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SMALL DEVIATION PROBABILITIES FOR POSITIVE RANDOM VARIABLES

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We deduce two lemmas that seem to be useful while studying small deviation probabilities for positive random variables. As an example, the so-called small balls problem is examined. Bibliography: 11 *titles.*

1. Results

Assume that a nonnegative random variable Z takes values in any neighborhood of zero with positive probability not equal to one.

For $u \geq 0$, set

$$\Lambda(u) = \mathbf{E}e^{-uZ}, \quad m(u) = -\left(\log\Lambda(u)\right)', \quad \sigma^2(u) = \left(\log\Lambda(u)\right)'', \\ Q(u) = -um(u) - \log\Lambda(u). \tag{1.1}$$

Recall that

$$m(u) = \mathbf{E}Z(u), \quad 0 < \sigma^2(u) = \mathbf{Var}Z(u), \tag{1.2}$$

where the distribution of the random variable $Z(u) \geq 0$ is conjugate to that of Z, i.e.,

$$\mathbf{P}(Z(u) \leq r) = \int_0^r e^{-uy} d\mathbf{P}(Z \leq y)/\Lambda(u), \quad r \geq 0. \tag{1.3}$$

Note (see [1, Lemma 2.1]) that $m(u)$ is a monotone decreasing function on $(0,\infty)$, $Q(u)$ is a monotone increasing function on $(0,\infty)$, and

$$m(0) = \mathbf{E}Z \leq \infty, \quad m(\infty) = 0, \quad Q(0) = 0, \quad Q(\infty) = -\log\mathbf{P}(Z=0). \tag{1.4}$$

Lemma 1. *For* $0 < r \leq \mathbf{E}Z$,

$$e^{-Q(h)} \geq \mathbf{P}(Z \leq r) \geq \frac{1}{2} e^{-2a(1+\sqrt{1+2Q(h)/a})}\, e^{-Q(h)}, \tag{1.5}$$

where $a = \sup\limits_{u>0} \frac{u^2\sigma^2(u)}{Q(u)}$ *and* h *is the unique solution of the equation*

$$m(h) = r. \tag{1.6}$$

Remark 1. Denote $\rho(r) = \sup\limits_{u\geq 0}(-ur - \log\Lambda(u))$. We have $\rho(r) = Q(h)$ provided that $0 < r \leq \mathbf{E}Z < \infty$ and equality (1.6) holds; $\rho(0) = Q(\infty)$; $\rho(r) = +\infty$ for $r < 0$ and $\rho(r) = 0$ for $r > \mathbf{E}Z$. Thus, using Lemma 1, we obtain the following inequality, which is now valid for all r:

$$e^{-\rho(r)} \geq \mathbf{P}(Z \leq r) \geq \frac{1}{2} e^{-2a(1+\sqrt{1+2\rho(r)/a})}\, e^{-\rho(r)}.$$

From (1.5) it follows (provided that $a \neq \infty$) that the values $\mathbf{P}(Z \leq r)$ and $1/Q(h)$ are small simultaneously; moreover, if $Q(h)$ tends to infinity, then $\log\mathbf{P}(Z \leq r)$ is *approximately* equal to $-Q(h)$. In other words, Lemma 1 allows one to find, under some additional assumptions, the asymptotics of the logarithm of the probability $\mathbf{P}(Z \leq r)$. The following result is a nice basis for the analysis of the asymptotic behavior of the probability itself.

Let (see (1.2)) $Z_0(u) = \frac{Z(u) - \mathbf{E}Z(u)}{\sqrt{\mathbf{Var}Z(u)}}$, $u > 0$. Denote

$$\delta_\varepsilon(u) = \int_0^{1/\varepsilon} |\mathbf{E}e^{itZ_0(u)} - e^{-t^2/2}|\, dt, \quad \varepsilon > 0. \tag{1.7}$$

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Lemma 2. *For any positive* $r,\ h,\ s,$ *and* ε,

$$\mathbf{P}(r-s<Z\le r)=\Lambda(h)e^{hr}\,\frac{1-e^{-h\,s}}{\tau\sqrt{2\pi}}\,\left(e^{-\beta^2/2}+\theta\left(\beta\,e^{-\beta^2/2}/\tau+1/\tau^2+\rho_\varepsilon(h,s)\right)\right), \tag{1.8}$$

where $|\theta|$ *is bounded by some absolute constant,*

$$\tau=h\,\sigma(h),\quad \beta=\frac{r-m(h)}{\sigma(h)},\quad \rho_\varepsilon(h,s)=\delta_\varepsilon(h)+(1+\delta_\varepsilon(h))(1+\tfrac{1}{h\,s})\,\tau\varepsilon. \tag{1.9}$$

