.62648	2.39	0.24967	-59.19
20		0.0005	1500	0.598	20	0.66271	0.64815	-2.20	0.52537	-20.72
10		0.001	1500	0.578	20	0.63051	0.64243	1.89	0.26908	-57.32
20		0.001	1500	0.598	20	0.67797	0.66466	-1.96	0.56315	-16.93
10		0.0015	1500	0.578	20	0.68475	0.65410	-4.47	0.27810	-59.39
20		0.0015	1500	0.598	20	0.73051	0.67674	-7.36	0.58516	-19.90
45		0	69.036	0.455	40	0.42113	0.45500	8.04	0.44563	5.82
50		0	69.036	0.456	40	0.42400	0.45600	7.55	0.45057	6.27
45		0.002	69.036	0.455	40	0.45612	0.48878	7.16	0.44568	-2.29
Hemmat Esfe et al. [63]	50	0.002	69.036	0.456	40	0.46128	0.48985	6.19	0.45071	-2.29
	45	0.005	69.036	0.455	40	0.46530	0.50420	8.36	0.44586	-4.18
	50	0.005	69.036	0.456	40	0.46989	0.50531	7.54	0.45099	-4.02
	45	0.0075	69.036	0.455	40	0.47333	0.51311	8.41	0.44602	-5.77
	50	0.0075	69.036	0.456	40	0.48078	0.51424	6.96	0.45106	-6.18
	45	0.01	69.036	0.455	40	0.48595	0.52040	7.09	0.44617	-8.18
	50	0.01	69.036	0.456	40	0.49742	0.52154	4.85	0.45123	-9.29
	45	0.015	69.036	0.455	40	0.50201	0.53224	6.02	0.44647	-11.06
	50	0.015	69.036	0.456	40	0.52381	0.53341	1.83	0.45146	-13.81
	45	0.02	69.036	0.455	40	0.54388	0.54192	-0.36	0.44678	-17.85
50	0.02	69.036	0.456	40	0.55535	0.54312	-2.20	0.45187	-18.63	
45	0.03	69.036	0.455	40	0.55593	0.55767	0.31	0.44737	-19.53	
50	0.03	69.036	0.456	40	0.57142	0.55889	-2.19	0.45240	-20.83	

Table 4 (continued)

