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COMPUTER SIMULATION SYSTEM FOR OSCILLATIONS OF MAGNETIZABLE MICRODROPLETS TAKING INTO ACCOUNT THE DEPENDENCE OF THE INTERFACIAL TENSION ON THE STRENGTH OF AN EXTERNAL MAGNETIC FIELD

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Abstract. We present a computer system for simulation of forced oscillations of magnetized microdroplets in alternating magnetic fields taking into account the dependence of the interfacial tension on the external magnetic field obtained from an experiment. Results of computing experiments allow one to identify features of deformation of microdroplets in increasing magnetic fields.

***Keywords and phrases*:** magnetizable microdroplets, oscillations of microdroplets, computer simulation system, magnetic field.

***AMS Subject Classification*:** 97M50; 68-04

1. Introduction

At present, the use of modern computer modeling technologies and software and hardware of computer technology not only allows one to perform the contactless measurement of physical characteristics of liquids based on the analysis of parameters of oscillation of their droplets, but can also serve as the basis for real-time measurement automation. At the same time, the choice of mathematical models, calculation methods, and measurement techniques depends on the size of droplets and their physical characteristics, which can change under the influence of environmental parameters.

The study of oscillations of droplets of real liquids taking into account their viscosity under the action of various forces began with the classical Rayleigh problem on small free oscillations of a droplet of an ideal fluid (see [23]), which led to the development of a new direction in the measurement technique, in particular, viscosity and interfacial tension of liquid droplets according to their oscillation parameters. However, methods for determining the viscosity and interfacial tension of liquid droplets by their oscillations are not universal since mathematical models and calculation methods developed for aerosol droplets of size $<10^{-6}$ m (see [4], droplets of dielectrics (see [6, 15, 16]), liquid metals (see [17]), charged and magnetized droplets (see [3, 7, 9, 26]) are significantly different from each other. The measurement techniques for various liquids are also different; for example, they involve furnaces for producing droplets of molten metals, microscopes and holographic methods for microscopic droplets, etc. However, in many cases, the only possible contactless method for measuring physical characteristics of liquids is the method based on the oscillations of droplets of a test fluid. Contactless methods of measurement of parameters of magnetic liquids used in various technical devices, technological processes, and control measurement systems and also liquids used as magnetosensitive media for studying magnetic fields of scattering of magnetized microscopic objects of complicated shape (see [22]) are of great practical and scientific interest. In magnetic liquids, under the influence of various factors, microdroplets and their aggregates appear whose magnetic permeability μ_i ($\mu_i > 20$) is an order of magnitude higher than the magnetic permeability μ_e of the surrounding liquid (see [2, 8, 12]). Such

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aggregates can be used as sensors of magnetic field (see [10, 11]); therefore, the study of their stability under changes in the external parameters of the medium is of great interest.

For example, a change in the concentration of magnetic particles at the interface of polarized and magnetizable liquids under the influence of an external magnetic field can have a significant impact on interfacial tension (see [24, 27]) and the stability of the interface of magnetized media with different magnetic properties. A large number of works are devoted to the study of the influence of magnetic fields on interfacial tension, but the results of these studies are contradictory. So, results were presented in [13, 24, 25] that allow one to conclude that the coefficient of interfacial tension depends on the orientation of the magnetic field relative to the interfacial boundary. Results of magnetic measurements and experimental data obtained in hydrostatic experiments with droplets of magnetic liquid were compared in [1, 5]. In [5], a conclusion was made that the surface tension of magnetized droplets is independent of the field and in [1] results were obtained on a significant change in surface tension in magnetic fields with strength greater than 10 kA/m.

For microdroplet aggregates of size 1–10 micrometers, which appear in some magnetic liquids, the interface between two magnetizable liquids separates microdroplets with a high concentration of magnetic particles inside and the surrounding liquid with a low concentration of magnetic particles; the magnitude of the interfacial tension at the interface is several orders of magnitude less than that of macroscopic droplets (see [2, 8, 12]). The difficulty in describing the hydrostatics of such aggregates is relate to the problem of simultaneously determining magnetic and surface properties and the limits of applicability of the theory of linear magnetization. In particular, in [14], the conclusion on inapplicability of the linear theory for the study of the influence of magnetic fields on the surface interfacial tension of magnetized microdroplets in fields stronger than 1 kA/m was made.

