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WEIGHT-VIBRATION PARETO OPTIMIZATION OF A DUAL MASS FLYWHEEL

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By using the methodology of the multiobjective optimal design of engineering systems, we consider the problem of weight-vibration Pareto optimization of a dual mass flywheel with an aim to study the feasibility of its application in heavy-duty truck powertrains. The results obtained show the following: the solution of the considered optimization problem does exist; the mass inertia, stiffness, and damping parameters of the absorber optimized in an operating engine speed range of 600–2000 rpm exist and provide the best attenuation of the torque oscillation at the transmission input shaft. Finally, the obtained results show the feasibility evidence for the application of weight-vibration optimized dual mass flywheels in heavy-duty truck drivetrain systems.

Keywords: torsional vibration absorber, dual mass flywheel, drivetrain system of a heavy-duty truck, global sensitivity analysis, weight-vibration Pareto optimization.

Introduction

An engineering system must meet numerous requirements, e.g., system's quickness and accuracy, safety and user friendliness, noiseless and low level of vibrations, environmental friendliness, and cost efficiency. These are some of constraints to be satisfied in the process of design of modern engineering products, which make the design of engineering systems very complicated.

In the present paper, the methodology of multiobjective optimal design of engineering systems is presented. This methodology is based on the global sensitivity analysis (GSA) and Pareto optimization techniques. It was implemented in the SAMO computer toolbox developed at the Mechanical Systems, Division of Dynamics, Chalmers University of Technology [4]. The methodology and SAMO toolbox were successfully used for the optimal design of engineering systems with different applications [5–7]. In what follows, we apply the methodology to solve the problem of weight-vibration Pareto optimization of the design of dual mass flywheels intended for application in torsional vibration attenuation in heavy-duty truck powertrains. A dual mass flywheel (DMF) is a well-known design of torsional vibration absorbers. It was a subject for extensive research [1, 3, 8–10]. The research is ongoing to understand whether this concept of absorber is suitable for the attenuation of torsional vibrations in the powertrains of heavy-duty trucks [1, 9, 10].

The outline of the paper is as follows: In Section 1, the global sensitivity analysis and Pareto optimization problems are formulated for the mathematical model of a generic engineering system. The formulations of these problems, together with the outline of the algorithm of GSA and the structure of the SAMO toolbox, constitute the basis of the methodology aimed at designing optimal engineering products. The results of weight-vibration Pareto optimal design of torsional vibration absorbers intended for application in heavy-duty truck powertrains are presented in Sections 2 and 3. The paper is finalized with conclusions and the outline of future research.

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Some results of the paper were presented at the 10th International Conference on the Mathematical Problems of Mechanics of Nonhomogeneous Structures, September 17–20, 2019, Lviv, Ukraine [2].

1. Sensitivity Analysis and Pareto Optimization

Consider an engineering system that consists of a number of functional components representing mass inertia, stiffness, and damping characteristics of the system. Let $\mathbf{q} = [q_1, q_2, \ldots, q_n]^\top$ be a vector of generalized coordinates, $\mathbf{T} = [T_1, T_2, \ldots, T_m]^\top$ be the vector of external loads, e.g., forces or/and torques acting upon the system, and $\mathbf{d} = [d_1, d_2, \ldots, d_k]^\top$ be the vector of design parameters representing the mass inertia, stiffness, and damping characteristics of all functional components of the system.

The following expression is used to represent the set of operational scenarios $(\mathbf{OS}s)$ of the analyzed generic engineering system:

$$\mathbf{OS}s = \{\mathbf{T}(t), \mathbf{q}(t), \mathbf{d}, t \in [t_0, t_f], \mathbf{d} \in \Omega\}. \tag{1}$$

In expression (1), t_0 and t_f are the initial and final instants of time and Ω is the domain of feasible values for the vector of design parameters.

For any feasible vector of design parameters $\mathbf{d} = [d_1, d_2, \ldots, d_k]^\top \in \Omega$ and given external loads $\mathbf{T} = [T_1, T_2, \ldots, T_m]^\top$, the vector of generalized coordinates $\mathbf{q} = [q_1, q_2, \ldots, q_n]^\top$ satisfies the equation

$$\mathbf{L}\big[\mathbf{q}(t), \dot{\mathbf{q}}(t), \ddot{\mathbf{q}}(t), \mathbf{T}(t), \mathbf{d}\big] = \mathbf{0}, \tag{2}$$

where $\mathbf{L}$ is an operator that, together with a given initial state of the system

$$\mathbf{q}(0) = \mathbf{q}^0, \qquad \dot{\mathbf{q}}(0) = \dot{\mathbf{q}}^0, \tag{3}$$

determine the system performance (response), i.e., the vector $\mathbf{q}[t, t_0, \mathbf{q}^0, \dot{\mathbf{q}}^0, \mathbf{T}(t), \mathbf{d}]$ for all $t \in [t_0, t_f]$.

Equation (2) and the initial state (3) form the mathematical model of a generic engineering system and allow us to obtain all its feasible operational scenarios.

