



# Potential energy and atomic stability of H<sub>2</sub>O/CuO nanoparticles flow and heat transfer in non-ideal microchannel via molecular dynamic approach: the Green–Kubo method

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## Abstract

It is interesting to investigate the number of nanoparticle (NP) and temperature effects on H<sub>2</sub>O/CuO nanofluid thermal conductivity and the atomic manner in a non-ideal microchannel. The outcomes of the physical features of these structures were supposed using molecular dynamic (MD) method and LAMMPS simulation package. For the study of dynamic properties of nanofluid microchannel system, parameters such as temperature profiles, velocity, density, and potential energy of H<sub>2</sub>O/CuO atomic structures were calculated. Furthermore, the thermal conductivity of these structures was estimated by the Green–Kubo method in the final step. This simulation shows that nanoparticle number is a crucial parameter in nanofluid movement in a microchannel. Theoretically, via adding CuO nanoparticle to H<sub>2</sub>O fluid, the maximum rate of velocity, density, temperature, and thermal conductivity of base fluid increases to 0.106 g cm<sup>-3</sup>, 29.810 Å ps<sup>-1</sup>, 549.217 K, and 0.81 W mK<sup>-1</sup> rates, respectively. Moreover, the temperature increase of Cu microchannel increases the rate of density, velocity, temperature, and thermal conductivity of CuO nanofluids.

**Keywords** MD · Water/copper oxide · Microchannel · Green–Kubo method

## Introduction

In engineering and physics, fluid dynamics is a subcategory of fluid mechanics which explains the fluids flow. It contains some categories, such as aerodynamics and hydrodynamics

[1–4]. Fluid dynamics contains a large variety of uses, like computing moments and forces on aircraft, verifying the mass flow rate of petroleum by pipelines, forecasting the patterns of weather, knowing about nebulae in interstellar space, and modeling fission weapon explosion [5–8]. Fluid dynamics proposes a well-organized structure which contains empirical and semiempirical laws that came from flow measurement and used for solving practical problems [9, 10]. The way to solve the problem of fluid dynamics usually entails the calculation of different features of the fluid, like pressure, flow velocity, density, temperature, and thermal conductivity, as functions of time and space [11, 12]. More, the nanofluid, described as a fluid with nanosized solids, the “nanoparticles (NPs).” The NP colloidal suspensions are formed by these liquids [13, 14]. CNTs, silicon carbides, metals, and oxides are the elements of NPs that were used in nanofluids. Water molecules and oil are typical cases of base fluids [15]. Ingenious aspects of nanofluids make them conceivably potent for lots of operations in heat transmitting, as in machining, chillers, fridges, electronic cooling systems, microelectronics, and fuel cells [16, 17]. Nanofluids, compared to typical liquids, point out better thermic power

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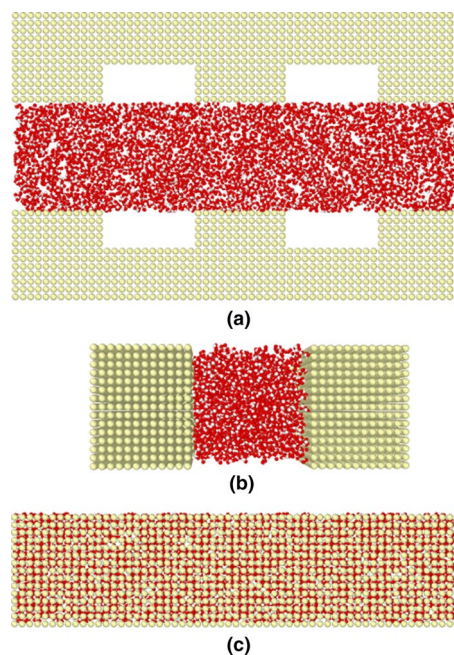
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of convected and conducting heat transmitting efficiency [18–20]. The primary researches demonstrate that appreciable increase in nanofluid thermal features than the base fluid, especially the heat transmission coefficient, has been mainly discredited. In spite of definitive experimental investigations, theoretical papers keep on following the claim of anomalous enhancement, especially by Brownian and thermophoretic mechanisms, as proposed by Buongiorno [21–27]. The molecular dynamic (MD) technique is another way to nanofluid studies. The MD technique is the greatest crucial class of computer simulation which is able to predict the atomic manner of framework assortment [28–30]. Today, this estimation technique is broadly applied in the thermal study of atomic structures [31, 32]. In this work, theoretical calculations were performed to predict nanoparticle number and microchannel temperature effect on the dynamic manner of H<sub>2</sub>O/CuO nanofluid.