In particular, if $r=m(h)$, *then* $\beta=0$ *and*

$$\mathbf{P}(r-s<Z\le r)=e^{-Q(h)}\,\frac{1-e^{-h\,s}}{\tau\sqrt{2\pi}}\,\left(1+\theta\left(1/\tau^2+\rho_\varepsilon(h,s)\right)\right). \tag{1.10}$$

Note that (1.8) implies, besides (1.10), that for $0<r<\mathbf{E}Z$, h satisfying (1.6), and any δ,

$$\mathbf{P}(Z\le r+\delta/h)=e^{-Q(h)}\,\frac{e^{\delta}}{\tau\sqrt{2\pi}}\,\left(e^{-\delta^2/2\tau^2}+\theta\left((1+\delta\,e^{-\delta^2/2\tau^2})/\tau^2+\rho_\varepsilon(h,\infty)\right)\right). \tag{1.11}$$

Remark 2. If $\delta_0(h)=\lim\limits_{\varepsilon\searrow 0}\delta_\varepsilon(h)<\infty$, then (1.8) implies that for all positive s

$$\mathbf{P}(r-s<Z\le r)=\Lambda(h)e^{hr}\,\frac{1-e^{-h\,s}}{\tau\sqrt{2\pi}}\,\left(e^{-\beta^2/2}+\theta\left(\beta\,e^{-\beta^2/2}/\tau+1/\tau^2+\delta_0(h)\right)\right) \tag{1.12}$$

(and similar corollaries of (1.10), (1.11)). In particular,

$$\frac{d\mathbf{P}(Z\le r)}{dr}=\Lambda(h)e^{hr}\,\frac{h}{\tau\sqrt{2\pi}}\,\left(e^{-\beta^2/2}+\theta\left(\beta\,e^{-\beta^2/2}/\tau+1/\tau^2+\delta_0(h)\right)\right),$$

i.e., Lemma 2 also allows one to investigate the probabilities of small deviations in the local setting.

Now let us demonstrate that if the parameter $\tau=h\,\sigma(h)$ is bounded, the function $e^{-Q(h)}/\tau\sqrt{2\pi}$ from (1.10) can give an approximation of the probability $\mathbf{P}(Z\le r)$ that is not quite satisfactory.

Let Z_j, $j=1,2,\ldots,n$, be independent random variables with distribution functions $F_j(x)$ such that

$$F_j(x)=l_j(x)x^{\alpha_j},\quad x\searrow 0, \tag{1.13}$$

where α_j are some positive numbers and $l_j(x)$ are functions slowly varying at zero. Then for a fixed n,

$$P_n(r)=\mathbf{P}(Z_1+\cdots+Z_n\le r)\sim k_n\prod_{j=1}^{n}F_j(r),\quad r\searrow 0, \tag{1.14}$$

where $k_n=\prod\limits_{j=1}^{n}\Gamma(1+\alpha_j)/\Gamma(1+\alpha)$ and $\alpha=\alpha_1+\cdots+\alpha_n$. (One can prove (1.14) by induction, using the properties of slowly varying functions.)

On the other hand, if $Z=Z_1+\cdots+Z_n$, then (see (1.1))

$$\Lambda(u)=\prod_{j=1}^{n}\Lambda_j(u),\quad m(u)=\sum_{j=1}^{n}m_j(u),\quad \sigma^2(u)=\sum_{j=1}^{n}\sigma_j^2(u),\quad Q(u)=\sum_{j=1}^{n}Q_j(u), \tag{1.15}$$

where

$$\Lambda_j(u)=\mathbf{E}e^{-uZ_j},\quad m_j(u)=-\left(\log\Lambda_j(u)\right)',\quad \sigma_j^2(u)=\left(\log\Lambda_j(u)\right)'',$$
$$Q_j(u)=-um_j(u)-\log\Lambda_j(u).$$

If condition (1.13) holds, then

$$\Lambda_j(u) \sim \Gamma(1+\alpha_j)F_j(1/u), \quad m_j(u) \sim \alpha_j/u, \quad \sigma_j^2(u) \sim \alpha_j/u^2, \quad u \to \infty, \tag{1.16}$$

whence

$$um(u) \to \alpha, \quad u^2\sigma^2(u) \to \alpha, \quad u \to \infty.$$

Taking into consideration (1.15), (1.16), and the relation $h \sim \alpha/r$, $r \searrow 0$, following from (1.6), we obtain

$$e^{-Q(h)}/(h\sigma(h)\sqrt{2\pi}) = e^{hr} \prod_{j=1}^{n} L_j(h)/(h\sigma(h)\sqrt{2\pi}) \sim \omega_\alpha P_n(r), \quad r \searrow 0.$$

Here $\omega_\alpha = \left(\frac{e}{\alpha}\right)^\alpha \frac{\Gamma(1+\alpha)}{\sqrt{2\pi\alpha}} > 1$ for any finite $\alpha > 0$ (and $\omega_\alpha \to 1$ as $\alpha \to \infty$). Thus the leading term in the right-hand side of (1.10) and the asymptotics from (1.14) are indeed different.

