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AXISYMMETRIC RESONANT VIBRATIONS AND VIBRATION HEATING OF AN INELASTIC CYLINDRICAL SHELL COMPLIANT TO SHEAR WITH PIEZOELECTRIC ACTUATORS AND RIGIDLY FIXED END FACES

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We consider the problem of axisymmetric resonance vibrations of an inelastic cylindrical shell compliant to shear with piezoelectric actuators under the action of electromechanical monoharmonic loading. We take into account transverse shear strains, rotation inertia of a normal element, and temperature dependence of complex modules of piezopassive and piezoactive materials. The nonlinear problem is solved by using the iterative approach with respect to time within the framework of which the system of ordinary differential equations for vibrations is reduced to the integration of nonstationary heat-conduction equation by the finite-difference method. By using piezoelectric actuators, we analyze the influence of shear strains and the temperature of vibration heating on the amplitude and temperature-frequency characteristics and active damping of the resonance vibrations of the shell.

Keywords: resonance vibrations, dissipative heating, cylindrical shell, piezoactuator, shear strains.

Introduction

In the contemporary engineering, for the purposes of damping of forced vibrations of thin-walled structural elements, parallel with passive methods, it is also customary to use the methods of active damping with the help of piezoelectric actuators [11, 15, 16]. In order to increase or decrease the amplitude of mechanical vibrations of the most power-consuming modes, the difference of electric potentials with the corresponding amplitude and phase and a frequency of mechanical loading is applied to the electrodes of these actuators. In the course of operation of thin-walled elements, in particular, made of viscoelastic polymeric and composite materials with low shear stiffness, vibration processes are accompanied by dissipative heating caused by the hysteresis losses.

As important factors that may strongly affect the characteristics of damping of vibrations of thin-walled elements, we can mention the susceptibility of structural materials to transverse shear strains, the temperature dependences of their viscoelastic characteristics, and the conditions of mechanical fastening and heat exchange.

The electrothermomechanical models are constructed within the framework of nonclassical and refined statements, the solutions of specific problems of forced vibrations and dissipative vibration heating of thin-walled layered beams, plates, and shells are found, and their damping with the help of piezoactive components is analyzed in the monographs [3, 6] and numerous works [7–10, 14, 15, etc.]. The results of major part of the-

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se works are presented in the surveys [4, 5, 13]. In particular, the problems of axisymmetric resonance vibrations of viscoelastic cylindrical shells with piezoelectric actuators in the classical statement were studied in [7–9].

In the present paper, within the framework of the refined statement, we solve the problem of forced vibrations and dissipative heating of a viscoelastic shell compliant to shear with piezoactive actuators and rigidly fixed end faces. We analyze the influence of transverse shear strains, temperature dependences of the electromechanical properties of piezopassive and piezoactive materials, and the sizes of actuators on the amplitude and temperature frequency characteristics of vibrations of the shell.

1. Statement of the Problem. Main Equations

Consider a three-layer cylindrical shell formed by a passive (without piezoeffect) transversely isotropic viscoelastic layer of thickness h_0 and piezoelectric layers of thickness h_1 rigidly fastened to its surface. The shell of length ℓ is referred to an orthogonal coordinate system α, θ, z with coordinate $z = 0$ on the middle surface of passive layer of radius R. The piezolayers (actuators) are made of viscoelastic piezoelectric ceramics with the same properties oppositely polarized across the thickness, except the piezoelectric moduli of the opposite signs. Assume that the inner piezolayer $z \le -h_0/2$ has the piezoelectric modulus $+d_{31}$ and that the outer piezolayer $z \ge -h_0/2$ is characterized by the piezoelectric modulus $-d_{31}$. Solid electrodes are applied to the surfaces of the piezolayers. These electrodes are separated into sections by infinitely thin circular cuts with longitudinal coordinates α_1 and α_2. The electrodes operating in contact with surfaces of the passive layer are kept at the electric potential equal to zero: $\varphi(\pm h_0/2) = 0$.