## Computational method

In the current numerical study, the MD method was used to study H<sub>2</sub>O/CuO nanofluid inside a non-ideal microchannel. The MD method is a computational approach to trace dynamic movements of atoms. In this method, the interaction of atoms with one another is permitted, so it gives the system's mechanical development concept. In the present study, every MD simulation was done by applying the LAMMPS package released by laboratories [33–36]. This software expansion started during the 1990s was supported by a unified research and expansion concurrence (CRADA) between Dupont firms, Bristol Myers Squibb, and Sandia and LLNL laboratories and Cray. For using this simulation package to predict H<sub>2</sub>O/CuO nanofluid, the first Cu non-ideal microchannel was simulated at  $T = 300$  K. In the next step, H<sub>2</sub>O fluid with differing numbers of CuO NPs was simulated and have these atomic structures settled in the microchannel center [37–43]. The primary atomic pattern was fulfilled by Packmol software [44]. Figure 1 indicates a simulation box at perspective, front, and top faces that was seen by OVITO (Open Visualization Tool) software [45]. OVITO is a scientific visualization and analysis package for molecule- and atom-based simulation data [46, 47].

In current MD simulations, periodic boundary statuses were fulfilled in whole 3 directions. To create the first status, the Nose–Hoover thermostat was used for atoms in the simulation box for fixed temperature at 300 K with one femtosecond time step [48, 49]. In the 2nd phase, extrinsic force equaled with  $F_{\text{ext}} = 0.02 \text{ eV \AA}^{-1}$  entered to fluid and the microcanonical ensemble was applied for estimating the dynamic manner of H<sub>2</sub>O in Cu microchannel [50]. Numerical researches for H<sub>2</sub>O molecules usually use simple experiential models [51–54]. Then, simulations are able to



**Fig. 1** Schematic picture of Cu microchannel and H<sub>2</sub>O fluid designed by Packmol software at **a** perspective, **b** front, and **c** top views

calculate features of H<sub>2</sub>O molecules that are used to compare with experiential reports. The most usual procedures for the MD simulation of H<sub>2</sub>O molecules include SPC, TIP3P, and TIP4P types. In total, 3–5 interaction spots were fulfilled in these models [55–57]. At SPC type 3, sites that were fulfilled for positive charges and electrostatic interactions on H atoms were balanced via a properly negative charge that added to oxygen atom [58]. Moreover, via a Lennard-Jones (LJ) potential with only one lone interaction point per molecule centered on the O atom, the no bonded interaction between 2 H<sub>2</sub>O molecules was simulated. The Lennard-Jones potential can be defined as a mathematically uncomplicated formula that estimates the interaction between particles (molecules or atoms) [59]. John Lennard-Jones was the first person who suggested a form of this interatomic potential in 1924 [60]. The commonest formula for LJ potential is:

$$U(r) = 4\epsilon \left[ \left( \frac{\sigma}{r_{ij}} \right)^{12} - \left( \frac{\sigma}{r_{ij}} \right)^6 \right] \quad r \ll r_c \quad (1)$$