As an example of mathematical model (2), (3), we can consider a matrix equation

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{U}[t, \mathbf{T}(t)]. \tag{4}$$

Together with the initial state (3), this equation governs the motion of an n-degree-of-freedom mechanical system with linear stiffness and damping functional components. Here, $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are, respectively, the mass inertia, damping, and stiffness matrices and $\mathbf{U}$ is the vector of generalized forces.

1.1. Global Sensitivity Analysis and the Formulations of Pareto Optimization Problems. As the first step in the optimal design of an engineering system, it is important to study the sensitivity of the system response to the variation of its design parameters. The analysis of sensitivity of an engineering system to the varying parameter d_i can be carried out either locally or globally. In the local sensitivity analysis, the effects of design input d_i on the system response is approximated by the partial derivative of an objective function used as the measure of system response to the design parameter d_i, which is taken around a fixed point d_i^0. This approach only considers the variations of an objective function with respect to a single design parameter at a time. Furthermore, the domain of input design variables might not be appropriately scanned by the local method.

The global sensitivity analysis is one of the most prominent approaches in the design of engineering systems that can provide informative insight into the design process. To determine global sensitivity indices, it is necessary to evaluate multilayer integrals. This process demands heavy computational efforts. In what follows, the multiplicative dimensional reduction method proposed in [11] is briefly described. This method is used in the SAMO computer toolbox [4] and can approximate global sensitivity indices in the efficient and accurate manner.

An objective function can be expressed as a function of the set of independent random variables, i.e., the design parameters

$$\mathbf{d} = [d_1, d_2, \ldots, d_k]^\top \in \Omega,$$

through the respective deterministic functional relationship $F = F(\mathbf{d})$. It is proposed to approximate the function F as follows:

$$F(\mathbf{d}) \approx [F(\mathbf{c})]^{1-k} \prod_{i=1}^{k} F(d_i, \mathbf{c}_{-i}), \tag{5}$$

where $F(\mathbf{c})$ is a constant and $F(d_i, \mathbf{c}_{-i})$ denotes the value of the function in the case where all inputs except d_i are fixed at their respective cut point coordinates $\mathbf{c} = [c_1, \ldots, c_k]^\top$. Expression (5) is capable to approximate the function F with a satisfactory level of accuracy and is particularly useful for approximating the integrals required to compute sensitivity indices [11]. By using this approach, primary and higher-order sensitivity indices can be approximated as follows:

$$S_i \approx \frac{\left(\frac{\beta_i}{\alpha_i^2} - 1\right)}{\left(\prod_{j=1}^{k} \frac{\beta_j}{\alpha_j^2}\right) - 1}, \qquad S_{i_1 \ldots i_s} \approx \frac{\prod_{j=1}^{s} \left(\frac{\beta_{ij}}{\alpha_{ij}^2} - 1\right)}{\left(\prod_{j=1}^{k} \frac{\beta_j}{\alpha_j^2}\right) - 1}. \tag{6}$$

The coefficients α_j and β_j are defined as the mean and mean square of the jth univariate function, respectively, and can be represented as

$$\alpha_j \approx \sum_{\ell=1}^{N} w_{j\ell} F(d_{j\ell}, \mathbf{c}_{-j\ell}), \qquad \beta_j \approx \sum_{\ell=1}^{N} w_{j\ell} F^2(d_{j\ell}, \mathbf{c}_{-j\ell}). \tag{7}$$

Here, N is the total number of integration points, $d_{j\ell}$ and $w_{j\ell}$ are the ℓth Gaussian integration abscissas and the corresponding weight, respectively.

Finally, the total sensitivity index corresponding to the parameter d_i can be expressed as

$$S_i^{\mathrm{T}} \approx \frac{1-\dfrac{\alpha_i^2}{\beta_i}}{1-\left(\prod_{j=1}^{k}\dfrac{\alpha_j^2}{\beta_j}\right)}. \tag{8}$$

It should be noted that the total number of objective function evaluations required to compute the sensitivity indices by using this method is only $k \times N$, where k is the number of design parameters.

To accomplish the sensitivity analysis of the system output, a suitable cut point must be chosen together with a probability distribution. Equations (6)–(8) are then utilized to attain sensitivity indices. More details on the multiplicative dimensional reduction method for the global sensitivity analysis can be found in [11].

Let the following functionals be chosen to measure the quality of performance of the engineering system in question:

$$F_1[\mathbf{q}(t),\mathbf{d}],\ldots,F_{nF}[\mathbf{q}(t),\mathbf{d}]. \tag{9}$$

The following problem of the global sensitivity analysis for a generic engineering system is formulated:

Problem GSA. *Let* $\mathbf{d}=[d_1,d_2,\ldots,d_k]^{\top}$ *be the vector of design parameters of the generic engineering system in question. It is required for a given feasible operational scenario*

$$\mathbf{O\tilde{S}} \in \mathbf{OS}s \tag{10}$$

to determine, by using equation (8), the total sensitivity indices

$$S_i^{\mathrm{T}}(F_j), \qquad i=1,\ldots,k, \qquad j=1,\ldots,nF, \tag{11}$$

of functionals (9) for all varying design parameters d_i *satisfying equation (2), the initial state (3) and the restriction*

$$\mathbf{d} = [d_1,d_2,\ldots,d_k]^{\top} \in \Omega. \tag{12}$$

The solution of the problem GSA provides mapping between the values of the total sensitivity indices (11) and the design parameters (12) of the generic engineering system.

