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DYNAMICS OF FLEXIBLE ELEMENTS OF A DRIVE UNDER THE ACTION OF IMPULSIVE PERTURBATIONS

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For flexible elements of drives characterized by a constant speed of longitudinal motion, we propose a method for the analytic investigation of vibrations caused by the action of a periodic system of impulsive perturbations. On the basis of this method, we obtain analytic relations for the description of the determining parameters of nonlinear vibrations in the analyzed class of systems for both nonresonance and resonance cases. It is shown that, in the resonance case, the value of the amplitude of transition through the resonance strongly depends on the speed of longitudinal motion of the flexible element.

Keywords: impulsive perturbation, flexible element, resonance phenomenon, amplitude, frequency.

Introduction

The process of trouble-free operation of drives with flexible elements (belt transmissions, rope pulls, etc.) is often affected by various disturbances. They may have either periodic or random nature, act at specific points and at certain moments of time and may be caused by the interaction, e.g., with external objects from which or to which the motion is transferred. The action of some of these objects upon flexible drives can be described by using periodic or random functions, whereas the action of the other objected can be simulated by the Dirac delta-function δ [5]. These disturbances may affect the process of normal trouble-free operation of the considered elements of drives (and, hence, of the systems containing these elements as components) even if their magnitude is low. A detailed analysis of the dynamic processes in these flexible elements can be carried out by using adequate mathematical models characterized by the presence of the longitudinal component of the velocity of motion, which gives a qualitatively new character to the mathematical model, and the discrete character of disturbances. This means that it is necessary to generalize the existing methods used for the investigation of the systems characterized by a constant speed of motion [4, 6, 8–13] in order to take into account the discrete action of the disturbances of motion.

This problem is partly solved in the present paper. Its solution is based on the principle of single-frequency oscillations in nonlinear systems with many degrees of freedom and distributed parameters [2, 3]; on the generalization of the main ideas of the asymptotic methods of nonlinear mechanics to dynamic systems characterized by a constant speed of motion [8], and on the basic statements of the theory of oscillating processes in systems with instantaneous disturbances [1, 3, 5]. The outlined procedure enables us to determine basic relations used to describe the determining parameters of vibration of the analyzed element of a flexible drive in the presence of periodic impulse disturbances in both resonance and nonresonance cases.

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1. Statement of the Problem

The transverse vibration of flexible elements of drive systems characterized by a constant speed V of motion is described by a differential equation [6, 8]

$$u_{tt} + 2Vu_{xt} - \left(\alpha^2 - V^2\right)u_{xx} = \varepsilon f(u, u_x, u_t), \tag{1}$$

where $u(x,t)$ is a transverse displacement of the cross section of the flexible element with an Euler coordinate x at any time t, an analytic function $f(u, u_x, u_t)$ is an approximation of the entire set of nonlinear forces, and a small parameter ε reflects a low magnitude of nonlinear forces as compared with the magnitude of the tensile force $\left(\alpha^2 = \frac{T}{\rho},\ T \text{ is a tensile force, and } \rho \text{ is the linear mass density of the flexible element}\right)$. In the case where a low-magnitude periodic impulsive force with period τ is additionally applied to the element of a drive at a point with the coordinate x_0, equation (1) can be transformed as follows:

$$u_{tt} + 2Vu_{xt} - (\alpha^2 - V^2)u_{xx}$$
$$= \varepsilon f(u, u_x, u_t) + \varepsilon\delta(x - x_0)\sum_{i=1}\delta(t - (i-1)\tau)g_i(u, u_x, u_t), \tag{2}$$

where

$$\varepsilon g_i\big(u(x_0, (i-1)\tau),\ u_x(x_0, (i-1)\tau),\ u_t(x_0, (i-1)\tau)\big)$$

is the magnitude of the impulsive force acting at the time $i\tau$, $i = 1, 2, \ldots$ and $\delta(\cdot)$ is the Dirac delta-function.

The boundary conditions for equation (2) are classical conditions of the first kind:

$$u(0,t) = u(\ell,t) = 0, \tag{3}$$

where $\ell = \text{const}$ is the length of the element. The problem is to construct and analyze the solution of equation (2) with boundary conditions (3).

