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ACTIVE DAMPING OF FORCED RESONANCE VIBRATIONS OF AN ISOTROPIC SHALLOW VISCOELASTIC CYLINDRICAL PANEL UNDER THE ACTION OF AN UNKNOWN MECHANICAL LOAD

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Using a new approach, we consider the problem of active damping of the forced resonance vibrations of a viscoelastic isotropic cylindrical panel with simply supported edge faces. The mechanical load is assumed to be unknown; it can be found from the experimental data of a sensor. The problem is solved by the Bubnov–Galerkin method. A relation was obtained for the potential difference that has to be supplied to an actuator for the damping of resonance vibrations of the panel. We also study the influence of sizes of sensors and actuators, the dissipative properties of materials, and temperature on the efficiency of active damping of the forced resonance vibrations of a cylindrical panel.

Introduction

Thin isotropic cylindrical panels are widely used in many branches of present-day science and engineering, namely, in space engineering, aircraft technology, automobile production, shipbuilding, machine building, radio electronics, etc. Very often such panels are subjected to the action of nonstationary and harmonic-in-time mechanical loads. Here, the most dangerous situation is connected with resonance vibrations, where the frequency of force, harmonic in time, coincides with the natural frequency of vibrations of the element under consideration. Hence, the problem of damping of the forced resonance vibrations of thin cylindrical panels is urgent. For this purpose, it is customary to use the passive methods of damping, where components with high hysteresis losses are included to the structure of the element. The problems of passive damping of the vibrations of thin-walled elements were studied in numerous works of both Ukrainian and foreign scientists in mechanics, a review of which can be found in [6, 7]. In recent years, scientists and engineers began to apply active methods of damping. They are based on the inclusion of piezoelectric components in the structure of a passive (without piezoelectric effect) thin-walled element made of metallic, polymeric, or composite materials [8–10]. Some piezoelectric components serve as sensors, which give information on the mechanical state of the body, and the others work as so-called actuators. There exist two main approaches to the active damping of vibrations. In the first, one applies, for the damping of vibrations, piezoelectric inclusions that serve as an actuator. The basic problem lies here in the calculation of the potential difference that has to be supplied to the actuator for compensating the resonance component of external mechanical load. If the load is known, then, by choosing the corresponding potential difference, one can completely damp a certain (e.g., the first) mode, and, as a result, the vibration amplitude on this mode will be equal to zero. In the second approach, besides actuators, piezoelectric sensors are used additionally. One supplies to the actuators a potential difference proportional to the indications of the sensor, namely, to the current or the first time derivative of the potential difference taken from the sensors. The coefficient of proportionality is called the feedback factor. As a result, the dissipative characteristics of the plate change, which leads to a decrease in the vibration amplitude. In using this approach, it is possible to de-

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crease substantially the level of resonance vibrations by choosing the feedback factor mentioned above. In both methods, it is necessary to know the external mechanical load.

In the present work, we consider the problem of active damping of forced resonance vibrations of an isotropic viscoelastic cylindrical panel with the help of the combined use of sensors and actuators for the case where the external load is unknown. We apply here a new approach, the essence of which lies in the following: using the indications of the sensor (the charge or potential difference), we reconstruct the external load. After that, we use the first approach described above, where a potential difference calculated according to experimental data of the sensor is supplied to the actuator. In what follows, we call this approach the third approach.

Thus, in the case of using the third approach, the potential difference supplied to the actuator for the damping of vibrations on the corresponding mode is calculated from experimental data of the sensor, namely, by the current or potential difference depending on the type of electrical boundary conditions. The vibration amplitude on the considered mode becomes equal to zero. This fact distinguishes in principle the third approach from the second described above: in the last case, only a decrease in the amplitude of forced resonance vibrations takes place due to an increase in the damping factor.

Statement of the Problem

Consider a cylindrical panel of size $a \times b$, subjected to the action of a pressure $p(x,y)e^{i\omega t}$ varying in time according to the harmonic law with a frequency close to resonance. To model vibrations of the panel, we use the Kirchhoff–Love hypotheses, supplemented with hypotheses adequate to them and describing the distribution of electric field quantities [1–4]. The passive layers can be metallic, polymeric, or composite. We consider them as isotropic. The piezoactive layers are assumed to be transversally isotropic and polarized over the plate thickness. If there are no electrodes between the layers, then a perfect mechanical and electric contact takes place at the boundaries between them. The dissipative properties of materials of the passive and piezoactive layers are taken into account on the basis of the concept of complex characteristics [5]. The main relations of the theory of shells with distributed sensors and actuators can be found in [5, 8, 10]. We present here those relations that are used in what follows. We restrict ourselves to the case of a three-layer cylindrical panel; its middle layer of thickness h_0 is manufactured of a passive isotropic viscoelastic material, and the two external layers of equal thicknesses h_1 are manufactured of piezoelectric transversally isotropic viscoelastic materials with opposite polarization direction. Then the complex constitutive equations for forces and moments have the following form [5]:

$$N_1 = D_N(\varepsilon_1 + \nu_N \varepsilon_2), \quad N_2 = D_N(\varepsilon_2 + \nu_N \varepsilon_2), \quad S = \frac{1-\nu_N}{2} D_N \varepsilon_{12},$$

$$M_1 = D_M(\kappa_1 + \nu_M \kappa_2) + M_0, \quad M_2 = D_M(\kappa_2 + \nu_M \kappa_2) + M_0,$$

$$H = \frac{1-\nu_M}{2} D_M \kappa_{12}. \tag{1}$$

The total panel thickness is $h = h_0 + 2h_1$.

The quantity $\tilde{M}_0 = M_0 e^{i\omega t}$ plays the key role in the damping of resonance vibrations. Precisely due to the corresponding choice of this quantity, the mechanical load is compensated when using the first approach presented above.

Expressions for the rigid characteristics of the equations of state (1) are given in [5] for the case of layered piezoelectric thin-walled elements of arbitrary structure. For example, if the piezoactive layers of a three-layer piezopanel have the same thickness and identical properties, except the fact that they possess opposite polarization $(\overset{2}{d}_{31} = -\overset{1}{d}_{31})$, then the complex electromechanical characteristics in the constitutive equations (1) are given by:

– in the presence of internal electrodes to which potential differences $V_1 = -V_2 = \frac{1}{2}V_0$ are supplied:

$$D_N = h_0 \overset{0}{B}_{11} + 2h_1 \overset{1}{B}_{11}, \quad \nu_N = \left(h_0 \overset{0}{B}_{12} + 2h_1 \overset{1}{B}_{12}\right)\frac{1}{D_N},$$

$$D_M = \frac{h_0^3}{12}\overset{0}{B}_{11} + \frac{2}{3}\Bigg\{\overset{1}{B}_{11} + \left(1+\overset{1}{\nu}\right)\overset{1}{B}_{11}\, k_p^2 \frac{1}{2(1-k_p^2)}\Bigg[1 - \frac{3}{4h_1}\Bigg[\left(\frac{h_0}{2}+h_1\right)^2$$

$$-\left(\frac{h_0}{2}\right)^2\Bigg]^2\Bigg[\left(\frac{h_0}{2}+h_1\right)^3 - \left(\frac{h_0}{2}\right)^3\Bigg]^{-1}\Bigg]\Bigg\}\Bigg[\left(\frac{h_0}{2}+h_1\right)^3 - \left(\frac{h_0}{2}\right)^3\Bigg],$$

$$\nu_M = \Bigg\{\frac{h_0^3}{12}\overset{0}{B}_{12} + \frac{2}{3}\Bigg\{\overset{1}{B}_{12} + \left(1+\overset{1}{\nu}\right)\overset{1}{B}_{12}\, k_p^2 \frac{1}{2(1-k_p^2)}\Bigg[1 - \frac{3}{4h_1}\Bigg[\left(\frac{h_0}{2}+h_1\right)^2$$

$$-\left(\frac{h_0}{2}\right)^2\Bigg]^2\Bigg[\left(\frac{h_0}{2}+h_1\right)^3 - \left(\frac{h_0}{2}\right)^3\Bigg]^{-1}\Bigg]\Bigg\}\Bigg[\left(\frac{h_0}{2}+h_1\right)^3 - \left(\frac{h_0}{2}\right)^3\Bigg]\frac{1}{D_M},$$

$$M_0 = \frac{1}{2}\overset{1}{\gamma}_{31}(h_1 + h_0)V_0; \tag{2}$$

– in the absence of internal electrodes:

$$D_N = h_0 \overset{0}{B}_{11} + 2h_1\left(\overset{1}{B}_{11} + \frac{1+\overset{1}{\nu}}{2}\overset{1}{B}_{11}\left(\overset{1}{k}_p\right)^2\left[1-\left(\overset{1}{k}_p\right)^2\right]^{-1}\right),$$