Ref	Temperature/ $^{\circ}\text{C}$	Nanoparticle volume concentration	Nanoparticle thermal conductivity/ W mK^{-1}	Base fluid thermal conductivity/ W mK^{-1}	Nanoparticle mean diameter/ nm	Measured thermal conductivity/ W mK^{-1}	Predicted thermal conductivity/ W mK^{-1}	Deviation (%)	Correlated thermal conductivity/ W mK^{-1} by Ref	Deviation/%
	45	0.002	38	0.266	5	0.26932	0.29568	9.79	0.26799	-0.49
	50	0.002	38	0.263	5	0.26676	0.29235	9.59	0.26505	-0.64
	45	0.005	38	0.266	5	0.27501	0.30924	12.45	0.26803	-2.54
	50	0.005	38	0.263	5	0.27182	0.30575	12.48	0.26509	-2.48
	45	0.0075	38	0.266	5	0.28355	0.31707	11.82	0.26806	-5.46
	50	0.0075	38	0.263	5	0.28139	0.31350	11.41	0.26513	-5.78
	45	0.01	38	0.266	5	0.29095	0.32347	11.18	0.26809	-7.86
	50	0.01	38	0.263	5	0.29096	0.31983	9.92	0.26517	-8.87
	45	0.02	38	0.266	5	0.31429	0.34239	8.94	0.26822	-14.66
	50	0.02	38	0.263	5	0.31742	0.33853	6.65	0.26532	-16.41
	45	0.03	38	0.266	5	0.33250	0.35623	7.14	0.26836	-19.29
	50	0.03	38	0.263	5	0.33824	0.35221	4.13	0.26547	-21.51
	45	0.04	38	0.266	5	0.34275	0.36754	7.23	0.26850	-21.66
	50	0.04	38	0.263	5	0.34443	0.36339	5.51	0.26563	-22.88
	25	0.0025	18	0.167	30	0.17148	0.17861	4.16	0.16760	-2.26
Karimipour et al. [71]	40	0.0025	18	0.164	30	0.18746	0.17540	-6.44	0.17866	-4.69
	25	0.005	18	0.167	30	0.17678	0.18243	3.20	0.16732	-5.35
	40	0.005	18	0.164	30	0.19228	0.17915	-6.83	0.17858	-7.13
	25	0.015	18	0.167	30	0.18006	0.19122	6.20	0.16766	-6.89
	40	0.015	18	0.164	30	0.19893	0.18779	-5.60	0.17886	-10.09
	25	0.025	18	0.167	30	0.18734	0.19687	5.09	0.16769	-10.49
	40	0.025	18	0.164	30	0.20918	0.19334	-7.57	0.17897	-14.44
	25	0.04	18	0.167	30	0.19988	0.20323	1.68	0.16789	-16.00
	40	0.04	18	0.164	30	0.22329	0.19958	-10.62	0.17910	-19.77
	33	0.02	38	0.618	36	0.71265	0.72750	2.08	0.72097	1.17
Li and Peterson [62]	35	0.02	38	0.621	36	0.73419	0.73103	-0.43	0.75043	2.21
	35	0.02	38	0.622	36	0.76644	0.73221	-4.47	0.76006	-0.83
	33	0.02	38	0.618	36	0.71166	0.72750	2.23	0.72083	1.29
	35	0.02	38	0.621	36	0.73227	0.73103	-0.17	0.75000	2.42
	35	0.02	38	0.622	36	0.76418	0.73221	-4.18	0.75996	-0.55
	29	0.04	18	0.613	29	0.82073	0.74711	-8.97	0.83482	1.72
	29	0.04	18	0.613	29	0.81876	0.74711	-8.75	0.83460	1.93

Table 4 (continued)

Ref	Temperature/°C	Nanoparticle volume concentration	Nanoparticle thermal conductivity /W mK ⁻¹	Base fluid thermal conductivity/W mK ⁻¹	Nanoparticle mean diameter/nm	Measured thermal conductivity/W mK ⁻¹	Predicted thermal conductivity/W mK ⁻¹	Deviation (%)	Correlated thermal conductivity/W mK ⁻¹ by Ref	Deviation/%
Fakoor Pakdaman et al. [59]	31	0.04	18	0.615	29	0.84978	0.74955	-11.80	0.86321	1.58
	40	0	1500	0.125	20	0.13250	0.12500	-5.67	0.13117	-1.00
	50	0	1500	0.124	20	0.13208	0.12400	-6.12	0.13205	-0.03
	40	0.001	1500	0.125	20	0.13890	0.13893	0.03	0.13208	-4.91
	50	0.001	1500	0.124	20	0.14143	0.13782	-2.55	0.13322	-5.80
	40	0.002	1500	0.125	20	0.14185	0.14352	1.18	0.13311	-6.16
	50	0.002	1500	0.124	20	0.14487	0.14237	-1.72	0.13457	-7.11
	50	0.004	1500	0.124	20	0.14733	0.14842	0.74	0.13787	-6.42
	60	0.004	1500	0.1228	20	0.14936	0.14698	-1.60	0.14002	-6.26
	50	0.005	38	0.263	150	0.27422	0.28160	2.69	0.28045	2.27
Patel et al. [61]	50	0.01	38	0.263	150	0.27743	0.28772	3.71	0.28717	3.51
	50	0.02	38	0.263	150	0.28368	0.29586	4.29	0.29637	4.47
	50	0.03	38	0.263	150	0.29186	0.30181	3.41	0.30329	3.91
	50	0.005	38	0.263	11	0.29960	0.29825	-0.45	0.29515	-1.48
	50	0.01	38	0.263	11	0.30478	0.30986	1.67	0.30750	0.89
	50	0.02	38	0.263	11	0.32875	0.32528	-1.05	0.32452	-1.29
	50	0.03	38	0.263	11	0.34721	0.33656	-3.07	0.33727	-2.86
	20	0.00495	18	0.136	31	0.14257	0.14841	4.10	0.14368	0.78
	30	0.010057	18	0.136	31	0.15138	0.15260	0.81	0.14935	-1.34
	40	0.030122	18	0.136	31	0.16200	0.16205	0.03	0.16208	0.05
	50	0.005021	18	0.136	31	0.14902	0.14848	-0.36	0.14876	-0.18
	20	0.005032	38	0.598	150	0.62118	0.64041	3.09	0.61726	-0.63
	30	0.004792	38	0.614	150	0.62903	0.65668	4.40	0.63796	1.42
	40	0.005032	38	0.628	150	0.64670	0.67253	4.00	0.65716	1.62
	50	0.005032	38	0.64	150	0.66769	0.68539	2.65	0.67341	0.86
	50	0.001	413	0.64	80	0.67529	0.68085	0.82	0.67489	-0.06
	50	0.005	413	0.64	80	0.72263	0.71909	-0.49	0.71399	-1.20
	30	0.01	413	0.614	80	0.70921	0.71485	0.80	0.68898	-2.85
30	0.02	413	0.614	80	0.75343	0.74804	-0.71	0.71782	-4.73	
30	0.03	413	0.614	80	0.76717	0.77232	0.67	0.73969	-3.58	
20	0.009993	237	0.598	80	0.70054	0.68738	-1.88	0.64867	-7.40	
30	0.005028	237	0.64	80	0.70476	0.71215	1.05	0.68822	-2.35	
40	0.010064	237	0.628	80	0.72752	0.72213	-0.74	0.70498	-3.10	