Thus, the question of the influence of magnetic fields on the surface tension of macroscopic magnetized droplets and the interfacial tension of microdroplets needs further research. The complexity of conducting experimental studies of oscillations and deformations of microdroplet aggregates gives rise to problems in the automation of measurements and in the development of a computer simulation systems.

1.1. Description of the object and research methods. In this paper, we present results of numerical experiments obtained by using a model and a computer simulation system developed for the study of forced oscillations of magnetized microdroplets in variable magnetic fields taking into account the dependence $\sigma(H)$.

For determining the form of the function $\sigma(H)$, we used experimental results presented in [21]. For approximation of experimental data by the least square method, we take the function $f(x, a, b) = a \cdot \exp(b \cdot x)$. As a result, the mathematical model (see [18]) obtained under the assumption that microdroplets have an ellipsoidal shape and obey the linear magnetization law (see [20]) was supplemented by equations that represent $\sigma(H)$ for various initial values of $\sigma(H)$ (see [19]):

$$
\begin{aligned}
\frac{4\pi R^2}{135}\left(\rho_1 + \frac{\rho_2}{2}\right) \cdot \Bigg[\frac{d^2 q(t)}{dt^2}\left(q(t)^{-8/3} + 2q(t)^{-2/3}\right) \\
- \frac{2}{3} \cdot \left(\left(\frac{dq(t)}{dt}\right)^2 \cdot \left(2q(t)^{-11/3} + q(t)^{-5/3}\right)\right)\Bigg] + \frac{16}{9}\eta\pi q^{-2}\frac{dq(t)}{dt} + \frac{\pi\sigma(H)\, q^{1/3}}{3R\left(q(t)^2 - 1\right)} \\
\times \left(\frac{q(t)^2 - 4}{\sqrt{q(t)^2 - 1}} \arcsin\frac{\sqrt{q(t)^2 - 1}}{q(t)} + 2q(t)^{-2} + 1\right) - \frac{2\pi\mu_0\left(\mu_i - \mu_e\right) H^2 \left(q(t)^2 - 1\right)^2}{3}
\end{aligned}
$$

$$\times \frac{\left[\left(\ln\left(2q(t)^2+2q(t)\sqrt{q(t)^2-1}-1\right)-2\sqrt{1-q(t)^{-2}}\right)\cdot\left(2q(t)^2+1\right)\cdot\left(q(t)^2-1\right)^{-3/2}-2q(t)^{-1}\right]}{\left[q(t)\left(\frac{\mu_i}{\mu_e}-1\right)\cdot\left(\ln\left(2q(t)^2+2q(t)\sqrt{q(t)^2-1}-1\right)-2\sqrt{1-q(t)^{-2}}\right)+2\cdot\left(q(t)^2-1\right)^{3/2}\right]^2}=0, \tag{1}$$

where

$$\sigma(H)=\begin{cases}1{,}5e^{5{,}7\cdot10^{-3}\cdot|H|}\cdot10^{-7} & \text{for } 10^{-7}\le\sigma(0)<10^{-6},\\ 0{,}6e^{1{,}5\cdot10^{-3}\cdot|H|}\cdot10^{-5} & \text{for } 10^{-5}\le\sigma(0)<10^{-4},\\ 0{,}3e^{0{,}7\cdot10^{-3}\cdot|H|}\cdot10^{-4} & \text{for } 10^{-4}\le\sigma(0)<10^{-3},\end{cases}$$
$$H(t)=H_0+H_a\sin(\omega t+\varphi),$$

where R, ρ_1, and μ_i are respectively the radius, the density, and the magnetic permeability of a highly concentrated microdroplet of a magnetic liquid; ρ_2 and μ_e are respectively the density and the magnetic permeability of a weakly concentrated magnetic liquid surrounding the droplet; $q(t)$ is a generalized coordinate, which is the ratio of the semiaxes of the ellipsoid $q(t)=a(t)/b(t)$, $a(t)>b(t)$; $dq(t)/dt$ is the derivative of the generalized coordinate; η is the viscosity of the microdroplet; $\sigma(H)$ is the interfacial tension; $\omega=2\pi f$; f and φ are respectively the frequency and the phase of the variable magnetic field; and $H(t)=H_0+H_a\sin(\omega t+\varphi)$ is the strength of the external magnetic field.