As a result of the solution of the problem GSA, we identify the vector of the most important design parameters

$$\mathbf{d}_s = [d_{s1},d_{s2},\ldots,d_{sk}]^{\top} \in \Omega, \qquad 1 \le sk \le k, \tag{13}$$

and the most sensitive functionals $F_j[\mathbf{q}(t),\mathbf{d}]$, $1 \le j \le nF_1 \le nF$. Thus, the Pareto optimization problem can be now stated as follows:

Problem PO. *For given feasible operational scenario (10), it is required to determine the design parameters*

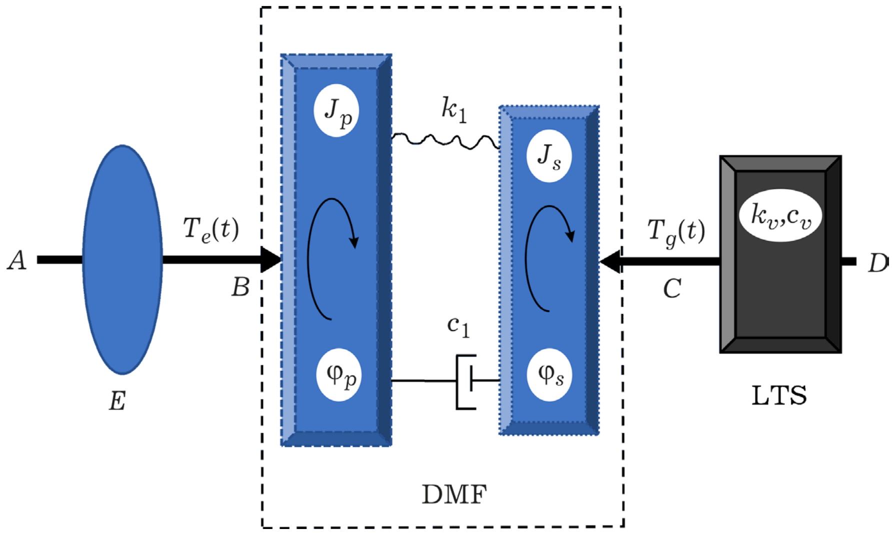

Fig. 1. Sketch of a generic drivetrain system equipped with a dual mass flywheel.

$$\mathbf{d}_s = \mathbf{d}_s^* = [d_{s1}^*, d_{s2}^*, \dots, d_{sk}^*]^\top, \qquad sk \in [1, \dots, k],$$

and the vector of generalized coordinates $\mathbf{q}(t) = \mathbf{q}^*(t)$ *that completely satisfy the system of variational equations*

$$\min_{\mathbf{d}_s \in \Omega} (F_j[\mathbf{q}(t), \mathbf{d}_s]) = F_j[\mathbf{q}^*(t), \mathbf{d}_s^*], \qquad j = 1, \dots, nF_1,$$

subject to the mathematical model (2)–(3) and restriction (13).

In [4], the SAMO computer code developed at Chalmers University of Technology was presented as an efficient toolbox for the optimal design of engineering systems. In this stage, the SAMO toolbox includes two modules: SAMO-GSA and SAMO-PO. The SAMO-GSA module is based on the multiplicative version of the dimensional reduction method [11] aimed at solving the above-formulated problem GSA. In the SAMO-GSA module, an efficient approximation is used to simplify the computation of variance-based sensitivity indices associated with a general function of n-random varying parameters. Then the results of solution of the problem GSA might be used as an input of the SAMO-PO module for multiobjective optimization (the above-formulated problem PO). The operation of the SAMO-PO module is based on the genetic algorithm (GA). The GA settings include lower and upper bounds for the variations of the design parameters, population size, number of generations, elite count, and Pareto fraction settings. The results of the SAMO-PO module are presented in terms of the Pareto fronts and the corresponding Pareto sets for the subsequent analysis and decision-making by the user. More details on the SAMO toolbox and the link to the corresponding computer codes for different examples can be found in [4].

2. Weight-Vibration Pareto Optimization of a Dual Mass Flywheel

In this section, we apply the methodology presented in Section 1 to solve the problem of weight-vibration Pareto optimization of the design of a dual mass flywheel intended for application to torsional vibration attenuation in heavy-duty truck powertrains.