Note that the analyzed boundary-value problem (1), (3) also describes the longitudinal vibration of a moving homogeneous elastic body if, in equation (1), we set $\alpha^2 = \frac{E}{\rho}$, where E is the modulus of elasticity and ρ is the linear mass density of the flexible element.

2. Procedure of Investigations

As already indicated, the maximum value of the right-hand side of equation (2) is small as compared with the tensile force in the flexible element and, therefore, in order to construct the solution of the boundary-value problem (2), (3), we can use the general concepts of the theory of integration of boundary-value problems that describe oscillating processes in systems with distributed parameters. Prior to adapting these concepts to the analyzed problem, we slightly modify the right-hand side of equation (2); more precisely, its part, which takes into account the effect of impulsive disturbances on the flexible element. It is easy to see that the complete normalized system of functions $\left\{\sin\frac{s\pi x}{\ell}\right\}$ satisfies the conditions

$$\left\{\sin\frac{s\pi x}{\ell}\right\}\Big|_{x=0,\,x=\ell} = 0,$$

which are, in a certain sense, related to the boundary conditions (3). This enables us to represent the Dirac delta-function $\delta(x-x_0)$ in the form of a series:

$$\delta(x-x_0) = \frac{2}{\ell}\sum_{s=1}\sin\frac{s\pi x_0}{\ell}\sin\frac{s\pi x}{\ell}.$$

As follows from the characteristics of the δ-function, the accuracy of the solution does not change if the function $\sum_{i=1}\delta(t-(i-1)\tau)g_i(u,u_x,u_t)$ is replaced by the function [7]:

$$\sum_{i=1}\delta(t-i\tau)g_i(u,u_x,u_t) = \cos\theta\sum_{i=1}\delta\left(\frac{\theta}{\mu}-\frac{2(i-1)\pi}{\mu}\right)g_i(u,u_x,u_t), \tag{4}$$

where

$$\theta=\mu t, \quad \mu=\frac{2\pi}{\tau}, \quad \text{and} \quad \delta(t-(i-1)\tau) = \cos\theta\,\delta\left(\frac{\theta}{\mu}-\frac{2(i-1)\pi}{\mu}\right).$$

Finally, the differential equation (2) takes the form:

$$u_{tt} + 2Vu_{xt} - (\alpha^2-V^2)u_{xx}$$
$$= \varepsilon f(u,u_x,u_t) + \varepsilon\frac{2}{\ell}\cos\theta\sum_{s=1}\sin\frac{s\pi x_0}{\ell}\sin\frac{s\pi x}{\ell}\sum_{i=1}\delta\left(\frac{\theta}{\mu}-\frac{2(i-1)\pi}{\mu}\right)g_i(u,u_x,u_t). \tag{5}$$

The second term on the right-hand side of equation (5) can be regarded as a periodic disturbance with certain frequency Ω. This serves as a basis for representing the first approximation to the single-frequency solution of equation (5) in the form

$$u(x,t) = au_0(x,\varphi) + \varepsilon U_1(a,x,\varphi,\theta), \tag{6}$$

where $au_0(x,\varphi) = a(\cos(\kappa x-\varphi)-\cos(\chi x+\varphi))$ is the solution of the unperturbed $(\varepsilon=0)$ boundary-value problem [2, 3] in which $\varphi = \omega t+\varphi_0$, $\kappa = \frac{k\pi}{\alpha\ell}(\alpha+V)$, $\chi = \frac{k\pi}{\alpha\ell}(\alpha-V)$, and $\omega = \frac{k\pi}{\alpha\ell}(\alpha^2-V^2)$, while a and

φ_0 are, respectively, the amplitude and initial phase of the unperturbed motion. Since the function $U_1(a, x, \varphi, \theta)$ in equation (6) must take into account the influence of nonlinear forces and impulsive periodic disturbances, we conclude that

first, it must satisfy the boundary conditions, which follow from conditions (3):

$$U_1(a, x, \varphi, \theta)\big|_{x=0} = U_1(a, x, \varphi, \theta)\big|_{x=\ell} = 0, \tag{7}$$

second, it should not contain the terms proportional to the first modes of the oscillation phase φ in its expansions:

$$\int_0^{2\pi} U_1(a, x, \varphi, \theta) \begin{Bmatrix} \cos\varphi \\ \sin\varphi \end{Bmatrix} d\varphi = 0. \tag{8}$$