In conclusion, we give an example of using relation (1.12).

Let $\{X\}$ be a centered Gaussian vector taking values in a separable Hilbert space H. Then $X = \sum \lambda_j^{1/2}\xi_j e_j$, where $\{e_j\}$ is an orthogonal basis in H, $\{\lambda_j\}$ is a sequence of nonnegative nonincreasing numbers with finite sum, and ξ_j are independent standard normal random variables.

Let $a = \sum \alpha_j e_j \in H$, where $\sum \alpha_j^2 < \infty$, and $r > 0$. We are interested in estimates of the probabilities $\mathbf{P}(r - s < Z \le r)$, where $Z = ||X - a||^2 = \sum(\lambda_j^{1/2}\xi_j - \alpha_j)^2$.

For $\gamma > 0$, denote (see [2])

$$\begin{aligned} \Lambda(\gamma) &= \mathbf{E}\exp(-\gamma Z) = \prod (1+2\gamma\lambda_j)^{-1/2}\exp(-\gamma\alpha_j^2/(1+2\gamma\lambda_j)), \\ m(\gamma) &= \sum_j \left(\frac{\lambda_j}{1+2\gamma\lambda_j} + \frac{\alpha_j^2}{(1+2\gamma\lambda_j)^2}\right), \\ \sigma^2(\gamma) &= \sum_j \sigma_j^2 = \sum_j \left(\frac{2\lambda_j^2}{(1+2\gamma\lambda_j)^2} + \frac{4\lambda_j\alpha_j^2}{(1+2\gamma\lambda_j)^3}\right). \end{aligned} \tag{1.17}$$

Theorem 1. *Let $\lambda_3 > 0$. Then for any r, $s > 0$ and $\gamma > 0$,*

$$\mathbf{P}(r - s < ||X-a||^2 \le r) = \Lambda(\gamma)e^{\gamma r}\frac{1-e^{-\gamma s}}{\gamma\,\sigma(\gamma)\sqrt{2\pi}}\left(e^{-\beta^2/2} + \theta\left((\gamma\,\sigma(\gamma))^{-1} + (\gamma\,\sigma(\gamma))^{-2}\right)\right), \tag{1.18}$$

where $\beta = (r - m(\gamma))/\sigma(\gamma)$, $|\theta| \le c\sqrt{\lambda_1/\lambda_2}\,(1 + \log(\lambda_2/\lambda_3))$, and c is an absolute constant.

Recall that if γ is the solution of the equation $m(\gamma) = r$, then $\beta = 0$.

Note that Theorem 1 allows one to refine known general results on the asymptotic behavior of the probability that a Gaussian vector from a Hilbert space hits a sphere of small radius (see, for example, [3–9], and also Theorem 4 of [2]).

Proof of Theorem 1. Let $Z = ||X - a||^2$ in (1.12). Then (see (1.2), (1.3), (1.7), and [2, p. 441]) the random variable $Z(\gamma)$ has the same distribution as $\sum(\beta_j^{1/2}\xi_j - \omega_j)^2$ with $\beta_j = \lambda_j/(1+2\gamma\lambda_j)$, $\omega_j = \alpha_j/(1+2\gamma\lambda_j)$. Hence

$$\mathbf{E}e^{itZ(\gamma)} = \prod (1 - 2it\beta_j)^{-1/2}\exp(it\omega_j^2/(1-2it\beta_j)). \tag{1.19}$$