Table 4 (continued)

Ref	Temperature/°C	Nanoparticle volume concentration	Nanoparticle thermal conductivity/W mK ⁻¹	Base fluid thermal conductivity/W mK ⁻¹	Nanoparticle mean diameter/nm	Measured thermal conductivity/W mK ⁻¹	Predicted thermal conductivity/W mK ⁻¹	Deviation (%)	Correlated thermal conductivity/W mK ⁻¹ by Ref	Deviation/%
Zadkhast et al. [68]	50	0.005024	237	0.614	80	0.68785	0.68320	-0.68	0.67585	-1.74
	25	0.002	1500	0.606	15	0.70389	0.70225	-0.23	0.59300	-15.75
	25	0.00399	1500	0.606	15	0.73538	0.73394	-0.20	0.60048	-18.34
	30	0.00101	1500	0.614	15	0.68513	0.68776	0.38	0.59775	-12.75
	30	0.00152	1500	0.614	15	0.70055	0.70127	0.10	0.60118	-14.18
	35	0.00099	1500	0.621	15	0.69226	0.69482	0.37	0.60726	-12.28
	35	0.00154	1500	0.621	15	0.70879	0.70957	0.11	0.61102	-13.79
	40	0.00052	1500	0.628	15	0.68234	0.68563	0.48	0.61247	-10.24
	40	0.001	1500	0.628	15	0.70545	0.70305	-0.34	0.61715	-12.52
	45	0.00051	1500	0.634	15	0.69552	0.69160	-0.56	0.62112	-10.70
	45	0.00099	1500	0.634	15	0.72192	0.70936	-1.74	0.62592	-13.30
	50	0.00051	1500	0.64	15	0.70650	0.69814	-1.18	0.63000	-10.83
	50	0.00101	1500	0.64	15	0.72906	0.71688	-1.67	0.63509	-12.89
	50	0.005	0.58	0.64295	10	0.68133	0.68639	0.74	0.68185	0.08
	Karimi et al. [70]	55	0.005	0.58	0.64849	10	0.69525	0.69231	-0.42	0.69033
60		0.005	0.58	0.65371	10	0.70032	0.69788	-0.35	0.69832	-0.28
35		0.01	0.58	0.62434	10	0.67822	0.68040	0.32	0.67134	-1.01
40		0.01	0.58	0.63087	10	0.68714	0.68752	0.06	0.68237	-0.69
45		0.01	0.58	0.63707	10	0.70046	0.69428	-0.88	0.69286	-1.09
30		0.02	0.58	0.61747	10	0.68583	0.69118	0.78	0.68105	-0.70
35		0.02	0.58	0.62434	10	0.69476	0.69886	0.59	0.69400	-0.11
40		0.02	0.58	0.63087	10	0.70992	0.70617	-0.53	0.70615	-0.53
35		0.03	0.58	0.62434	10	0.70949	0.71236	0.40	0.70928	-0.03
40		0.03	0.58	0.63087	10	0.72152	0.71981	-0.24	0.72202	0.07
45		0.03	0.58	0.63707	10	0.73136	0.72689	-0.61	0.73389	0.35
35		0.04	0.58	0.62434	10	0.71867	0.72339	0.66	0.72098	0.32
40		0.04	0.58	0.63087	10	0.73313	0.73096	-0.30	0.73419	0.14
45		0.04	0.58	0.63707	10	0.74263	0.73815	-0.60	0.74639	0.51
35		0.005	0.635	0.62434	15	0.65833	0.66313	0.73	0.65369	-0.70
40	0.005	0.635	0.63087	15	0.67335	0.67006	-0.49	0.66357	-1.45	
45	0.005	0.635	0.63707	15	0.68088	0.67665	-0.62	0.67287	-1.18	
55	0.005	0.635	0.64849	15	0.69448	0.68878	-0.82	0.69021	-0.62	