The shell is loaded by the surface pressure $q_z = q_0(\alpha)\cos\omega t$ varying as a harmonic function of time t with an angular frequency ω close to the resonance frequency. To neutralize the action of this load, the voltage with an amplitude $\pm V_a$, the same frequency ω, and the opposite phase is applied to the circular sections of the electrode-containing surfaces $z = \pm(h_1 + h_0/2)$ whose width is equal to $\Delta_\alpha = \alpha_2 - \alpha_1$. In the sections $\alpha < \alpha_1$ and $\alpha > \alpha_2$, the electrodes are grounded $(V_a = 0)$. The ends of the shell are free in the tangential direction and rigidly fixed in the transverse direction. On the boundary surfaces of the shell, we impose the conditions of convective heat transfer with the ambient medium whose temperature is T_c. We simulate the viscoelastic behavior of passive and piezoactive materials with the help of the concept of complex moduli whose components depend on the temperature of vibration heating.

We describe the electromechanical behavior of the analyzed shell by using the Timoshenko-type theory of layered shells that takes into account the transverse shear strains and inertia of rotation of the normal element. As for the electric characteristics of piezolayers, we assume that the components of the vector of induction D_α and D_θ can be neglected. Moreover, it follows from the equations of electrostatics that the normal component in the piezolayers is independent of the thickness coordinate z, i.e., $D_z = \text{const}$ [3]. The components E_α and E_θ of the vector of electric-field strength are determined from the trivial determining equations $D_\alpha = 0$ and $D_\theta = 0$ for the piezoceramics polarized along the thickness. Assume that the temperature of dissipative heating of the shell is constant across the thickness of the entire set of layers.

In view of the accepted assumptions imposed on the complex electromechanical characteristics, the problem of forced harmonic vibrations of the analyzed shell is reduced to:

– *the following equations of motion* (the factor $e^{i\omega t}$ is omitted):

$$\frac{dN_\alpha}{d\alpha} + \rho_\bullet \omega^2 u = 0, \quad \frac{dQ_\alpha}{d\alpha} - \frac{N_\theta}{R} + \rho_{\bullet\bullet}\omega^2 w + q_z = 0,$$

$$\frac{dM_\alpha}{d\alpha} - Q_\alpha + \rho_{\bullet\bullet}\omega^2 \psi_\alpha = 0; \tag{1}$$

– *determining relations for the forces and moments:*

$$N_\alpha = C_{11}\varepsilon_\alpha + C_{12}\varepsilon_\theta, \quad N_\theta = C_{12}\varepsilon_\alpha + C_{11}\varepsilon_\theta, \quad Q_\alpha = k_s C_{44}\varepsilon_{\alpha z},$$

$$M_\alpha = D_{11}\kappa_\alpha + M_E, \quad M_\theta = D_{12}\kappa_\alpha + M_E; \tag{2}$$

– *relationships between the amplitudes of strains and displacements:*

$$\varepsilon_\alpha = \frac{du}{d\alpha}, \quad \varepsilon_\theta = \frac{w}{R}, \quad \kappa_\alpha = \frac{d\psi_\alpha}{d\alpha}, \quad \vartheta_\alpha = -\frac{dw}{d\alpha}, \quad \varepsilon_{\alpha z} = \psi_\alpha - \vartheta_\alpha; \tag{3}$$

– *expressions for the electric induction in the inner and outer piezolayers:*

$${}^{1,2}D_z = -b_{33}\frac{V_a}{h_1} \pm b_{31}(\varepsilon \mp h_1\kappa_\alpha), \quad z \le -\frac{h_0}{2}, \quad z \ge \frac{h_0}{2}; \tag{4}$$

– *heat-conduction equation averaged over the period of vibrations and across the thickness of the shell:*

$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial \alpha^2} - \frac{2\tilde{\alpha}_s}{\lambda H}(T - T_c) + \frac{\omega}{2\lambda H}\langle W \rangle \tag{5}$$

with a dissipative function

$$\langle W \rangle = N''_\alpha \varepsilon'_\alpha - N'_\alpha \varepsilon''_\alpha + N''_\theta \varepsilon'_\theta - N'_\theta \varepsilon''_\theta + M''_\alpha \kappa'_\alpha - M'_\alpha \kappa''_\alpha$$

$$+ Q''_\alpha \varepsilon'_{\alpha z} - Q'_\alpha \varepsilon''_{\alpha z} + ({}^1D''_z + {}^2D''_z)V'_\alpha - ({}^1D'_z + {}^2D'_z)V''_\alpha. \tag{6}$$