Set (see (1.17)) $f(t) = \mathbf{E}\exp(it(Z(\gamma) - m(\gamma))/\sigma(\gamma))$ and $\tau_j = \gamma\beta_j$. Keeping in mind that in the case under consideration $\mathbf{E}Z(\gamma) = m(\gamma)$ and $\mathbf{Var}Z(\gamma) = \sigma^2(\gamma)$, (1.19) implies, by standard arguments, that

$$\log f(t) = -t^2/2 + \theta(t/\tau)^3 \sum \tau_j\gamma^2\sigma_j^2 = -t^2/2 + \theta t^3/\tau, \quad |t| \le \varepsilon\tau,$$

where $\tau = \gamma\,\sigma(\gamma)$, $|\theta| \le c$, and c, ε are absolute positive constants. It follows that

$$\int_0^{\varepsilon\tau} |f(t) - e^{-t^2/2}|\,dt \le c_1/\tau.$$

Now let us estimate $I = \int\limits_{\varepsilon\tau}^{\infty} |f(t)|\, dt$. We have $I = \tau \int\limits_{\varepsilon}^{\infty} \prod \xi_j(u)\, du$, where

$$\xi_j(u) = (1 + 4u^2\tau_j^2)^{-1/4} \exp\left(-2\, u^2\tau_j\gamma\omega_j^2/(1 + 4u^2\tau_j^2)\right).$$

Using the equality $\sigma_j^2 = 2\beta_j^2 + 4\beta_j\omega_j^2$ (see (1.17)), it is easy to prove that for $\delta = 1/10$,

$$\begin{aligned} \xi_j(u) &\le \exp\left(-\delta u^2\gamma^2\sigma_j^2\right), \quad u\tau_j \le 1, \\ \xi_j(u) &\le (e/(1 + 4u^2\tau_j^2))^{-1/4}\ \exp\left(-\delta\gamma^2\sigma_j^2/\tau_j^2\right), \quad u\tau_j \ge 1. \end{aligned} \tag{1.20}$$

Choose a positive integer n satisfying the condition

$$\sum_{j=1}^{n} \sigma_j^2 \ge \sigma^2/2 > \sum_{j=1}^{n-1} \sigma_j^2$$

and set

$$I = \left(\tau \int\limits_{\varepsilon}^{1/\tau_n} + \tau \int\limits_{1/\tau_n}^{\infty}\right) \prod \xi_j(u)\, du = I_1 + I_2.$$

With the help of (1.20) we obtain

$$I_1 \le \tau \int\limits_{\varepsilon}^{1/\tau_n} \prod_{j\ge n} \xi_j(u)\, du \le \tau \int\limits_{\varepsilon}^{\infty} e^{-\delta u^2\gamma^2\sigma^2/2} du \le c/\tau^2.$$

Now let us estimate I_2. We have to consider three cases: $n = 1,\ n = 2$, and $n \ge 3$. Let us restrict ourselves to the most general first case. Bearing in mind (1.20) and the inequality $\tau_j \ge \tau_n$ for $j \le n$, we obtain

$$I_2 \le \tau \int\limits_{1/\tau_3}^{\infty} \xi_1(u)\xi_2(u)\xi_3(u)\, du + \tau \int\limits_{1/\tau_2}^{1/\tau_3} \xi_1(u)\xi_2(u)\, du + \tau \int\limits_{1/\tau_1}^{1/\tau_2} \xi_1(u)\, du = I_{21} + I_{22} + I_{23},$$

where

$$I_{21} \le \tau e^{1-\delta\tau^2/2\tau_1^2} \int\limits_{1/\tau_3}^{\infty} \prod_{j=1}^{3} (1 + 4u^2\tau_j^2)^{-1/4}\, du$$

and

$$\int\limits_{1/\tau_3}^{\infty} \prod_{j=1}^{3} (1 + 4u^2\tau_j^2)^{-1/4}\, du \le \frac{1}{\tau_1}\sqrt{\tau_1/2\tau_2} \le \frac{1}{\tau_1}\sqrt{\lambda_1/2\lambda_2};$$

$$I_{22} \le \tau/\tau_1\, e^{1-\delta\tau^2/2\tau_1^2} \sqrt{\lambda_1/2\lambda_2}\, \log \lambda_2/\lambda_3; \quad I_{23} \le \tau/\tau_1\, e^{1-\delta\tau^2/2\tau_1^2} \sqrt{2\lambda_1/2\lambda_2}.$$

Theorem 1 follows from (1.12) and the above calculations. □

2. Proofs of Lemmas 1 and 2.

The proof of Lemma 1 essentially repeats the proof of Theorem 1 from [10]. However, we present it, because the computations are rather short.

For any $u \ge 0$,

$$\mathbf{P}(Z \le r) = \Lambda(u)\mathbf{E}e^{u\bar{Z}}\mathbf{I}[\bar{Z} \le r], \tag{2.1}$$

where $\bar{Z} = Z(u)$ and the random variable $Z(u)$ has the distribution (1.3). Hence

$$\mathbf{P}(Z \le r) = e^{-Q(h)}\, I(u), \quad I(u) = \mathbf{E}e^{u\sigma(u)Z_0(u)}\mathbf{I}[Z_0(u) \le \frac{r - m(u)}{\sigma(u)}] \tag{2.2}$$

($Z_0(u)$ is defined before relation (1.7)).