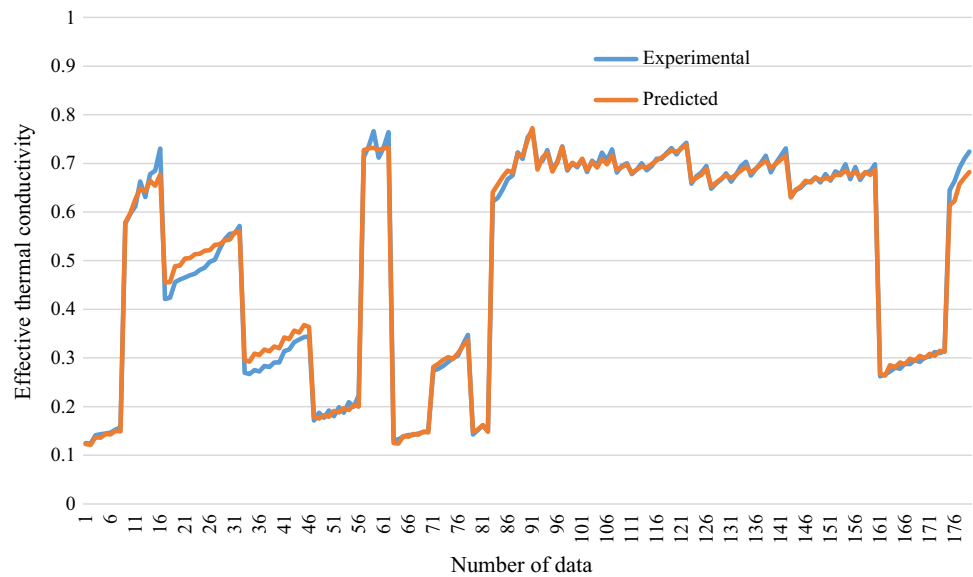
Table 4 (continued)