The system (1) allows one to describe the dynamics of magnetized microdroplets taking into account the dependence $\sigma(H)$ of the interfacial tension on the magnetic field for various initial values of the interfacial tension: $10^{-7}\,\mathrm{N}\cdot\mathrm{m}^{-1}\le\sigma(0)\le10^{-3}\,\mathrm{N}\cdot\mathrm{m}^{-1}$.

The model proposed was implemented in MATLAB and Simulink software systems. To develop a system of computer simulating of forced oscillations of magnetized microdroplets, we used the second-order system of differential equations (1) presented in the equivalent form of a system of first-order differential equations:

$$\frac{dV_q(t)}{dt}=\frac{2\left(q(t)^2+2\right)}{3q(t)\left(2q(t)^2+1\right)}V_q(t)^2-\frac{60\eta(t)q(t)^{2/3}}{r^2\left(\rho_1+\frac{\rho_2}{2}\right)\left(2q(t)^2+1\right)}V_q(t)$$
$$+\frac{45\sigma(t)q(t)^3}{4r^3\left(\rho_1+\frac{\rho_2}{2}\right)\left(2q(t)^2+1\right)\left(q(t)^2-1\right)}\left(\frac{q(t)^2-4}{\sqrt{q(t)^2-1}}\sin^{-1}\left(\frac{\sqrt{q(t)^2-1}}{q(t)}\right)+2q(t)^{-2}+1\right)$$
$$-\frac{45\mu_0(\mu_i-\mu_e)^2H(t)^2\left(q(t)^2-1\right)^2}{2r^2\left(\rho_1+\frac{\rho_2}{2}\right)\left(q(t)^{-8/3}+2q(t)^{-2/3}\right)}$$
$$\times\frac{\left[\left(\ln\left(2q(t)^2+2q(t)\sqrt{q(t)^2-1}-1\right)-2\sqrt{1-q(t)^{-2}}\right)\cdot\left(2q(t)^2+1\right)\left(q(t)^2-1\right)^{-3/2}-2q(t)^{-1}\right]}{q(t)\left(\frac{\mu_i}{\mu_e}-1\right)\left(\ln\left(2q(t)^2+2q(t)\sqrt{q(t)^2-1}-1\right)-2\sqrt{1-q(t)^{-2}}+2\left(q(t)^2-1\right)^{3/2}\right)^2}=0, \tag{2}$$

where

$$\frac{dq(t)}{dt}=V_q(t),$$

$$\sigma(H) = \begin{cases} 1{,}5e^{5{,}7\cdot 10^{-3}\cdot|H|}\cdot 10^{-7} & \text{for } 10^{-7}\le \sigma(0) < 10^{-6}, \\ 0{,}6e^{1{,}5\cdot 10^{-3}\cdot|H|}\cdot 10^{-5} & \text{for } 10^{-5}\le \sigma(0) < 10^{-4}, \\ 0{,}3e^{0{,}7\cdot 10^{-3}\cdot|H|}\cdot 10^{-4} & \text{for } 10^{-4}\le \sigma(0) < 10^{-3}, \end{cases}$$

$$H(t) = H_0 + H_a \sin(2\pi f t + \varphi).$$

The numerical simulation system developed within the Simulink software environment is presented in Fig. 1. In contrast to a system described earlier in [20], the module for determining $\sigma(H)$ is now included.