The mechanical boundary conditions have the form

$$N_\alpha = 0, \quad w = 0, \quad \psi_\alpha = 0. \tag{7}$$

We represent the boundary and initial conditions for the heat-conduction equation in the form

$$\lambda\frac{\partial T}{\partial \alpha} = \pm\tilde{\alpha}_{0,\ell}(T - T_c), \quad \alpha = 0, \ell,$$

$$T = T_0, \quad t = 0. \tag{8}$$

In relations (1)–(8), we introduce the following notation:

$$C_{1n} = c_{1n}h_0 + 2c_{1n}^E h_1, \quad C_{44} = G_{\alpha z}h_0 + 2c_{44}^E h_1,$$

$$D_{1n} = \frac{c_{1n}h_0^3 + 2c_{1n}^E \bar{h}_{13} + 2\gamma_{33}h_1^3}{12}, \quad c_{11} = \frac{E}{1-\nu^2}, \quad c_{12} = \nu c_{11},$$

$$c_{11}^E = \frac{1}{s_{11}^E(1-\nu_E^2)}, \quad c_{12}^E = \nu_E c_{11}^E, \quad \nu_E = -\frac{s_{12}^E}{s_{11}^E},$$

$$c_{44}^E = \frac{1}{s_{44}^E - d_{15}^2/\varepsilon_{11}^T}, \quad b_{31} = \frac{d_{31}}{s_{11}^E(1-\nu_E)}, \quad b_{33} = \varepsilon_{33}^T(1-k_p^2),$$

$$k_p^2 = \frac{2d_{31}b_{31}}{\varepsilon_{33}^T}, \quad \gamma_{33} = \frac{b_{31}^2}{b_{33}}, \quad \rho_\bullet = 2\rho_1 h_1 + \rho_0 h_0,$$

$$\rho_{\bullet\bullet} = \frac{2\rho_1\bar{h}_{13} + \rho_\bullet h_0^3}{12}, \quad \bar{h}_{13} = 4h_1^3 + 6h_1{}^2 h_0 + 3h_1 h_0^2, \quad \bar{h}_1 = \frac{h_0 + h_1}{2},$$

$$H = 2h_1 + h_0, \quad M_E = -2h_1 b_{31} V_\alpha, \quad \varepsilon = \varepsilon_\alpha + \varepsilon_\theta.$$

Here, $s_{kk}^E = s'_{kk}(1 - i\delta_{kk}^s)$, $d_{ik} = d'_{ik}(1 - i\delta_{ik}^d)$, and $\varepsilon_{kk}^T = \varepsilon'_{kk}(1 - i\delta_{kk}^\varepsilon)$ are, respectively, the temperature-dependent complex compliances, piezoelectric moduli, and dielectric permittivities of piezoceramics; $\nu = \text{const}$ is Poisson's ratio, $E = E' + iE''$ and $G_{\alpha z} = G'_{\alpha z} + iG''_{\alpha z}$ are, respectively, temperature-dependent complex Young's modulus and the transverse-shear modulus of the passive material; k_s is the transverse shear coefficient; $w = w' + iw''$, $u = u' + iu''$, and $\psi_\alpha = \psi'_\alpha + i\psi''_\alpha$ are, respectively, the complex amplitudes of deflections, longitudinal displacements, and the angle of rotation of the nondeformed normal element; N_α, N_θ, Q_α and M_α, M_θ are the similar amplitudes of forces and bending moments; ρ_0 and ρ_1 are the specific densities of the passive layer and piezoactuators; λ and a are, respectively, the averaged thermal conductivity and thermal diffusivity coefficients;

$$\tilde{\alpha}_s = \frac{\tilde{\alpha}_+ + \tilde{\alpha}_-}{2};$$

$\tilde{\alpha}_\pm$ and $\tilde{\alpha}_{0,\ell}$ are the heat-exchange coefficients on the corresponding surfaces and ends of the shell, and T_0 is the initial temperature of the shell. Here and in what follows, we use the standard notation of complex quantities, i.e., $a = a' + ia''$, $|a| = (a'^2 + a''^2)^{1/2}$, $i = \sqrt{-1}$.

