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TO THE CALCULATIONS OF SCATTERING AMPLITUDES IN DIFFRACTION PROBLEMS FOR ELONGATED BODIES OF REVOLUTION

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This paper is a complement to the article "Scattering amplitudes in a neighborhood of limit rays in short-wave diffraction by elongated bodies of revolution". It contains discussions of some points of the article, which worth of more detailed considerations, such as the influence of the integration limits on the computation result of scattering amplitudes and the estimation of permissible values of scattering angle intervals as functions of parameters of the problems. Bibliography: 4 titles.

In paper [1] we obtained formulas for scattering amplitudes of the plane wave for smooth elongated bodies of revolution (axially symmetric case) in the direction of limit rays and used them to perform calculations. This note represents an addition to the aforementioned paper and contains a more detailed discussion of items, which, in our opinion, were not examined properly in that paper. In particular, we added Figs. 1–4.

Let us briefly dwell upon key principles of our approach. The diffraction problems are considered in the short-wave approximation, where the length of the incident wave is much smaller than the geometric dimensions of the scatterer. It turns out in this case that the Green formula for the wave field in the body exterior leads to quickly oscillating integrals, which obviously perform an essential contribution to the extent of this field only at the critical (stationary) points of the corresponding phase functions. At that, in the vicinity of the light-shadow boundary, where the wave field slides along the scatterer boundary, the stationary points correspond to limit rays, i.e., to such rays that touch the scatterer surface at points of the light-shadow boundary (equator). This circumstance enables us to obtain formulas for the scattering amplitudes in the direction of the limit rays in the form of integrals of the wave field current on the part of the scatterer boundary in a neighborhood of the equator where a specific boundary layer appears. In this boundary layer a smooth transition of the wave field from the illuminated to the shadowed part of the body takes place. Thus we arrive at the Fock problem [2] of finding the wave field current exactly within this boundary layer (the principle of short-wave approximation locality).

The calculation of the current in the vicinity of the equator is implemented in our papers by numerical methods. Let us note, first of all, that, from the mathematical point of view, the problem that appears within the boundary layer constitutes a scattering problem for Schrödinger type equations, where the incident wave field is defined by its own ray asymptotics in the illuminated part of the scatterer (see, for example, [3, 4]).

In the case of a strongly convex body of revolution, where the curvature of its surface in the direction of the incident rays does not vanish at the equator points, we arrive at the boundary layer and the Fock problem, which can be calculated precisely by means of the variable separation method. This allows us to obtain analytic expressions for scattering amplitudes in the form of not simple integrals, which contain Airy functions and their derivatives. In what follows, we intend to examine these integrals with the aim of simplifying them.

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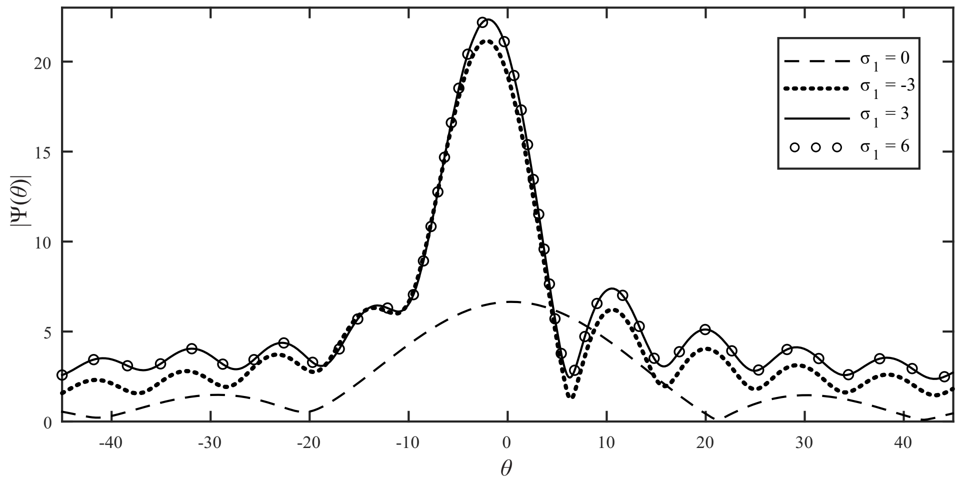

Fig. 1. The dependence of the scattering amplitude on the value of σ_1 for the case of a Fock boundary layer with Dirichlet conditions on the scatterer boundary.