We consider the first approximation to the asymptotic solution in the form close to the first mode of vibrations of the unperturbed motion, i.e., in the dependences for the wave numbers κ and χ and also for the frequency of oscillations ω, we assume that the parameter $k = 1$. In addition, if, for the unperturbed motion, the amplitude and frequency of oscillations are constant, then, for the disturbed motion they are variable. The regularities of their variations are determined not only by nonlinear forces and impulsive actions, but also by the relationships between the frequency (period) of natural oscillations and the frequency (period) of impulsive disturbances. If the indicated quantities do not satisfy the relation $m\omega \neq n\Omega$ (this case is called nonresonance), then the regularities of variation of the parameters a and φ for the first approximation are determined by the following ordinary differential equations:

$$\frac{da}{dt} = \varepsilon A_1(a) \quad \text{and} \quad \frac{d\varphi}{dt} = \omega + \varepsilon B_1(a). \tag{9}$$

We seek the unknown functions $A_1(a)$ and $B_1(a)$ on the right-hand sides of relations (9) in order to guarantee that the representation of the solution in the form (6) satisfies the original equation (5) with the required accuracy in the case where the functions a and φ in representation (6) are replaced by the functions of time expressed by using dependence (9). Differentiating the asymptotic representation with respect to the independent variables, substituting the obtained expressions in equation (5), and then equating the coefficients of ε on the right- and left-hand sides of the obtained dependence, we can write

$$\begin{aligned} \bar{L}(U_1) = {} & f_1(a, x, \varphi) \\ & + \frac{2}{\ell}\cos\theta \sum_{s=1} \sin\frac{s\pi x_0}{\ell} \sin\frac{s\pi x}{\ell} \sum_{i=1} \delta\left(\frac{\theta}{\mu} - \frac{2(i-1)\pi}{\mu}\right) g_1(a, x, \varphi) \\ & + \rho(x, \varphi) A_1(a) + a h(x, \varphi) B_1(a), \end{aligned} \tag{10}$$

where

$$\bar{L}(U_1) = \omega^2 \frac{\partial^2 U_1}{\partial \varphi^2} + \mu^2 \frac{\partial^2 U_1}{\partial \theta^2} + 2\mu\omega \frac{\partial^2 U_1}{\partial \varphi \partial \theta}$$

$$+ 2V\left(\frac{\partial^2 U_1}{\partial \varphi \partial x}\omega + \frac{\partial^2 U_1}{\partial \theta \partial x}\mu\right) - (\alpha^2 - V^2)\frac{\partial^2 U_1}{\partial x^2},$$

$$\rho(x,\varphi) = 2\left[(\omega + \kappa V)\sin(\kappa x + \varphi) + (\omega - \chi V)\sin(\chi x - \varphi)\right],$$

$$h(x,\varphi) = 2[(\omega + \kappa V)\cos(\kappa x + \varphi) - (\omega - \chi V)\cos(\chi x - \varphi)],$$

the functions $f_1(a,x,\varphi)$ and $g_1(a,x,\varphi)$ correspond to the values of the functions $f(u,u_x,u_t)$ and $g(u,u_x,u_t)$ under the condition that $u(x,t)$ and all its derivatives are determined according to the principal part in representation (6).