2. Solution of the Problem

After necessary transformations, we represent the equations of harmonic vibrations (1)–(3) for the numerical solution of the posed problem in the form of ordinary differential equations of the normal form for the complex quantities N_α, Q_α, M_α, u, w, and ψ_α:

$$\frac{dN_\alpha}{d\alpha} = -\rho_\bullet \omega^2 u, \qquad \frac{dQ_\alpha}{d\alpha} = \frac{1}{R}\left(\tilde{C}_{12} N_\alpha - \tilde{C}_{11}\frac{W}{R}\right) - q_z - \rho_\bullet \omega^2 w,$$

$$\frac{dM_\alpha}{d\alpha} = Q_\alpha - \rho_{\bullet\bullet}\omega^2 \psi_\alpha, \qquad \frac{du}{d\alpha} = J_C N_\alpha - \tilde{C}_{12}\frac{W}{R}, \tag{9}$$

$$\frac{d\psi_\alpha}{d\alpha} = J_D (M_\alpha - M_E), \qquad \frac{dw}{d\alpha} = -\psi_\alpha + J_{SD} Q_\alpha,$$

where

$$J_C = \frac{1}{C_{11}}, \quad J_D = \frac{1}{D_{11}}, \quad \nu_C = \frac{C_{12}}{C_{11}}, \quad \tilde{C}_{11} = C(1-\nu^2), \quad J_{SD} = \frac{1}{k_s C_{44}}.$$

In view of the dependence of electromechanical properties of materials on the temperature of dissipative heating, equation (9) of forced vibrations of the shell and the heat-conduction equation (5), (6) are coupled and nonlinear. For the solution of these equations, we use the method of step-by-step time integration [3, 6]. In each time step Δt, we integrate the complex-valued system of equations of electromechanics (9) with the boundary conditions (7) by the numerical method of discrete orthogonalization [2] with the help of typical software intended for the solution of ordinary differential equations. In the first step, we solve the problem for isothermal characteristics of the materials $(T = T_0)$. In the second step, we compute the dissipative function (6) and solve the heat conduction problem (5), (8) by the finite-difference method according to the explicit scheme. Further, on the basis of the obtained temperature distribution, we determine the stiffness characteristics and repeat the outlined procedure in the next time step. In the numerical realization of the proposed approach, we use the dimensionless space $x = \alpha/\ell$ and time $\tau = at/\ell^2$ coordinates and the Biot parameters of heat exchange $(\gamma_s)_{0,\ell} = (\tilde{\alpha}_s)_{0,\ell}/\lambda$.

3. Numerical Results and Their Analysis

In our numerical calculations, we restrict ourselves to the case of harmonic loading of the shell by a constant pressure with amplitude $q_z(\alpha) = q_0$. The amplitude of electric potential used for the compensation of pressure q_0 is given by the formula [7]

$$V_\alpha = \kappa_\alpha(\Delta_\alpha) q_0, \tag{10}$$

where κ_α is a control coefficient.

The value of κ_α corresponding to the maximal value of the amplitude of mechanical vibrations is determined as follows:

$$\kappa_\alpha = \frac{\left|w^1_{q\max}\right|}{\left|w^1_{E\max}\right|},$$

where $\left|w^1_{q\max}\right|$ and $\left|w^1_{E\max}\right|$ are the maximum amplitudes of deflections at the frequency of linear resonance computed on the basis of the solution of standard problems for $q_0 = 1$ Pa, $V_\alpha = 0$ and $q_0 = 0$, $V_\alpha = 1$ V, respectively. The opposite phases of the electric potential applied to the electrodes of the actuator is taken into account by the formula $V_\alpha \cos(\omega t + \pi) = -V_\alpha \cos\omega t$. Note that, under the analyzed loading, we observe the realization mainly of bending vibrations. Therefore, the numerical analysis is performed in the vicinity of the first frequency of the most power-consuming mode of bending vibrations of the shell. The passive layer is made of a polymeric composite [8], while the piezoactuators are made of the TsTStBS-2 viscoelastic piezoelectric ceramics [1]. The experimental temperature dependences of their electromechanical characteristics are approximated by the following expressions:

$$E' = (1672 - 118.6T)\cdot 10^6 \ \text{[Pa]}, \quad E'' = (15.01 - 1.205T)\cdot 10^6 \ \text{[Pa]},$$

$$G_{\alpha z} = \frac{E}{2(1+\nu)}, \quad \bar{T} = T - T_0, \quad \nu_E = 0.37, \quad \nu = 0.3636,$$

$$s'_{11} = 12.5(1 + 0.377\cdot 10^{-3}\bar{T})\cdot 10^{-12} \ [\text{m}^2/\text{N}],$$

$$s'_{44} = 39.7(1 + 0.5458\cdot 10^{-3}\bar{T})\cdot 10^{-12} \ [\text{m}^2/\text{N}],$$

$$\delta^s_{11} = 0.16(1 + 0.6155\cdot 10^{-3}\bar{T} + 0.4158\cdot 10^{-4}\bar{T}^2),$$

$$\delta^s_{44} = 0.14(1 + 8.33\cdot 10^{-3}\bar{T})\cdot 10^{-2},$$

$$d'_{31} = -1.6(1 + 0.219\cdot 10^{-3}\bar{T})\cdot 10^{-10} \ \text{[K/m]},$$

$$\delta^d_{31} = 0.4(1 + 1.198\cdot 10^{2}\bar{T} + 1.823\cdot 10^{-4}\bar{T}^2)\cdot 10^{-2},$$

$$d'_{15} = 4.5(1 + 0.9722\cdot 10^{-3}\bar{T})\cdot 10^{-10} \ \text{[K/m]},$$

$$\delta^d_{15} = 0.35(1 + 0.3571\cdot 10^{-2}\bar{T})\cdot 10^{-2},$$

$$\varepsilon'_{11} = 18.5\varepsilon_0(1 + 0.4505\cdot 10^{-2}\bar{T})\cdot 10^{2},$$

$$\delta^\varepsilon_{11} = 0.5(1 + 0.015\bar{T})\cdot 10^{-2},$$

$$\varepsilon'_{33} = 21\varepsilon_0(1 + 0.111\cdot 10^{-3}\bar{T} + 0.8426\cdot 10^{-4}\bar{T}^2)\cdot 10^{2},$$

$$\varepsilon_0 = 8.854\cdot 10^{-12} \ \text{F/m},$$

$$\delta'_{33} = 0.35(1 + 0.0119\cdot\bar{T} + 0.119\cdot 10^{-3}\bar{T}^2)\cdot 10^{-2},$$