Ref	Temperature/°C	Nanoparticle volume concentration	Nanoparticle thermal conductivity /W mK ⁻¹	Base fluid thermal conductivity/W mK ⁻¹	Nanoparticle mean diameter/nm	Measured thermal conductivity/W mK ⁻¹	Predicted thermal conductivity/W mK ⁻¹	Deviation (%)	Correlated thermal conductivity/W mK ⁻¹ by Ref	Deviation/%
	20	0.01	0.635	0.60276	15	0.64767	0.65254	0.75	0.63404	-2.10
	25	0.01	0.635	0.61028	15	0.65881	0.66068	0.28	0.64737	-1.74
	30	0.01	0.635	0.61747	15	0.66701	0.66847	0.22	0.65950	-1.13
	35	0.01	0.635	0.62434	15	0.67934	0.67589	-0.51	0.67125	-1.19
	20	0.02	0.635	0.60276	15	0.66234	0.66893	0.99	0.65212	-1.54
	25	0.02	0.635	0.61028	15	0.67759	0.67727	-0.05	0.66701	-1.56
	30	0.02	0.635	0.61747	15	0.69486	0.68525	-1.38	0.68091	-2.01
	35	0.02	0.635	0.62434	15	0.70348	0.69287	-1.51	0.69406	-1.34
	20	0.03	0.635	0.60276	15	0.67485	0.68091	0.90	0.66477	-1.49
	25	0.03	0.635	0.61028	15	0.68895	0.68940	0.07	0.68077	-1.19
	30	0.03	0.635	0.61747	15	0.70149	0.69752	-0.56	0.69559	-0.84
	35	0.03	0.635	0.62434	15	0.71599	0.70528	-1.50	0.70915	-0.95
	20	0.04	0.635	0.60276	15	0.68176	0.69070	1.31	0.67488	-1.01
	25	0.04	0.635	0.61028	15	0.70075	0.69932	-0.20	0.69152	-1.32
	30	0.04	0.635	0.61747	15	0.71563	0.70756	-1.13	0.70675	-1.24
	35	0.04	0.635	0.62434	15	0.73074	0.71542	-2.10	0.72085	-1.35
	40	0	1.4	0.63040	40.77	0.63049	0.63040	-0.01	0.63040	0.0149
Ranjbarzadeh et al. [72]	55	0	1.4	0.64570	40.77	0.64489	0.64570	0.12	0.64570	-0.1248
	45	0.001	1.4	0.63450	40.77	0.64998	0.65273	0.42	0.63470	2.4063
	55	0.001	1.4	0.64570	40.77	0.66014	0.66425	0.62	0.64599	2.1905
	45	0.0025	1.4	0.63450	40.77	0.66268	0.66105	-0.25	0.63494	4.3692
	50	0.0025	1.4	0.64380	40.77	0.67115	0.67074	-0.06	0.64433	4.1622
	40	0.005	1.4	0.63040	40.77	0.66099	0.66546	0.68	0.63104	4.7454
	45	0.005	1.4	0.63450	40.77	0.67793	0.66979	-1.20	0.63529	6.7117
	35	0.01	1.4	0.62150	40.77	0.66438	0.66745	0.46	0.62241	6.7433
	40	0.01	1.4	0.63040	40.77	0.68386	0.67700	-1.00	0.63155	8.282
	35	0.015	1.4	0.62150	40.77	0.67793	0.67577	-0.32	0.62278	8.8561
	40	0.015	1.4	0.63040	40.77	0.69826	0.68544	-1.84	0.63203	10.48
	30	0.02	1.4	0.61560	40.77	0.66777	0.67609	1.25	0.61684	8.2561
	35	0.02	1.4	0.62150	40.77	0.69233	0.68257	-1.41	0.62313	11.106
	25	0.025	1.4	0.60600	40.77	0.66607	0.67126	0.78	0.60708	9.7175
	30	0.025	1.4	0.61560	40.77	0.68132	0.68189	0.08	0.61710	10.407
	25	0.03	1.4	0.60600	40.77	0.68386	0.67633	-1.10	0.60726	12.614

Table 4 (continued)