$$D_M = \frac{h_0^3}{12}\overset{0}{B}_{11} + \frac{2}{3}\Bigg\{\overset{1}{B}_{11} + \left(1+\overset{1}{\nu}\right)\overset{1}{B}_{11}\, k_p^2 \frac{1}{2(1-k_p^2)}\Bigg[1 - \frac{3}{4h_1}\Bigg[\left(\frac{h_0}{2}+h_1\right)^2$$

$$-\left(\frac{h_0}{2}\right)^2\Bigg]^2\Bigg[\left(\frac{h_0}{2}+h_1\right)^3 - \left(\frac{h_0}{2}\right)^3\Bigg]^{-1}\Bigg]\Bigg\}\Bigg[\left(\frac{h_0}{2}+h_1\right)^3 - \left(\frac{h_0}{2}\right)^3\Bigg]\frac{2h_1\overset{0}{\gamma}_{33}}{h_0\overset{1}{\gamma}_{33} + 2h_1\overset{0}{\gamma}_{33}},$$

$$\nu_M = \left\{ \frac{h_0^3}{12} \overset{0}{B}_{12} + \frac{2}{3} \left\{ \overset{1}{B}_{12} + \left(1 + \overset{1}{\nu} \right) \overset{1}{B}_{12}\, k_p^2 \frac{1}{2(1-k_p^2)} \left[1 - \frac{3}{4h_1} \left[\left(\frac{h_0}{2} + h_1 \right)^2 \right.\right.\right.$$

$$\left.\left.\left. - \left(\frac{h_0}{2} \right)^2 \right]^2 \left[\left(\frac{h_0}{2} + h_1 \right)^3 - \left(\frac{h_0}{2} \right)^3 \right]^{-1} \right] \right\} \right\} \left[\left(\frac{h_0}{2} + h_1 \right)^3 - \left(\frac{h_0}{2} \right)^3 \right] \frac{2h_1 \overset{0}{\gamma}_{33}}{h_0 \overset{1}{\gamma}_{33} + 2h_1 \overset{0}{\gamma}_{33}} \frac{1}{D_M},$$

$$M_0 = \frac{\overset{0}{\gamma}_{31} h_1 (h_0 + h_1) \overset{0}{\gamma}_{33}}{h_0 \overset{1}{\gamma}_{33} + 2h_1 \overset{0}{\gamma}_{33}} V_0 . \tag{3}$$

Here, for a passive isotropic material, we have

$$\overset{0}{B}_{11} = \overset{0}{B}_{22} = \frac{\overset{0}{E}}{1-\nu^2}, \quad \overset{0}{B}_{12} = \nu \overset{0}{B}_{11}$$

and, for a piezoelectric transversally isotropic material,

$$\overset{k}{B}_{11} = \overset{k}{B}_{22} = \left\{ \overset{k}{S}_{11} \left[1 - \overset{k}{\nu}{}^2 \right] \right\}^{-1}, \quad \overset{k}{B}_{12} = \overset{k}{\nu} \overset{k}{B}_{11} ,$$

$$\overset{k}{B}_{66} = \frac{1}{2} \left[1 - \overset{k}{\nu} \right] \overset{k}{B}_{11}, \quad \overset{k}{\gamma}_{31} = \overset{k}{d}_{31} \left\{ \overset{k}{S}{}_{11}^{E} \left[1 - \overset{k}{\nu}{}^2 \right] \right\}^{-1}, \quad \overset{k}{\gamma}_{33} = \overset{k}{\varepsilon}{}_{33}^{T} \left[1 - \overset{k}{k}{}_p^2 \right],$$

$$\overset{k}{k}{}_p^2 = 2 \overset{k}{d}{}_{31}^2 \left\{ \overset{k}{\varepsilon}{}_{33}^{T} \overset{k}{S}{}_{11}^{E} \left[1 - \overset{k}{\nu} \right] \right\}^{-1}, \quad \overset{1}{d}_{31} = - \overset{2}{d}_{31} > 0, \quad \overset{k}{\nu} = - S_{12}^{E} \frac{1}{\overset{k}{S}{}_{11}^{E}} .$$

Equations of the forced vibrations of a shallow cylindrical panel in a Cartesian coordinate system have the form [1, 2, 5]

$$\frac{\partial N_1}{\partial x} + \frac{\partial S}{\partial y} = 0, \quad \frac{\partial S}{\partial x} + \frac{\partial N_2}{\partial y} = 0 ,$$

$$\frac{\partial^2 M_1}{\partial x^2} + 2 \frac{\partial^2 H}{\partial x \partial y} + \frac{\partial^2 M_2}{\partial y^2} + \frac{1}{R} N_2 + p(x,y) + \tilde{\rho} \omega^2 w = 0 , \tag{4}$$

where $\tilde{\rho}$ is the reduced density.