In current equation, the depth of potential is shown by  $\epsilon$ , and  $\sigma$  is the distance, when potential is zero, and also, the distance between 2 particles is  $r_{ij}$ . In the MD simulations, the two  $\sigma$  and  $\epsilon$  factors are depended on the kind of atoms in the box. The following simple harmonic oscillator equations show angle stretch and bond in this atomic pattern [61]:

$$E_r = \frac{1}{2} k_r (r - r_0) \quad (2)$$

$$E_{\theta} = \frac{1}{2}k_{\theta}(\theta - \theta_0) \tag{3}$$

where  $r_0$  and  $\theta_0$  are the atomic bond length and angles balanced value, individually.  $K_r$  and  $K_{\theta}$  are harmonic oscillator constants. At the SPC pattern, the  $\theta_0$  of HOH angle equals 109.47°, while the  $r_0$  of OH bond is equivalent to 1.0 Å, and. Furthermore, all interatomic interactions involving the hydrogen atoms are ignored. This model and other parameters for H<sub>2</sub>O molecule simulation are presented in Table 1 [58].

To study nanoparticles added to base fluid, CuO nanoparticles are simulated. The atomic structure of H<sub>2</sub>O/CuO nanofluid atomic structures is depicted in Fig. 2. To simulate CuO nanoparticles, there are also several atomic potentials such as UFF, dreiding, and Tersoff potentials [62]. Among the interatomic potentials, the best selection for simulating metal oxide structures is Tersoff potential. The most familiar interpretation of the Tersoff potential is [63],

$$E = \frac{1}{2} \sum_i \sum_{i \neq j} V_{ij} \tag{4}$$

$$V_{ij} = f_C(r_{ij}) [f_R(r_{ij}) + b_{ij}f_A(r_{ij})] \tag{5}$$

In this equation,  $f_R$  is a two-body term, and  $f_A$  includes three-body interactions. The summations in the formula are over whole neighbors'  $J$  and  $K$  of atom  $I$  within a cutoff distance  $= R + D$  [64]. In the last part of the simulations, after determining the interatomic potentials and atomic structure, to investigate the flow mechanical behavior in the Copper microchannel, temperature profiles, velocity, and the density are calculated. In the end, the thermal conductivity of atomic structures by the Green–Kubo method was calculated [65].

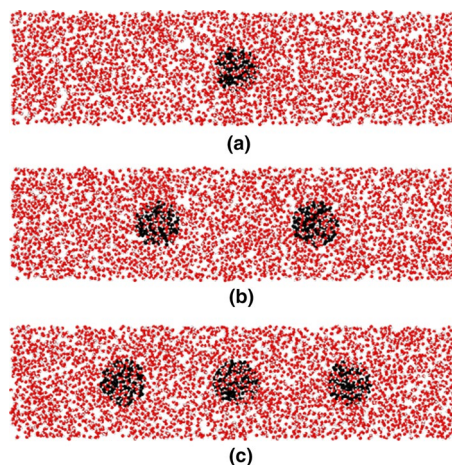
## Results and discussion

### Potential energy of atomic structures

In the initial phase of the molecular dynamic simulation, the atomic patterns' potential energy was examined to ensure the stability of this MD simulation. Figures 3 and 4 show that one ns time is sufficient for equilibration of fluid/nanofluid and microchannel structures. Figure 3 shows that the potential energy and atomic stability of the simulated system

**Table 1** Length and energy specifications for Lennard-Jones potential for the SPC pattern of H<sub>2</sub>O molecules [58]

Element	$\epsilon/\text{kcal mol}^{-1}$	$\sigma/\text{Å}^{-1}$
O	0.1553	3.166
H	0.0000	0.000