The subsystem 1 contains four modules 2–5 for determining forces of various nature that affect the character of oscillations. The module 2 is intended for the computation of the coefficient

$$E(q) = \frac{2\left(q(t)^2 + 2\right)}{3q(t)\left(2q(t)^2 + 1\right)}$$

of $V_q(t)^2$ in the first term of the right-hand side of the first differential equation of the system (2); similarly, the purpose of the following three modules 3–5 is the computation of the second, third, and fourth terms of the right-hand side of the first equation of the system (2), respectively. These coefficients characterize the action of the following forces: viscous and inertia forces $E(\eta)/E(\rho)$ (the module 3), surface and inertia forces $E(\sigma)/E(\rho)$ (the module 4), and magnetic and inertia forces $E(\mu)/E(\rho)$ (the module 5). The subsystem 6 is intended for computation of the external magnetic field acting on a magnetized droplet: $H(t) = H_0 + H_a \sin\left(\omega t + \varphi\right)$.

Results of computations performed by the subsystem 6 and the modules 2–5 are stored by the unit Scope involved in the subsystem 1. The units XY Graph and XY Graph 1 serve for constructing phase portraits of the dynamic system considered, which describes forced oscillations of a magnetized droplet in an external magnetic field, and for constructing the dependence of the generalized coordinate $q(t) = a(t)/b(t)$, $a(t) > b(t)$, which is the ratio of the axes of the ellipsoid describing the shape of the droplet as a function of the external magnetic field $q(H)$.

1.2. Presentation and analysis of results. Forced oscillations of magnetized droplets in variable magnetic fields were modeled by using the Mathcad software (the numerical solution of the system (1)) and a computer simulation system developed in the MATLAB and Simulink packages (the numerical solution of the system (2)).

In Figs. 2–4, we present the results of a computer experiment for simulating forced oscillations of a microdroplet with radius $r = 5\,\mu\text{m}$ obtained by using the computer simulation system (see Fig. 1) with the following parameters:

$$\rho_1 = 1900\,\text{kg/m}^3,\quad \rho_2 = 1060\,\text{kg/m}^3,\quad \mu_e = 1,\quad \mu_i = 50,\quad \sigma(0) = 1{,}5\cdot 10^{-7}\,\text{N/m},\quad \eta = 0{,}226\,\text{Pa}\cdot\text{s},$$

under the action of a variable magnetic field with the parameters

$$H_0 = 0,\quad H_a = 200\,\text{A/m},\quad f = 0{,}01\,\text{Hz}.$$

Results were stored by the unit Scope.

The curve 1 corresponds to values $q(t)$; the curve 3 shows the change in the external magnetic field $H(t)$; the curves 2, 4, and 5 demonstrate respectively $E(q)$ and the time dependencies of the ratios $E(\mu)/E(\rho)$, $E(\sigma)/E(\rho)$, and $E(\eta)/E(\rho)$. The function $q(t)$ is periodic: the microdroplet manages to change its shape during the half period of the oscillations of the external field. The curves 4, 5, and 6 characterize the influence of forces of various nature (magnetic, surface, and viscous) on the oscillatory process.