Conditions (8) imposed on the function $U_1(a,x,\varphi,\theta)$ enable us to deduce from equation (10) the following relations specifying the regularities of variations of the amplitude and frequency of the disturbed motion in the following form:

$$A_1(a) = \frac{\varepsilon}{\pi\ell[(\omega+\kappa V)^2 + (\omega-\chi V)^2]} \int_0^{\ell}\int_0^{2\pi} \Bigg\{ f_1(a,x,\varphi)$$

$$+ \frac{1}{\pi\ell\mu}\sum_{i=1}\int_0^{2\pi}\cos\theta \sum_{s=1}\sin\frac{s\pi x_0}{\ell}\sin\frac{s\pi x}{\ell}\,\delta\left(\frac{\theta}{\mu} - \frac{2(i-1)\pi}{\mu}\right) g_i(a,x,\varphi)\,d\theta \Bigg\}$$

$$\times\left[\bar{\rho}(x)\cos\varphi + \bar{h}(x)\sin\varphi\right] d\varphi\, dx, \tag{11}$$

$$B_1(a) = \frac{\varepsilon}{a\pi\ell\left[(\omega+\kappa V)^2 + (\omega-\chi V)^2\right]} \int_0^{\ell}\int_0^{2\pi} \Bigg\{ f_1(a,x,\varphi)$$

$$+ \frac{1}{\pi\ell\mu}\sum_{i=1}\int_0^{2\pi}\cos\theta \sum_{s=1}\sin\frac{s\pi x_0}{\ell}\sin\frac{s\pi x}{\ell}\,\delta\left(\frac{\theta}{\mu} - \frac{2(i-1)\pi}{\mu}\right) g_i(a,x,\varphi)\,d\theta \Bigg\}$$

$$\times\left[\bar{\rho}(x)\sin\varphi - \bar{h}(x)\cos\varphi\right] d\varphi\, dx,$$

where

$$\bar{\rho}(x) = (\omega + \kappa V)\sin\kappa x + (\omega - \chi V)\sin\chi x,$$

$$\bar{h}(x) = (\omega + \kappa V)\cos\kappa x - (\omega - \chi V)\cos\chi x.$$

As a special case of the facts presented above, we can mention the results concerning the action of a periodic (with constant magnitude) force F_i, i.e., $g_i(u, u_x, u_t) \equiv F_i$, upon a flexible element at a fixed point. Thus, we get

$$A_1(a) = \frac{\varepsilon}{\pi\ell\left[(\omega+\kappa V)^2+(\omega-\chi V)^2\right]}\int_0^{\ell}\int_0^{2\pi} f_1(a,x,\varphi) \times \left[\bar{\rho}(x)\cos\varphi + \bar{h}(x)\sin\varphi\right] d\varphi\, dx,$$

$$B_1(a) = \frac{\varepsilon}{a\pi\ell\left[(\omega+\kappa V)^2+(\omega-\chi V)^2\right]}\int_0^{\ell}\int_0^{2\pi} f_1(a,x,\varphi) \times \left[\bar{\rho}(x)\sin\varphi - \bar{h}(x)\cos\varphi\right] d\varphi\, dx. \tag{12}$$

It follows from relations (12) that, in the nonresonance case, periodic impulsive disturbances affect, in the first approximation, neither the amplitude of the process, nor its frequency. The influence of periodic disturbances manifests itself in partial changes in the form of the dynamic process, i.e., in the nonresonance case, periodic disturbances affect the form of the function $U_1(a, x, \varphi, \theta)$. In order to describe this effect, it is necessary to expand the right-hand side of equation (10) and the unknown function U_1, in Fourier series and equate the coefficients of the same harmonics.

As already indicated, the resonance case where $m\omega \approx n\Omega$ is much more difficult for investigation. It is known [2, 3] that, in the nonlinear systems, the dynamic process strongly depends on the phase difference between the natural and forced oscillations, i.e., on the parameter

$$\gamma = \varphi - \frac{m}{n}\theta.$$

Therefore, unlike the nonresonance case considered above, we assume that the regularities of variations of the basic parameters a and γ specifying the dynamic process in representation (6) have a somewhat more complicated form than in the nonresonance case:

$$\frac{da}{dt} = \varepsilon A_1(a,\gamma) + \ldots \quad \text{and} \quad \frac{d\gamma}{dt} = \omega - \frac{m}{n}\Omega + \varepsilon B_1(a,\gamma) + \ldots. \tag{13}$$