Kinematic relations in these coordinates look like

$$\varepsilon_1 = \frac{\partial u}{\partial x}, \quad \varepsilon_2 = \frac{\partial v}{\partial y} - \frac{1}{R} w, \quad \varepsilon_{12} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x},$$

$$\kappa_1 = -\frac{\partial^2 w}{\partial x^2}, \quad \kappa_2 = -\frac{\partial^2 w}{\partial y^2}, \quad \kappa_{12} = -2\frac{\partial^2 w}{\partial x \partial y}. \tag{5}$$

For a shallow cylindrical panel, the equation of strain compatibility has the form

$$\frac{\partial^2 \varepsilon_1}{\partial y^2} + \frac{\partial^2 \varepsilon_2}{\partial x^2} - \frac{\partial^2 \varepsilon_{12}}{\partial x \partial y} = -\frac{1}{R}\frac{\partial^2 w}{\partial x^2}. \tag{6}$$

The functions of forces

$$N_1 = h\frac{\partial^2 \Phi}{\partial y^2}, \quad N_2 = h\frac{\partial^2 \Phi}{\partial x^2}, \quad S = -h\frac{\partial^2 \Phi}{\partial x \partial y} \tag{7}$$

satisfy identically the first two equations in (4).

Using the equations of state (1), we find

$$\varepsilon_x = \frac{N_1 - \nu_N N_2}{(1-\nu_N^2)D_N}, \quad \varepsilon_y = \frac{N_2 - \nu_N N_1}{(1-\nu_N^2)D_N}, \quad \varepsilon_{xy} = \frac{2S}{(1-\nu_N)D_N}. \tag{8}$$

Substituting relations (7) in Eqs. (8) and the result obtained in the equation of strain compatibility (6), we come to the first resolving equation:

$$\frac{h}{(1-\nu_N^2)D_N}\Delta\Delta\Phi + \frac{1}{R}\frac{\partial^2 w}{\partial x^2} = 0. \tag{9}$$

Further, using the third equation of motion (4), the equations of state (1), and relations (7), we obtain the second resolving equation:

$$D_M \Delta\Delta w - \frac{h}{R}\frac{\partial^2 \Phi}{\partial x^2} - \tilde{\rho}\omega^2 w - q(x,y) = 0, \tag{10}$$

where

$$q = p + \Delta M_0, \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

Thus, the complex resolving system of equations for a shallow isotropic cylindrical panel has the form (9), (10).

We add to this system the boundary conditions corresponding to the case of a panel with simply supported edge faces:

$$w=0,\quad M_x=0,\quad \Phi=0,\quad \frac{\partial^2\Phi}{\partial x^2}=0,\quad x=0,a\,,$$

$$w=0,\quad M_y=0,\quad \Phi=0,\quad \frac{\partial^2\Phi}{\partial y^2}=0,\quad y=0,b\,. \tag{11}$$

Equations (9)–(11) enable us to study the active damping of a cylindrical panel subjected to the action of an unknown mechanical load. The problem lies in the calculation of the potential difference that has to be supplied to the actuator for compensating the mechanical load. Since the load is unknown, it is necessary to reconstruct it using experimental data of the sensor.

Analytical Solution of the Problem

Consider the case of a cylindrical panel with simply supported edge faces under the action of harmonic mechanical and electrical loads. We search for the solution of the problem of its forced resonance vibrations on a certain mode in the form

$$w=w_{mn}\sin k_m x\sin p_n y,\quad \Phi=\Phi_{mn}\sin k_m x\sin p_n y\,, \tag{12}$$

where

$$k_m=\frac{m\pi}{a}\qquad\text{and}\qquad p_n=\frac{n\pi}{b}.$$

The resonance components of mechanical and electrical loads are also represented as

$$p_0=q_{mn}\sin k_m x\sin p_n y,\quad M_0=M_{mn}\sin k_m x\sin p_n y. \tag{13}$$

Suppose that the center of the actuator is placed at a point (ξ,η), and its sizes are (c,d). Then we obtain

$$M_{mn}=\frac{16M_0\varphi(\xi,\eta,c,d)}{abk_m p_n},$$

$$\varphi(\xi,\eta,c,d)=\sin k_m\xi\sin p_n\eta\sin\frac{k_n c}{2}\sin\frac{p_n d}{2}. \tag{14}$$