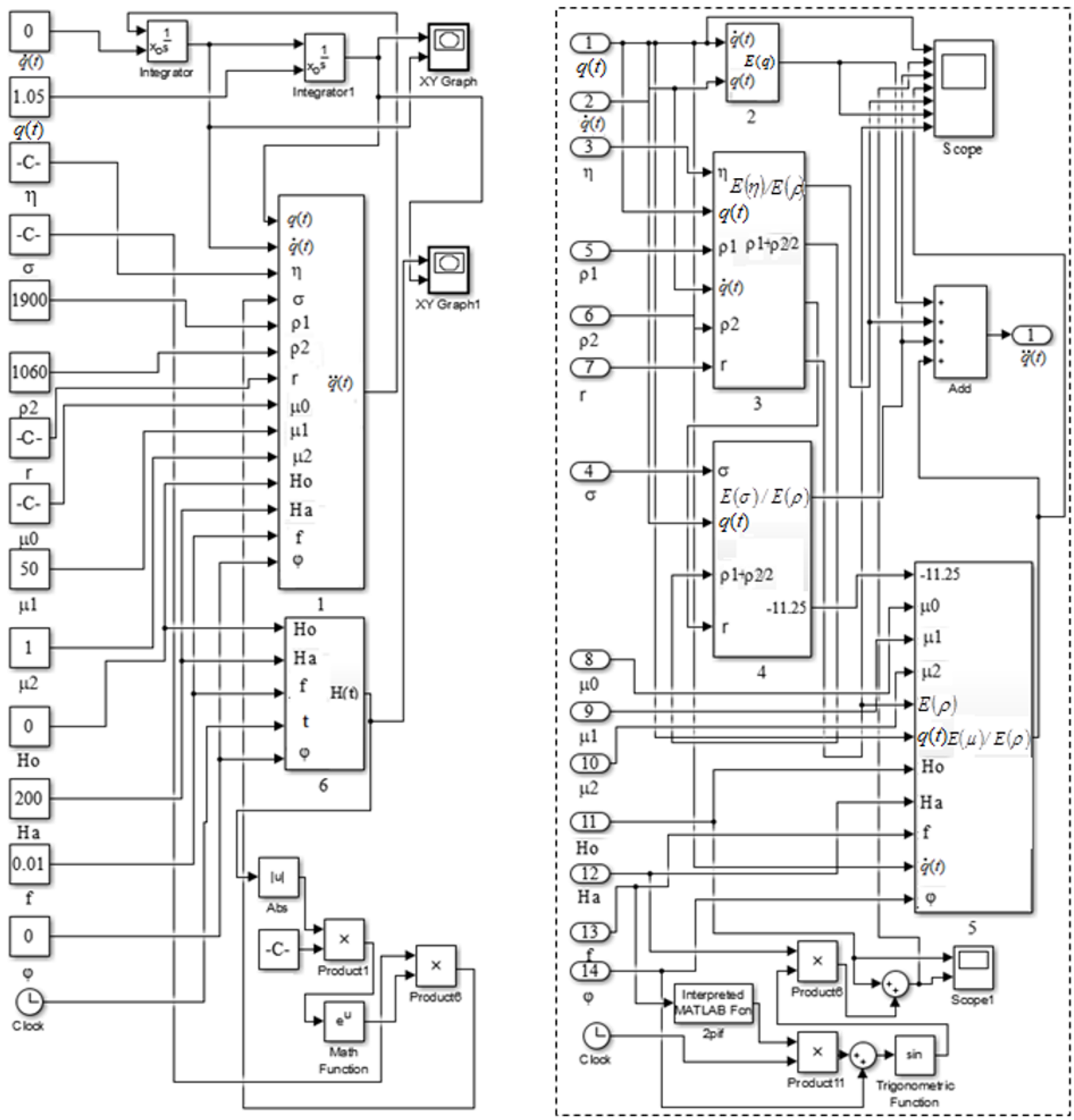

Fig. 1. Structure on the numerical simulation system with the subsystem 1 containing modules 2–5 and the subsystem 6

Figure 3 shows the dependence of instantaneous values of the ratio of the semiaxes of microdroplets $q(t)$ on instantaneous values of the sinusoidal magnetic field, which is recorded by the unit XY Graph. As can be seen from Fig. 3, the nature of the dependence $q(H)$ is hysteretic, and the rapid increase in$q(t)$ corresponds to outer fragments of the graphs and occurs when the external field exceeds the threshold field of the jump elongation H_1. With a decrease in the external field, the change in $q(t)$ is slower and corresponds to internal fragments of the graph.

In Fig. 4, the phase portrait of the dynamical system that describes forced oscillations of magnetized droplets in external magnetic field is shown.