Thus, our aim is to find the functions $A_1(a,\gamma)$ and $B_1(a,\gamma)$ satisfying, together with the asymptotic representation (6), the basic equation (2) with the required accuracy provided that, in this equation, we replace a and φ by the functions of time specified by dependence (13). By analogy with the nonresonance case, the differential equation of the first approximation, which relates the required quantities, takes the following form:

$$\bar{L}(U_1) = f_1(a,x,\varphi) + \frac{2}{\ell}\cos\theta$$

$$\times \sum_{s=1} \sin \frac{s\pi x_0}{\ell} \sin \frac{s\pi x}{\ell} \sum_{i=1} \delta\left(\frac{\theta}{\Omega} - \frac{2(i-1)\pi}{\Omega}\right) g_1(a, x, \varphi)$$

$$+ \rho(x, \varphi) A_1(a, \gamma) + a h(x, \varphi) B_1(a, \gamma)$$

$$+ \left[\wp(x, \varphi) \frac{\partial A_1(a, \gamma)}{\partial \gamma} + a \hbar(x, \varphi) \frac{\partial B_1(a, \gamma)}{\partial \gamma}\right]\left(\omega - \frac{s}{n}\mu\right), \tag{14}$$

where

$$\wp(x, \varphi) = -[\cos(\kappa x + \varphi) - \cos(\chi x - \varphi)],$$

$$\hbar(x, \varphi) = \sin(\kappa x + \varphi) + \sin(\chi x - \varphi).$$

Conditions (8) imposed on the function $U_1(a, x, \varphi, \theta)$ enable us to obtain the following system of equations for the unknown functions:

$$\bar{\rho}(x) A_1(a, \gamma) + a \bar{h}(x) B_1(a, \gamma)$$

$$+ \left[\bar{\wp}(x) \frac{\partial A_1(a, \gamma)}{\partial \gamma} + a \bar{\hbar}(x) \frac{\partial B_1(a, \gamma)}{\partial \gamma}\right]\left(\omega - \frac{s}{n}\mu\right)$$

$$= \frac{\varepsilon}{2\pi^2} \sum_q \exp(inq\gamma) \int_0^{2\pi}\int_0^{2\pi} \left\{ f_1(a, x, \varphi) + \frac{2}{\ell}\cos\theta \right.$$

$$\left. \times \sum_{s=1} \sin \frac{s\pi x_0}{\ell} \sin \frac{s\pi x}{\ell} \sum_{i=1} \delta\left(\frac{\theta}{\mu} - \frac{2(i-1)\pi}{\mu}\right) g_1(a, x, \varphi) \right\}$$

$$\times \exp\left(-inq\left(\varphi - \frac{s}{n}\theta\right)\right) \cos\varphi \, d\varphi \, d\theta,$$

$$- \bar{h}(x) A_1(a, \gamma) + a \bar{\rho}(x) B_1(a, \gamma)$$

$$+ \left[\bar{\hbar}(x) \frac{\partial A_1(a, \gamma)}{\partial \gamma} - a \bar{\wp}(x) \frac{\partial B_1(a, \gamma)}{\partial \gamma}\right]\left(\omega - \frac{s}{n}\mu\right)$$

$$= \frac{\varepsilon}{2\pi^2} \sum_q \exp(inq\gamma) \int_0^{2\pi}\int_0^{2\pi} \left\{ f_1(a, x, \varphi) + \frac{2}{\ell}\cos\theta \right.$$

$$\times \sum_{s=1} \sin\frac{s\pi x_0}{\ell} \sin\frac{s\pi x}{\ell} \sum_{i=1} \delta\left(\frac{\theta}{\mu} - \frac{2(i-1)\pi}{\mu}\right) g_1(a, x, \varphi)\Bigg\}$$

$$\times \exp\left(-inq\left(\varphi - \frac{s}{n}\theta\right)\right)\cos\varphi \, d\varphi \, d\theta. \tag{15}$$

As follows from equations (15), in the general case of periodic disturbances, it is impossible to find, in the closed form, the relations specifying $\frac{da}{dt}$ and $\frac{d\varphi}{dt}$ in the first approximation. However, the right-hand sides of the indicated relations are periodic functions of the phase difference γ and amplitude a, i.e., they are sums of the form $\sum_q \tilde{f}_q(a)\exp(inq\gamma)$. Therefore, the solution for the functions $A_1(a, \gamma)$ and $B_1(a, \gamma)$ should also be sought in the form of similar sums. All calculations performed to determine the functions $A_1(a, \gamma)$ and $B_1(a, \gamma)$ in the indicated way are reduced to operations with trigonometric functions. The obtained dependences make it possible to study the dynamic process both directly in the resonance region and on approaching this region.