2.1. Drivetrain System Equipped with a Dual Mass Flywheel. Consider a system depicted in Fig. 1. The system comprises an engine (*E*), a torsional vibration absorber, (DMF), and a load transmission system (LTS). Assume that the vibration absorber (Fig. 1) consists of two rigid bodies called the primary flywheel (PFW) and the secondary flywheel (SFW). These wheels are connected by a massless linear torsional spring and a massless linear torsional viscous damper. The engine output shaft *AB* and the transmission input shaft *CD* are assumed to be rigid and rigidly connected to the PFW and to the SFW, respectively. The torque $T_e(t)$ rotates the primary flywheel about the shaft *AB*.

In Fig. 1, φ_p and φ_s are the absolute angles of rotation of the PFW and the SFW, respectively, J_p and J_s are the torsional moments of inertia of the PFW and the SFW, respectively, and k_1 and c_1 are, respectively, the coefficients of torsional stiffness and torsional damping.

The equations of torsional vibration dynamics of the drivetrain system equipped with a DMF can be represented in the matrix form (4) with

$$\mathbf{q} = [\varphi_p, \varphi_s]^\top, \quad \dot{\mathbf{q}} = [\dot{\varphi}_p, \dot{\varphi}_s]^\top, \quad \ddot{\mathbf{q}} = [\ddot{\varphi}_p, \ddot{\varphi}_s]^\top,$$

$$\mathbf{U}[t, \mathbf{T}(t)] = [T_e(t), -T_g(t)]^\top, \tag{14}$$

$$\mathbf{M} = \begin{pmatrix} J_p & 0 \\ 0 & J_s \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} c_1 & -c_1 \\ -c_1 & c_1 \end{pmatrix}, \quad \mathbf{K} = \begin{pmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{pmatrix}.$$

Equations (4) and (14), together with the following initial state

$$\varphi_p(t_0) = \varphi_p^0, \quad \varphi_s(t_0) = \varphi_s^0, \quad \dot{\varphi}_p(t_0) = \dot{\varphi}_p^0, \quad \dot{\varphi}_s(t_0) = \dot{\varphi}_s^0, \tag{15}$$

constitute a mathematical model of drivetrain system equipped with a DMF.

2.2. Global Sensitivity Analysis of a Drivetrain System Equipped with a DMF. The set of operational scenarios (1) for the system in question is described by the following expressions:

$$\begin{gathered} \mathbf{OS}s = \{\mathbf{T}(t) = [T_e(t), -T_g(t)]^\top, \quad \mathbf{q}(t) = [\varphi_p(t), \varphi_s(t)]^\top, \\ \mathbf{d} = [J_p, J_s, k_1, c_1]^\mathrm{T}, \quad t \in [t_0, t_f], \quad \mathbf{d} \in \Omega\}, \end{gathered} \tag{16}$$

where

$$T_e(t) = T_m + a_e \sin(\omega_{n_0} t), \quad \omega_{n_0} = n_0 \omega, \quad \omega = \frac{2\pi n_e}{60}, \tag{17}$$

$$T_g(t) = k_v(\varphi_s - \varphi_v) + c_v(\dot{\varphi}_s - \dot{\varphi}_v), \quad \varphi_v(t) = \omega_v t. \tag{18}$$

Here, in expressions (17), the engine input torque $T_e(t)$ is modeled by a constant torque T_m plus a harmonic function, ω_{n_0} is the n_0-order engine vibration frequency, i.e., n_0 times the angular velocity ω, and n_e is the engine speed (in rpm). The torque at the transmission input shaft $T_g(t)$ is modeled by expressions (18), k_v and c_v are, respectively, the equivalent torsional stiffness and damping coefficients of the load transmission system, and φ_v, ω_v are, respectively the absolute angle of rotation and the angular velocity of the transmission input shaft.

Consider a vector

$$\mathbf{d} = [d_1, d_2, d_3, d_4]^\top = [J_p, J_s, k_1, c_1]^\top \in \Omega \tag{19}$$

and the following functionals:

$$F_1(\mathbf{d}) = \int_{600}^{2000} \operatorname{std}(T_g[\mathbf{q}(t), \mathbf{d}, n_e])\, dn_e\,, \tag{20}$$

$$F_2(\mathbf{d}) = J_p + J_s\,, \tag{21}$$

$$F_3(\mathbf{d}) = \int_{600}^{2000} \operatorname{std}(T_f[\mathbf{q}(t), \mathbf{d}, n_e])\, dn_e \tag{22}$$

as the vector of design parameters and the quality measures of the performance of drivetrain system equipped with a DMF. Here, the function T_f represents the friction torque in the stiffness-damping interface of the DMF and is defined as follows:

$$T_f = k_1(\varphi_p - \varphi_s) + c_1(\dot{\varphi}_p - \dot{\varphi}_s)\,. \tag{23}$$

The functionals $F_1(\mathbf{d})$ and $F_3(\mathbf{d})$ characterize the oscillations of torque at the transmission input shaft and the energy dissipating in the DMF within the operating engine speed range $600\,\text{rpm} \le n_e \le 2000$ rpm, respectively. The functional $F_2(\mathbf{d})$ characterizes the mass inertia properties of the DMF and is well relevant for estimating the total weight of the absorber.