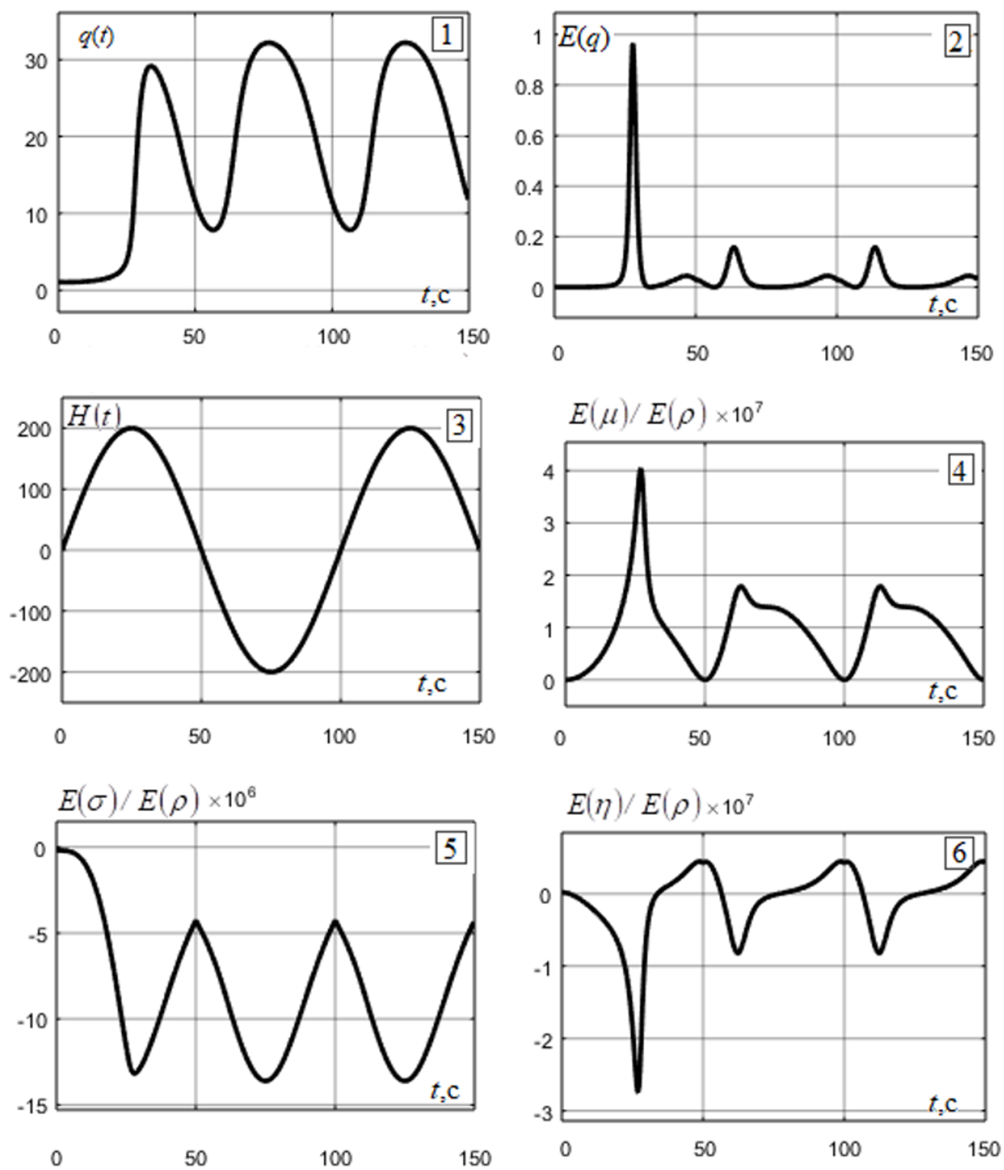

Fig. 2. Time dependencies: curve 1: $q(t)$; curve 2: $E(q)$; curve 3: $H(t)$; curve 4: $E(\mu)/E(\rho)$; curve 5: $E(\sigma)/E(\rho)$; curve 6: $E(\eta)/E(\rho)$.

Figures 3 and 4 show that in the steady state, the nature of forced oscillations of magnetized microdroplets is hysteretic.