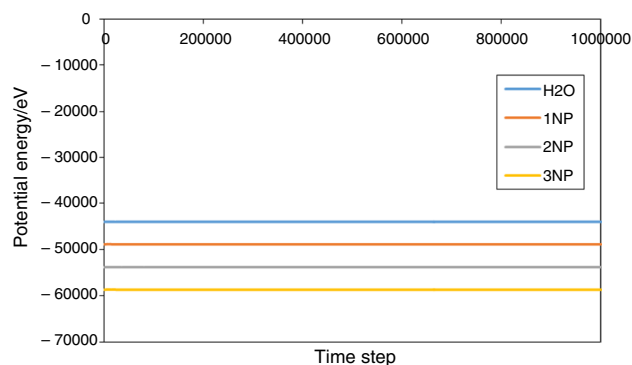


**Fig. 2** Schematic of H<sub>2</sub>O/CuO nanofluid with **a** one, **b** two, and **c** three CuO NPs

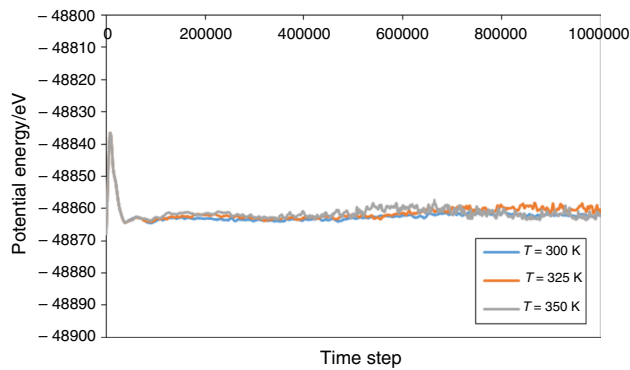
increases by adding nanoparticle to the base fluid. Furthermore, the temperature of microchannel increasing does not have an appreciable effect on the stability of atomic structures. After verifying the potential energy equilibration, temperature profiles, velocity, and the density of fluid/nanofluid structures in the next steps were reported.

### Density profile of atomic structures

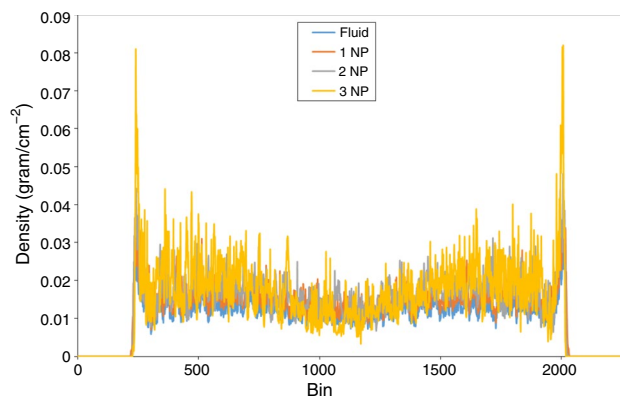
The density profiles indicate fluid–particle interactions with the microchannel walls, so this parameter is so important. To calculate the density profile, the microchannel is divided into 2226 bins along the  $y$  direction, and every bin has  $L_x * L_z * \Delta y$  vol., where the microchannel lengths are  $L_z$  and  $L_x$  together with  $z$  and  $x$  directions, and  $L_y/N_{\text{bin}}$  refers to  $\Delta y$ . Figure 5 displays the dispersion of fluid atoms in the microchannel at which the temperature of simulated walls is fixed at 300 K. It is obvious that the fluid atoms are absorbed



**Fig. 3** Potential energy of H<sub>2</sub>O molecules and H<sub>2</sub>O/CuO nanofluid with a varied number of NPs being the simulation time phase function



**Fig. 4** Potential energy of H<sub>2</sub>O/CuO nanofluid with one nanoparticle being the microchannel temperature and simulation time phase function