It is known [2, 3] that the dynamic process running in nonlinear systems eventually approaches either a steady-state process determined by the equations

$$A_1(a, \gamma) = 0 \quad \text{and} \quad \omega - \frac{m}{n}\mu + \varepsilon B_1(a, \gamma) = 0, \tag{16}$$

or a periodic process.

In the first case, the frequency of natural vibrations is in a simple rational relationship with the frequency of forced oscillations and, hence, this dynamic process corresponds to synchronous oscillations of the drive element. In the second case, the solution of equations

$$\frac{da}{dt} = A_1(a, \gamma) \quad \text{and} \quad \frac{d\gamma}{dt} = \omega - \frac{m}{n}\mu + \varepsilon B_1(a, \gamma) \tag{17}$$

eventually approaches the periodic solution and the dynamic process in the flexible drive element is realized with its natural frequency of vibrations and in the form of vibrations with a frequency

$$\Delta\omega = \omega - \frac{m}{n}\mu.$$

This case corresponds to asynchronous oscillations of the system. Note that, at first sight, the equations used to describe the regularities of changes in the amplitude-frequency characteristics of the dynamic process in the system with regard for the action of nonlinear and periodic forces in the resonance case are quite cumbersome. However, for some specific values of the force, they become much simpler. We now show this by analyzing an example of resonance vibration of a flexible element of the drive under the conditions of periodic action at a fixed point of an impulsive disturbance whose frequency is close to the frequency of its natural vibrations. In this case, equation (17) takes the form

$$\frac{da}{dt} = \frac{\varepsilon}{\pi\ell\left[(\omega+\kappa V)^2+(\omega-\chi V)^2\right]}$$

$$\times\left\{\int_0^{\ell}\int_0^{2\pi} f_1(a,x,\varphi)\left[\bar{\rho}(x)\cos\varphi+\bar{h}(x)\sin\varphi\right]d\varphi\,dx\right.$$

$$+\frac{1}{\pi\ell\mu}\sum_{i=1}F_i\int_0^{\ell}\int_0^{2\pi}\cos\theta\left[\sum_{s=1}\sin\frac{s\pi x_0}{\ell}\sin\frac{s\pi x}{\ell}\,\delta\left(\frac{\theta}{\mu}-\frac{2(i-1)\pi}{\mu}\right)\right]$$

$$\left.\times\left[\bar{\rho}(x)\cos(\gamma+\theta)+\bar{h}(x)\sin(\gamma+\theta)\right]d\theta\,dx\right\},$$

(18)

$$\frac{d\gamma}{dt} = \omega-\mu+\frac{\varepsilon}{\pi a\ell\left[(\omega+\kappa V)^2+(\omega-\chi V)^2\right]}$$

$$\times\left\{\int_0^{\ell}\int_0^{2\pi} f_1(a,x,\varphi)\left[\bar{\rho}(x)\sin\varphi-\bar{h}(x)\cos\varphi\right]d\varphi\,dx\right.$$

$$+\frac{1}{\pi\ell\mu}\sum_{i=1}F_i\int_0^{\ell}\int_0^{2\pi}\cos\theta\left[\sum_{s=1}\sin\frac{s\pi x_0}{\ell}\sin\frac{s\pi x}{\ell}\,\delta\left(\frac{\theta}{\mu}-\frac{2(i-1)\pi}{\mu}\right)\right]$$

$$\left.\times\left[\bar{\rho}(x)\sin(\gamma+\theta)-\bar{h}(x)\cos(\gamma+\theta)\right]d\theta\,dx\right\}.$$