Ref	Temperature/°C	Nanoparticle volume concentration	Nanoparticle thermal conductivity/W mK ⁻¹	Base fluid thermal conductivity/W mK ⁻¹	Nanoparticle mean diameter/nm	Measured thermal conductivity/W mK ⁻¹	Predicted thermal conductivity/W mK ⁻¹	Deviation (%)	Correlated thermal conductivity/W mK ⁻¹ by Ref	Deviation/%
Keyvani et al. [73]	30	0.03	1.4	0.61560	40.77	0.69826	0.68704	-1.61	0.61734	13.107
	45	0	11.71	0.268	30	0.262	0.26800	2.29	0.27510	5.00
	50	0	11.71	0.264	30	0.266	0.26400	-0.75	0.27377	2.92
	45	0.0025	11.71	0.268	30	0.272	0.28532	4.90	0.27961	2.80
	50	0.0025	11.71	0.264	30	0.28	0.28106	0.38	0.27821	-0.64
	45	0.005	11.71	0.268	30	0.278	0.29103	4.69	0.28412	2.20
	50	0.005	11.71	0.264	30	0.288	0.28668	-0.46	0.28265	-1.86
	45	0.01	11.71	0.268	30	0.287	0.29860	4.04	0.29314	2.14
	50	0.01	11.71	0.264	30	0.296	0.29415	-0.63	0.29154	-1.51
	45	0.015	11.71	0.268	30	0.292	0.30415	4.16	0.30216	3.48
	50	0.015	11.71	0.264	30	0.301	0.29961	-0.46	0.30042	-0.19
	45	0.02	11.71	0.268	30	0.303	0.30868	1.87	0.31117	2.70
	50	0.02	11.71	0.264	30	0.312	0.30407	-2.54	0.30930	-0.86
Afrand et al. [74]	40	0.025	11.71	0.27	30	0.31	0.31491	1.58	0.31975	3.14
	45	0.025	11.71	0.268	30	0.314	0.31258	-0.45	0.32019	1.97
	20	0.001044	0.58	0.59803	30	0.64539	0.614269	-4.82237	0.6141	-4.8491
	25	0.001044	0.58	0.606	30	0.6629	0.622455	-6.1009	0.62392	-5.8806
	35	0.000986	0.58	0.621	30	0.69091	0.637471	-7.73491	0.63611	-7.9322
	45	0.000986	0.58	0.634	30	0.71017	0.650816	-8.35754	0.6486	-8.669
	55	0.000986	0.58	0.645	30	0.72417	0.662108	-8.57058	0.65915	-8.9792

Fig. 7 The accuracy of proposed correlation



Conclusions

This paper takes a different look at the published mathematical model of the effective thermal conductivity of nanofluids. There are a couple of correlations considering different variables with different roles. Nevertheless, there is no a global agreement on the role of different variables on the nanofluids thermal conductivity. We started with the models which driven from an experimental study. Our sensitivity analysis demonstrated that in most published models, the role of nanofluids bulk temperature is negligible. For detailed investigation, we extracted a big data bank from the models in the valid ranges of variables. The next step was performing statistical analysis of the variances and means of different datasets (data populations). The results showed that there is no significant statistical difference between different datasets. It means that changing considerably the nanofluids temperature doesn't affect considerably the nanofluids thermal conductivity.

We don't claim that nanofluids bulk temperature really doesn't affect the effective thermal conductivity. Nonetheless, the proposed mathematical models for effective thermal conductivity of nanofluids didn't devote a considerable role for temperature. Since we believe that the process of developing a correlation should be more detailed and the resulting correlation must satisfy physical aspects of phenomena. Moreover, it was shown that the most of proposed correlations for effective thermal conductivity are dimensionally inconsistent.

We introduced a replacing simpler correlation (neglecting the nanofluids temperature) for each published model with a low average absolute error from experimental data. At the end, we proposed a comprehensive correlation to estimate all available experimental data. In this correlation, the role of

nanofluids bulk temperature has been ignored. However, the predicted values of the effective thermal conductivity are in a good agreement with experimental results. Future studies can be focused on investigation of effect of temperature on all effective parameters of nanofluids. One also can experimentally measure the thermal conductivity of nanofluids with and without considering temperature to examine the actual effect of temperature change on thermal conductivity of nanofluids.

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