**Fig. 5** Density profile of fluid/nanofluid being the CuO nanoparticles number function

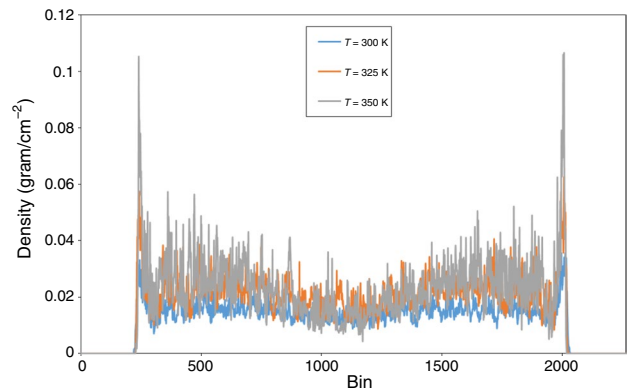
via the nanochannel surface and decrease the density profile of these atomic patterns in the middle of Cu microchannel. Via adding CuO NPs to H<sub>2</sub>O liquid, the density profile of structure rises. Further, it can be concluded that the nanoparticle numbers have important influence on the distributions of nanofluid atom positions and nanofluid density increases from 0.030 to 0.081 g cm<sup>-3</sup> by adding 3 CuO nanoparticles to H<sub>2</sub>O fluid (see Table 2). Furthermore, by increasing temperature from 300 to 325 K and 350 K, the density maximum rate reaches 0.062 g cm<sup>-3</sup> and 0.106 g cm<sup>-3</sup>, individually (Fig. 6 and Table 3). Therefore, the temperature of Cu microchannel is another important parameter for the dynamic manner of nanofluid, which should be considered in practical applications.

### Velocity profile of atomic structures

The velocity profiles of nanofluid structure indicate how these atomic structures move being the simulation time function. To calculate the velocity dispersion of fluid/

**Table 2** Density maximum rate in fluid/nanofluid by a varied number of CuO NPs

Sample	Density/g cm <sup>-3</sup>
H <sub>2</sub> O fluid	0.030
H <sub>2</sub> O fluid with 1 NP	0.036
H <sub>2</sub> O fluid with 2 NPs	0.048
H <sub>2</sub> O fluid with 3 NPs	0.081

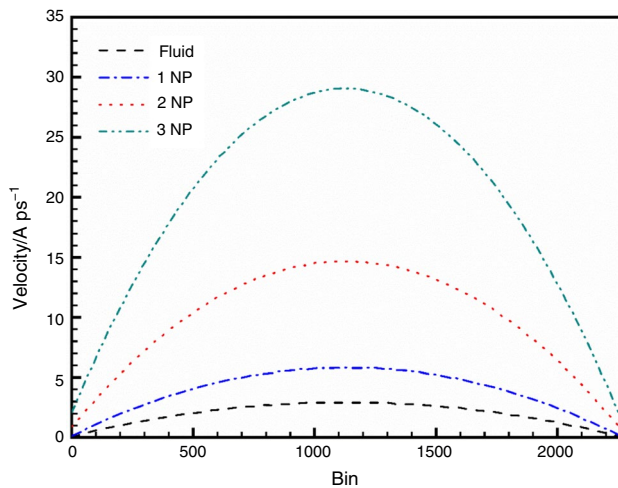


**Fig. 6** Density profile of H<sub>2</sub>O/CuO nanofluid with one nanoparticle as a function of Cu microchannel temperature

**Table 3** Density maximum rate in H<sub>2</sub>O/CuO nanofluid with one nanoparticle at various temperatures

Temperature/K	Density/g cm <sup>-3</sup>
300	0.036
325	0.062
350	0.106

nanofluid atoms in a microchannel, the simulation box is divided into 2226 bins in the y direction. Figure 7 displays the velocity distribution of fluid and nanofluid with various numbers of nanoparticles in Cu microchannel. It can be seen that the velocity rate is minimum near the microchannel wall surface. It happens because of the strong interactions between wall atoms and nanofluid particles. Furthermore, this physical parameter becomes maximized in the middle region of microchannel, which shows the Poiseuille flow manner of fluid/nanofluid particles in a non-ideal microchannel. Based on these calculated results, the maximum rate of fluid velocity is 3.609 Å ps<sup>-1</sup>, and by adding 3 NPs to fluid–structure, this parameter rises to 27.100 Å ps<sup>-1</sup> (Table 4). Microchannel temperature increasing has an identical effect on the dynamic manner of nanofluid. By temperature increasing from 300 to 350 K, the maneuverability of the atomic structure grows