Comparison of the results of numerical simulation of oscillations of magnetized droplets based on the systems (1) and (2) performed by two computer mathematical systems (Simulink/MATLAB and Mathcad) shows their coincidence with good accuracy. Results of nature experiments and calculations obtained with $\sigma(0) = 1{,}5 \cdot 10^{-7}$ N/m were also compared; a good consistence of simulation results with experimental results was found. Oscillations of microdroplets with other values of $\sigma(0)$ have not been experimentally studied.

1.3. Conclusion. The computer simulation system developed by the authors allows one to simulate forced oscillations of magnetized microdroplets based on the dependencies of the interfacial tension

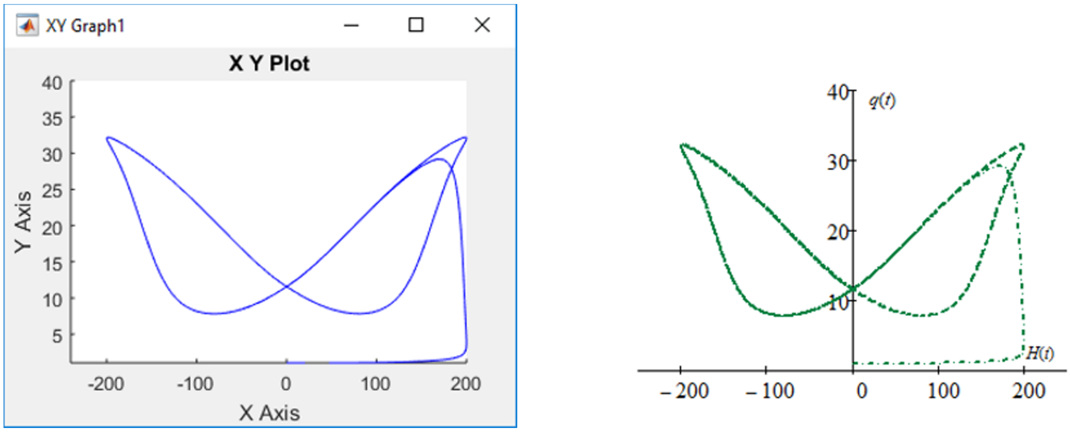

Fig. 3. The dependence $q(H)$ for $\sigma(0) = 1{,}5{\cdot}10^{-7}$ N/m in Matlab (a) and Mathcad (b)

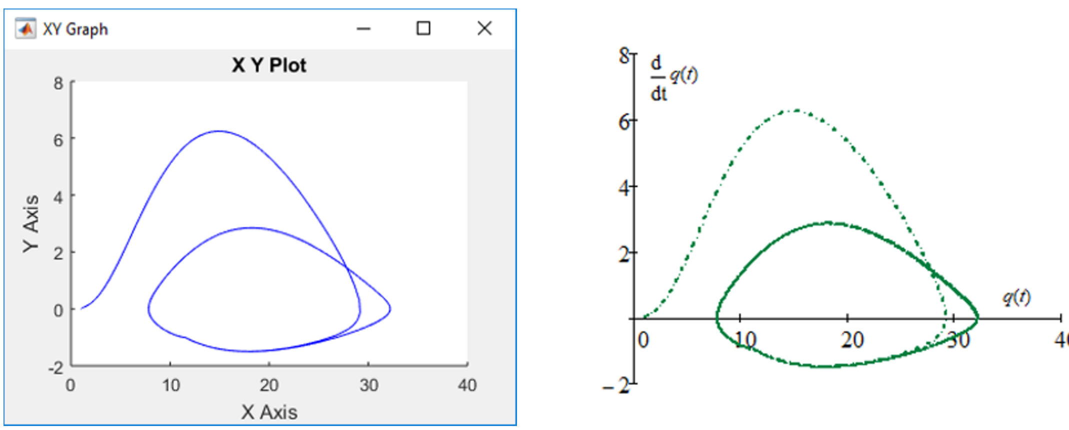

Fig. 4. The phase portrait in Matlab (a) and Mathcad (b)

on the external magnetic field obtained from nature experiments. Simulation results are consistent with experimental results and can be useful in the development of devices that use magnetic liquids containing magnetized microdroplets.

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