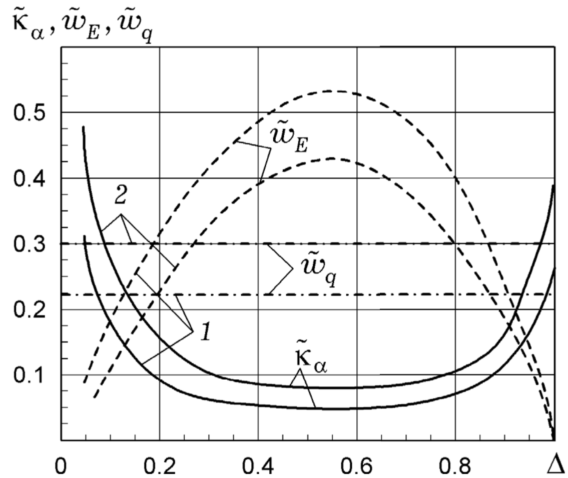


Fig. 1

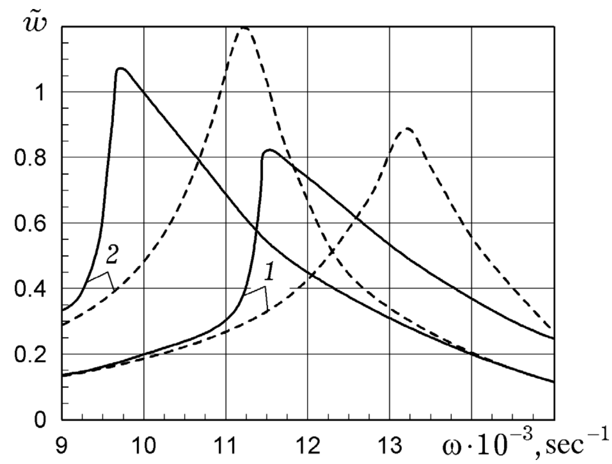


Fig. 2

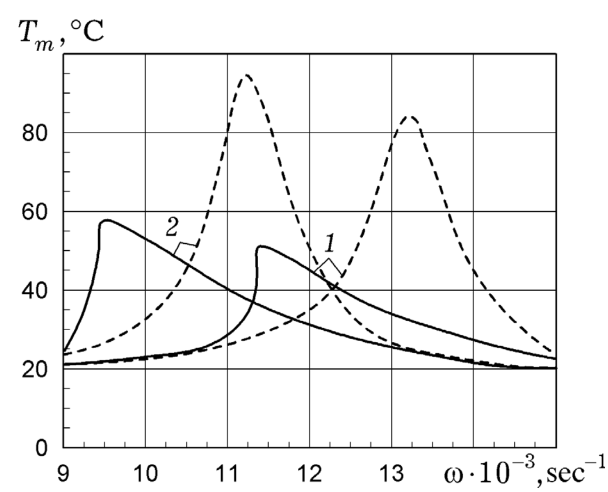
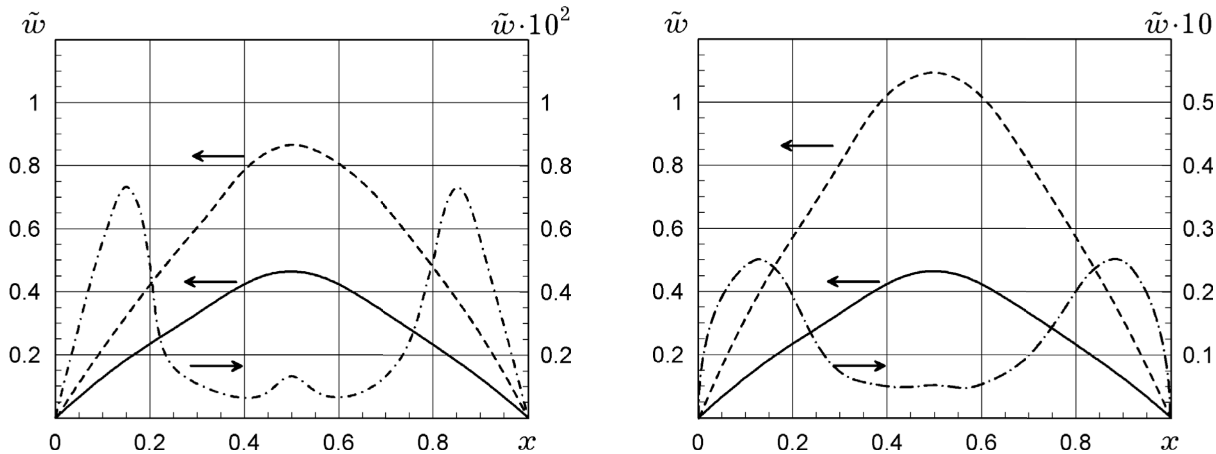


Fig. 3



(a) **(b)**

Fig. 4

$$\rho_0 = 929 \text{ kg/m}^3, \quad \rho_1 = 7520 \text{ kg/m}^3, \quad \lambda = 0.45 \text{ W/(m}\cdot{}^\circ\text{C)}.$$

The coefficient of transverse shear $k_s = 5/6$ [2]. The shell has the following geometric parameters: $R = \ell = 0.2$ m, $h_0 = 0.04$ m, and $h_1 = 0.1 \cdot 10^{-4}$ m.