To determine w_{mn} and Φ_{mn}, one can use variational methods or the Bubnov–Galerkin method. According to the last, we substitute (12), (13) in Eqs. (9), (10), multiply the equations obtained by the shape function, and integrate over the shell area. As a result, we come to the following relations:

$$w_{mn} = \frac{\Delta_{1mn}}{\Delta_{2mn}}, \quad \Phi_{mn} = \frac{k_m^2}{hR\Delta_{mn}} w_{mn}, \tag{15}$$

where

$$\Delta_{1mn} = p_{mn} - (k_m^2 + p_n^2) M_{mn}, \quad \Delta_{2mn} = D_M (k_m^2 + p_n^2)^2 + \frac{k_m^4}{R^2 \Delta_{mn}} - \tilde{\rho}\omega^2,$$

$$\Delta_{mn} = h(k_m^2 + p_n^2)^2 \frac{1}{(1-\nu_N^2) D_N}. \tag{16}$$

To realize the described approach to the damping of vibrations for an unknown mechanical load, it is necessary to have expressions for indications of the sensor in the case of action of only the mechanical load on the panel. For short-circuited electrodes, the charge on the sensor is given by

$$Q = -\gamma_{31}(h_0 + h_1) \iint_{(S_1)} (\kappa_1 + \kappa_2)\, dx\, dy. \tag{17}$$

For disconnected electrodes, the potential difference can be determined as

$$V_S = \frac{h_1 Q}{S_1 \gamma_{33}}. \tag{18}$$

To determine indications of the sensor for the case of vibrations on a certain mode (m, n), we substitute (12) in relations (17), (18) and calculate integrals over the sensor area. We assume that the piezoelectric inclusions play the role of sensor and actuator simultaneously. Therefore, the sensor is placed in the same way as the actuator. After the corresponding calculations, we obtain the following expression for the indications of the sensor corresponding to vibrations of the cylindrical panel on the mode (m, n):

$$Q_{mn} = -4\gamma_{31}(h_0 + h_1) \frac{\varphi(\xi, \eta, c, d)}{k_m p_n} w_{mn}. \tag{19}$$

The potential difference of the sensor for mechanical vibrations on a certain mode is given by

$$V_{Smn} = \frac{h_1 Q_{mn}}{S_1 \gamma_{33}}. \tag{20}$$

The solution of the problem of resonance mechanical vibrations of a panel on the mode (m, n) has the form

$$w_{mn} = \frac{p_{mn}}{\Delta_{2mn}}. \tag{21}$$

The natural frequency of resonance vibrations can be determined as

$$\omega_{mn} = \sqrt{\frac{1}{\tilde{\rho}}\left[D'_M (k_m^2 + p_n^2)^2 + \frac{k_m^4 \Delta'_{mn}}{R^2 (\Delta'^2_{mn} + \Delta''^2_{mn})}\right]}. \tag{22}$$

In the case of resonance vibrations, it is assumed that the frequencies of mechanical and electrical loads are close to frequency ω_{mn}. Vibrations of the panel take place precisely in a neighborhood of this frequency. Substituting (21) in (19) and (20), we obtain the following relation between indications of the sensor and the load:

$$p_{mn} = -\frac{Q_{mn}\Delta_{2mn}}{4\gamma_{31}(h_0 + h_1)\varphi(\xi,\eta,c,d)}, \tag{23}$$

$$p_{mn} = -\frac{S_1 \gamma_{33} V_{Smn} \Delta_{2mn}}{4\gamma_{31} h_1 (h_0 + h_1)\varphi(\xi,\eta,c,d)}. \tag{24}$$

These results enable us to realize the third of the approaches described above.

Analysis of the Solution

As follows from relations (15) and (16), for compensation of the external mechanical load, it is necessary to supply to the actuator a potential difference determined from

$$V_A = \frac{ab}{16 f_1} \frac{k_m p_n p_{mn}}{(k_m^2 + p_n^2)} \frac{1}{\varphi(\xi,\eta,c,d)}. \tag{25}$$

Here, according to (2) and (3),

$$f_1 = \frac{1}{2} \overset{1}{\gamma}_{31} (h_1 + h_0)$$

or

$$f_1 = \frac{\overset{0}{\gamma}_{31} h_1 (h_0 + h_1) \overset{0}{\gamma}_{33}}{h_0 \overset{1}{\gamma}_{33} + 2 h_1 \overset{0}{\gamma}_{33}}$$

depending on the presence or absence of internal electrodes.