**Fig. 7** Velocity profile of fluid/nanofluid being the CuO NP number function

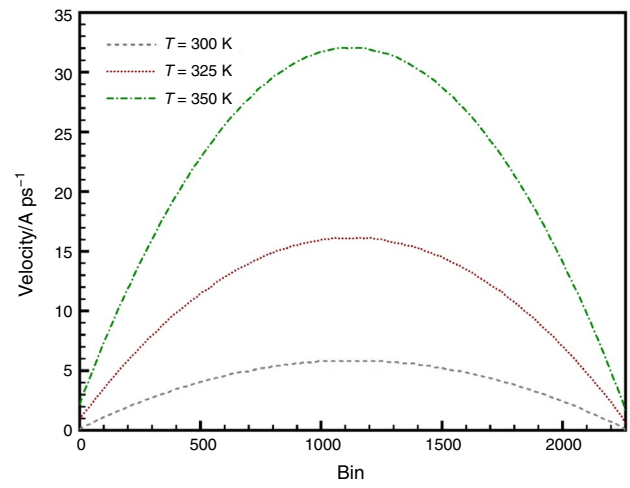
**Table 4** Atomic velocity maximum rate in fluid/nanofluid by a varied number of CuO NPs

Sample	Velocity/A ps <sup>-1</sup>
H <sub>2</sub> O fluid	3.609
H <sub>2</sub> O fluid with 1 NP	7.218
H <sub>2</sub> O fluid with 2 NPs	13.678
H <sub>2</sub> O fluid with 3 NPs	27.100

by  $\frac{1}{2}mv^2 = 3/2kT$  equation. Numerically, the atomic velocity maximum rate grows from 7.218 to 27.100 A ps<sup>-1</sup>. Figure 8 shows the velocity variation of fluid/nanofluid structures as a function of simulation temperature (Table 5). Based on this figure, it can be said that, by increasing the temperature, the effective force enforced to nanofluid from microchannel walls grow less, and the atomic velocity of H<sub>2</sub>O/CuO nanofluids increases.

**Temperature profile of atomic structures**

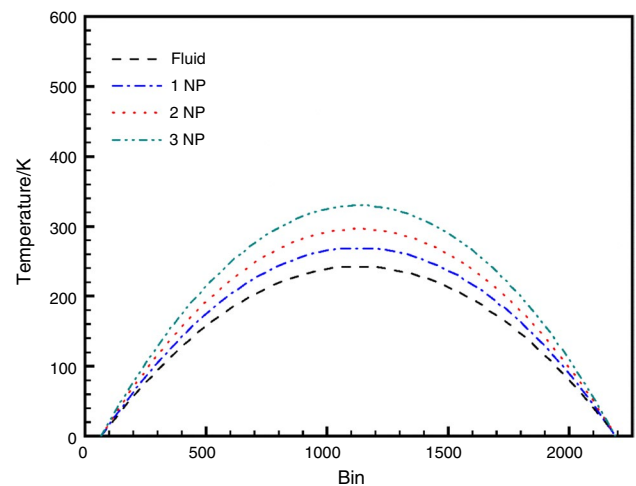
The thermodynamic features of atomic patterns have a direct impact on the function in practical use. In this part, it was examined how the fluid/nanofluid atom temperature is dispersed in simulated Cu microchannels. To attain the temperature profile, the average temperature is computed in every 2226 bin and in every 100-time steps. Figures 9 and 10 show the effects of nanoparticle and microchannel temperature on fluid/nanofluid temperature profiles along the width of the microchannel. As presented in these figures, the Poiseuille flow has a quadratic temperature profile, and the temperature maximum rate happens in the center of a microchannel. Figure 9 demonstrates the atomic temperature