In curves *1* and *2* (Fig. 1), we present the dependences of the maximal amplitudes of deflections on the width $\Delta = \Delta_x / \ell$ of a ring actuator. The dash-dotted lines correspond to the deflection $\tilde{w}_q = |w_q^1(0.5)| \cdot 10^8$ m attained under mechanical loading with an amplitude $q_0 = 1$ Pa for $V_\alpha = 0$, whereas the dashed lines illustrate the deflection $\tilde{w}_E = w_E^1(0.5) \cdot 10^6$ m realized in the case where the electric potential $V'_\alpha = 1$ V, $V''_\alpha = 0$ with $q_0 = 0$ is applied to the actuator. The solid lines correspond to the control coefficient $\kappa_\alpha = 10\kappa_\alpha$. Curves *1* correspond to the resonance frequency $\omega_{\text{p}} = 0.132 \cdot 10^5 \text{ sec}^{-1}$ in the classical statement of the problem. Curves *2* describe the results obtained at a frequency $\omega_{\text{p}} = 0.113 \cdot 10^5 \text{ sec}^{-1}$ with regard for transverse shear strains. The width of the actuator Δ is such that its midpoint coincides with the coordinate $x = 0.5$ of the maximum normal deflections of the shell.

The frequency dependences of the maximum values of amplitudes of deflections $\tilde{w} = w(0.5) \cdot 10^5$ m and the temperature of vibration heating $T_m = T(0.5)$ (dashed lines) and the plots obtained with regard for temperature dependences of the properties of materials (solid lines) in the neighborhood of the resonance frequency of the most power-consuming mode of bending vibrations of the shell are depicted in Figs. 2 and 3, respectively. Curves 1 correspond to the solution of the problem in the classical statement. At the same time, curves 2 correspond to the statement of the problem with regard for the transverse shear strains for the shell mechanically loaded with an amplitude $q_0 = 0.4 \cdot 10^4$ Pa and the of heat-conduction coefficient $\gamma_3 = 0.4$.

The distributions of the amplitude of deflections $\tilde{w} = |w| \cdot 10^5$ m along the generatrix of the shell for the indicated load are depicted in Fig. 4a and Fig. 4b for the classical and refined statements, respectively. In these figures, the dashed curves correspond to the isothermal electromechanical characteristics, while the solid lines take into account the temperature dependences of these characteristics. The dash-dotted lines correspond to the joint antiphase action of the mechanical loading $q_0 = 0.4 \cdot 10^4$ Pa and the electric potentials $V_a = 16.7$ V

(Fig. 4a) and $V_a = 28$ V (Fig. 4b).

The analysis of the curves presented in Figs. 1–4 and the data of numerical analyses enable us to conclude that, for the investigated shell with rigidly fixed ends, there exists an actuator with the optimal sizes $\Delta = 0.5$, $\alpha_1 = 0.215\ell$, and $\alpha_2 = 0.785\ell$ for which the deflections of the bending mode of vibrations are maximum for the minimum drop of electric potentials on the electrodes of the actuator. This actuator proves to be most efficient for the compensation of forced mechanical vibrations of the shell. The influence of transverse shear strains and temperature dependences of the electromechanical characteristics of piezopassive and piezoactive materials (curves 2) results in a shift of the amplitude and temperature (Fig. 3) frequency characteristics toward lower resonance frequencies and also to an increase in deflections and temperature of vibration heating at the corresponding refined resonance frequency of bending mechanical vibrations of the shell. As a result of the joint antiphase action of the surface pressure and the compensating drop of electric potentials at the frequencies of classical (Fig. 4a) and refined (Fig. 4b) resonances, the amplitudes of mechanical deflections of an undamped shell decrease by about two orders of magnitude. Moreover, the temperature of vibration heating remains close to the initial temperature.

CONCLUSIONS

We consider the coupled problem of forced axisymmetric vibrations and dissipative heating of a rigidly fixed viscoelastic cylindrical shell with piezoactuators subjected to electromechanical monoharmonic loading. We take into account the transverse shear strains and temperature dependences of the complex characteristics of piezopassive and piezoactive materials. The nonlinear problem is solved by the method of step-by-step time integration with the help of the procedure of discrete orthogonalization for the integration of the equations of mechanics and the explicit scheme of the finite-difference method for the solution of the heat-conduction equation. We study the influence of transverse shear strains, geometric sizes of the actuator, and temperature dependences of the properties of materials on the frequency characteristics of the amplitudes of deflections and the temperature of vibration heating and on the active damping of forced vibrations of the shell.

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