The problem of global sensitivity analysis formulated in Section 1 was solved for the drivetrain system equipped with a DMF by using the differential equation of motion (4) with (14), the initial state (15), the vector of the design parameters (19), and the functionals (20)–(22). The feasible operational scenario (10) was given by the torques $T_e(t)$ and $T_g(t)$ determined by expressions (17), (18).

The third-order engine vibration harmonic is in the focus of analysis as one of the most significant contributions to the oscillatory response [1, 9], i.e., in all simulations, the engine order vibration frequency n_0 is chosen to be equal to 3. The remaining values of the parameters for the torque $T_e(t)$ are as follows: the mean value of the engine input torque $T_m = 300$ Nm; the amplitude of the engine torque harmonic excitation $a_e = 500$ Nm, and the engine speed n_e was chosen within the range $600-2000$ rpm. The values of the parameters of the torque $T_g(t)$ at the transmission input shaft are: $k_v = 10^5$ Nm/rad, $c_v = 0.1$ Nms/rad, and $\omega_v = \omega_{n_0}/3$.

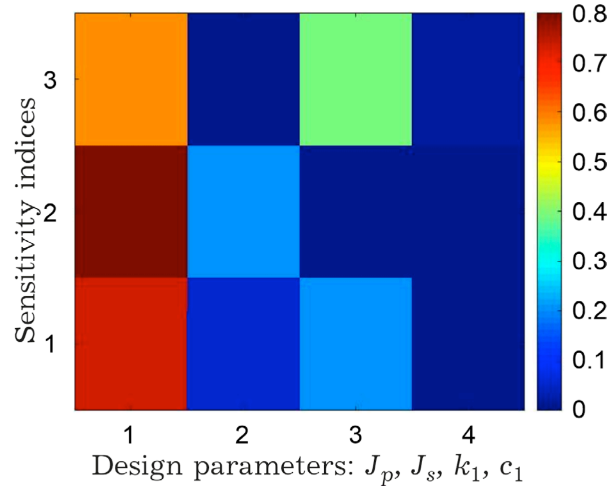

Fig. 2. Sensitivity indices of the objective functions $F_1(\mathbf{d})$, $F_2(\mathbf{d})$, and $F_3(\mathbf{d})$ for the DMF within the operating engine speed range $600\,\text{rpm} \le n_e \le 2000$ rpm.

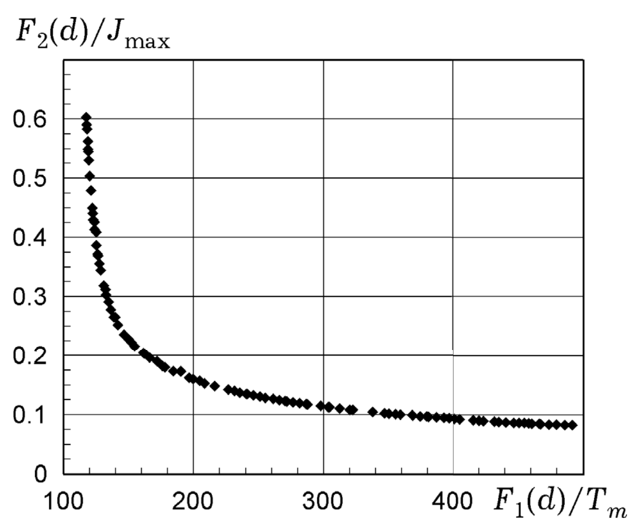

Fig. 3. Pareto front of weight-vibration Pareto optimization for the DMF within the operating speed range $600\,\text{rpm} \le n_e \le 2000$ rpm; $T_m = 300$ Nm, $J_{max} = 4.05\,\text{kgm}^2$.

Table 1. Settings for the GSA and Pareto Optimization of a Drivetrain System with DMF

Design parameter, **d**	J_p, kgm^2	J_s, kgm^2	k_1, Nm/rad	c_1, Nms/rad
Nominal values, **d**	1.8	0.9	12,732	30
Lower bounds, **d**	0.2	0.1	2000	0
Upper bounds, **d**	2.4	1.2	26,242	150

The results of the GSA of drivetrain systems with respect to variations of the design parameters (19) have been obtained for engine speeds within the range of $600-2000\,\text{rpm}$ by using the SAMO computer code with settings given in Table 1. Here, the nominal values of the design parameters of the DMF are chosen to be feasible for application in heavy-duty truck drivetrain systems. The analysis was performed for the normal distribution of varying parameters and the coefficient of variation equal to 0.15.

The solution of the global sensitivity problem for engine speeds within the range $600-2000$ rpm is depicted in Fig. 2. This solution is presented by means of a mapping between the design parameters

$$d_1 = J_p, \quad d_2 = J_s, \quad d_3 = k_1, \quad d_4 = c_1,$$

and the values of total sensitivity indices of the objective functions (20)–(22).