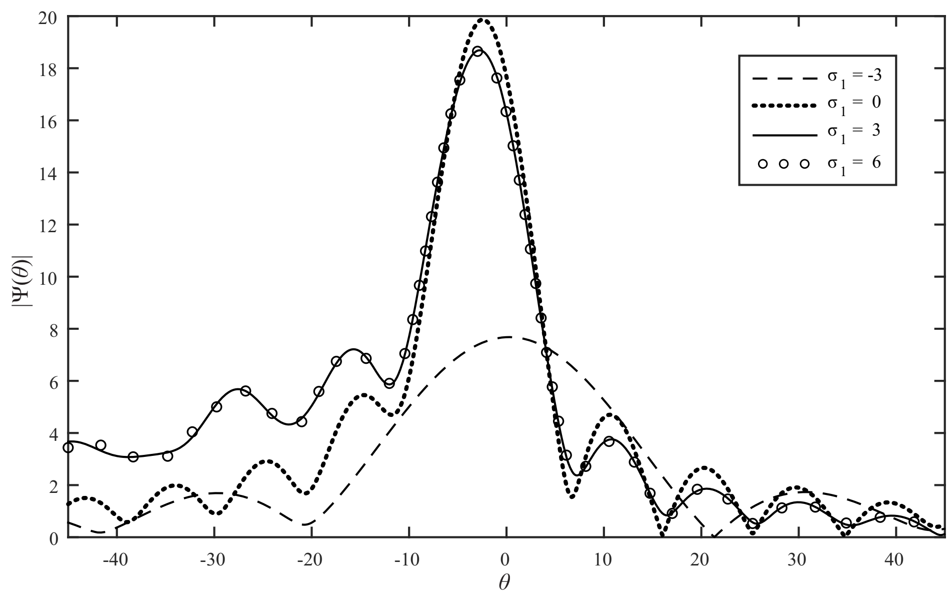

Fig. 2. The dependence of the scattering amplitude on the value of σ_1 for the case of a Fock boundary layer with Neumann conditions on the scatterer boundary.

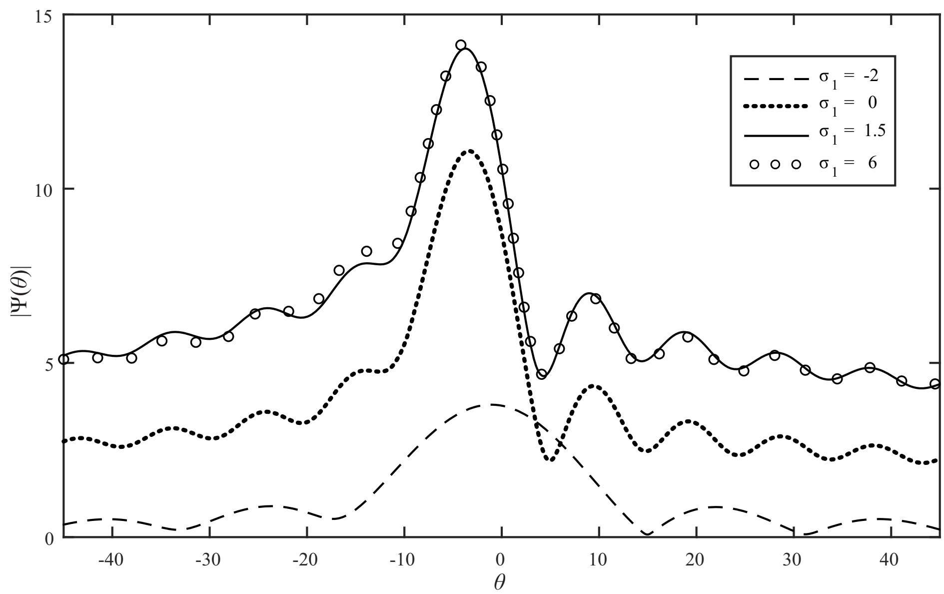

Fig. 3. The dependence of the scattering amplitude on the value of σ_1 for the case of a strongly elongated body with Dirichlet conditions on the scatterer boundary.

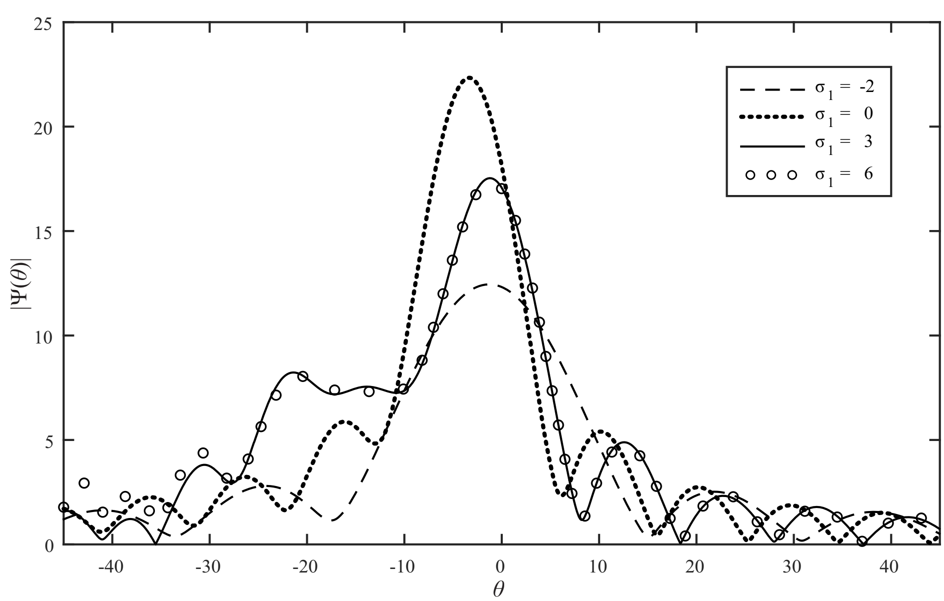

Fig. 4. The dependence of the scattering amplitude on the value of σ_1 for the case of a strongly elongated body with Neumann conditions on the scatterer boundary.