If condition (25) is satisfied, the amplitude of flexural vibrations on the considered mode will be equal to zero. Here, using the second relation (15), we have $\Phi_{mn} = 0$.

Relation (25) holds also in the case where the properties of the materials depend on the temperature, e.g., the temperature of dissipative heating, and even with regard for the physical nonlinearity of materials. As follows from this relation, if the properties of the active material are independent of temperature or strain amplitudes, then the potential difference necessary for compensating the principal mode of vibrations is independent of the properties of the passive material, so that, in such a case, neither temperature nor physical nonlinearity affects this quantity. This fact is quite important because it enables one to calculate the potential difference mentioned above by the simplest linear theory of viscoelasticity. On the other hand, if the properties of the piezomaterial depend on temperature or strain amplitudes, then we see from relation (25) that the potential difference can change substantially depending on the sensitivity of γ_{31} to changes in temperature or vibration amplitude.

The main shortcomings of the approach based on relation (25) are:

(i) free vibrations are not damped;

(ii) it is necessary to know the external mechanical load.

The first shortcoming is removed by the presence of internal losses in the materials. To remove the second shortcoming, we use the third approach described above and based on relations (23)–(25).

We substitute the load found from (22) and (23) in relation (25) for the potential of the actuator compensating this load. As a result, we obtain the following expressions for this potential:

$$V_A = -\frac{ab}{64 f_1} \frac{(k_m p_n)^2 \Delta_{2mn} Q_{mn}}{\gamma_{31}(h_0 + h_1)(k_m^2 + p_n^2)\varphi(\xi, \eta, c, d)}, \tag{26}$$

$$V_A = -\frac{ab}{64 f_1} \frac{(k_m p_n)^2 \Delta_{2mn} V_{Smn} \gamma_{33} S_1}{\gamma_{31} h_1 (h_0 + h_1)(k_m^2 + p_n^2)\varphi(\xi, \eta, c, d)}. \tag{27}$$

We see that, using the proposed approach, the potential difference, determined according to experimental indications of the sensor by formulas (26) and (27), is supplied to the actuator.

The efficiency of active damping of the forced resonance vibrations of a panel depends on the efficiency of work of sensors and actuators. If the load is assigned, the actuator that works more efficiently is that to which it is necessary to supply a lower potential difference for compensating this load. It is usually thought that, for a given mechanical load, the sensor that shows greater indications works more efficiently. As is seen from relations (19), (20), and (25), the efficiency of piezoinclusions depends on their placing and sizes, which, in turn, depend on the vibration mode. Analysis of the influence of these factors is reduced to analysis of the function $\varphi(\xi, \eta, c, d)$ for the extremum. Such analysis for a rectangular plate is presented in [5]. Since the functions $\varphi(\xi, \eta, c, d)$ for panels and plates are identical, the results given in [5] are also valid for a cylindrical panel.

Vibrations on the most power-intensive minimal frequency are the most dangerous for the serviceability of structures. To find this frequency, one should test for the minimum of the expression under the root in relation (22). It is easy to show that, for a shallow cylindrical panel over a wide range of its geometrical parameters and rigid characteristics, the minimum frequency corresponds to $m = n = 1$. For this case, the efficiency of work of piezoinclusions will be the highest if the panel is completely covered with sensors and actuators.

CONCLUSIONS

We have solved the problem of active damping of the forced resonance vibrations of an isotropic viscoelastic cylindrical panel with the help of piezoelectric inclusions for the case where the external mechanical load is unknown. To solve this problem, we use the Bubnov–Galerkin method. We have obtained relations for reconstructing the mechanical load from the experimental data of a sensor, namely, the charge or potential difference.

After determining the load, for the damping of vibrations, we supply to the actuator a potential difference that compensates the action of the external load. Owing to this, the vibration amplitude on the corresponding mode becomes equal to zero. The relation for the potential difference mentioned above contains the experimental indications of the sensor. We have studied the influence of placing of sensors and actuators and their sizes, viscosity, and temperature on the efficiency of work of sensors and actuators as well as on the efficiency of active damping with their help. The relations obtained show that the presence of viscosity in the material of the passive layer is a necessary condition for efficient damping of forced resonance vibrations according to the proposed method. In the absence of viscosity, indications of the sensor tend to infinity upon approaching the resonance frequency. If the viscosity of the material is low, the control of vibrations becomes very sensitive to measurement errors.

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