The properties of the delta-function and the periodicity of impulsive disturbances enable us to represent relation (18) in the form

$$\frac{da}{dt} = \frac{\varepsilon}{\pi\ell[(\omega+\kappa V)^2+(\omega-\chi V)^2]}\left[A_1(a)+\frac{\cos\gamma}{\pi\ell\Omega}\beta_1\cos\gamma+\beta_2\sin\gamma\right],$$

(19)

$$\frac{d\gamma}{dt} = \omega-\mu+\frac{\varepsilon}{\pi a\ell[(\omega+\kappa V)^2+(\omega-\chi V)^2]}\left[B_1(a)+\beta_1\sin\gamma-\beta_2\cos\gamma\right],$$

where

$$\beta_1 = \frac{1}{\pi\ell\mu}\sum_{i=1}F_i\sum_{s=1}\sin\frac{s\pi x_0}{\ell}\int_0^{\ell}\bar{\rho}(x)\sin\frac{s\pi x}{\ell}dx,$$

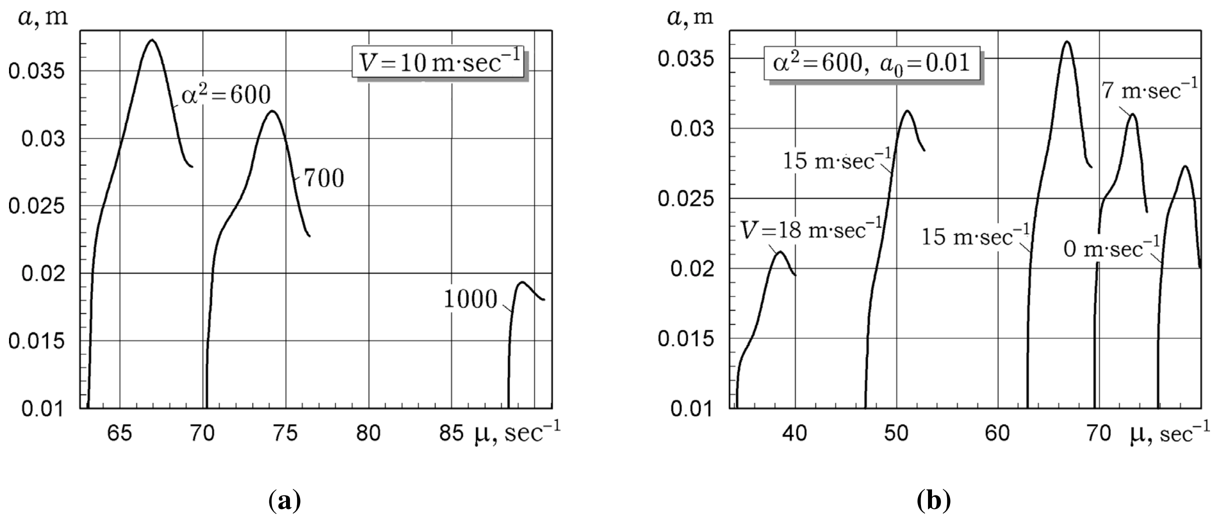

Fig. 1. Resonance values of the amplitude for various values of the parameters.