2.3. Pareto Optimization of the Drivetrain System Equipped with a DMF. The multi-objective optimization problem formulated in Section 1 is now considered for the drivetrain system equipped with a DMF. The problem is stated as follows: for the feasible operational scenario given by expressions (16)–(18), it is required to determine the vector of design parameters of the DMF

$$\mathbf{d} = [J_p^*, J_s^*, k_1^*, c_1^*]^\top = \mathbf{d}^* \in \Omega$$

and the torsional vibration dynamics $\mathbf{q}(t) = \mathbf{q}^*(t)$ satisfying the variational equations

$$\min_{\mathbf{d}\in\Omega}\left\{\int_{600}^{2000} \operatorname{std}(T_g[\mathbf{q}(t),\mathbf{d},n_e])\,dn_e\right\} = \int_{600}^{2000} \operatorname{std}(T_g[\mathbf{q}^*(t),\mathbf{d}^*,n_e])\,dn_e\,,$$

$$\min_{\mathbf{d}\in\Omega}\{J_p + J_s\} = J_p^* + J_s^*,$$

subject to the differential equation (4) with (14), the initial state (15), and the restrictions imposed on the design parameters by the lower and upper bounds in Table 1.

This problem was solved by using the SAMO computer code for the same operational scenarios as the problem of global sensitivity analysis. The corresponding system of differential equations was solved by using a MATLAB subroutine ode45 with absolute and relative tolerances equal to $1\text{e}-5$. The setting of the genetic algorithm was as follows: population size $=100$; number of generations $=100$; elite count $=4$; and Pareto fraction $=1$.

The Pareto front, i.e., the best trade-off relationship between (20) and (21) obtained for engine speeds within the range of $600-2000$ rpm, is shown in Fig. 3.

Every point of the Pareto front corresponds to the set of values of the design parameters of DMF. The values of the design parameters J_p^*, J_s^*, k_1^*, c_1^* minimizing the objective function (20) are as follows:

$$[J_p^*, J_s^*, k_1^*, c_1^*]^\top = [2.34, 0.1, 3938, 30]^\top. \tag{24}$$

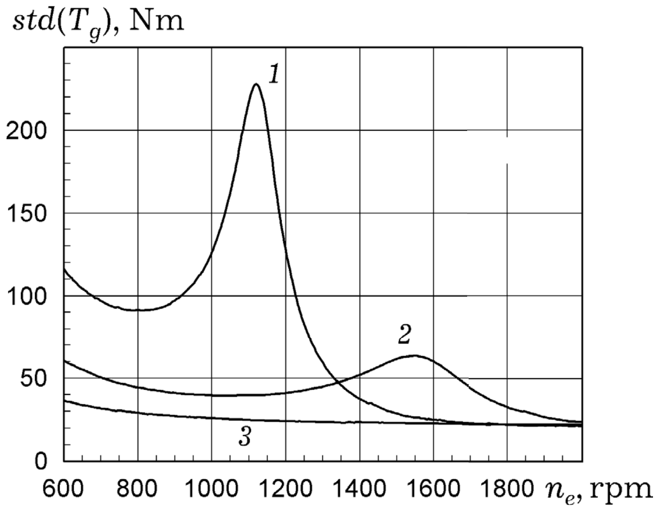

Fig. 4. Standard deviation of the torques at the transmission input shaft within the operating engine speed range $600\text{ rpm} \le n_e \le 2000\text{ rpm}$ for the DMF with the nominal design parameters (26) (curve *1*) and the weight-vibration optimized parameter (24) (curve *3*), as well as with the energy-vibration optimized parameters (27) for the DMF (curve *2*).

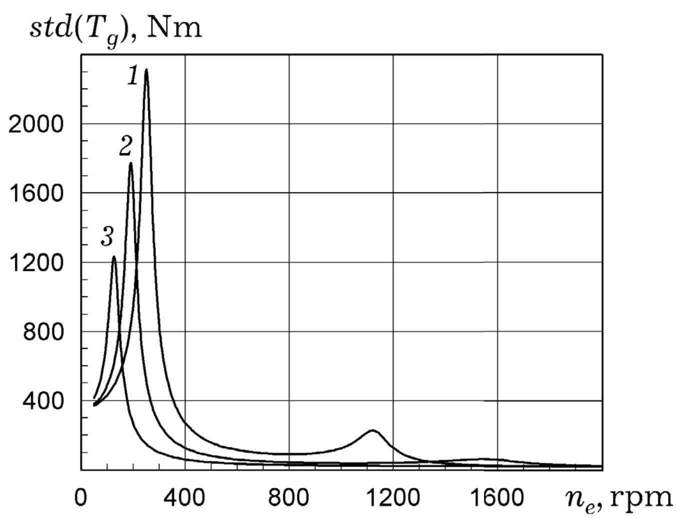

Fig. 5. Standard deviation of the torques at the transmission input shaft within the operating engine speed range $50\text{ rpm} \le n_e \le 2000\text{ rpm}$ for the DMF with the nominal design parameters (26) (curve *1*) and with the weight-vibration optimized parameter (24) (curve *3*), as well as with the energy-vibration optimized parameters (27) for the DMF (curve *2*).