Let h be the solution of Eq. (1.6). Then $I(h) \le 1$, and (2.2) with $u = h$ implies

$$\mathbf{P}(Z \le r) \le e^{-Q(h)}. \tag{2.3}$$

If $a = \infty$, the lower bound is obvious. Now let $a < \infty$. If $m(u) < r$ (or $u > h$), then

$$I(u) \ge e^{u\,(m(u)-r)}\mathbf{P}\left(|Z_0(u)| \le \frac{r - m(u)}{\sigma(u)}\right) \ge e^{u\,(m(u)-r)}\left(1 - \frac{\sigma^2(u)}{(r - m(u))^2}\right). \tag{2.4}$$

For $t \ge h$, put $\xi(t) = t\,(r - m(t))$. Obviously,

$$Q(u) - Q(h) = -\int\limits_h^u tm'(t)dt = \xi(u) + \int\limits_h^u (m(t) - m(h))dt \le \xi(u), \quad h \le u < \infty. \tag{2.5}$$

Denote $\nu = a + \sqrt{a^2 + 2aQ(h)}$. Let u be the solution of the equation $\xi(u) = \nu$ (such a solution exists and is unique, because the function $\xi(t)$ monotonically grows from zero to infinity on $[h, \infty)$). Then, by (2.5) and the choice of ν,

$$Q(u) \le Q(h) + \xi(u) = Q(h) + \nu, \quad u\,(r - m(u)) = \xi(u) = \nu,$$
$$\frac{\sigma^2(u)}{(r - m(u))^2} = \frac{u^2\sigma^2(u)}{\xi^2(u)} \le \frac{a\,Q(u)}{\nu^2} = \frac{a(Q(h) + \nu)}{\nu^2} = \frac{1}{2}.$$

This, along with (2.2)–(2.4), implies Lemma 1. □

Proof of Lemma 2. Let $h > 0$. From (2.1) and (2.2) it follows that

$$\mathbf{P}(r - s < Z \le r) = e^{-Q(h)} \int\limits_\gamma^\beta e^{\tau t}\, F^{(h)}(dt), \tag{2.6}$$

where $\beta = (r - m(h))/\sigma(h)$, $\gamma = (r - s - m(h))/\sigma(h)$, $\tau = h\sigma(h)$, and $F^{(h)}(t)$ is the distribution function of $Z_0(h)$. Denote $\Delta(t) = F^{(h)}(t) - \Phi(t)$ (where $\Phi(t)$ is the standard normal distribution function). We have

$$\int\limits_\gamma^\beta e^{\tau t}\, F^{(h)}(dt) = \int\limits_\gamma^\beta e^{\tau t}\, \Phi(dt) + \int\limits_\gamma^\beta e^{\tau t}\, \Delta(dt) = I + J. \tag{2.7}$$

As in [11, (2.4)–(2.8)], we obtain

$$|J| \le c\,\frac{e^{\tau\beta} - e^{\tau\gamma}}{\tau}\,(\delta_\varepsilon(h) + \varepsilon\,(1 + \delta_\varepsilon(h))\,(\tau + 1/(\beta - \gamma)). \tag{2.8}$$

Now let us consider the integral I. We have

$$\sqrt{2\pi}\, I = \frac{1}{\tau} e^{\tau\beta} \int\limits_{-\mu}^0 e^{u - (\beta + u/\tau)^2/2}\, du, \tag{2.9}$$

where $\mu = \tau(\beta - \gamma) = hs$. Further,

$$e^{-(\beta+u/\tau)^2/2} = e^{-\beta^2/2}(1 - 4\beta/\tau) + \frac{1}{2}(u/\tau)^2 (e^{-t^2/2})''\Big|_{t=\beta+\theta u/\tau}, \quad 0 < \theta < 1,$$

whence

$$\int\limits_{-\mu}^{0} e^{u-(\beta+u/\tau)^2/2}\,du = e^{-\beta^2/2}\Big(1-e^{-\mu}-\frac{\beta}{\tau}\int\limits_{-\mu}^{0} te^t\,dt\Big)+\frac{\theta_1}{2\tau^2}\int\limits_{-\mu}^{0} t^2e^t\,dt$$
$$= (1-e^{-\mu})\big(e^{-\beta^2/2}(1+\theta_2\beta/\tau)+\theta_3/\tau^2\big), \qquad (2.10)$$

where $-1 < \theta_1 < 2e^{-3/2}$, $0 < \theta_2 < 1$, $|\theta_3| < 1$.

Lemma 2 follows from (2.6)–(2.10). $\square$

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