In the case of a strongly elongated body, the variables in the emerging equations are not separated, and it only remains for us to apply numerical techniques to get a solution. In the internal stretched variables σ, ν of the boundary layer, where σ is a dimensionless arc length, counted from the equator in the direction of rays, and ν represents a dimensionless external normal to the scatterer surface, the scattering problem is set in the half-plane $-\infty < \sigma < +\infty$, $\nu \geq 0$. As $\sigma \to -\infty$, the field of the incident and the reflected wave is defined by its ray asymptotics. For $\nu = 0$, we set Neumann and Dirichlet boundary conditions. The situation concerning the condition as $\nu \to +\infty$ is more complicated, because neither incident nor reflected waves decrease, i.e., do not belong to $L_2(0, \infty)$, to be more precise, but stay restricted.

We solve this problem by numerical methods in the rectangle $-|\sigma_0| \leq \sigma \leq \sigma_1$, $0 \leq \nu \leq \nu_*$, where the boundaries are subject to tentative selection. We set a Cauchy condition for $\sigma = \sigma_0$ and σ_0 is selected based on the fitting condition for the currents, generated by the ray asymptotics and the grid solution.

As $\nu = \nu_*$ we introduce a fictitious boundary, where the complete field vanishes, which gives rise to a reflected wave, fictitious as well. At that the quantity ν_* is selected with the aim to minimize the contribution of this wave to the current on the boundary $\nu = 0$ within the interval $\sigma_0 \leq \sigma \leq \sigma_1$ (see [3, 4] for more detail).

Figures 1–4 display the influence of the boundaries of the rectangle $\sigma = \sigma_1$ on the scattering amplitudes depending on the boundary conditions at $\nu = 0$ for Fock boundary layers and with flattening points on the equator in the case of strongly elongated bodies. The value of the angle θ on them is plotted on the x-coordinate axis in degrees, the same as in [1]. We use the same angle interval for all calculations, albeit [1] draws attention to the fact that the definition of angle smallness depends on the values of the large parameters M_0 and M; see the following paragraph of this paper. The calculations in Figs. 1–4 were accomplished for the same value of M_0 and M parameters. In the case of Dirichlet conditions, the current of the wave field decreases more swiftly, in contrary to the Neumann condition, however the plot stability for the amplitudes is achieved practically at the same values of σ_1. It may be noted as well that in the case of the Neumann condition, the wave field in the shadowed zone proves to be greater than for the Dirichlet condition.

Let us dwell on the question of the smallness of θ angles for which the formulas for the scattering amplitudes turn out to be justified; see relations (14) and (17) in [1].

The restrictions on the angles θ are derived from the requirement, natural for the asymptotic expansion, that in the principal term of the asymptotics, all the summands should be of the same order and, in particular, both summands under the exponent in relations (14), (17) from [1], i.e., with the requirement $\theta M_0 = O(1)$ to be fulfilled for the Fock boundary layer, and for the case of a strongly elongated body $\theta M^3 = O(1)$ as $M_0 \to \infty$ and $M \to \infty$, respectively. This ensues from the fact that the dimensionless extended coordinates σ and ν have order of $O(1)$. Let us recall that the symbol $O(1)$ allows for the existence of a certain, generally speaking, undefined constant, which for its part does not depend on the large parameter anymore. If we set this constant to one, then for the maximal angle $\theta_{\max}$, measured obviously in radians, we arrive at $|\theta_{\max}| = M_0^{-1}$ for elongated bodies and $|\theta_{\max}| = M^{-3}$ for strongly elongated ones.

Let us recall that for the calculations in Figs. 1–4 and in [1], we took the angle θ interval independent of the large parameter of the boundary layer and angles θ were measured in degrees. Let us give intervals of acceptable rays, associated with θ measured in degrees as well, for various values of the large parameters M_0 and M, which correspond to Figs. 3, 4 and 7, 8 from [1]. In the case of the Fock boundary layer with $M_0 = 2.2$ and $M_0 = 2.9$, we obtain

$|\theta_{\max}| \approx 26^\circ$ and $|\theta_{\max}| \approx 20^\circ$, respectively. Thus, the acceptable values of the angles in Figs. 3 and 4 are described by the inequalities $-26^\circ \le \theta \le 26^\circ$ and $-20^\circ \le \theta \le 20^\circ$, respectively.

In the case of a strongly elongated body (consider the boundary layer with flattening points on the equator) for the values $M = 1.8$ and $M = 3.0$ the angle intervals in Figs. 7 and 8 are described by the inequalities $-9.7^\circ \le \theta \le 9.7^\circ$ and $-2.1^\circ \le \theta \le 2.1^\circ$, respectively.

Therefore, the acceptable angles θ depend on the large parameter of the boundary layer and decrease (get narrower) with its rise, while the maximum of the emanated wave field approaches the limit ray $\theta = 0$, which represents the geometric boundary of the shadow.

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