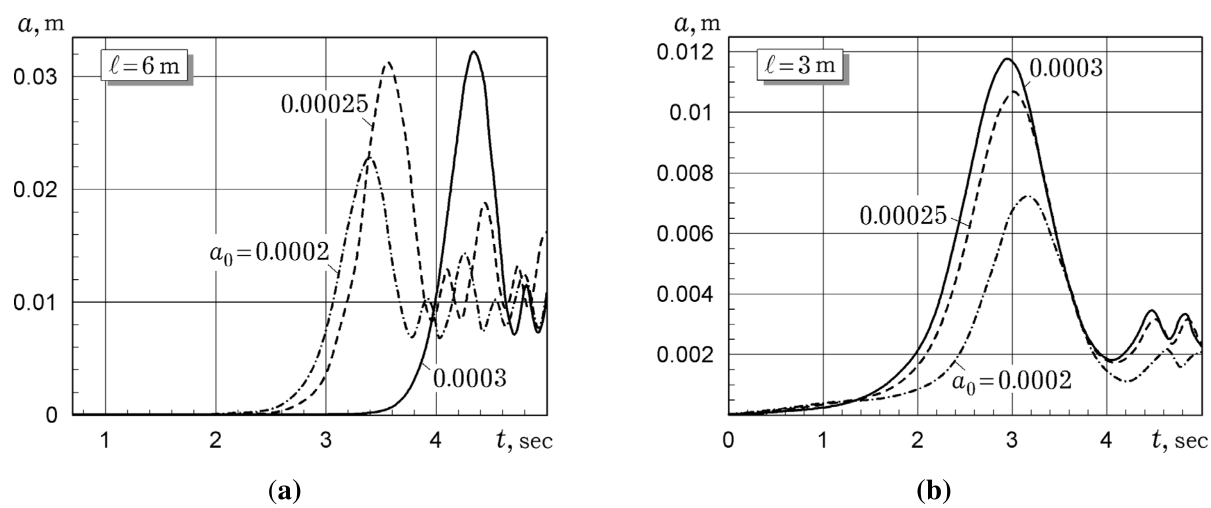

Fig. 2. Behavior of the amplitude of longitudinal vibration of the drive element in passing through the main resonance caused by the action of the periodic system of impulsive forces acting at the ends of the element.

$$\beta_2 = \frac{1}{\pi\ell\mu}\sum_{i=1} F_i \sum_{s=1} \sin\frac{s\pi x_0}{\ell}\int_0^{\ell}\overline{h}(x)\sin\frac{s\pi x}{\ell}\,dx.$$

In Fig. 1, by using dependences (19) for the case

$$f(u, u_x, u_t) = -\delta u_t + k_1 (u_{xx})^3,$$

we present the computed resonance values of the amplitude a for various values of the tensile force α^2 and various values of the speed V of motion of the flexible element.

All calculations are performed for $m = 30$ kg/m, $F_i = 200$ H, $E = 2\cdot 10^{11}$ H/m^2, and $A = 4\cdot 10^{-6}$ m^2.

In Fig. 2, we display the behavior of the amplitude of longitudinal vibrations of an elastic element caused by the action of a periodic system of impulsive forces acting at its ends, in passing through the main resonance for different values of the amplitude a_0 [m] of entering the resonance and the length of the flexible element ℓ [m].

CONCLUSIONS

For flexible drive elements characterized by a constant speed of longitudinal motion, we propose a method for the analytic investigation of transverse vibrations caused by the action of a periodic system of impulsive disturbances. In mechanical systems, the resonance processes prove to be most dangerous because they are responsible for significant dynamic loads acting upon on objects. Therefore, our main attention in the present work is focused on the analysis of the influence of periodic impulsive disturbances.

We construct the first asymptotic approximation for the corresponding boundary-value problems of perturbed motion and obtain analytic relations that describe the determining parameters of nonlinear vibrations in the analyzed class of systems for both nonresonance and resonance cases. As a separate case, we consider the oscillations of flexible drive elements under the action of impulsive disturbances with constant magnitude. It is shown that, in the nonresonance case, disturbances of this kind partly affect the shape of vibrations. As for the resonance vibrations, which are of high theoretical and applied importance, we conclude that:

- for higher speeds of longitudinal motion of a flexible element of the drive system, the natural frequency of its vibrations decreases and, at the same time, the period of impulsive disturbance in which the resonance occurs increases;

- the resonance value of the amplitude first increases with the speed of longitudinal motion but then decreases;

- as the speeds of motion of flexible elements of the drive systems approaches the critical value, the resonance values of the amplitudes strongly depend on the initial value of the amplitude and the magnitude of the harmonic disturbance.

The results obtained in the present work may serve as a basis for choosing the operating parameters of systems containing flexible drive elements with an aim either to avoid resonance phenomena in these systems or to pass through the resonance with the lowest possible amplitude. The method described in the present work can be applied for the investigation of systems whose mathematical models are similar to the considered models, in particular, in the case of heterogeneous boundary conditions.

The algorithm used for the construction and investigation of the first asymptotic approximation for the analyzed problem can be extended to the second and subsequent asymptotic approximations. It is clear that, in this case, the calculations become more cumbersome. At the same time, it might be possible to discover some new features or effects.

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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