These values correspond to the highest point of the Pareto front. The DMF with the design parameters (24) performs the best attenuation of the torsional oscillation of the torque at the transmission input shaft with the value of the objective function (20) equal to $35{,}250$ Nm. The obtained design of the DMF is characterized by the feasible total mass inertia $J_p^* + J_s^* = 2.44\ \text{kgm}^2$.

The values of the design parameters $J_p^*,\ J_s^*,\ k_1^*,\ c_1^*$ minimizing the objective function (21) are

$$[J_p^*, J_s^*, k_1^*, c_1^*]^\top = [0.23, 0.1, 4010, 14]^\top. \tag{25}$$

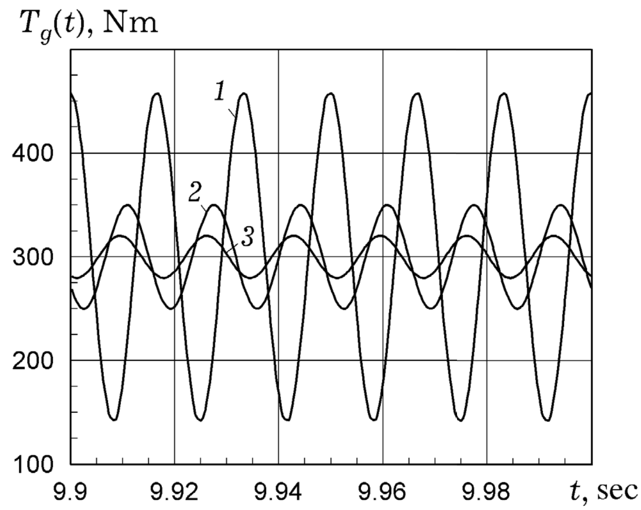

Fig. 6. The torques at the transmission input shaft for an engine speed of 1200 rpm in the presence of DMF with the nominal design parameters (26) (curve *1*) and with the weight-vibration optimized parameters (24) (curve *3*), as well as with the energy-vibration optimized parameters (27) for the DMF (curve *2*).

These values correspond to the lowest point of the Pareto front in Fig. 3. The DMF with the design parameters (25) is characterized by lowest feasible total mass inertia $J_p^* + J_s^* = 0.33 \text{ kgm}^2$. However, the attenuation of torsional oscillation of the torque at the transmission input shaft with this design of the DMF is much worse than in the case of using the design parameters (24). The value of the objective function (20) for the obtained design parameters (25) is about 150,000 Nm.

3. Results and Discussion

The application of the global sensitivity analysis and Pareto optimization provides deep insight into the torsional vibration dynamics of generic drivetrain systems with DMF. The chosen functionals (20)–(22) are appropriate to focus the design process for the vibration absorber on the best attenuation of the oscillation of torque at the transmission input shaft in order to minimize its weight as well as to decrease the energy dissipation in the stiffness-damping interface of the absorber.

The results of global sensitivity analysis of the drivetrain system with respect to the design parameters of the DMF presented in Section 2 (Fig. 2) make it possible to conclude the following: For the drivetrain system equipped with the DMF in the operating engine speed range $600 \text{ rpm} \le n_e \le 2000$ rpm, the moment of inertia of the primary flywheel J_p, as well as the stiffness of the absorber, k_1, mostly affect the vibration attenuation and the energy efficiency of the design of vibration absorber. The weight of the absorber, as expected, depends only on the mass inertia parameters.

The solution of the Pareto optimization problem presented in Section 2 shows that there exist a clear trade-off between the measure of the oscillation attenuation of the torque at the transmission input shaft and the total mass inertia characteristics (weight) of the optimized DMF of drivetrain system within the operating engine speed range $600 \text{ rpm} \le n_e \le 2000$ rpm (Fig. 3).

The standard deviations of the torques at the transmission input shaft as a function of the engine speed for the DMF with nominal design parameters

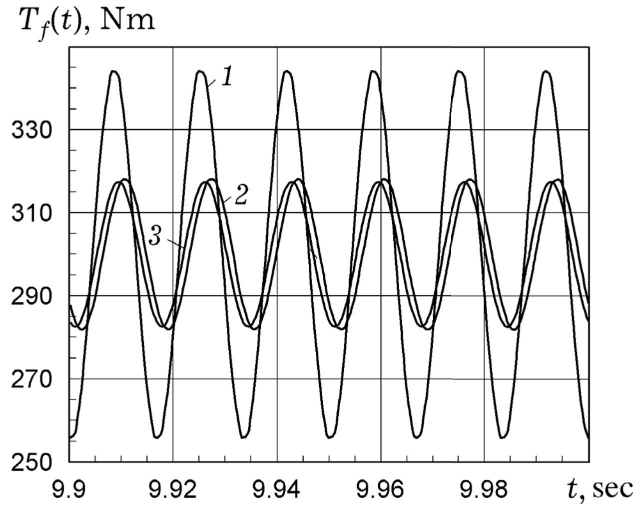

Fig. 7. The friction torques at the stiffness-damping interface of the DMF for an engine speed of 1200 rpm for the nominal design parameters (26) (curve *1*) and the weight-vibration optimized parameters (24) (curve *3*), as well as for the energy-vibration optimized parameters (27) in the presence of a DMF (curve *2*).