**Fig. 8** Velocity profile of H<sub>2</sub>O/CuO nanofluid with one nanoparticle as a function of Cu microchannel temperature

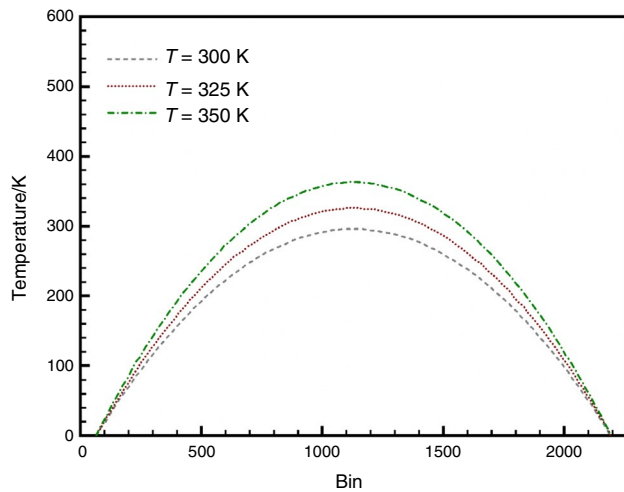
**Table 5** Atomic velocity maximum rate in H<sub>2</sub>O/CuO nanofluid with one nanoparticle at various temperatures

Temperature/K	Velocity/A ps <sup>-1</sup>
300	7.218
325	15.046
350	29.810



**Fig. 9** Temperature profile of fluid/nanofluid as a function of CuO nanoparticles number

of fluid/nanofluid structures grow via CuO nanoparticle adding to H<sub>2</sub>O molecules. This phenomenon occurs because of nanoparticle high thermal conductivity rather than base fluid. From numerically view, the maximum rate of temperature differs from 407.345 to 499.289 K via growth in CuO



**Fig. 10** Temperature profile of H<sub>2</sub>O/CuO nanofluid with one nanoparticle as a function of temperature

**Table 6** Atomic temperature maximum rate in fluid and nanofluid by varied number of CuO nanoparticles

Sample	Fluid/nanofluid temperature/K
H <sub>2</sub> O fluid	366.610
H <sub>2</sub> O fluid with 1 NP	407.345
H <sub>2</sub> O fluid with 2 NPs	448.080
H <sub>2</sub> O fluid with 3 NPs	499.289

**Table 7** Atomic temperature maximum rate in nanofluid at varied temperatures

Sample temperature/K	Nanofluid temperature/K
300	448.080
325	492.887
350	549.217

NPs from 1–3 numbers, as given in Table 6. Microchannel temperature increasing has an alike impact on fluid/nanofluid atomic temperature profile, being presented in Fig. 10. Via growth in microchannel wall temperature from 300 to 350 K, the nanofluid temperature maximum rate grows from 448.080 to 549.217 K (Table 7). The fast transfer of temperature from the microchannel walls to the simulated nanofluids at higher temperatures causes this phenomenon.

### Thermal conductivity of atomic structures

At the final step, the MD simulations are used for estimating the thermal conductivity of H<sub>2</sub>O/CuO nanofluid. Before

**Table 8** Thermal conductivity of fluid and nanofluid by various CuO nanoparticle numbers

Sample	Thermal conductivity/W mK <sup>-1</sup>
H <sub>2</sub> O fluid	0.62
H <sub>2</sub> O fluid with 1 NP	0.68
H <sub>2</sub> O fluid with 2 NPs	0.71
H <sub>2</sub> O fluid with 3 NPs	0.75