$$[J_p, J_s, k_1, c_1]^\top = [1.8, 0.9, 12732, 30]^\top \tag{26}$$

(curve *1*) and for the absorber with the weight-vibration optimized design parameters (24) (curve *3*) are depicted in Figs. 4 and 5 for different ranges of the engine speeds. The analysis of Figs. 4, 5 shows that the efficiency of attenuation of the oscillation of torque at the transmission input shaft by using the weight-vibration Pareto optimized DMF significantly increases as compared with the performance of the DMF with nominal design parameters. Thus, for the engine speed $n_e = 1200$ rpm, the standard deviation of the torque at the transmission input shaft $(\mathrm{std}[T_g(t)])$ of the drivetrain system with nominal design parameters of the DMF is equal to 114 Nm and decreased down to $\mathrm{std}[T_g(t)] = 24$ Nm in the case of using the DMF with obtained weight-vibration optimized parameters (24). As follows from Figs. 4, 5, both resonance peaks of curves *1* significantly decrease in case of using the DMF with weight-vibration optimized parameters.

In Figs. 6 and 7, we present the time history of the torques at the transmission input shaft (18), as well as the time history of the friction torques (23) illustrating how much the DMF with optimized design parameters (24) can enhance the attenuation of the oscillations of torques as compared with the DMF with the nominal design parameters (26).

The choice of objective functions is an important step in the design optimization of engineering systems. Earlier, in [1], various functionals were proposed for the design optimization of vibration absorbers for heavy-duty truck drivetrain systems. As in Section 2.3, the Pareto optimization problem was formulated and solved for the drivetrain system equipped with the DMF for the case of minimizing the objective functions (20), (22) satisfying the differential equation (4) with (14), the initial state (15), and the restrictions on the design parameters provided by the lower and upper bounds in Table 1.

Functionals (20), (22) characterize the energy of oscillations of the torque at the transmission input shaft and the energy dissipating in the DMF within the operating engine speed range $600\,\mathrm{rpm} \le n_e \le 2000$ rpm. It is believed that, by minimizing these functionals at the same time, the obtained design parameters increase the energy efficiency of the vibration absorber.

The obtained values of the design parameters of the DMF minimizing the objective function (20) in the Pareto optimization problem (20), (22) are as follows [1]:

$$[J_p^*, J_s^*, k_1^*, c_1^*]^\top = [2.7, 0.45, 10967, 41]^\top . \tag{27}$$

These values correspond to the highest point of the obtained Pareto front in the bi-objective optimization problem that was solved by using functionals (20) and (22). The DMF with the design parameters (27) realizes the best attenuation of the torsional oscillation of the torque at the transmission input shaft of the drivetrain system within the operating engine speed range $600\,\text{rpm} \le n_e \le 2000$ rpm.

In Figs. 4–7, curve *2* represents the corresponding characteristics obtained by solving the energy-vibration Pareto optimization problem (20), (22) for the drivetrain system equipped with the DMF [1]. The analysis of Figs. 4–7 shows that, within the framework of considered assumptions, the weight-vibration Pareto optimized DMF attenuates the oscillations of torques at the transmission input shaft much better than the operation of the DMF with energy-vibration optimized design parameters (27) studied earlier in [1].

The quantitative analyses of values of the nominal design parameters (26), the weight-vibration optimized parameters (24), and the energy-vibration optimized parameters (27) of the DMF show that the solution of the weight-vibration Pareto optimization problem results in the lowest total mass inertia of the vibration absorber. This can be a significant advantage of the weight-vibration optimized DMF for its implementation in the actual drivetrain systems.

Conclusions and Outlook

The following concluding remarks can be made:

- The methodology of multiobjective optimal design of engineering systems based on the global sensitivity analysis and Pareto optimization is proven to be efficient for the advanced analysis and designing of torsional vibration absorbers for drivetrain systems.

- There exists a clear trade-off between the measure of oscillation attenuation of the torque at the transmission input shaft and the measure of total weight in designing the DMF for heavy-duty truck drivetrain systems.

- For a heavy-duty truck drivetrain system equipped with a DMF, there exists the weight-vibration bi-objective optimized mass inertia, stiffness, and damping parameters providing the best attenuation of oscillation of the torque at the transmission input shaft within the operating engine speed range $600-2000$ rpm, when the third-order engine vibration harmonic is in focus.

- The obtained results reveal the feasibility of application of the weight-vibration optimized dual mass flywheels in heavy-duty truck drivetrain systems.

The verification and validation of the results obtained by using the complete model of drivetrain system of a heavy-duty truck [9], as well as the experimental data are important next steps of the study [10].

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