**Table 9** Thermal conductivity of nanofluid structure with one CuO nanoparticle at various temperatures

Sample temperature/K	Thermal conductivity/W mK <sup>-1</sup>
300	0.68
325	0.74
350	0.81

calculating nanofluid thermal conductivity, the thermal conductivity of H<sub>2</sub>O molecules is calculated. Thermal conductivity of simulated structures is estimated via Green–Kubo formalism based on the fixed number of energy condition, volume, and atoms [31, 66]:

$$k = \frac{1}{3k_B VT^2} \int_0^{\infty} \langle S(t) \cdot S(0) \rangle dt \quad (6)$$

where  $T$  is temperature,  $V$  is the simulation volume,  $k_B$  is the Boltzmann constant, and  $S$  is the heat current vector with elements in  $X$ – $Y$ – $Z$  directions. The heat current  $S$  is estimated in every time phase by applying velocity and the stress tensor of every atom in LAMMPS.  $\langle S(t) \cdot S(0) \rangle$  is named the function of heat current autocorrelation, which is the correlation of the heat current at different times. The calculated rate for the thermal conductivity of H<sub>2</sub>O molecules is 0.62 W mK<sup>-1</sup>, where this rate is appropriate for experimental value (0.59 W mK<sup>-1</sup>) [32]. Table 8 shows that the thermal conductivity of H<sub>2</sub>O/CuO nanofluid increases by inserting CuO nanoparticle to base fluid. Numerically, by CuO nanoparticle adding to H<sub>2</sub>O molecules, the thermal conductivity of nanofluid rises to 0.75 W mK<sup>-1</sup>. Furthermore, the thermal conductivity of H<sub>2</sub>O/CuO nanofluid increases from 0.68 to 0.81 W mK<sup>-1</sup> by microchannel temperature increasing from 300 to 350 K (Table 9). This thermal manner of nanofluid arises from the increase in the mobility of nanofluid atoms by microchannel temperature rising.

## Conclusions

In the present study, the MD simulation is used for estimating the dynamic and thermal manner of H<sub>2</sub>O/CuO nanofluid in the non-ideal microchannel at 300 K, 325 K, and 350 K temperatures. In current simulations, the number of CuO NPs differs from 1 to 3 NPs [67, 68]. These conclusions from stated simulations are:

- (A) The number of CuO NPs inside the base fluid is a prominent factor in H<sub>2</sub>O/CuO nanofluid density. In the simulated models, the density of H<sub>2</sub>O/CuO nanofluid increased from 0.036 to 0.081 g cm<sup>-3</sup> with an increase in the number of nanoparticles from 1–3, respectively.
- (B) The MD results propose that the atomic density of H<sub>2</sub>O/CuO density is increased by increasing the microchannel temperature from 300 to 350 K.
- (C) In simulated structure, the atomic velocity of nanofluid has a straight connection with CuO NP number. The maximum rate of velocity in H<sub>2</sub>O/CuO nanoparticle raises to 27.100 Å ps<sup>-1</sup> via adding 3 NPs to base fluid.
- (D) Maximum velocity of simulated nanofluid increases from 7.218 to 29.810 Å ps<sup>-1</sup> via temperature growth in microchannel temperature from 300 to 350 K.
- (E) By CuO nanoparticle adding to base fluid, the atomic temperature of nanofluid grows. Theoretically, the maximum temperature of nanofluid increases from 407.345 to 499.289 K by 1–3 CuO nanoparticle, adding to H<sub>2</sub>O molecules.
- (F) By temperature increasing in simulated Cu microchannel, the maximum rate of nanofluid temperature reaches 549.217 K.
- (G) Thermal conductivity of H<sub>2</sub>O/CuO nanofluid increases to 0.75 W mK<sup>-1</sup> by 3 CuO nanoparticle, adding to H<sub>2</sub>O molecules.
- (H) The thermal conductivity of H<sub>2</sub>O/CuO nanofluid changes from 0.68 to 0.81 W mK<sup>-1</sup> by temperature increasing from 300 to 350 K.

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